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ORIGINAL ARTICLE

Symmetric and asymmetric 3:1 resonant periodic orbits with an application to the 55Cnc extra-solar system

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Abstract We study the dynamics of 3:1 resonant motion for planetary systems with two planets, based on the model of the general planar three body problem. The exact mean motion resonance corresponds to periodic motion (in a rotating frame) and the basic families of symmetric and asymmetric periodic orbits are computed. Four symmetric families bifurcate from the family of circular orbits of the two planets. Asymmetric families bifurcate from the symmetric families, at the critical points, where the stability character changes. There exist also asymmetric families that are independent of the above mentioned families. Bounded librations exist close to the stable periodic orbits. Therefore, such periodic orbits (symmetric or asymmetric) determine the possible stable configurations of a 3:1 resonant planetary system, even if the orbits of the two planets intersect. For the masses of the system 55Cnc most of the periodic orbits are unstable and they are associated with chaotic motion. There exist however stable symmetric and asymmetric orbits, corresponding to regular trajectories along which the critical angles librate. The 55Cnc extra-solar system is located in a stable domain of the phase space, centered at an asymmetric periodic orbit.

Keywords Extra-solar planetary systems · Resonances · Periodic orbits

1 Introduction

The general three body problem can be considered as a good model for studying the dynamics of two-planet extra-solar planetary systems. To date, about 12% of the observed extra-solar systems have two or more planets (e.g., see Schneider 2005). In many cases (but few confirmed) the two planets are in mean motion resonance, with relatively large eccentricity values.

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An efficient way for studying the resonant dynamics of a planetary system and detecting stable and unstable regions in phase space is to determine the periodic configurations of the system, i.e. the periodic orbits in an appropriate rotating frame of reference, and the corresponding stability (e.g. Hadjidemetriou 2002; Psychoyos and Hadjidemetriou 2005a). In this way, we obtain a chart of the phase space indicating where a resonant planetary system could exist. These are the stable configurations to which a planetary system could be trapped under a migration process (Lee and Peale 2002; Ferraz-Mello et al., 2003, Beauge et al. 2006). Such stable configurations are associated with stable apsidal corotations and stable asymmetric librations as has been shown first by Beaugé et al. (2003).

There are three topologically different resonances: (1) The first order resonances $(n+1):n$, i.e. 2:1, 3:2, …,(2) The resonances $(2n+1):(2n-1)$, i.e. 3:1, 5:3, …and (3) all other resonances, i.e. 5:2, 7:3, 8:3,…. In the first-order resonances the unperturbed circular periodic orbits (zero planetary masses) are not continued to m_1, $m_2 > 0$, and we have a gap on the family of circular periodic orbits, from which two resonant families of elliptic periodic orbits are generated. At the 5:2 resonance (and all similar resonances, i.e. 7:3, 8:3, etc.) the continuation of the family of circular orbits, from zero to nonzero masses, is possible. The stability is preserved and we have two points on the perturbed circular family from which there bifurcate families of resonant 5:2 elliptic periodic orbits. Voyatzis and Hadjidemetriou (2005) studied the periodic orbits and examined the dynamics of the 2:1 resonant motion. Psychoyos and Hadjidemetriou (2005b) studied the 5:2 mean motion resonance. The 3:1 resonant planetary dynamics is different from the above two types of resonant motion. The continuation is possible, but now an unstable region appears on the perturbed circular family of periodic orbits close to the 3:1 resonance. At this region we have a bifurcation of four 3:1 resonant families of elliptic periodic orbits, as we shall show in the following.

In the present paper, we study the 3:1 resonant dynamics by considering the approach of determining bifurcations and periodic orbits as mentioned above. We mainly use in our computations the values for the planetary masses that correspond to the companions B and C of the extra-solar system 55Cnc (Marcy et al., 2002; McArthur et al., 2004), which is at the 3:1 resonance. Numerical simulations that indicate the 3:1 resonant dynamics of the system can be found in Ji et al. (2003), Zhou et al. (2004) and Marzari et al. (2005). Stable symmetric and asymmetric configurations have been calculated by Beaugé et al. (2003) and possible resonance capture is shown in Ferraz-Mello et al. (2003). In the next section we discuss briefly the model and the possible periodic configurations. In Section 3, we present the families of periodic orbits and discuss the dynamics of the phase space at the 3:1 resonance, for the masses that correspond to the 55Cnc planetary system. We also study how the 3:1 resonant stable periodic configurations are affected when the planetary masses are changed. Finally, in Section 4, we study the stability of the 55Cnc system.

2 The model and the periodicity conditions

The dynamical model that we use is the general planar three-body problem of planetary type. The system consists of a star of mass m_0 and two planets, denoted as $\mathcal{P}_1$ and $\mathcal{P}_2$, of masses $m_1 \ll m_0$ and $m_2 \ll m_0$, respectively. The short-term evolution of the system is described by nearly Keplerian motion of the two planets, with semimajor axes a_i, eccentricities e_i, mean anomalies M_i, longitudes of pericenter ϖ_i and periods T_i (the index $i = 1, 2$ refers to the planet $\mathcal{P}_i$). The $\mathcal{P}_1$ is the "inner" planet and $\mathcal{P}_2$ is the "outer" planet in the sense that $T_1 < T_2$, although the planetary orbits may intersect.

The center of mass of the planetary system is considered as fixed in an inertial frame, and the study is made in a *nonuniformly rotating* frame of reference xOy, whose x-axis is the line defined by the star and the inner planet $\mathcal{P}_1$. The origin O is the center of mass of these two bodies and the y-axis is perpendicular to the x-axis. In this rotating frame $\mathcal{P}_1$ moves on the x-axis and $\mathcal{P}_2$ in the xOy plane. The system can be studied as a system of three degrees of freedom (Hadjidemetriou, 1975) and possesses the fundamental symmetry Σ:

$$x \rightarrow x, \quad y \rightarrow -y, \quad t \rightarrow -t.$$

We fix the units of mass, length and time by considering normalized values of the masses such that $m_0 + m_1 + m_2 = 1$, the gravitational constant, G, is equal to unity and also by keeping a fixed value of the angular momentum of the system.

Our study contributes to the dynamics of the 3:1 mean motion resonance. In this case it is $T_2/T_1 \approx 3$ and the corresponding resonant or critical angles are defined as

$$\theta_1 = \lambda_1 - 3\lambda_2 + 2\varpi_1, \qquad \theta_2 = \lambda_1 - 3\lambda_2 + 2\varpi_2,$$

where λ_i is the mean longitude of the planet $\mathcal{P}_i$. Since the motion is studied in a rotating frame, it is useful to consider the half of the difference of the critical angles, i.e. the angle $\Delta\varpi = \frac{1}{2}(\theta_1 - \theta_2) = \varpi_1 - \varpi_2$, which provides directly the angle between the apsidal lines of the two planets.

The periodic orbits, which are studied, are periodic with respect to the rotating frame defined above. Such a periodic orbit corresponds to a motion in the inertial plane where the *relative* configuration of the planets is periodically repeated. The system *is not*, in general, periodic in the inertial frame.

The periodic orbits that are invariant under the symmetry Σ are called symmetric. In a symmetric periodic orbit the planet $\mathcal{P}_2$ starts from the x-axis perpendicularly and at that time $\dot{x}_1 = 0$, and after some time $t = T/2$, T being the period, the planet $\mathcal{P}_2$ crosses again the x-axis perpendicularly and at that time it is $\dot{x}_1 = 0$. The symmetry implies that $\Delta\varpi = 0°$ or $180°$, i.e. the lines of apsides are either aligned or antialigned, respectively. Additionally, there exists a moment that when one planet is at perihelion (or aphelion) the other planet is also at perihelion or aphelion, i.e. is $\theta_i = 0°$ or $180°$, $i = 1, 2$.

An asymmetric periodic orbit is not invariant under Σ. It is mapped by Σ to another periodic orbit, called *mirror image*, which has the same elements a_i and e_i as the original one but opposite values in M_i and ϖ_i. The periodicity conditions can be defined in two different, but equivalent, options. In the first option the planet $\mathcal{P}_2$ starts from the x-axis (nonperpendicularly) and the planet $\mathcal{P}_1$ is not at rest on the x-axis. After a time $t = T$, when $\mathcal{P}_2$ crosses again the x-axis, the planets $\mathcal{P}_1$,$\mathcal{P}_2$ have the same initial position and velocity as at $t = 0$. In the second option we start, at $t = 0$, at the moment when $\mathcal{P}_1$ has zero velocity on the x-axis, which means that $\mathcal{P}_1$ is either at perihelion or at aphelion ($\mathcal{P}_2$ is not on the x-axis), and after a time $t = T$, when $\mathcal{P}_1$ has again zero velocity on the x-axis, the planets $\mathcal{P}_1$,$\mathcal{P}_2$ have the same initial position and velocity as at $t = 0$. In both of the above cases the critical arguments θ_1 and θ_2, cannot take the value $0°$ or $180°$ at the same moment, i.e. the lines of apsides of the planets do not coincide and/or the planets do not pass from the pericenter or apocenter position simultaneously.

3 Families of 3:1 resonant periodic orbits

In our study, we shall consider the 55Cnc planetary system for the star and the companion planets B and C. We use the following normalized values for the masses (Schneider 2005):

$$m_0 = 0.99903, \qquad m_1 = 0.00078, \qquad m_2 = 0.00019.$$

The periods of the two companions were estimated to be 14.7 and 43.9 days, respectively, i.e. they are in a 3:1 mean motion resonance. The above values are slightly different from those given by McArthur et al. (2004) in the sense that they do not cause structural changes in the dynamics of the system.

Our study starts with the computation of the family of circular periodic orbits of the system and the determination of the critical periodic orbits with respect to the linear stability. These critical periodic orbits are the bifurcation points for the families of resonant elliptic periodic orbits.

3.1 The family of circular orbits

The unperturbed motion with zero masses and zero eccentricities of both planets is periodic in the rotating frame for *any* value of the radii. Thus by fixing the radius of the inner planet we can form a family of periodic orbits with parameter the radius of the outer planet. The ratio n_2/n_1 of the mean motion of the two planets varies along the family. If we switch on the masses, these periodic orbits are continued as periodic orbits in the rotating frame, which correspond to almost circular orbits in the inertial frame and form the *family C*. The continuation is not possible at the resonances 2:1, 3:2, ..., where a gap appears. In the particular study, we focus in the part of the *family C* where $n_2/n_1 \approx 3$. This is presented in Figure 1.

It can be proved (Hadjidemetriou 1982), that a region of instability appears on the family of circular orbits, close to the 3:1 resonance. We remark that, we have three degrees of freedom (in the rotating frame) and consequently, we have three pairs of eigenvalues. The orbit is stable only in the case where all the eigenvalues are on the unit circle, in the complex plane. One pair of eigenvalues is always on the unit circle, at +1, due to the existence of the energy (Jacobi) integral. The other two pairs are free to move on the unit circle, preserving the stability, or to move outside the unit circle and generate instability. The section of the family C cloce to the 3:1 resonance starts as stable, as we see in Figure 1. As we proceed along the family, from $n_1/n_2 = 3.008$ to $n_1/n_2 = 3.024$, one pair of eigenvalues goes outside

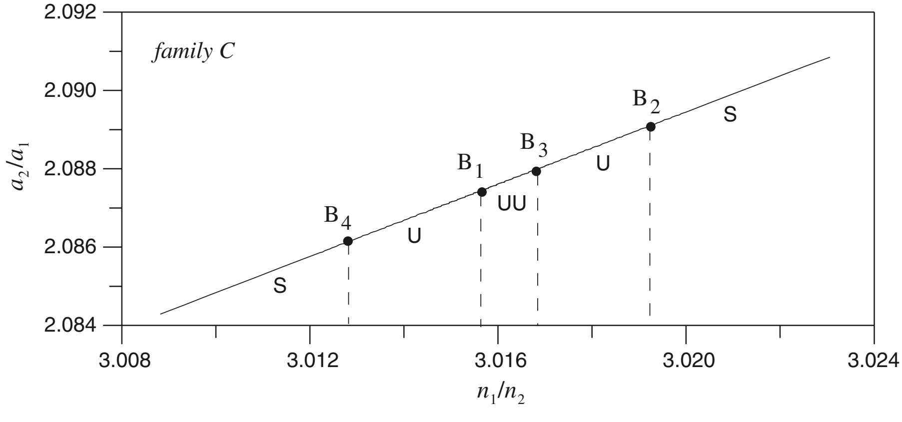

Fig. 1 The *family C* of circular periodic orbits near the 3:1 mean motion resonance. It is presented in the projection plane where the horizontal and vertical axis correspond to the ratio of mean motions and the ratio of semimajor axes of the planets, respectively. The type of linear stability is denoted by the symbols "S" for stable orbits, "U" for simply unstable orbits and "UU" for doubly unstable orbits. The four critical orbits (or bifurcation points) are denoted by $B_1, \ldots, B_4$

the unit circle at the point B_4, thus generating instability, while the other pair is still on the unit circle. As we proceed further along the family, the second pair of eigenvalues goes also outside the unit circle, starting from point B_1, and ending at the point B_3, where again in goes back on the unit circle. The first pair is still outside, up to the point B_2, where it goes back on the unit circle, thus restoring the stability. Conclusively, the region B_1B_3 is doubly unstable (two pairs of eigenvalues outside the unit circle) and the regions B_4B_1 and B_2B_3 are simply unstable (only one pair of eigenvalues outside the unit circle). The points B_1, B_2, B_3, and B_4 are the critical points, where the stability changes character, and it is from each of these points that we have bifurcation of a family of elliptic periodic orbits. All these orbits are symmetric.

3.2 The families of symmetric periodic orbits

There are four families of resonant elliptic periodic orbits, that bifurcate from the critical points B_1, B_2, B_3, and B_4, called *families* S_i, $i = 1, \dots, 4$, where i denotes the corresponding bifurcation point B_i of the *family C*. The configuration of each of these families at $t = 0$ and at $t = T/2$ is given in Table 1. The characteristic curves of these families are presented in the space of the "signed eccentricities" $e_1^* = e_1 \sin \Delta\varpi$ and $e_2^* = e_2 \sin \theta_1$ (note that $\Delta\varpi$ and θ_1 are either 0° or 180°). In this presentation, shown in Figure 2, each one of the four families is placed in a particular quarter of the plane $e_1^* - e_2^*$.

The *family* S_1 is unstable. It starts with simply unstable orbits. The orbits become doubly unstable in the segment between the points B_{12} and B_{13}, which denote critical orbits. Starting with initial conditions close to an orbit of the *family* S_1 we have chaotic evolution and the critical angles $\Delta\varpi$ and θ_1 rotate. Chaos becomes more obvious as the planetary eccentricities increase.

The *family* S_2 starts from the circular family and consists initially of simply unstable orbits. The particular configuration permits the planets to come very close to each other and we have a collision orbit. After the collision the family is continued with stable periodic orbits, where the corresponding planetary orbits intersect.

The *family* S_3 starts with unstable orbits. The type of stability changes at the high-eccentricity value $e_1 = 0.74$ (and $e_2 = 0.59$) and the periodic orbits of the family become stable after this point. Around these stable orbits regular librating motion, with respect to the critical angles, is obtained. The point B_{31} where the stability changes is a bifurcation point for a family of stable asymmetric orbits (see paragraph 3.3).

The *family* S_4 is the only family that starts with stable orbits. The stable segment extends up to the point B_{41}, at $e_1 = 0.085$ and $e_2 = 0.110$. For higher eccentricities the orbits become unstable. The phase space region around the low-eccentricity stable periodic orbits consists of invariant tori, the orbits are regular and the critical angles $\Delta\varpi$ and θ_1 librate around the value of 180°.

Table 1 Configurations at $t = 0$ and $t = T/2$ obeyed by the 3:1 resonant symmetric periodic orbits of the families S_1–S_4

S_1:	$\mathcal{P}_2$(per)	–	Star	–	$\mathcal{P}_1$(apo)	→	$\mathcal{P}_1$(per)	–	Star	–	$\mathcal{P}_2$(apo)
S_2:	Star	–	$\mathcal{P}_1$(per)	–	$\mathcal{P}_2$(apo)	→	$\mathcal{P}_2$(per)	–	$\mathcal{P}_1$(apo)	–	Star
S_3:	Star	–	$\mathcal{P}_1$(apo)	–	$\mathcal{P}_2$(apo)	→	$\mathcal{P}_2$(per)	–	$\mathcal{P}_1$(per)	–	Star
S_4:	$\mathcal{P}_1$(per)	–	Star	–	$\mathcal{P}_2$(per)	→	$\mathcal{P}_2$(apo)	–	Star	–	$\mathcal{P}_1$(apo)

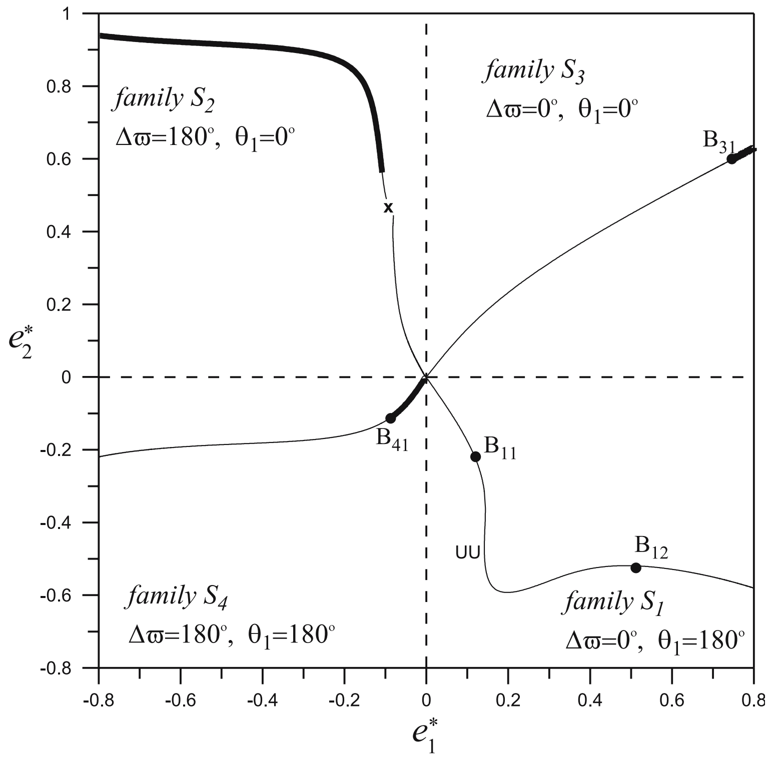

Fig. 2 The four families of symmetric periodic orbits presented in the plane of the "signed" eccentricities $e_1^*-e_2^*$. All families start from circular orbits at (0,0). The value of the critical angles $\Delta\varpi$ and θ_1 along each family is indicated. The bold line segments indicate stable orbits while the thin ones correspond to unstable orbits. The orbits between the critical orbits B_{11} and B_{12} are doubly unstable. The symbol $\times$ denotes a collision orbit

3.3 The families of asymmetric periodic orbits

The critical orbits B_{11}, B_{12}, B_{31}, and B_{41} on the symmetric families S_i, shown in Figure 2, are bifurcation points for new families. We found that all families that bifurcate from the above critical orbits consist of *asymmetric* periodic orbits. The corresponding characteristic curves are presented in the plane of eccentricities e_1–e_2 shown in Figure 3.

The asymmetric *family* A_1 starts from the critical point B_{11} of the symmetric family S_1 and terminates at the critical point B_{12} of the same family (Figure 3a). All orbits of the *family* A_1 are doubly unstable and the orbits close to the asymmetric periodic orbits are chaotic. The critical angle $\Delta\varpi$ corresponds to the value 0° at the critical points while it varies along the family increasing up to 35°. The critical angle θ_1 is 180° at the edges of the family and along the family reaches the maximum value of 222° (Figure 4a).

The second family of asymmetric orbits, the *family* A_3, bifurcates from the critical point B_{31} of the *family* S_3 (Figure 3b). This family starts as stable and is directed towards to lower values of the eccentricity e_1. At $e_1 \approx 0.13$ the family turns towards to higher eccentricity values e_1 and e_2. The stable character of the asymmetric orbits holds up to the critical orbit B_{32}. At this point the orbits become unstable and a new family of asymmetric orbits, the *family* A'_{31} bifurcates from this critical point. All orbits of A'_{31} are unstable. The critical angles

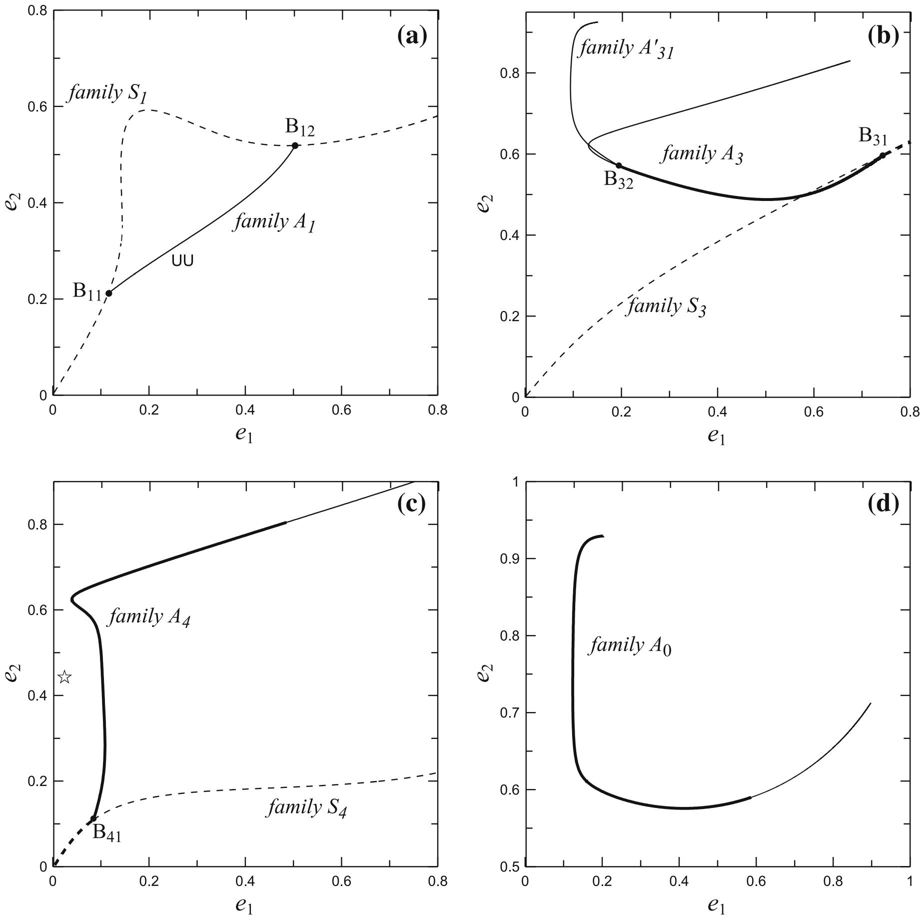

Fig. 3 The families of asymmetric periodic orbits (solid curves) presented in the plane of eccentricities e_1-e_2. The associated symmetric families are indicated by dashed curves. The bold phase segments indicate stable orbits while the thin ones correspond to unstable orbits. The star in (c) indicates the position of the 55Cnc planetary system

$\Delta\varpi$ and θ_1 that correspond to the bifurcation point B_{31} are equal to 0°, while at about the bifurcation point B_{32} they take their maximum value, 93° and 121°, respectively (Figure 4b). We found that a relatively large region of phase space extended around the stable asymmetric periodic orbits shows regular motion. Along these orbits the critical angles librate around values different than 0° or 180°.

The asymmetric *family* A_4 starts from the critical orbit B_{41} of the family S_4. Initially and up to $e_1 \approx 0.5$, it is $e_2 \approx$ const. (see Figure 3c). The apsidal difference $\Delta\varpi$ is almost constant (~250°) along this part of the *family* A_4. This is shown in Figure 4c, which presents the variation of the critical angles along the family. The major part of the family consist of stable orbits and the phase space consist of regular librating resonant motion. The family becomes unstable for high eccentricity values ($e_1 \approx 0.5$ and $e_1 \approx 0.8$) but we have not succeed in calculating the family that bifurcates from this critical point.

Finally, we present the *family* A_0 of asymmetric periodic orbits in Figure 3d. This family does not bifurcate from a symmetric family. It consists of one segment of unstable orbits and

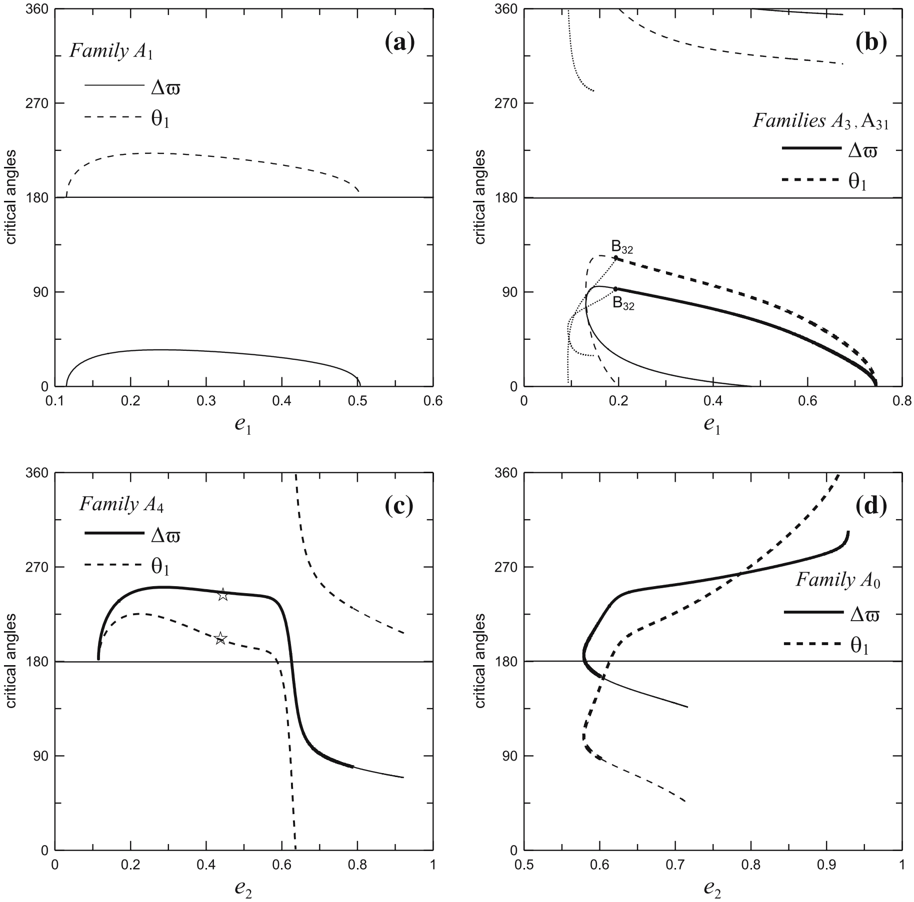

Fig. 4 The variation of the critical angles $\Delta\varpi$ (solid curves) and θ_1 (dashed curves) along the *families* of asymmetric periodic orbits presented in Figure 3. It is convenient to use as family parameter the value e_1 for the cases (a) and (b) and the value e_2 for the cases (c) and (d). Thin and bold lines indicate unstable and stable periodic orbits, respectively. The star in panel (c) indicates the position of the 55Cnc planetary system

one segment of stable orbits. It is remarkable that there exist stable orbits for $e_2 > 0.9$. Such a kind of family has been also found in the 2:1 resonant motion (Voyatzis and Hadjidemetriou 2005).

3.4 The families S_4 and A_4 for various mass values

The S_4 and A_4 are the only families of periodic orbits associated with stable resonant librations for low and moderate values of the planetary eccentricities. Also, as it is discussed in the next section, these families are associated with the dynamics of 55Cnc system. Our computations showed that these families exist for a large range of planetary masses. In the following, we present two numerical experiments in order to clarify the dependence of the families on the planetary masses.

In the first numerical experiment we multiply the planetary mass values, given in the beginning of Section 2, by a factor a and we recalculate the families. As it is shown in Figure 5a, when the planetary masses decrease, the families are not affected significantly with respect

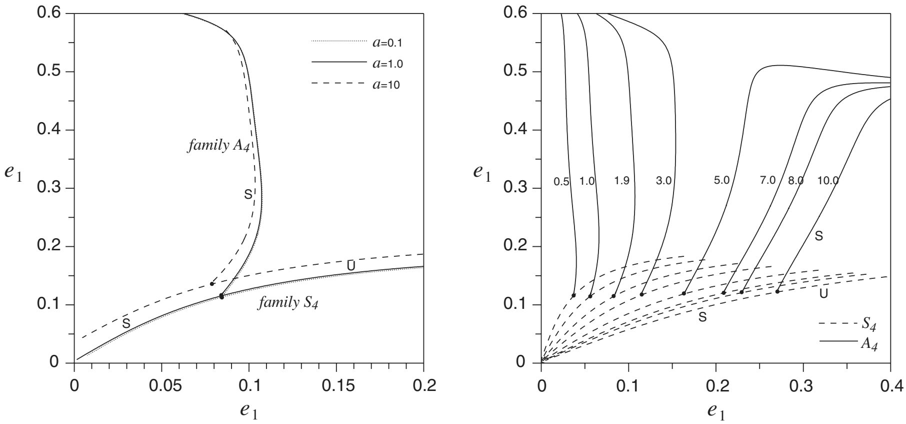

Fig. 5 Characteristic curves of the symmetric *family* S_4 and the asymmetric one A_4 for various values of planetary masses. (a) constant mass ratio, $m_1 = a \times 8 \times 10^{-4}$, $m_2 = a \times 2 \times 10^{-4}$. (b) $m_1 = 8 \times 10^{-4}$ = const. and $m_2 = \mu \times 10^{-4}$, where the value of μ is indicated beside each curve

to their location in the plane of eccentricities and their stability. When the planetary masses increase, the bifurcation point for the asymmetric family shifts only slightly to a lower value for the eccentricity e_1 and to a higher value for the eccentricity e_2. The most important consequence is that the unstable part of the family A_4 (see Figure 3c) increases and extends towards to lower eccentricity values as the masses increase. Thus, by increasing the masses by a factor of 10, the asymmetric orbits of the *family* A_4 become unstable for $e_2 > 0.5$.

In the second numerical experiment we fix the mass of the inner planet to the value $m_1 = 8 \times 10^{-4}$ and we set $m_2 = \mu \times 10^{-4}$, where we let μ to vary. The computed families for various values of μ are shown in Figure 5b. When the mass of the outer planet decreases, the *family* A_4 starts from lower e_1 values. The opposite case holds if m_2 increases. Note that the asymmetric *family* A_4 still exists for $m_2 > m_1$. These results are in a good qualitative agreement with those obtained from the averaged model and the migration scenario proposed by Beaugé et al. (2003) and Ferraz-Mello et al. (2003), respectively. It should be noted that the bifurcation point for the asymmetric family is located to $e_2 \approx 0.11$ for any value of μ or, equivalently, any planetary mass ratio. This characteristic feature for the 3:1 resonance has been indicated first by Ferraz-Mello et al. (2003).

4 Dynamical aspects and stability of the 55Cnc system

The observed position of the 55Cnc planetary system (Schneider 2005) is indicated by a star in Figures 3c and 4a and corresponds to $e_1 = 0.02$, $e_2 = 0.44$, $\Delta\varpi = 245°$, and $\theta_1 = 200°$. These elements are close to those given by McArthur et al. (2004). It is evident that this position is associated with the asymmetric periodic orbits of the *family* A_4 and, in particular, it seems to be located near the asymmetric orbit at $e_1 = 0.1$, $e_2 = 0.44$. The family A_4 is linearly stable at the particular domain, suggesting that a six-dimensional region of the phase space nearby the orbits of the family A_4 corresponds mainly to quasiperiodic motion or, equivalently, to regular trajectories for longterm evolution.

The width of the stable region can be determined numerically by examining a large number of trajectories. Particularly, for the initial conditions of the 55Cnc system, mentioned

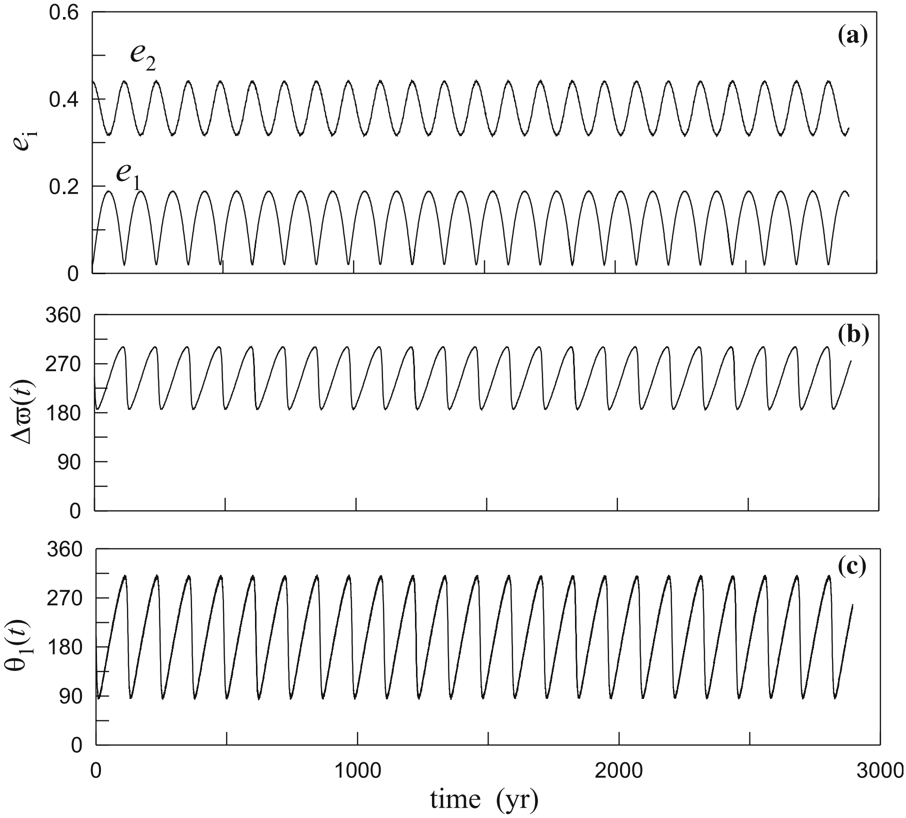

Fig. 6 The variation of the eccentricity values e_1 and e_2 (panel (a)) and the critical angles $\Delta\varpi$ and θ_1 (panels (b),(c)) along the trajectory associated with the 55Cnc planetary system. The initial conditions are $e_1(0) = 0.02$, $e_2(0) = 0.44$, $\Delta\varpi(0) = 245°$ and $\theta_1(0) = 200°$

above, and in the framework of the three body problem, we obtain regular evolution. The variation of the planetary eccentricities and the critical angles are presented in Figure 6. The semimajor axes of the planets are almost constant during the trajectory evolution but, as it is shown in Figure 6a, the eccentricities oscillate with relatively large amplitude. The large amplitude oscillations of the eccentricities have been also obtained and discussed by Zhou et al. (2004) and Beaugé et al. (2006). The critical angles $\Delta\varpi$ and θ_1 librate around the values 250 and 200°, respectively (Figure 6b, c). The period of libration is about 125 years for the particular case. Such stable librations have been also observed in the numerical simulations by Ji et al. (2003) and correspond to the stable motion of type '**a**' found by Zhou et al. (2004).

By examining a large number of trajectories, using Poincaré sections, we found that regular motion exists in a wide region of initial conditions around the stable asymmetric orbits of family A_4. In Figure 7, we present Poincaré maps on the surface of section $y_2 = 0$, $\dot{y}_2 > 0$, projected on the plane $x_2 - \dot{x}_2$. The panel (a) corresponds to the initial conditions of the 55Cnc system, mentioned above. The motion is regular on a torus presented in the section by two "islands". The corresponding periodic orbit close to this quasiperiodic orbit is of multiplicity two, i.e. intersects the plane of section twice per period and these fixed points are the centers of the "islands" in the four-dimensional space of the Poincaré map. In the

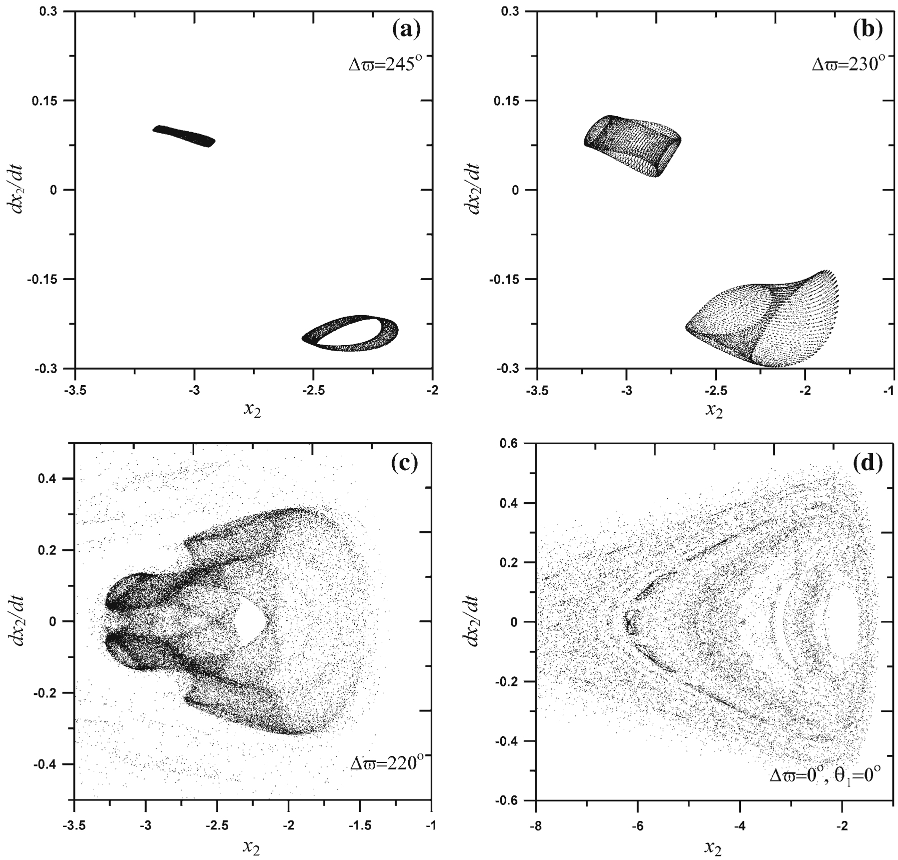

Fig. 7 Projection planes $x_2 - \dot{x}_2$ of the Poincare section $y_2 = 0$, $\dot{y}_2 > 0$ of the trajectory with initial conditions $e_1(0) = 0.02$, $e_2(0) = 0.44$ and (a) $\Delta\varpi(0) = 245°$, $\theta_1(0) = 200°$, (b) $\Delta\varpi(0) = 230°$, (c) $\Delta\varpi(0) = 220°$, $\theta_1(0) = 200°$ and (d) $\Delta\varpi(0) = 0°$, $\theta_1(0) = 0°$

panels (b) and (c), we present the trajectories corresponding to the same initial conditions as in panel (a), but we change the initial apsidal angle $\Delta\varpi$. For $\Delta\varpi = 230°$ (i.e. 15° deviation from the value of the corresponding periodic orbit) we observe still regular islands and librations of the critical angles. For larger deviations ($\Delta\varpi < 225°$) the trajectories show stickiness. Namely, the planetary orbits seem to evolve regularly in the resonant region for a long time interval. After this interval, the trajectory enters a wide chaotic region and the semimajor axis and the eccentricity of the outer planet increase rapidly. Such a trajectory is shown in Figure 7c and corresponds to $\Delta\varpi = 220°$. The points of the Poincaré map spread irregularly in a wide domain after about 15 K years. The evolution of the semimajor axes and eccentricities along this trajectory is shown in Figure 8a and illustrates the stickiness of the trajectory and the escape of the outer planet $\mathcal{P}_2$. The stickiness time decreases as $\Delta\varpi$ decreases and for $\Delta\varpi < 200°$ the chaotic motion becomes apparent after few iterations of the Poincaré map.

From the above results, we can see that the asymmetry, which is imposed by the periodic orbits of the family A_4, stabilizes the planetary system. Sufficient deviations from such an asymmetric configuration result to strongly chaotic motion. In the particular case the sym-

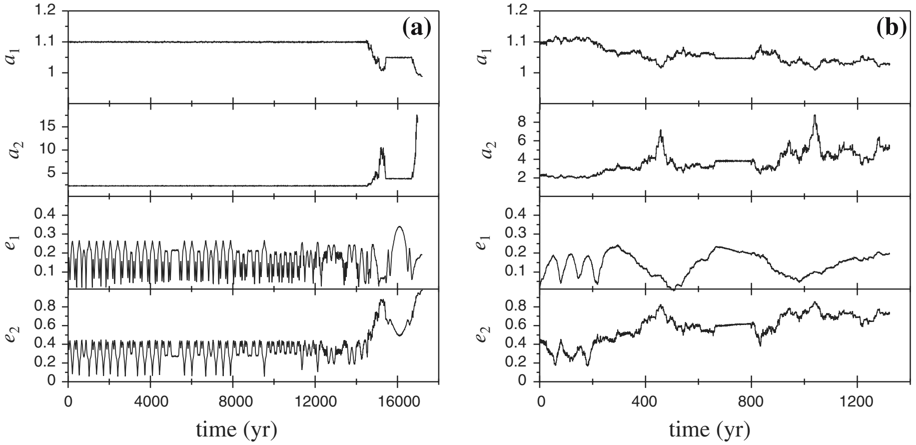

Fig. 8 (a) The evolution of the semimajor axes and eccentricities of the planetary orbits that corresponds to the sticky trajectory shown in Figure 7(c). (b) The same as in (a) for the chaotic trajectory of Figure 7(d), which corresponds to the symmetric initial configuration $\Delta\varpi(0) = \theta_1(0) = 0°$.

metric configurations are also unstable. All the trajectories with initial elements $e_1 = 0.02$, $e_2 = 0.44$, $\Delta\varpi(0) = 0$ or $180°$ and $\theta_1(0) = 0$ or $180°$, correspond to chaotic motion. An example is presented in Figure 7(d) and 8(b) for $\Delta\varpi(0) = \theta_1(0) = 0°$.

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