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ADDITIONS TO THE THEORY OF THE ROTATION OF EUROPA

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Abstract. In a previous paper (*The Rotation of Europa*, Henrard, *Celest. Mech. Dyn. Astr.*, **91**, 131–149, 2005) we have developed a semi-analytical theory of Europa, one of the Galilean satellites of Jupiter. It is based on a synthetic theory of the orbit of Europa and is developed in the framework of Hamiltonian formalism. It was assumed that Europa is a rigid body and Jupiter a point mass. Several additional effects should be investigated in order to complete the theory. The present contribution considers the effect of the shape of Jupiter and of the gravitational pull of Io. The sensitivity of the main theory to a change in the values of the moments of inertia of Europa is also considered.

Key words: Europa, natural satellite, oblateness of the planet, synchronous rotation, third body perturbations

1. Introduction

Like the Moon, the Galilean satellites present the same face to their planet. Cassini (1730) showed, for the Moon, how this peculiar feature corresponds to an equilibrium, *a Cassini's state*, of a simplified model of the rotation and how perturbations from this model lead not to the destabilization of this equilibrium but to the excitation of *librations* around it. The fact that so many satellites are found in this special state is due to internal dissipation of energy in the satellites which drives them to it (Goldreich and Peale, 1966).

The case of Europa is particularly interesting as the quasi-certitude of the existence of an ocean below its icy crust raises many questions. In order to answer them, it is necessary to obtain a good knowledge of its rotation state.

In a previous paper (Henrard, 2005 – which we will call Paper I) we have developed an analytical theory of the rotation of a rigid Europa perturbed by a Jupiter considered as a point mass. We were able to use the synthetic theory of Lainey (2002); Lainey et al. (2004a,b) to represent the orbit of Europa around Jupiter. This theory is based on a numerical integration of the orbits of the Galilean satellites and a frequency analysis of the result. It presents itself as Fourier series in the angular variables, λ_i the mean longitudes of the four Galilean satellites, ϖ_i the longitudes of their pericenters, Ω_i, the longitudes of their nodes plus λ_{Sun}, the mean longitude of the Sun and ℓ the Laplacian libration. The frequencies and phases of these angular variables are given by the theory. The equator of Jupiter is taken as an inertial plane and the epoch of the theory is the Julian day 2433282.5 (01/01/1950 at 0h00). The accuracy is somewhat better than 100 km for Europa.

We ended paper I by stating that several additions to our analytical representation of the rotation of Europa should be worked out. The fact that Jupiter is not a point mass, the direct effect of the other Galilean satellites on the rotation (the indirect effect has already be included by taking a perturbed orbit for Europa) should be taken into account. Due to the fact that the values of the moments of Inertia are not known with high precision, we mentioned also that the sensibility of the theory to a change in these parameters should be evaluated. These additions are the subject of this contribution.

We believe that no other source of perturbation on a rigid Europa should reach the value of 10^{-7} radian (0.02″) which is the truncation level of our theory; but of course the existence of a core, the presence of the ocean, the non-rigidity of the body are expected to have an important influence on the rotation. We hope to be able to address these questions in the future.

2. Sensitivity of the Theory to the Parameters δ_1 and δ_2

The values of the moments of inertia of Europa, which are of prime importance in the analysis of the rotation, are embedded in the parameters δ_1 and δ_2:

$$\delta_1 = -\frac{3}{2}\frac{n^{*2}}{n_2^2}\frac{2C - A - B}{2C}, \qquad \delta_2 = -\frac{3}{2}\frac{n^{*2}}{n_2^2}\frac{B - A}{2C}, \tag{1}$$

where $n^* = \sqrt{GM_J/d_0^3}$, where d_0 is the mean distance between Europa and Jupiter. The value 0.9997 of the ratio n^*/n_2, where n_2 is the frequency of λ_2, is obtained from (Lainey, 2002). The solution given in paper I is based on values of δ_1 and δ_2 corresponding to $J_2 = 4.35 \times 10^{-4}$, $C_2^2 = 1.31 \times 10^{-4}$ and $C/MR^2 = 0.346$:

$$\delta_1 = -1.885 \times 10^{-3}, \qquad \delta_2 = -1.135 \times 10^{-3}. \tag{2}$$

We have evaluated the sensitivity of the theory of paper I to a change in these parameters by recomputing it for four other sets of values: $[\delta_1^m = 0.95\delta_1, \delta_2], [\delta_1^p = 1.05\delta_1, \delta_2], [\delta_1, \delta_2^m = 0.95\delta_2]$, and $[\delta_1, \delta_2^p = 1.05\delta_2]$. For each sets of parameters, we evaluate the corresponding series solutions, S_1^m, S_1^p, S_2^m, and S_2^p. For both parameters, we compute the series

$$\Delta_i^1 = (S_i^p - S_i^m)/2, \qquad \Delta_i^2 = (S_i^p - 2S^0 + S_i^m)/2, \tag{3}$$

where S^0 is the nominal solution of paper I. $\Delta_i^1/0.05$ is an approximation of the derivative of the series of paper I with respect to δ_i. Δ_i^2 is an indication of the accuracy by which Δ_i^1 approximates the derivative.

Table I shows the sensitivity of the Fourier series of the first two components (P_1 and P_2) of the unit vector perpendicular to the equator of Jupiter, Table II the sensitivity of the last two components (X_2 and X_3) of the unit vector pointing towards Jupiter and Table III the sensitivity of the first two components (Q_1 and Q_2) of the unit vector along the angular momentum. Each of these vectors are expressed in the body frame of Europa; the first axis is in the direction of the principal axis of least inertia, the last one in the direction of the principal axis of greatest inertia.

TABLE I

Sensitivity of the first two components of the unit vector perpendicular to the equator of Jupiter. (s) (resp. (c)) indicates that the sine (resp. the cosine) of the angular variable should be taken.

5% Change in δ_1	$10^4\Delta_1^1$ on P_1	$10^4\Delta_1^2$ on P_1	$10^4\Delta_1^1$ on P_2	$10^4\Delta_1^2$ on P_2
$\lambda_2 - \Omega_2$	(s) 0.3366	−0.0118	(c) 0.3366	−0.0118
$\lambda_1 - 2\lambda_2 + \Omega_2$	(s) 0.0146	0.0007	(c) −0.0601	−0.0013
$\lambda_2 - \Omega_3$	(s) 0.0035	−0.0001	(c) 0.0035	−0.0001
$\lambda_1 - 2\lambda_2$	(s) 0.0012	−0.0006	(c) −0.0021	−0.0001
$\lambda_1 - 2\lambda_2 + \Omega_3$	(s) 0.0006	0.0008	(c) −0.0014	−0.0001
$\lambda_l - \Omega_2$	(s) −0.0009	0.0010		
$2\lambda_1 - 3\lambda_2 + \Omega_2$	(s) 0.0009	−0.0005	(c) 0.0009	−0.0005
5% Change in δ_2	$10^4\Delta_2^1$ on P_1	$10^4\Delta_2^2$ on P_1	$10^4\Delta_2^1$ on P_2	$10^4\Delta_2^2$ on P_2
$\lambda_2 - \Omega_2$	(s) 0.2026	−0.0043	(c) 0.2025	−0.0042
$\lambda_1 - 2\lambda_2 + \Omega_2$	(s) −0.0043	−0.0002	(c) −0.0212	0.0003
$\lambda_2 - \Omega_2$	(s) 0.0021	−0.0001	(c) 0.0021	0.0000
$\lambda_1 - \lambda_2 + \varpi_3 - \Omega_3$	(c) −0.0015	−0.0015	(s) 0.0015	0.0015
$\lambda_l - \Omega_2$	(s) −0.0010	0.0009		
$\lambda_1 - 2\lambda_2 + \Omega_3$			(c) −0.0028	−0.0002
$\varpi_3 - \Omega_3$			(s) 0.0011	−0.0003

TABLE II

Sensitivity of the last two components of the unit vector pointing towards Jupiter.

5% Change in δ_1	$10^4\Delta_1^1$ on X_2	$10^4\Delta_1^2$ on X_2	$10^4\Delta_1^1$ on X_3	$10^4\Delta_1^2$ on X_3
$\sin(\lambda_2-\Omega_2)$			−0.3371	0.0119
$\sin(\lambda_1-2\lambda_2+\Omega_2)$			−0.0115	−0.0008
$\sin(\lambda_1-\Omega_2)$			0.0044	−0.0014
$\sin(\lambda_2-\Omega_3)$			−0.0035	0.0002
$\sin(2\lambda_1-3\lambda_2+\Omega_2)$			−0.0015	0.0005
$\sin(\lambda_1-2\lambda_2)$			−0.0011	0.0006
$\sin(2\lambda_1-2\lambda_2+\Omega_3)$			−0.0006	−0.0008

5% Change in δ_2	$10^4\Delta_2^1$ on X_2	$10^4\Delta_2^2$ on X_2	$10^4\Delta_2^1$ on X_3	$10^4\Delta_2^2$ on X_3
$\sin(\lambda_1-\lambda_2)$	−0.0417	0.0002		
$\cos(\lambda_1-2\lambda_2+\varpi_3)$	0.0049	−0.0006		
$\cos(\lambda_1-2\lambda_2+\varpi_4)$	0.0027	−0.0001		
$\cos(\lambda_1-2\lambda_2+\varpi_2)$	0.0021	−0.0001		
$\cos(\lambda_2-\varpi_3)$	−0.0020	−0.0010		
$\sin(\lambda_2-\lambda_3)$	−0.0019	0.0000		
$\cos(2\lambda_1-3\lambda_2+\varpi_3)$	−0.0010	−0.0011		
$\sin(\lambda_2-\Omega_2)$			−0.2028	0.0042
$\sin(\lambda_1-2\lambda_2+\Omega_2)$			0.0060	0.0001
$\sin(\lambda_1-\Omega_2)$			0.0032	−0.0013
$\sin(\lambda_2-\Omega_3)$			−0.0021	0.0000
$\cos(\lambda_1-\lambda_2+\varpi_3-\Omega_3)$			0.0005	0.0005
$\sin(\lambda_1-2\lambda_2+\Omega_3)$			−0.0007	−0.0005

TABLE III

Sensitivity of the first two components of the unit vector along the angular momentum. (s) (resp. (c)) indicates that the sine (resp. the cosine) of the angular variable should be taken.

5% Change in δ_1	$10^4\Delta_1^1$ on Q_1	$10^4\Delta_1^2$ on Q_1	$10^4\Delta_1^1$ on Q_2	$10^4\Delta_1^2$ on Q_2
$\lambda_1-2\lambda_2+\Omega_2$	(s) 0.0149	0.0007	(c) −0.0601	−0.0013
$\lambda_1-2\lambda_2$	(s) 0.0011	−0.0006	(c) −0.0021	−0.0001
$\lambda_1-2\lambda_2+\Omega_3$	(s) 0.0006	0.0008	(c) −0.0014	−0.0001

5% Change in δ_2	$10^4\Delta_2^1$ on Q_1	$10^4\Delta_2^2$ on Q_1	$10^4\Delta_2^1$ on Q_2	$10^4\Delta_2^2$ on Q_2
$\lambda_1-2\lambda_2+\Omega_2$	(s) −0.0040	−0.0002	(c) −0.0214	0.0003
$\lambda_1-2\lambda_2+\Omega_3$	(s) 0.0008	0.0005	(c) −0.0028	−0.0002
$\varpi_3-\Omega_3$			(s) 0.0011	−0.0003

3. Shape of Jupiter

The oblateness of Jupiter contributes to the perturbation of the rotation of Europa. The *additional* potential of the effect on the second-order harmonics of Europa is computed in Appendix A and can be written as

$$V_s = \delta_s C n_2^2 \left[\frac{d_0}{d}\right]^5 [\delta_1(x^2+y^2)+\delta_2(x^2-y^2)], \tag{4}$$

where (x, y, z) are the coordinates in the body frame of the unit vector pointing to Jupiter and, where d is the distance between the centers of mass of Jupiter and Europa. The coefficient δ_s is given by

$$\delta_s = \frac{5}{2}\frac{(C'-A')}{M_J R_J^2}\left[\frac{R_J}{d_0}\right]^2 = 4.1872\times 10^{-4}, \tag{5}$$

where R_J is the mean radius of Jupiter. We have taken $(R_J/d_0)=0.106561$, and $J_2=(C'-A')/M_J R_J^2=0.01475$.

The Fourier series of this expression is computed by means of Lainey's expressions for longitude, latitude and distance of Europa as seen from Jupiter, taking into account the rotations needed to go from the inertial frame to the body frame of Europa. Furthermore the normal modes of vibration around the main Cassini's state are introduced as in Section 6 of paper I. Finally the Lie transformation defined in Section 7 of paper I is applied in order to express the additional perturbing potential in the final variables of the previous theory. The main terms of this additional potential are:

$$\begin{aligned} V_s = {} & \sqrt{2U}[-6.83\times 10^{-8}\cos(\lambda_1-\lambda_2+u)+6.83\times 10^{-8}\cos(\lambda_1-\lambda_2-u)] \\ & +(2U)[7.06\times 10^{-6}-7.06\times 10^{-6}\cos 2u] \\ & +(2V)[6.31\times 10^{-7}-6.31\times 10^{-7}\cos(2\lambda_2-2\Omega_2+2v)] \\ & +\sqrt{4VW}[-6.27\times 10^{-7}\cos(\lambda_2-\Omega_2+v+w) \\ & +6.27\times 10^{-7}\cos(\lambda_2-\Omega_2+v-w)]. \end{aligned} \tag{6}$$

A Lie transformation, up to order 3, is then computed in order to transform this function into a function which does not depend upon the angular variables. This transformation is conducted to the third-order. The main terms in the generating function W^J are given in Table IV.

The transformation generated by W^J is then applied to the functions defining the nominal solution described in paper I to yield the corrections shown in Table V.

TABLE IV

The largest terms in the various components of the generating function.

	Degree 1	Degree 2	Degree 3
W_1^J	$4.52\times 10^{-7}\sqrt{2v}\,\sin v$	$-5.24\times 10^{-5}(2U)\sin 2u$	$2.43\times 10^{-5}(2W)\sqrt{2U}\,\sin(u-2w)$
W_2^J		$2.19\times 10^{-8}(2U)\sin 2u$	
$\frac{1}{2}W_3^J$			

TABLE V

Corrections (truncated at 10^{-7}) due to the fact that Jupiter is not a point mass. (s) (resp. (c)) indicates that the sine (resp. the cosine) of the angular variable should be taken. The corrections to the unit vector along the angular momentum are below our truncation level of 10^{-8}.

Unit vector perpendicular to the equator of Jupiter	$10^4 P_1$	$10^4 P_2$	Frequency
$\lambda_2-\Omega_2$	(s) 0.0045	(c) 0.0045	1.00032171
$\lambda_2-\Omega_3$	(s) 0.0019	(c) 0.0019	1.00007060
λ_2	(s) −0.0014	(c) −0.0014	1.00000000
Unit vector pointing frequency towards Jupiter	$10^4 X_2$	$10^4 X_3$	Frequency
$\lambda_2-\Omega_2$		(s) −0.0045	1.00032171
$\lambda_2-\Omega_3$		(s) −0.0019	1.00007060
λ_2		(s) 0.0014	1.00000000

4. Direct Effect of Io on the Rotation of Europa

The additional potential due to Io, and acting on the rotation of Europa, is given by:

$$V^{\mathrm{Io}} = Cn_2^2\left[\frac{m_{\mathrm{Io}}}{M_{\mathrm{J}}}\right]\left[\frac{d_0}{d_{\mathrm{Io}}}\right]^3 [\delta_1(x_{\mathrm{Io}}^2+y_{\mathrm{Io}}^2)+\delta_2(x_{\mathrm{Io}}^2-y_{\mathrm{Io}}^2)], \tag{7}$$

where x_{Io}^2 and y_{Io}^2 are the first two coordinates of the unit vector in the direction of Io as seen from Europa, and d_{Io} is the distance between the two satellites.

In order to evaluate the size of this additional potential on the rotation of Europa, we assume that Io is on a circular orbit in the equatorial plane of Jupiter. As in Section 3, the Fourier series of this expression is computed and transformed in the angles-actions variables of paper I. The principal terms of this expansions are:

$$
\begin{aligned}
V_{\mathrm{Io}} = {} & 10^{-7}\sqrt{2U}\cos u \\
& \times [5.068 \sin(\lambda_1 - \lambda_2) + 4.684 \sin(2\lambda_1 - 2\lambda_2) \\
& + 1.965 \sin(3\lambda_1 - 3\lambda_2) + 1.547 \sin(4\lambda_1 - 4\lambda_2) \\
& + 1.166 \sin(5\lambda_1 - 5\lambda_2) + 0.856 \sin(6\lambda_1 - 6\lambda_2) \\
& + 0.613 \sin(7\lambda_1 - 7\lambda_2) + 0.433 \sin(8\lambda_1 - 8\lambda_2) \\
& + 0.302 \sin(9\lambda_1 - 9\lambda_2) + 0.207 \sin(10\lambda_1 - 10\lambda_2) \\
& + \cdots]. \qquad (8)
\end{aligned}
$$

As it can be seen the convergence of the Fourier series in the difference in longitude is very poor. Fortunately, the higher the harmonic, the higher the frequency and the integration brings them down. In any case the additional potential is so small that its effect upon the rotation will be smaller than our level of truncation. The main terms for the components of the unit vector perpendicular to the equator of Jupiter are:

$$
\begin{aligned}
P_1 &= -6.20 \times 10^{-10} \sin(\lambda_1 - 2\lambda_2 + \Omega_2) - 5.57 \times 10^{-10} \sin(\lambda_1 - \Omega_2), \qquad (9) \\
P_2 &= 6.20 \times 10^{-10} \cos(\lambda_1 - 2\lambda_2 + \Omega_2) - 5.57 \times 10^{-10} \cos(\lambda_1 - \Omega_2).
\end{aligned}
$$

The main term for the component of the unit vector pointing towards Jupiter are:

$$
\begin{aligned}
X_2 = {} & 10^{-7}[-1.302 \sin(\lambda_1 - \lambda_2) - 0.300 \sin(2\lambda_1 - 2\lambda_2) \\
& - 0.112 \sin(3\lambda_1 - 3\lambda_2) - 0.049 \sin(4\lambda_1 - 4\lambda_2) \\
& - 0.024 \sin(5\lambda_1 - 5\lambda_2) - 0.012 \sin(6\lambda_1 - 6\lambda_2) \\
& - 0.006 \sin(7\lambda_1 - 7\lambda_2) + \cdots], \qquad (10) \\
X_3 < {} & 10^{-10}.
\end{aligned}
$$

At the level of truncature of our computations (10^{-10}), Io has no effect upon the direction of the angular momentum. The other Galilean satellites should have a smaller effect.

The above estimates of the effect of Io are done under the assumption that the *free librations* have zero amplitudes, i.e. that the momenta, (U, V, W) vanish. If this assumption were not correct, the effect of Io could be much larger. Indeed the solution for the effects of Io includes the following terms:

$$
\begin{aligned}
P_1 = {} & 10^{-5}\sqrt{2W}[3.092 \sin(\lambda_1 - \lambda_2 - w) - 1.350 \sin(\lambda_1 - \lambda_2 + w)] \\
& + 10^{-5}\sqrt{2V}[1.575 \sin(2\lambda_1 - 3\lambda_2 + \Omega_2 - v) \\
& \qquad + 0.822 \sin(\lambda_1 - 2\lambda_2 + \Omega_2 - v)] + \cdots,
\end{aligned}
$$

$$P_2 = 10^{-5}\sqrt{2W}[3.082\cos(\lambda_1-\lambda_2-w) - 1.339\cos(\lambda_1-\lambda_2+w)]$$
$$+10^{-5}\sqrt{2V}[1.561\cos(2\lambda_1-3\lambda_2+\Omega_2-v)$$
$$-8.944\cos(\lambda_1-2\lambda_2+\Omega_2-v)]+\cdots,$$
$$X_2 = 4.294\times 10^{-5}\sqrt{U}\sin u+\cdots,$$
$$X_3 = -10^{-5}\sqrt{2W}[3.079\sin(\lambda_1-\lambda_2-w) - 1.363\sin(\lambda_1-\lambda_2+w)]$$
$$-10^{-5}\sqrt{2V}[1.660\sin(2\lambda_1-3\lambda_2+\Omega_2-v)$$
$$+0.842\sin(\lambda_1-2\lambda_2+\Omega_2-v)]+\cdots \quad (11)$$

Even a small amplitude of the free librations (a small value of $\sqrt{U}$, $\sqrt{V}$ or $\sqrt{W}$) would amplify considerably the effect of Io upon the rotation of Europa.

5. Conclusion

The sensitivity of the theory of the rotation of Europa to a change in the values of its moments of inertia has been evaluated. The non-linear effect of the oblateness of Jupiter has been analyzed and the additions to the theory have been computed. The effects of the other Galilean satellites and of the fourth harmonic (term in J_4) of the gravitational potential of Jupiter have been considered and found to be negligible at the order of 10^{-7} radian.

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Appendix A

The potential, dV, describing the gravitational effect of a volume element dW of density ρ inside Jupiter on the rotation of Europa, is given by Henrard (2005)

$$dV = Cn_2^2\left[\frac{d_0}{r}\right]^3\left[\delta_1\frac{(X_1^2+X_2^2)}{r^2}+\delta_2\frac{(X_1^2-X_2^2)}{r^2}\right]\frac{\rho dW}{M_J}, \quad (A1)$$

where (X_1, X_2, X_3) are the components in the body frame of Europa of the vector $\vec{X}$ pointing from the center of mass of Europa toward the volume element dW, and r is its norm. The vector $\vec{X}$ is the sum $\vec{X} = \vec{d} + \vec{s}$, where $\vec{d}$ points from the center of mass of Europa toward the center of mass of Jupiter and $\vec{s}$ from the center of mass of Jupiter toward the volume element dW.

Considering that:

$$X_1^2 = (\vec{X}|\vec{f}_1)^2 = (\vec{d}|\vec{f}_1)^2 + 2(\vec{d}|\vec{f}_1)(\vec{s}|\vec{f}_1) + (\vec{s}|\vec{f}_1)^2, \tag{A2}$$

$$X_2^2 = (\vec{X}|\vec{f}_2)^2 = (\vec{d}|\vec{f}_2)^2 + 2(\vec{d}|\vec{f}_2)(\vec{s}|\vec{f}_2) + (\vec{s}|\vec{f}_2)^2, \tag{A3}$$

$$r^{-5} = d^{-5}\left[1 - 5\frac{(\vec{d}|\vec{s})}{d^2} - \frac{5}{2}\frac{(\vec{s}|\vec{s})}{d^2} + \frac{35}{2}\frac{(\vec{d}|\vec{s})^2}{d^4} + \cdots\right], \tag{A4}$$

where $\vec{f}_i$ (for $i=1,2,3$) are the vectors forming the body frame of Europa, where d is the norm of the vector $\vec{d}$, and $\cdots$ stands for terms of degree higher than 2 in the components of the vector $\vec{s}$. We shall argue that it is enough to truncate the expansion of the potential field of Jupiter to the second harmonic and thus that terms of degree higher than 2 in the components of the vector $\vec{s}$ can be neglected.

In order to compute the potential $V = \int_W \mathrm{d}V$ on the rotation of Europa, due to the finite size of Jupiter, we ought to integrate over the volume of Jupiter the expressions (for $i=1,2$):

$$\begin{aligned} d^5\frac{(\vec{X}|\vec{f}_i)^2}{r^5} = {} & (\vec{d}|\vec{f}_i)^2\left[1 - 5\frac{(\vec{d}|\vec{s})}{d^2}\right] + 2(\vec{d}|\vec{f}_i)(\vec{s}|\vec{f}_i) \\ & + (\vec{d}|\vec{f}_i)^2\left[\frac{35}{2}\frac{(\vec{d}|\vec{s})^2}{d^4} - \frac{5}{2}\frac{(\vec{s}|\vec{s})}{d^2}\right] \\ & + (\vec{s}|\vec{f}_i)^2 - 10\frac{(\vec{d}|\vec{f}_i)(\vec{d}|\vec{s})(\vec{s}|\vec{f}_i)}{d^2}. \end{aligned} \tag{A5}$$

Because the origin of the vector $\vec{s}$ is at the center of mass of Jupiter, the integration of the first line reduces to the term $M_J(\vec{d}|\vec{f}_i)^2$. This is the term entering in the expression of the potential due to a point mass Jupiter. It will be neglected in what follows, as we are interested in the *additional* potential due to the finite size of Jupiter. In order to integrate the second and third line we need to evaluate:

$$\int_W (\vec{s}|\vec{s})\rho\,\mathrm{d}W = \int_W (\xi_1^2 + \xi_2^2 + \xi_3^2)\rho\,\mathrm{d}W = \frac{1}{2}(A' + B' + C'), \tag{A6}$$

where (ξ_1, ξ_2, ξ_3) are the coordinates of the volume element $\mathrm{d}W$ in the frame of the principal axes of inertia of Jupiter, and (A', B', C') the principal moments of inertia of the planet. We have also:

$$2\int_W (\vec{d}|\vec{s})^2\rho\,\mathrm{d}W = 2\int_W (Z_1\xi_1 + Z_2\xi_2 + Z_3\xi_3)^2\rho\,\mathrm{d}W$$
$$= Z_1^2[B'+C'-A'] + Z_2^2[A'+C'-B'] + Z_3^2[A'+B'-C']$$
$$= A'[Z_2^2+Z_3^2-Z_1^2] + B'[Z_1^2+Z_3^2-Z_2^2] + C'[Z_1^2+Z_2^2-Z_3^2]$$
$$= (A'+B'+C')d^2 - 2(A'Z_1^2 + B'Z_2^2 + C'Z_3^2), \tag{A7}$$

where (Z_1, Z_2, Z_3) are the coordinates of the vector $\vec{d}$ in the frame of the principal axes of inertia of Jupiter. Similarly we have:

$$2\int_W (\vec{f_i}|\vec{s})^2\rho\,\mathrm{d}W = (A'+B'+C') - 2(A'a_i^2 + B'b_i^2 + C'c_i^2), \tag{A8}$$

where (a_i, b_i, c_i) are the coordinates of the vector $\vec{f_i}$ in the frame of the principal axes of inertia of Jupiter. We have also:

$$2\int_W (\vec{d}|\vec{s})(\vec{f_i}|\vec{s})\rho\,\mathrm{d}W = 2\int_W (a_iZ_1\xi_1^2 + b_iZ_2\xi_2^2 + c_iZ_3\xi_3^2)\rho\,\mathrm{d}W$$
$$= A'(b_iZ_2 + c_iZ_3 - a_iZ_1)$$
$$+ B'(a_iZ_1 + c_iZ_3 - b_iZ_2) + C'(a_iZ_1 + b_iZ_2 - c_iZ_3)$$
$$= (A'+B'+C')(\vec{d}|\vec{f_i}) - 2(A'a_iZ_1 + B'b_iZ_2 + C'c_iZ_3). \tag{A9}$$

Collecting the results and assuming that Jupiter is axisymmetric (i.e. $A' = B'$), we obtain:

$$d^5\int_W \frac{X_i^2}{r^5}\rho\,\mathrm{d}W = \frac{(\vec{d}|\vec{f_i})^2}{d^2}\left\{-\frac{5}{4}(2A'+C') + \frac{35}{4}(2A'+C') - \frac{35}{2}A' - \frac{35}{2}(C'-A')\frac{Z_3^2}{d^2}\right\}$$
$$+\frac{1}{2}(2A'+C') - A' - (C'-A')c_i^2$$
$$-5\frac{(\vec{d}|\vec{f_i})^2}{d^2}C' + 10(C'-A')\frac{(\vec{d}|\vec{f_i})(\vec{e}_3|\vec{f_i})(\vec{e}_3|\vec{d})}{d^2}, \tag{A10}$$

$$d^5\int_W \frac{X_i^2}{r^5}\rho\,\mathrm{d}W = \frac{(\vec{d}|\vec{f_i})^2}{d^2}(C'-A')\left[\frac{5}{2} - \frac{35}{2}\frac{(\vec{d}|\vec{e}_3)^2}{d^2}\right]$$
$$+\frac{C'}{2} - (C'-A')(\vec{e}_3|\vec{f_i})\left\{(\vec{e}_3|\vec{f_i})] - 10\frac{(\vec{d}|\vec{f_i})(\vec{e}_3|\vec{d})}{d^2}\right\}. \tag{A11}$$

In the case of Europa, $(\vec{e}_3|\vec{d})/d$, the sine of the angle between the orbital plane and the equator of Jupiter, is smaller than 10^{-2}. Hence the ratio between the last term of the first line and the first term is smaller than 10^{-3}. As we shall see, the principal effect of the size of Jupiter is of the order of 10^{-6}, so that the effect of the last terms is well below our truncation level of 10^{-7}. The second line of (A11) can also be neglected; the first term because it is a constant and the second term because it is proportional to the square of the sine of the angle between the equators of Jupiter and of Europa. The effect of $J_4 \approx 10^{-3} J_2$ is also negligible. Hence we can assume that (for $i = 1, 2$):

$$d^5 \int_W \frac{X_i^2}{r^5} \rho \, \mathrm{d}W = \frac{5}{2}(C' - A') \frac{(\vec{d}|\vec{f}_i)^2}{d^2}. \tag{A12}$$

The additional potential due to the finite size of Jupiter is thus

$$V = \frac{5}{2} \frac{n_2^2 C}{M_J d_0^2} (C' - A') \left[\frac{d_0}{d}\right]^5 [\delta_1(x^2 + y^2) + \delta_2(x^2 - y^2)], \tag{A13}$$

where (x, y, z) are the coordinates, in the body frame of Europa, of the unit vector pointing to the center of mass of Jupiter.

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