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DYNAMICS OF 2/1 RESONANT EXTRASOLAR SYSTEMS APPLICATION TO *HD82943* AND *GLIESE876*

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Abstract. A complete study is made of the resonant motion of two planets revolving around a star, in the model of the general planar three body problem. The resonant motion corresponds to periodic motion of the two planets, in a rotating frame, and the position and stability properties of the periodic orbits determine the topology of the phase space and consequently play an important role in the evolution of the system. Several families of symmetric periodic orbits are computed numerically, for the 2/1 resonance, and for the masses of some observed extrasolar planetary systems. In this way we obtain a global view of all the possible stable configurations of a system of two planets. These define the regions of the phase space where a resonant extrasolar system could be trapped, if it had followed in the past a migration process.

The factors that affect the stability of a resonant system are studied. For the same resonance and the same planetary masses, a large value of the eccentricities may stabilize the system, even in the case where the two planetary orbits intersect. The phase of the two planets (position at perihelion or aphelion when the star and the two planets are aligned) plays an important role, and the change of the phase, other things being the same, may destabilize the system. Also, the ratio of the planetary masses, for the same total mass of the two planets, plays an important role and the system, at some resonances and some phases, is destabilized when this ratio changes.

The above results are applied to the observed extrasolar planetary systems *HD 82943*, *Gliese 876* and also to some preliminary results of *HD 160691*. It is shown that the observed configurations are close to stable periodic motion.

Key words: periodic orbits, resonances, extrasolar systems, HD 82943, GJ 876

1. Introduction

The position and the stability properties of the periodic orbits (or, equivalently, of the fixed points of the Poincaré map) play an important role in the study of the dynamical evolution of a planetary system, because they determine the topology of the phase space. In particular, the mean motion resonances of a planetary system correspond to a periodic motion, in a rotating frame. This is the reason why the resonances play an important role in the study of the long term evolution of a planetary system.

There are several papers on the dynamics of the 2/1 resonant planetary motion and on the mechanisms that stabilize the system, or generate chaotic motion and instability: Gozdjiewski and Maciejewski (2001), Kinoshita and

Nakai (2001), Laughlin and Chambers (2001), Lee and Peale (2001, 2002), Ferraz-Mello (2002), Ferraz-Mello et al. (2003), Ji et al. (2002, 2003a,b), Mahlhotra (2002a, b), Bois et al. (2003), Gozdjiewski et al. (2004), Beaugé et al. (2004). In these papers different methods have been applied (averaging method, direct numerical integrations of orbits, or various numerical methods which provide indicators for the exponential growth of nearby orbits), for a range of orbital parameters. In this way the regions where stable motion exists have been detected, in the orbital elements space.

In the present paper a complete study is made of the resonant motion of two planets revolving around a star, which will be called the *sun*, in the model of the general planar three body problem, by computing all the basic families of resonant periodic orbits. The families of periodic orbits are very useful in the study of the stability and the evolution of an extrasolar planetary system. This is so, because it is close to a stable periodic orbit that a stable system could exist. In addition, the motion close to a periodic motion is the motion with the smallest variation of the orbital elements, a condition which may play an important role in the appearance of life. Finally, the regions of phase space close to a stable periodic motion are the regions where a planetary system may be finally trapped, if it had followed a migration process in the past, before it settled down to its present position. The study of this process is however beyond the object of the present paper.

We consider symmetric periodic orbits, which in this case are the most important resonances, which means that the perihelia of the two planets are either in the same direction or in opposite directions (aligned or antialigned), when the two planets are in the same line with the sun. In this symmetric case, the line of apsides of the two planets precesses slowly, in such a way that $\Delta\omega = 0$ or 2π. Several families of periodic orbits are computed numerically, for the 2/1, and for the masses of the systems *HD 82943*, *HD 160961* and *Gliese 876*.

The factors that affect the stability of a resonant system are studied. For the same resonance and the same planetary masses, the value of the planetary eccentricities is in some cases important and a large value of the eccentricities may stabilize the system, which for smaller eccentricities is unstable. The phase of the two planets (position at perihelion or aphelion when the star and the two planets are aligned) plays an important role, and the change of the phase, other things being the same, may destabilize the system. Also, the ratio of the planetary masses, for the same total mass of the two planets, plays an important role and the system, at some resonances and some phases, is destabilized when this ratio changes. The stability analysis of a resonant planetary system by the method of periodic orbits, that we present in this paper, allows us to obtain a global view of the dynamics of all the observed planetary systems at the 2/1 resonance. In this way the stability of all these systems can bet treated in a unified way.

The above results are applied for the study of the observed extrasolar planetary systems *HD 82943*, *Gliese 876*, *HD 160691*, at the 2/1 resonance and all the possible configurations which lead to stable motion are found. The elements of the above systems, as obtained from the observations, given in the web site *http://www.obspm.fr/encycl/catalog.html* maintained by Jean Schneider. All the masses are multiplied by $\sin i$, where i is the inclination of the planetary orbit and are therefore the minimum masses. In the present study we considered $\sin i = 1$. In the case of HD160961 there is some ambiguity on the value of the elements (Gozdjiewski et al., 2003), but we used the elements published in the above mentioned site, in order to show that even for very high eccentricities, the system may be stable for a suitable phase.

In all the following the central star will be called the *sun*, the inner planet will be called P_1 and the outer planet P_2.

2. The Dynamical Model

2.1. THE EQUATIONS OF MOTION

The model we used in the study of periodic motion of the planetary system is the general three body problem, for planar motion. As we shall see in the following, the gravitational interaction between the two planets is important, even for small planetary masses.

The center of mass of the planetary system is considered as fixed in an inertial frame, and the study is made in a rotating frame of reference xOy, whose x-axis is the line *sun*-P_1, the origin O is the center of mass of these two bodies and the y-axis is perpendicular to the x-axis (Figure 1). In this rotating frame P_1 moves on the x-axis and P_2 on the xOy plane. The coordinates are the position x_1 of P_1, x_2, y_2 of P_2 and the angle θ between the x-axis and a fixed direction in the inertial frame. The coordinates x_1, x_2, y_2, define the position of the system in the rotating frame and the angle θ defines the orientation of the rotating frame, so these four coordinates determine the position of the system in the inertial frame. This is a system of four degrees of freedom, and the Lagrangian of the system is (Hadjidemetriou, 1975)

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}(m_1+m_0)\{q(\dot{x}_1^2+x_1^2\dot{\theta}^2)+\frac{m_2}{m}[\dot{x}_2^2+\dot{y}_2^2+\dot{\theta}^2(x^2+y^2)\\ &+2\dot{\theta}(x_2\dot{y}_2-\dot{x}_2y_2)]\}-V,\end{aligned} \tag{1}$$

where

$$V = -\frac{Gm_0m_1}{r_{01}}-\frac{Gm_0m_2}{r_{02}}-\frac{Gm_1m_2}{r_{12}}, \tag{2}$$

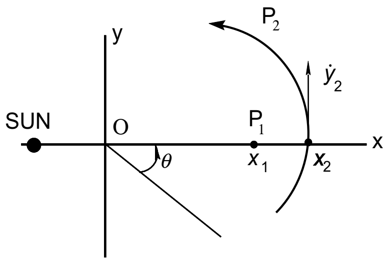

Figure 1. The rotating frame xOy.

and

$$m = m_0 + m_1 + m_2, \quad q = m_1/m_0. \tag{3}$$

G is the gravitational constant and r_{01}, r_{02} and r_{12} are the distances between the sun and P_1, the sun and P_2 and P_1P_2, respectively. We note that the angle θ is ignorable, so besides the energy (or Jacobi) integral there also exists the angular momentum integral, $L = \partial\mathcal{L}/\partial\dot{\theta} =$ constant,

$$L = (m_0 + m_1)\left\{\dot{\theta}\left[qx_1^2 + \frac{m_2}{m}(x_2^2 + y_2^2)\right] + \frac{m_2}{m}(x_2\dot{y}_2 - \dot{x}_2 y_2)\right\}. \tag{4}$$

By making use of this latter integral, we can eliminate the angle θ and thus reduce the system to a system of three degrees of freedom. This can be achieved by constructing the Routhian function, which is the new Lagrangian of the reduced three degrees of freedom system (Pars, 1965; Hadjidemetriou, 1975). The value of the angular momentum appears as a fixed parameter in the differential equations of motion in the rotating frame.

2.2. PERIODIC ORBITS

The differential equations of motion in the rotating frame xOy are invariant under the transformation

$$x_1 \to x_1, \quad x_2 \to x_2, \quad y_2 \to -y_2, \quad t \to -t,$$

which implies that if the planet P_2 starts perpendicularly from the x-axis ($y_2 = 0,\ \dot{x}_2 = 0$) and at that time $\dot{x}_1 = 0$, and after some time $t = T/2$ the planet P_2 crosses again the x-axis perpendicularly and at that time it is $\dot{x}_1 = 0$, the orbit is periodic with period equal to T, symmetric with respect to the x-axis. We remark that the second perpendicular crossing of P_2 from the x-axis may take place after several non perpendicular crossings.

From the above we see that the non zero initial conditions of a symmetric periodic orbit, in the rotating frame, are

$$x_{10}, \quad x_{20}, \quad \dot{y}_{20}. \tag{5}$$

This implies that a family of symmetric periodic orbits is represented by a smooth curve in the three dimensional space $x_{10},\ x_{20},\ \dot{y}_{20}$.

The periodic orbits that we will construct are in this rotating frame, which means that the *relative position* of the planetary system is repeated in the inertial frame. In order to avoid duplication of the results we fix the units of mass, length and time. This is achieved by taking the total mass of the system as the unit of mass, the gravitational constant equal to unity and also by keeping a fixed value of the angular momentum L for all the orbits of a family of periodic orbits. So, the normalizing conditions are

$$m_0 + m_1 + m_2 = 1, \quad G = 1, \quad L = \text{constant}.$$

In practice, we made the integration of the planetary system in the inertial frame (where the center of mass is fixed) and the reduction to three degrees of freedom in the rotating frame was made by a coordinate transformation. The method of integration was based on Taylor series expansion, and the accuracy was 10^{-14}.

We have computed all the basic families of periodic orbits of two planets in planar motion, in the 2/1 mean motion resonance. Along these orbits the resonance (or the ratio of the semi major axes) of the two planets is almost constant. The planetary orbits are perturbed ellipses and their eccentricity varies along the family, starting from zero values. Most of these families bifurcate from the family of circular orbits of the two planets, along which the orbits of the planets are almost circular, and the ratio T_1/T_2 of their periods starts from very large values and decreases along the family. At the 2/1 resonance a gap appears, for non zero planetary masses, and two distinct families of resonant elliptic orbits start from this gap (Hadjidemetriou, 2002; Hadjidemetriou and Psychoyos, 2003).

The periodic orbits that we computed are symmetric with respect to the rotating x-axis, which means that at $t = 0$, when the two planets are on the same line with the sun, the line of apsides are on this line and the position of the perihelia are either in the same direction or in opposite directions. Consequently, we have at $t = 0$ eight different phases, that are equivalent in pairs, and are given in Table I and Figure 2. All the possible initial phases of a periodic orbit, and the equivalent phase at $t = T/2$, are summarized in Table I.

Each of the symmetric families of periodic orbits that we present in the following sections belongs to a certain type, as described in Table I. In order to distinguish between the different families we will use the terminology:

– Family 1: Type 1 ($\omega_1 = \omega_2$), perihelia in the same direction.
– Family 2: Type 4 ($\omega_1 = \omega_2 + \pi$), perihelia in opposite directions.
– Family 3: Types 2 and 3. Starts as type 2 and ends as type 3, because the eccentricity of P_1 crosses zero and the planet goes from perihelion to aphelion.

As we shall see in the following, there are two basic families of periodic orbits, family 1 and family 2 (family 3 appears only in one case and is

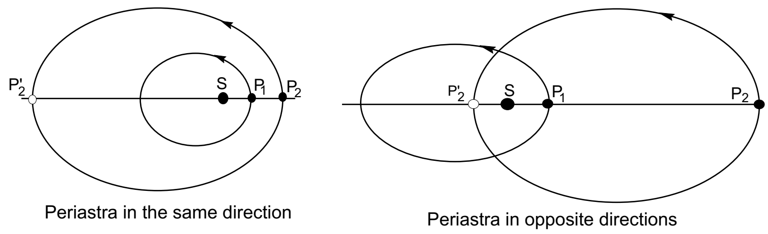

Figure 2. The four equivalent pairs of configurations at the 2/1 resonance.

TABLE I

All possible phases at $t = 0$ and $t = T/2$ for the 2/1 resonance

Type 1:	Sun – P_1(per) – P_2(per)	→	P_2 (ap) – Sun – P_1 (per)
Type 2:	Sun – P_1 (ap) – P_2 (ap)	→	P_2 (per) – Sun – P_1 (ap)
Type 3:	Sun – P_1 (per) – P_2 (ap)	→	P_2 (per) – Sun – P_1 (per)
Type 4:	Sun – P_1 (ap) – P_2 (per)	→	P_2 (ap) – Sun – P_1 (ap)

unstable). In family 1 the perihelia of the two planetary orbits are in the same direction and the two planets are both at perihelion at $t = 0$. In family 2, the perihelia are in opposite directions and at $t = 0$ P_1 is at aphelion and P_2 at perihelion (see Figure 2).

In the following we will present families of periodic orbits for the masses of the extrasolar planetary systems HD 82943, Gliese 876 and HD 160961. In order to have a better physical insight, we will not present the families of periodic orbits in the space of initial conditions, but in the space e_1e_2 of the planetary eccentricities. In order to avoid artificial discontinuities, we use the convention $e > 0$ if the planet is at aphelion and $e < 0$ if it is at perihelion.

3. The System HD 82943

3.1. FAMILIES OF PERIODIC ORBITS FOR THE MASSES OF THE SYSTEM HD 82943

We computed all the basic families of periodic orbits at the 2/1 resonance, for the masses of HD 82943, normalized to $m_0 + m_1 + m_2 = 1$. The basic results have been presented in Hadjidemetriou and Psychoyos (2003), but for reasons of completeness, and for comparison with further results on the 2/1 resonance, we will repeat them here. The basic families are given in Figure 3. The normalized masses (corresponding to the masses given by Israelian et al., 2001) are $m_0 = 0.9978$, $m_1 = 0.0008$, $m_2 = 0.0014$. Note that $m_1 < m_2$. There are three different families: *Family 1*, corresponding to the initial phase *sun – perihelion – perihelion*, *family 2*, corresponding to the initial phase *sun – aphelion – perihelion* and *family 3*, corresponding to the initial phase *sun –*

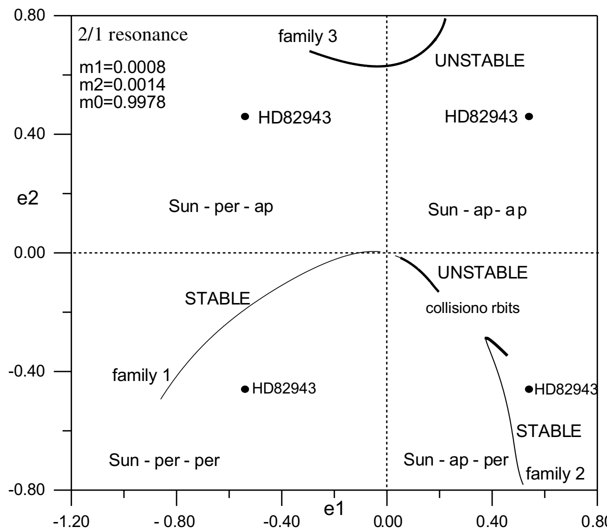

Figure 3. The symmetric families of periodic orbits at the 2/1 resonance, for the masses of HD 82943. The unstable parts are presented by a thicker line. The four possible positions of HD 82943 are also shown.

aphelion – aphelion → sun – aphelion – perihelion. In *family 1* the perihelia are in the same direction and in *family 2* they are in opposite directions. In *family 3* the perihelia are in the same direction at one end, but as e_1 decreases and passes from the value $e_1 = 0$, the perihelia shift to the opposite direction. All orbits of the family 1 are stable. The family 2 is unstable for small eccentricities. A gap appears on this family, due to close encounters between the two planets, and after the collision area the orbits on this family become stable, although the planetary orbits intersect. All orbits of family 3 are unstable. We remark that along the family 2, although all orbits have the same phase, the orbits with small planetary eccentricities are unstable, but the system is stabilized when the eccentricities are large. Four typical orbits on these families are presented in Figure 4.

The stability we mention above is the linear stability. In order to study the non linear stability, we considered two types of perturbations, that preserve the resonance: We shift the position of P_2 along its orbit to a new, non symmetric position, and also we rotate the orbit of P_2 by a certain angle. The evolution of the perturbed orbits were studied by computing the Poincaré map on the surface of section $y_2 = 0$. We found that in all cases the linearly stable orbits have a stable region, where bounded motion exists, with bounded variation of the orbital elements. The linearly unstable orbits become chaotic and in many cases one planet escapes. The details of the computations are in Hadjidemetriou and Psychoyos (2003).

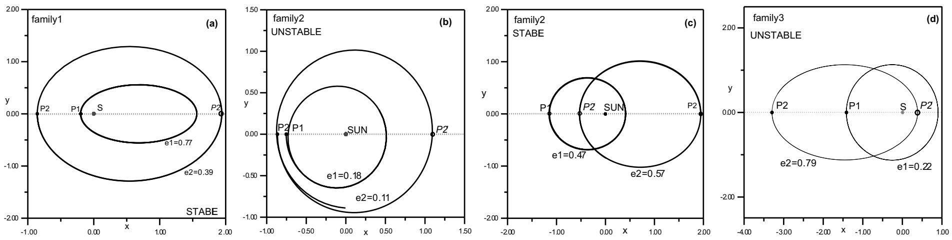

Figure 4. Four typical orbits on the families 1, 2 and 3. (a) family 1, $e_1 = 0.77, e_2 = 0.39$, stable. (b) family 2, $e_1 = 0.18, e_2 = 0.11$, unstable. (c) family 2, $e_1 = 0.47, e_2 = 0.57$, stable. (d) family 3, $e_1 = 0.22, e_2 = 0.79$, unstable.

3.2. THE PLANETARY SYSTEM HD 82943 WITH VARIABLE RATIO OF THE PLANETARY MASSES

In order to study the effect of the mass ratio of the planets on the stability of the system, we considered a planetary system with the same total mass of the planets as HD 82943, but with the masses reversed: $m_1 \to m_2$ and $m_2 \to m_1$. Now $m_1 = 0.0014, m_2 = 0.0008, m_1 > m_2$. The new families of periodic orbits are shown in Figure 5a. We note that the family 3 of Figure 3 no longer exists and a large part of the family 1 is now unstable. The stability of the family 2 is not affected by the inversion of the masses.

In Figure 5b we present the families 1 both for the true and the inverse masses, and we select two orbits, orbit 1 and orbit 2 on these two families. Orbit 1 is stable and orbit 2 is unstable. They both have the same eccentricity for P_2, $e_2 = 0.30$, and the eccentricities of P_1 are $e_1 = 0.06$ and 0.09 for the orbit 1 and orbit 2, respectively. In order to study the long term stability we computed the evolution of these two orbits by shifting the position of P_2 on its orbit by about 45°, by the Poincaré map on the surface of section $y_2 = 0$. The results are in Figure 6, for the evolution of the eccentricities and the semi major axes. We note that in both cases the system remains bounded, but there is an important qualitative difference between the linearly stable and unstable orbits. The variation of the eccentricities of the stable orbit 1 is very small, while it is large for the unstable orbit 2. The variation of the semimajor axes is small in both cases, but still there is a difference between the stable and the unstable orbit.

As we showed above, the whole family 1, corresponding to the phase *sun – perihelion – perihelion*, is stable when the mass of the inner planet is smaller than the mass of the outer planet, but the system is destabilized if the mass of the inner planet is larger than the mass of the outer planet. So the mass ratio m_1/m_2 plays an important role on the stability, for this phase.

We remark that a periodic orbit has two non unit pairs of eigenvalues (and one more pair which is always equal to unity, because of the existence of the energy integral). In the present case, where we have a nearly integrable

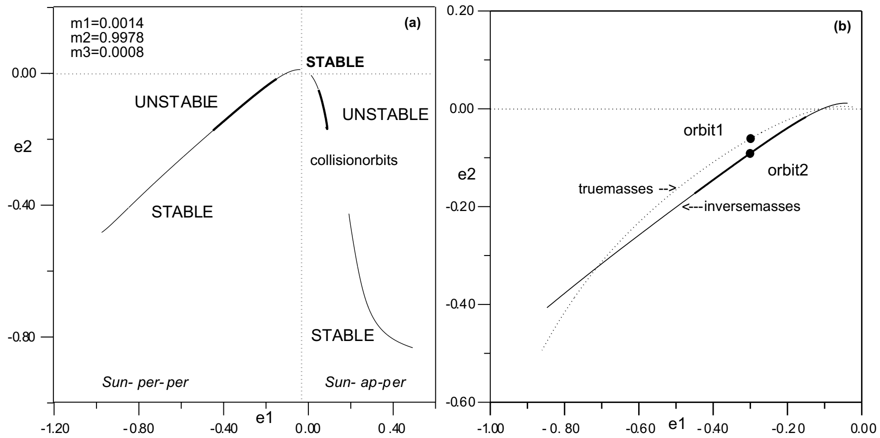

Figure 5. Families of periodic orbits for the inverse masses of HD 82943. (a) The families 1 and 2 with inverse masses. (b) The families 1 both for the true and the inverse masses, and the orbits 1 and 2 on these families, respectively. Note the unstable region on family 1.

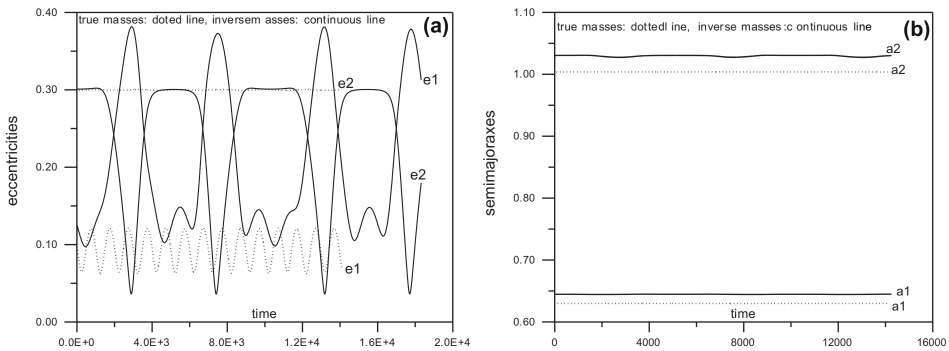

Figure 6. The evolution of the orbits 1 and 2 of the Figure 5b, when the position of the planet P_2 is shifted along its orbit by 45°. Dotted lines refer to the true masses and continuous lines to the inverse masses. (a) The evolution of the eccentricities. (b) The evolution of the semi major axes.

dynamical system (two weakly coupled Keplerian orbits), there exist two stability indices, which are close to the value -2 (corresponding to the two non unit pairs of eigenvalues). The orbit is stable if both stability indices are larger than -2 (and smaller than 2).

It turns out that along the family 1 the first stability index is in all cases larger than -2 (and close to this value), but the second stability index, which we will call b, may cross the value -2, $b < -2$, for a region of the family 1, and the system is destabilized. This is the mechanism how the system is destabilized at the 2/1 resonance.

In order to find the exact value of m_1/m_2 where instability appears for the first time on a region of the family 1, as m_1/m_2 varies (and $m_1 + m_2 = \text{constant}$),

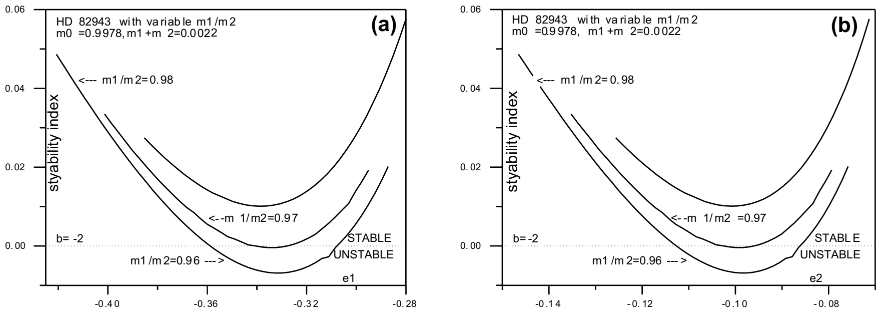

Figure 7. The stability along the *family 1* for three different mass ratios: (a) The stability vs. e_1. (b) The stability vs. e_2. Instability is generated when $m_1/m_2 < 0.97$.

we computed several families for different mass ratios and the results are shown in Figure 7a and b. In this figure we present the stability index b along the *family 1* for three different mass ratios, close to the transition value, which is equal to $m_1/m_2 = 0.97$. The x-axis in panel (a) is the eccentricity e_1, and in panel (b) the eccentricity e_2, which play the role of a parameter along the family. For the y-axis we used the value $10^3(2 + b)$, instead of the stability index b, so the transition value is zero, corresponding to $b = -2$.

From all the above we see that the system is stable, at the phase *sun – perihelion – perihelion*, if the mass ratio of the planets is $m_1/m_2 < 0.97$, provided that the mass of the sun is kept equal to $m_0 = 0.9978$ in normalized units.

In the case $m_1/m_2 > 0.97$ an unstable region appears on the family 1 (see Figure 5a), which increases as the ratio m_1/m_2 increases. At both ends of this unstable region there are critical points, with one stability index equal to $b = -2$, and we have a bifurcation of a new resonant 2/1 family of *non-symmetric* periodic orbits, from each critical point. It turns out that these two non symmetric families are connected, so in fact there is one non symmetric family of periodic orbits that starts from the first critical point and ends to the second critical point (Voyatzis and Hadjidemetriou, 2005). This is in agreement with the work of Beaugé et al. (2004), who found non symmetric periodic orbits in the 2/1 resonance.

3.3. THE EVOLUTION OF THE SYSTEM HD 82943

The elements of the system HD 82943 are (Israelinian et al., 2001): $m_0 = 1.05$ M_{SUN}, $m_1 \sin i = 0.88$ MJ, $m_2 \sin i = 1.63$ MJ, $a_1 = 0.73$ AU, $a_2 = 1.16$ AU, $T_1 = 221.6 \pm 2.7$ d, $T_2 = 444.6 \pm 8.8$ d, $e_1 = 0.54 \pm 0.05$, $e_2 = 0.41 \pm 0.08$, $\omega_1 = -$, $\omega_2 = 117.8 \pm 3.4$. This is a system very close to the 2/1 resonance.

In our numerical computations we considered four different cases, all with the same masses, semimajor axes and eccentricities, of the system HD 82943, given above, corresponding to the four different phases presented in Figure 1. These four positions are shown in Figure 2, where the families of periodic orbits are presented. It turned out that two phases, *sun – perihelion – perihelion* and *sun – aphelion – perihelion*, which are close to stable periodic orbits, are stable and the other two phases are unstable. The study was made by considering the Poincaré map on the surface of section $y_2 = 0$. This map is in the four dimensional phase space $x_1, \dot{x}_1, x_2, \dot{x}_2$ and in the above two stable phases the motion is clearly on a 4-torus. In the other two phases, close encounters between the two planets take place (line $x_1 = x_2$ in the Poincaré maps of Figure 8c and d) and the system is destabilized in a rather short time. Projections of the Poincaré map for all the above four cases are shown in Figure 8. In the first two cases, where we have bounded motion, the variation of the eccentricities is quite large, contrary to the semimajor axes whose variation has a small amplitude (Hadjidemetriou and Psychoyos, 2003).

Ji et al. (2003, submitted for publication) found, by a numerical exploration of the evolution of HD 82943 that this system is stabilized if, in addition to being in the 2/1 resonance, it is also in an apsidal resonance, with the axes of the two planets antialigned. This means, in fact, that the orbit should be close to a periodic orbit. This result coincides with our result, as presented in Figure 8b (phase: *sun – aphelion – perihelion*). Note that this phase is stable, despite the fact the the two planetary orbits intersect.

3.4. NEW ORBITAL VALUES FOR HD 82943

New values for the system HD 82943 were given recently by Mayor et al. (in preparation). The orbital elements and the values of the masses are quite different from those published before. The new values are: $m_0 = 1.05\ M_{\mathrm{SUN}}$, $m_1 \sin i = 1.85$ MJ, $m_2 \sin i = 1.84$ MJ, $a_1 = 0.75$ AU, $a_2 = 1.18$ AU, $T_1 = 219.4 \pm 0.2$ d, $T_2 = 435.1 \pm 1.4$ d, $e_1 = 0.38 \pm 0.01$, $e_2 = 0.18 \pm 0.04$, $\omega_1 = 124 \pm 3$, $\omega_2 = 237 \pm 13$. This is a system very close to the 2/1 resonance and although it is stated that the values of the eccentricities may be different from those published, we repeated the study of the evolution of this system, as in Section 3.2, using the new values. In Figure 9 we present the families of periodic orbits for the new masses of HD 82943 and we also show the position of the system corresponding to these new elements. We note that the phase *sun – perihelion – perihelion* is very close to the stable region of the family 1, but not far from the unstable region.

In Figure 10a–d we show the evolution of the system for the above four configurations, using the new elements. We note that the only stable phase is *sun – perihelion – perihelion*. Note that the phase *sun – aphelion – perihelion* is unstable, contrary to the case of Figure 8b (for the same phase), for the old

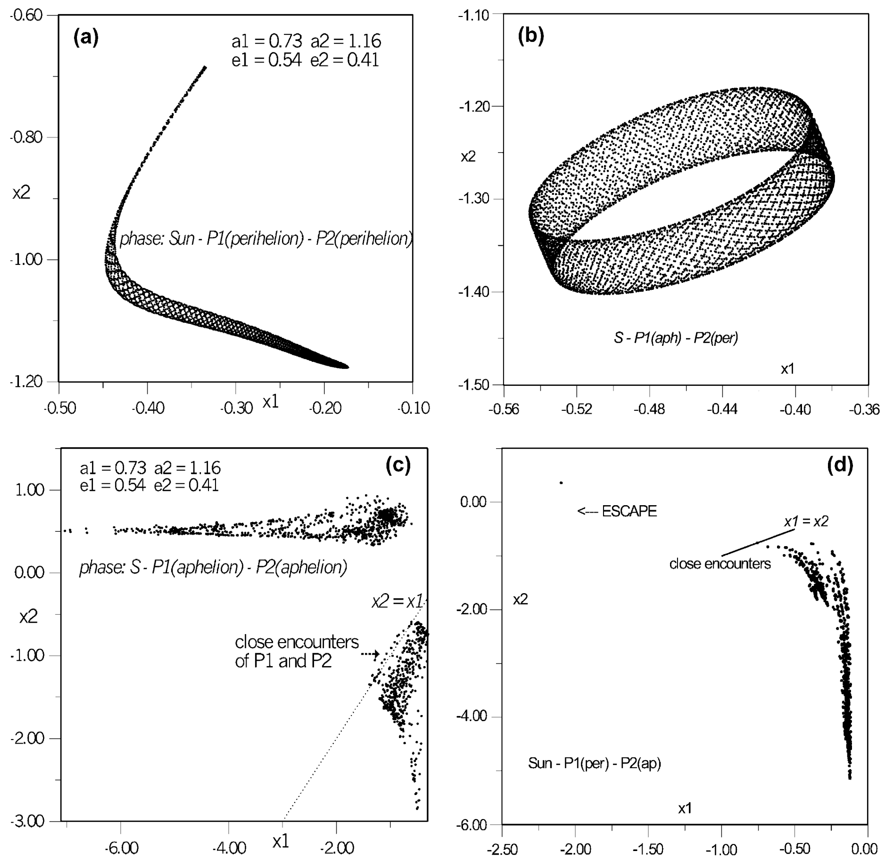

Figure 8. The evolution of HD 82943 for the four different possible phases. Projection of the Poincaré map on coordinate planes. (a) *sun – perihelion – perihelion*, (b) *sun – aphelion – perihelion*, (c) *sun – aphelion – aphelion*, (d) *sun – perihelion – aphelion*. In cases a and b the motion is bounded, on a 4-torus.

elements of HD 82943, which is stable. This is so because now this phase is close to the close encounter zone, because the values of the eccentricities are in the new case smaller, and the system is destabilized.

4. The System Gliese 876

4.1. FAMILIES OF PERIODIC ORBITS FOR THE MASSES OF GLIESE 876

We repeated the study of the families of periodic orbits for the masses of the system Gliese 876, as for the system HD 82943. In this case we also have

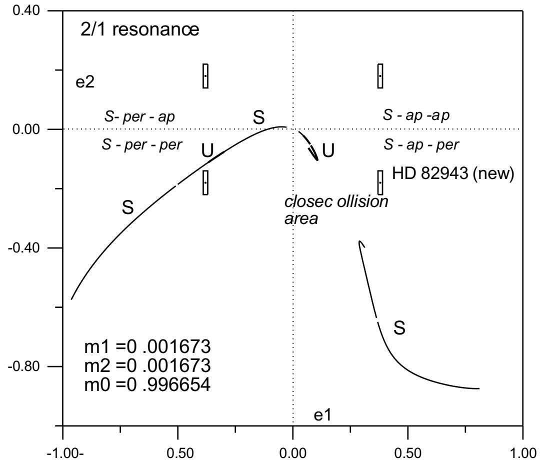

Figure 9. The symmetric families of periodic orbits at the 2/1 resonance, for the *new* masses of HD 82943. The unstable parts are presented by a thicker line. The four possible positions of HD 82943 for the new elements are also shown, together with the error bars. Compared with the figure 3, we note that a small unstable region appears on the family 1 (sun – perihelion – perihelion).

$m_1 < m_2$, but now the normalized planetary masses are larger, compared to the normalized masses of the system HD 82943. The normalized masses of Gliese 876 (corresponding to the masses given by Marcy et al., 2001) are $m_0 = 0.98275, m_1 = 0.00166, m_2 = 0.00559$. This means that the gravitational interaction between the two planets is stronger and in particular, for the phase *sun – aphelion – perihelion* and small planetary eccentricities, it dominates. As a consequence, the part of the family 2 corresponding to small eccentricities practically does not exist at all and we have in this region a much larger gap. We remind that we also had a gap for the masses of HD 82943 (Figure 3). In fact, a 2/1 resonant motion for this phase and small eccentricities cannot exist at all, because the two planets are trapped into a 1/1 resonance and the two planets revolve around the sun as a close binary, as we will show in the next section. (Such orbits do exist however, if the planetary masses are smaller, as we verified by numerical computations). In Figure 11 we present the families 1 and 2 of periodic orbits, for the masses of Gliese 876. In this figure we also indicate the position of Gliese 876, for the observed elements (*sun – perihelion – perihelion*) and for the three other possible phases.

From this figure we can find all the possible stable configurations that the system Gliese 876 could obtain. A different approach to this problem was made by Gozdjiewski et al. (2002), who made a complete stability investigation using the MEGNO technique, and they found estimations of the 2/1 mean motion resonance widths.

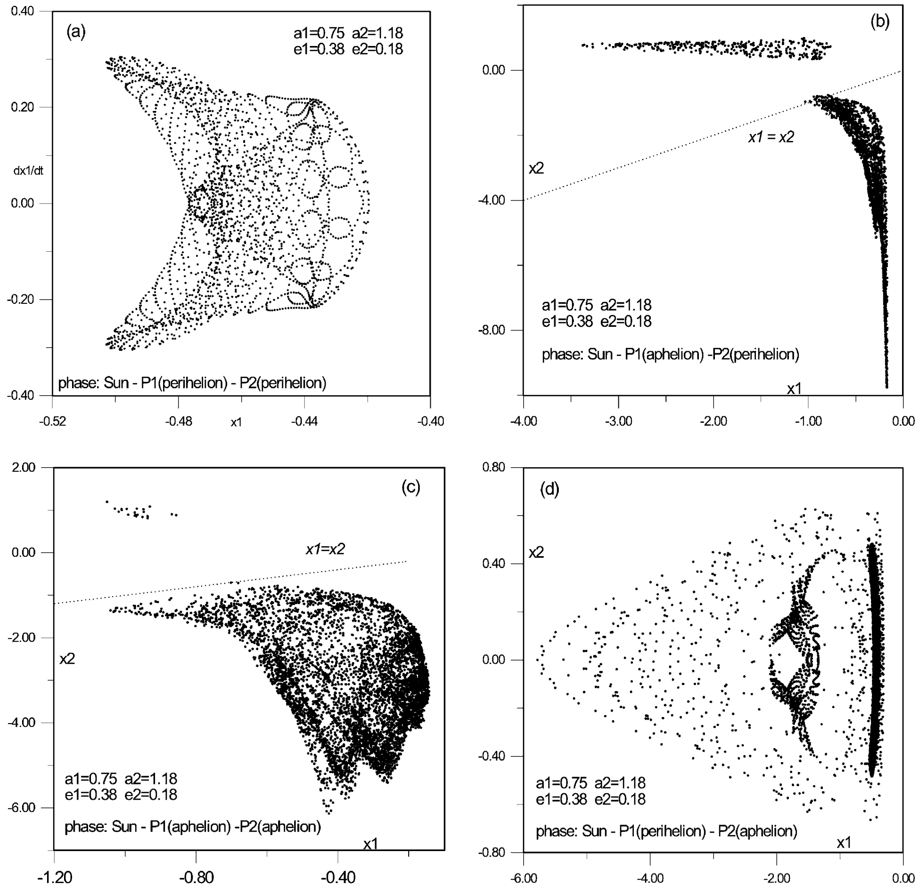

Figure 10. The evolution of HD 82943, new, for the four different possible phases. Projection of the Poincaré map on coordinate planes. (a) *sun – perihelion – perihelion*, (b) *sun – aphelion – perihelion*, (c) *sun – aphelion – aphelion*, (d) *sun – perihelion – aphelion*. The motion is bounded, on a 4-torus, only in the case (a). In the case (d) the motion is initially on a 4-torus (black region to the right), but later chaotic motion develops. In the cases (b) and (c) close encounters between P_1 and P_2 take place (points close to the line $x_1 = x_2$) and the system is destabilized.

4.2. THE EVOLUTION OF GLIESE 876

The observed system Gliese 876 corresponds to the phase where the line of apsides of both planets are almost on the same line and the perihelia in the same direction. The elements of this system are (Marcy et al., 2001; Laughlin and Chambers, 2001; Rivera and Lissauer, 2001): $m_0 = 0.32$ $M_{\rm SUN}$, $m_1 \sin i = 1.89$ MJ, $m_2 \sin i = 0.56$ MJ, $a_1 = 0.21$ AU, $a_2 = 0.13$ AU, $T_1 = 61.02$ d, $T_2 = 30.1$ d, $e_1 = 0.10$, $e_2 = 0.27$, $\omega_1 = 333$, $\omega_2 = 330$. This position is indicated in Figure 11, for $e_1 = -0.10, e_2 = -0.27$. We also show

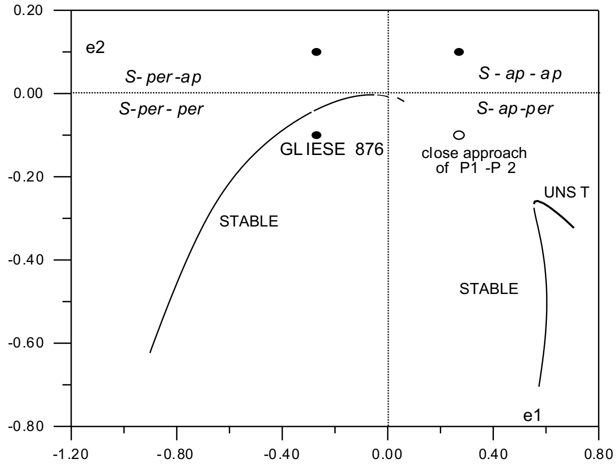

Figure 11. The families 1 and 2 of periodic orbits, for the masses of Gliese 876. Orbits with the phase *sun – aphelion – perihelion*, for small eccentricities do not exist, due to the strong gravitational interaction between the planets.

in this figure the other three possible phases with the same eccentricities. The phase $e_1 = +0.10, e_2 = -0.27$ (*sun – aphelion – perihelion*) is inside the close approach region, and is indicated in Figure 11 by an empty circle.

The evolution of the true system is studied by computing the Poincaré map on the surface of section $y_2 = 0$. In Figure 12a we show a projection of the Poincaré map on the plane $x_1, \dot{x}_1$ and in Figure 12b and c we present the evolution of the eccentricities and semimajor axes. We note that the system moves on a well defined 4-torus (a projection is in Figure 12a) and the amplitude of the variation of the eccentricities and semimajor axes is small.

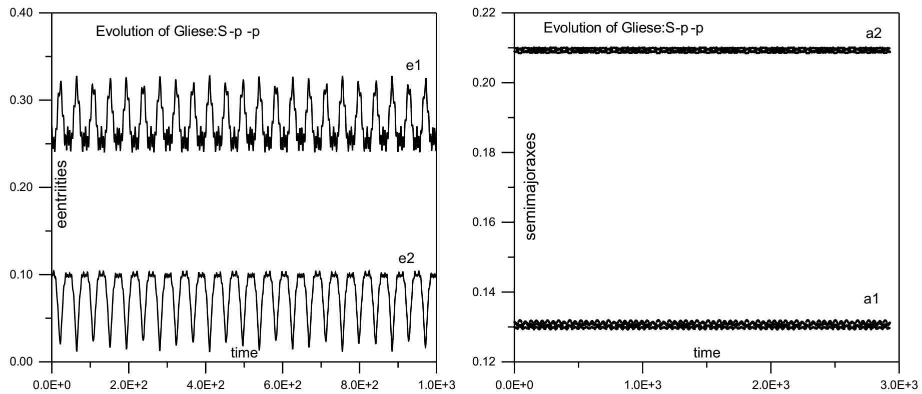

Figure 12. The Poincaré map of the true system Gliese 876, *sun – per – per*. (a) Projection on the $x_1\dot{x}_1$ plane. (b) Evolution of the eccentricities. (c) Evolution of the semimajor axes.

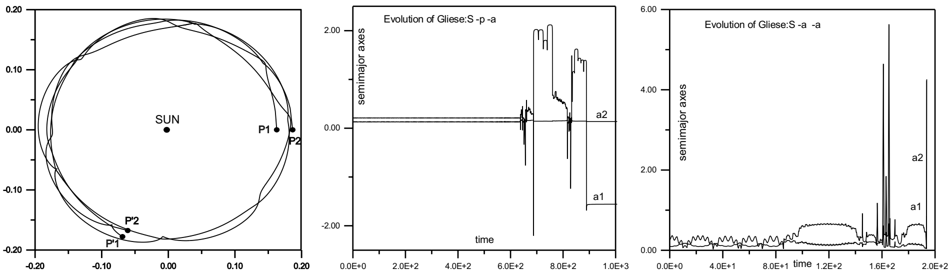

Figure 13. (a) The orbit of Gliese 876 for the phase *sun – aphelion – perihelion* inside the close approach zone. Due to the strong gravitational interaction between P_1 and P_2 the two planets are trapped into a close binary which revolves around the sun. (b) The evolution of the semimajor axes of Gliese 876 for the unstable phase *sun – aphelion – aphelion*. (c) The evolution of the semimajor axes of Gliese 876 for the unstable phase *sun – perihelion – aphelion*.

In Figure 13a we show the orbit of the system when initially the two planets are placed at the phase *sun – aphelion – perihelion*. The gravitational interaction between the two planets dominates the attraction from the sun and the system is trapped to a 1/1 resonance, forming a close binary which revolves around the sun. Note that the same phase gives a stable configuration for the system HD 82943 (Figure 8b), because the planetary eccentricities are larger, and this fact stabilizes the system, because the close encounters between the planets are avoided.

In Figure 13b and c we show the evolution of the eccentricities and semimajor axes for the phases *sun – aphelion – aphelion* and *sun – perihelion – aphelion*. In both cases the system is destabilized, due to close encounters between the two planets, as is verified from the projection of the Poincaré map on the x_1x_2 plane. Note that in both cases the system is at first trapped on a torus, but soon chaotic motion develops and the system is destabilized.

A stability analysis of the system Gliese 876 was made by Gozdziewski et al., 2002. They used the MEGNO technique and proved that the system is stable if, in addition to being at the 2/1 resonance, it is also in a $\omega_1 - \omega_2 \simeq 0$ secular resonance. This means that the system should be close to a periodic orbit, and this coincides with our results, as shown in Figure 12, which corresponds to the stable phase *sun – perihelion – perihelion*.

5. The System HD 160961

5.1. FAMILIES OF PERIODIC ORBITS FOR THE MASSES OF HD 160961

In this case we also computed the basic 2/1 resonant families of periodic orbits, corresponding to the masses of the system HD 160961, which is a system very close to the 2/1 resonance. The normalized masses (corre-

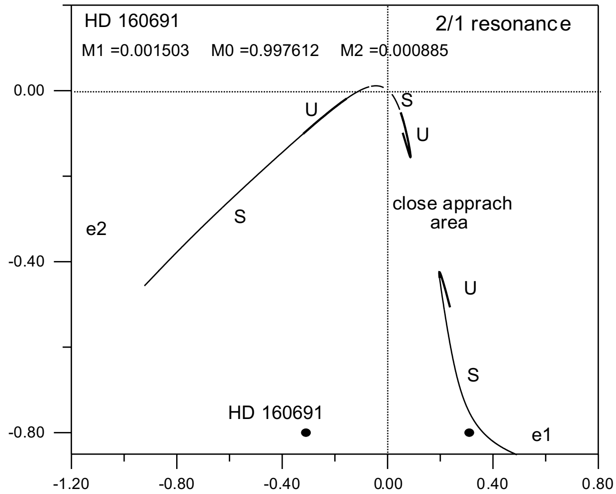

Figure 14. The families 1 and 2 of periodic orbits, for the masses of HD 160961. The position of the real system HD 160961 for two different phases are also shown.

sponding to the masses given by Jones et al., 2002) are $m_0 = 0.997612, m_1 = 0.001503, m_2 = 0.000885$. Two families, *family 1* and *family 2* are presented in Figure 14. Note that $m_1 > m_2$ and consequently an unstable region appears in family 1. In this figure we also present the position of the real system HD 160961, for two different phases: *sun – perihelion – perihelion* and *sun – aphelion – perihelion*. Two more possible phases (*sun – aphelion – perihelion* and *sun – aphelion – aphelion*) are not shown. Only one phase, namely *sun – aphelion – perihelion* is close to a periodic orbit, and it is the only stable configuration, as we will show in the following.

5.2. THE EVOLUTION OF THE SYSTEM HD 160961

The elements of the system HD 160961 are given (Jones et al., 2002): $m_0 = 1.08$, M_{SUN}, $m_1 \sin i = 1.7$ MJ, $m_2 \sin i = 1$ MJ(?), $a_1 = 1.48$ AU, $a_2 = 2.3$ AU(?), $T_1 = 637.3$ d, $T_2 = 1300$ d(?), $e_1 = 0.31$, $e_2 = 0.8$(?), Omega (deg):, $\omega_1 = 320$, $\omega_2 = 99$(?). Although the above values are not the only possible fits to the observational data and possibly they are not correct (Gozdjiewski et al., 2003), we computed the Poincaré map with these elements, for all four different phases, mentioned in the previous section. These are computed for the symmetric case. The only stable configuration is the one corresponding to the phase *sun – aphelion – perihelion* (perihelia in opposite directions) and its evolution is given in Figures 15 and 16. The motion is clearly on a 4-torus (Figure 16a) and the evolution of the eccentricities of the two planets is almost periodic. The same is true for the semimajor axes (not shown). We also note

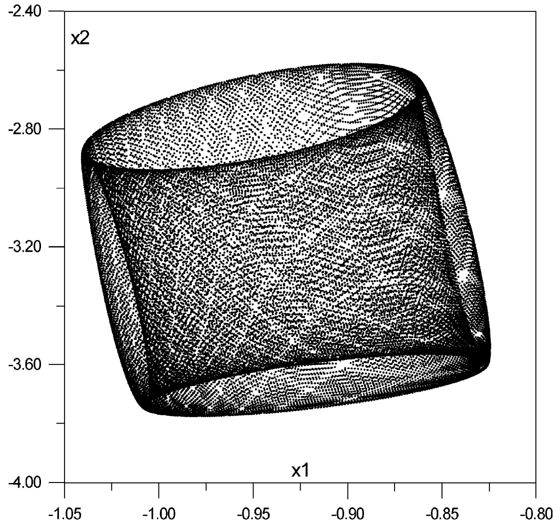

Figure 15. The Poincaré map of the system HD 160961, for the phase *sun – aphelion – perihelion*. Projection on the x_1x_2 plane. The motion is bounded, on a 4-torus.

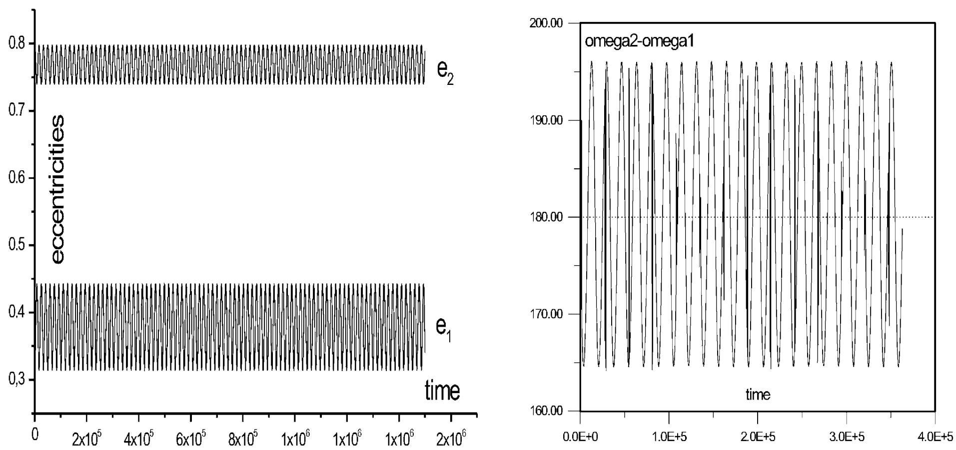

Figure 16. The evolution of the eccentricities and the angle $\omega_2 - \omega_1$, which librates around the value π, corresponding to the system HD 160961, for the phase of Figure 15.

that the angle $\omega_2 - \omega_1$ librates around 180 (Figure 16b). Note that this configuration is stable, despite the fact that the two planetary orbits intersect.

Starting from the above configuration, we extended the study of the evolution of the system, by changing the angle $\omega_2 - \omega_1$. We found that the system remains bounded up to $\omega_2 - \omega_1 = 45°$. Beyond this value, the system is chaotic.

In Figure 17 we present the evolution of the semimajor axes and the angle $\omega_2 - \omega_1$ of the system HD 160961 for the phase *sun – perihelion – aphelion*. The motion is clearly chaotic.

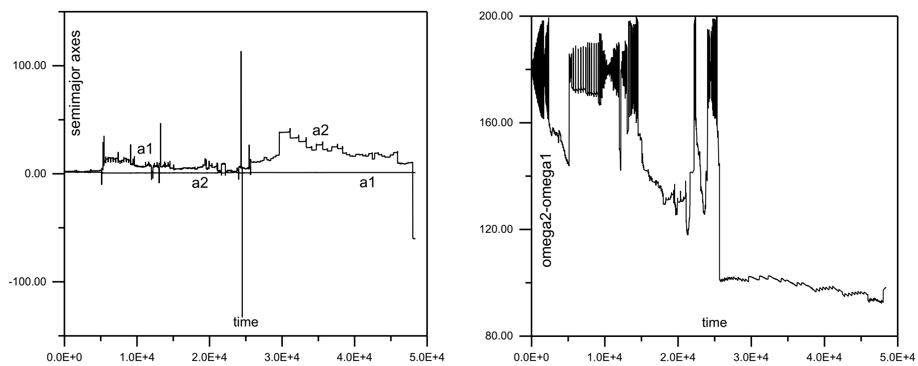

Figure 17. The evolution of the semimajor axes and the angle $\omega_2 - \omega_1$ corresponding to the system HD 160961, for the phase *sun – perihelion – aphelion*.

In Figure 18 we present the evolution of the semimajor axes of the system HD 160961, for the phases *sun – aphelion – aphelion* and *sun – perihelion – perihelion*. In this case also the motion is chaotic.

Bois et al. (2003), studied the stability of HD 160961 by the MEGNO technique. They found that the system is stable if it is in a 2/1 resonance, combined with an apsidal secular resonance, corresponding to the phase *aphelion – sun – aphelion* $\rightarrow$ *sun – aphelion – perihelion*. Their results are in complete agreement with the results that we obtained, as shown in Figures 15 and 16.

6. Conclusions

Several techniques have been used to study the dynamical evolution and the stability of an extrasolar planetary system. In the present work we present a

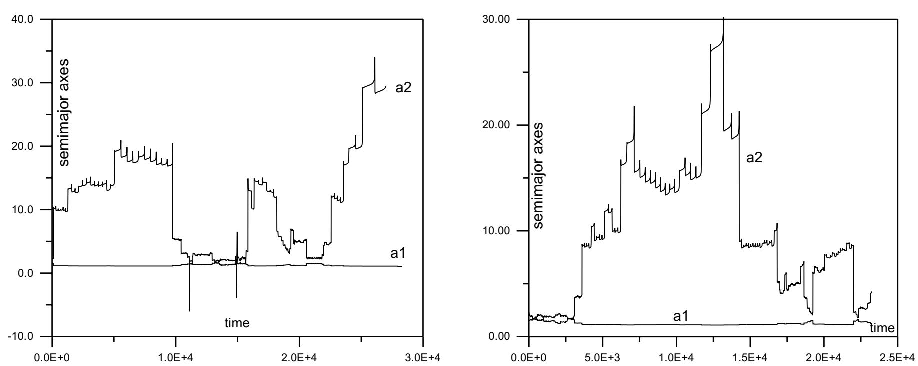

Figure 18. The evolution of the semimajor axes corresponding to the system HD 160961, for the phases *sun – aphelion – aphelion* (panel (a)) and *sun – perihelion – perihelion* (panel (b)).

method to obtain a global view of all the stable and unstable resonant configurations of a planetary system, by making a complete study of all the basic families of resonant periodic orbits.

The periodic orbits (or, equivalently, the corresponding fixed points in a Poincaré map on a surface of section) determine the topology of the phase space. In particular, close to a stable periodic orbit there exists a region where the orbit librates around the exact periodic orbit, and consequently stable, bounded, motion could be expected to exist in nature. On the contrary, the motion close to an unstable periodic orbit is chaotic, and in some cases one planet escapes. Consequently, a planetary system could not exist in nature at this region of the phase space. So, if we know the families of periodic orbits, we know in what regions of the phase space, or, equivalently, in the space of the orbital elements, a planetary system could exist in nature. These are the regions where a planetary system could be trapped in its present form, if it had followed a migration process in the past. The stable regions can also serve as a guide to select the best fits of elements in the observation of a new planetary system. These latter topics however, are beyond the scope of the present paper.

The periodic orbits that we study are in the model of the general planar three body problem and are periodic in a *rotating* frame. This means that the *relative* configuration is repeated in the inertial frame. The two planets revolve around the sun in elliptic orbits, which are perturbed because of their mutual gravitational interaction, and are in mean motion resonance. In addition, since the most important families are symmetric with respect to the rotating x-axis (Section 2), they are also in an apsidal secular resonance, which means that either $\omega_1 - \omega_2 = 0$ or $\omega_1 - \omega_2 = \pi$. This means that the apsidal lines are either aligned or antialigned.

The present study is restricted to the 2/1 resonance, and we found, in a global way, all the factors that stabilize a resonant planetary system. In this way, the study of the dynamics of all the observed 2/1 resonant planetary systems, HD 82943, Gliese 876 and HD 160961, can be made in a unified way.

We found that the phase of the two planets, that is, their position at perihelion or aphelion, when they are in the same line with the sun, plays an important role for the stability. The most stable phase is *sun – perihelion – perihelion*, for $m_1 < m_2$, which is equivalent to the phase *aphelion – sun – perihelion*. The perihelia are in this case aligned.

The value of the eccentricities of the planetary orbits is also an important stabilizing parameter, especially in the phase *sun – aphelion – perihelion*, which implies that the two perihelia are antialigned. For small eccentricities the system is chaotic, but if the eccentricities are large, the close encounters are avoided, and the system is ordered and stays bounded. This is clearly seen by comparing Figures 8b and 10b. Thus, the increase of the eccentricities plays a stabilizing role. We remark that in this latter phase the two planetary orbits intersect, but the resonance generates a phase protection mechanism, which does not allow the two planets to come close to each other.

The other two possible phases, *sun – perihelion – aphelion* and *sun – aphelion – aphelion* are always unstable. In some cases (Figure 13b and c) the system sticks on a 4-torus for a long time interval, but finally chaotic motion develops and the system is destabilized. The mechanism of generation of instability is the close approach between the two planets, as is seen in Figures 8c and d and 10c and d.

Another factor that plays an important role on the stability of a planetary system is the ratio m_1/m_2 of the planetary masses. We found that in the phase *sun – perihelion – perihelion* the system is stable if the mass of the inner planet is smaller than the mass of the outer planet and becomes unstable if $m_1 > m_2$. The change of the mass ratio however, does not affect the stability in the phase *sun – aphelion – perihelion*.

The above results, applied to the observed systems HD 82943, HD 160961 and Gliese 876, showed that the phase *sun – perihelion – perihelion* is stable in all cases. The real systems can be considered as perturbed periodic orbits corresponding to this resonance. The phase *sun – aphelion – perihelion* is stable only in the case HD 82943, old data, because the eccentricities are large. In all other systems, including HD 82943 with the new data, the system is unstable, because the eccentricities are small.

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