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DEVELOPING THE ANALYTIC HIERARCHY PROCESS UNDER COLLECTIVE DECISION-MAKING BASED ON AGGREGATED MATRICES OF PAIRWISE COMPARISONS

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Abstract. *An approach to collective decision-making based on the analytic hierarchy process is proposed. This approach is based on the mechanism of constructing aggregated matrices of pairwise comparisons. The key point of this mechanism is to reconcile the polar opinions of experts on the preference of alternatives. Such harmonization of opinions is implemented by choosing the most fair hypothesis. The basis for this choice is the degree of confidence in the validity of this hypothesis. The degree of confidence is calculated using the Shortliff combination function. Coordination of polar opinions of experts is a computational model of group choice, which is an independent component and can be used as a basis for the development of collective decision-making procedures. The proposed approach is quite natural and easy to use and harmoniously forms a single whole within the analytic hierarchy process.*

Keywords: *collective decision-making, ranking, expert, analytic hierarchy process, confidence coefficients.*

INTRODUCTION

In the practice of decision-making, the analytic hierarchy process (AHP) is widely used, which is one of the effective methods of system analysis and can be applied both by one person and by a group of experts depending on the stated problem complexity [1]. Moreover, such a group is a collective expert that makes a decision either as a result of consensus or, if a joint conclusion cannot be reached, uses some existing rule to reach a unified judgment.

In view of the above, various expansions of this method were proposed for the case of collective decision-making [2–15]. Hence, in [2], a model of the analytic hierarchy process of group decision-making (AHP-GDM) is proposed to reduce investment risk. In order to satisfy the properties of the inverse matrix, the method of least squares is used to adjust the matrix of group decisions.

In [3], modified AHP algorithms are considered, taking into account the judgments of several experts and problem situations. Here, the Boarda count is used as a group selection rule. Procedures for aggregating judgments expressed through matrices of pairwise comparisons are given in [4, 5]. In [6], the AHP application for group decision-making (AHP-GDM) taking into account the cognitive levels of various experts is considered. A fuzzy extension of the analytic hierarchy process to group decision-making is described in [7]. In [8, 9], methods for identifying differences in judgments and smoothing them out, resolving conflicts, and combining individual judgments to obtain a common group advantage are given. In [10], the Ramanathan and Ganesh's method of determining the priorities of decision-makers is considered. Its use in the process of aggregating group preferences of people whose judgments are unequal is shown. A comparison of aggregation methods is considered in [11–13]. In [14], the DS/AHP method that combines

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Dempster–Shafer evidence theory with the AHP is proposed. This method enables judgments to be made considering group alternatives, and it also offers a measure of uncertainty in the final results. In [15], the DS/AHP method is developed as an effective tool in group decision-making. Here, attention is paid to the totality of the testimony of individual members of the group, which is considered to be non-equivalent in its significance.

This article proposes an alternative approach to the AHP use in the case of group decision-making. It is based on the selection mechanism based on the principle of maximum confidence implemented by the Shortliff scheme.

CONSTRUCTING AGGREGATED MATRICES OF PAIRED COMPARISONS

The key point of the AHP in the case of group decision-making is the aggregation of individual judgments of group members. Individual judgments can be aggregated in different ways, for example, either by individual pairwise comparisons or by individual priorities. If group members act as individuals when making decisions, then it is better to aggregate their judgments at the level of individual priorities. In this case, it is possible to apply the Kemeny median or its modification [16] that takes into account the weight factors of group members. However, during group decision-making, group members usually act together and not as individuals, i.e., they communicate. Thus, it is more appropriate to use the first method [10].

Let the hierarchy of the problem of group choice $\mathcal{J}$ and the set E of experts ($|E| = r$) be given where r is odd. Let $M^l = \{M_k^l = \| m_{ij}^{lk} \| \mid k = \overline{1, r}\}$ be a set of matrices of individual pairwise comparisons of alternatives with respect to the lth criterion as well. Let us make a note on the construction of individual matrices of pairwise comparisons. If pairwise comparisons are carried out for a qualitative criterion, then experts evaluate only the dominance of alternatives as it is always possible to determine a slight advantage of one alternative over another even during physical measurements, not to mention the qualitative characteristics inherent to multifacetedness.

The elements of the aggregated matrix M_A^l are calculated for each pair of alternatives as follows: The judgments of experts regarding the absolute superiority of alternatives i and j are considered.

If the experts' points of viewcompletely coinside (for $\forall k$, there is either $m_{ij}^{lk} > 1$ or $m_{ij}^{lk} < 1$ or $m_{ij}^{lk} = 1$), then the aggregated element $\overline{m}_{ij}^l$ of the matrix M_A^l is calculated according to the following formula:

$$\overline{m}_{ij}^l = \prod_{k=1}^{r} (m_{ij}^{lk})^{\mu_k}, \tag{1}$$

where $\sum_{k=1}^{r} \mu_k = 1.$

If there are disagreements in the judgments of experts regarding the preference of the alternatives i and j that take the form $i \succ j$ and $i \prec j$, then the direct application of this formula is not feasible. If, for example, two equal experts indicate reciprocal evaluations during the comparison of these alternatives, a unit is obtained in the process of calculating the aggregated evaluation according to (1), which indicates their equivalence. Since the judgment is new, it cannot be the result of a mechanical averaging of scores. In this case, such differences are resolved either as a result of consensus or, if a joint conclusion cannot be reached, on the basis of the group selection principle that determines the rule for obtaining an agreed upon judgment.

Consider the following statements.

The subject of the decision is a person as they and only they makes the decision and are responsible for the consequences of its implementation. Therefore, the group choice mechanism should take into account its behavioral characteristics.

From the point of view of the psychology of decision-making, the main driving force at the moment when a choice is made in favor of one of the alternatives, i.e., when a certain decision is made, is confidence in the correctness of this choice. Let us use this statement as a principle of confidence in the process of group selection.

Next, let us call judgments $i \succ j$ and $i \prec j$ hypotheses and denote them by h_1 and h_2, respectively. Then, as a basis for choosing one of the hypotheses, the degree of confidence in its truth is used.

Let us introduce the function $f(h)$ that relates the hypothesis h to its degree of confidence η by taking into account the judgments of all experts regarding the hypothesis h. Then, the hypothesis of choice is the hypothesis h_{k_0} where $k_0 = \arg\max_k \eta_k$. If $\eta_1 = \eta_2$, then the choice of one of the hypotheses h_1 and h_2 is carried out according to the majority rule.

Let r_{k_0} be the number of experts who expressed the hypothesis h_{k_0}; then, the element $\overline{m}_{ij}^{\,l}$ of the matrix M_A^l is calculated by the following formula:

$$\overline{m}_{ij}^{\,l} = \prod_{k=1}^{r_{k_0}} (m_{ij}^{lk})^{\mu_k}. \tag{2}$$

In this formula, the condition $\sum_{k=1}^{r_{k_0}} \mu_k = 1$ is fulfilled for the weight coefficients. Aggregated matrices of pairwise comparisons of alternatives with respect to other criteria are constructed is a similar manner.

CALCULATING THE DEGREE OF CONFIDENCE IN THE HYPOTHESIS

From the point of view of psychology as stated in [17], an individual's confidence in a hypothesis is based on the belief that he trusts it. In turn, trust is based on the proven validity of the hypothesis. Thus, confidence is an expression of conviction obtained through the assessment of trust.

From the point of view of logic, a hypothesis as an assumption is a conclusion according to the schemes of a conditional-categorical syllogism with the true meaning of its premises being uncertain. To measure the degree of confidence in the conclusion of any production rule, Shortliff developed a scheme [18, 19] that is effective in practical applications. This scheme is based on so-called confidence coefficients that reflect the relationship between confidence and trust.

Let the expert e express the hypothesis h about the dominance of alternatives i and j. Then, there is a causal relationship between the alternatives, the expert, and the hypothesis that can be represented by the rule $i, j, e \to h$. For the sake of simplicity, we denote the antecedent i, j, e of this rule by the singleness e. Then, the coefficient of confidence in the hypothesis h expressed by the expert e is calculated according to the formula from [19] as follows:

$$CF\,[h, e] = \frac{MB\,[h, e] - MD\,[h, e]}{1 - \min\,(MB\,[h, e], MD\,[h, e])}, \tag{3}$$

where $MB\,[h, e]$ is the measure of trust and $MD\,[h, e]$ is a measure of mistrust to the hypothesis h. Here, a measure of trust $MB\,[h, e]$ is understood as the probability that the hypothesis h is true. Hence, $1 - MB\,[h, e]$ can be considered as a mistrust degree of $MD\,[h, e]$ to the truth h. To determine the measure of trust, let us proceed from the considerations following below.

According to the subjective probability theory, it can be argued that an expert's personal probability reflects his belief in a hypothesis at any given time. There are several methods to determine subjective probability [20]. The most accessible method connects the expert's subjective probability to his weight coefficient. If $\mu \in [0, 1]$ is the weight coefficient of the expert, then μ is his personal probability. Hence, if the expert expressed a hypothesis h, the probability of its truth is μ.

Next, if the ordinal judgments of several experts regarding the dominance of the alternatives i and j coincide, then these experts should collectively provide greater confidence in this judgment than each of them individually. Given the independence of experts' judgments, the given in what follows combination function is used to take into account the joint influence of their opinions.

Let $CF_1\,[h, e_1]$ and $CF_2\,[h, e_2]$ be the coefficients of confidence in the hypothesis h expressed by experts e_1 and e_2. Then, the coefficient $CF\,[h, e_1 \& e_2] = CF_2\,[h]$ of their joint support for the hypothesis h is calculated by the function from [19] as follows:

$$CF_2\,[h] = \begin{cases} CF_1[h, e_1] + CF_2[h, e_2](1 - CF_1[h, e_1]), & CF_1[h, e_1] > 0,\ CF_2[h, e_2] > 0, \\ CF_1[h, e_1] + CF_2[h, e_2](1 + CF_1[h, e_1]), & CF_1[h, e_1] < 0,\ CF_2[h, e_2] < 0, \\ \dfrac{CF_1[h, e_1] + CF_2[h, e_2]}{1 - \min\,(|\,CF_1[h, e_1]|, |\,CF_2[h, e_2]|)}, & CF_1[h, e_1] \cdot CF_2[h, e_2] < 0. \end{cases} \tag{4}$$

If the experts have opposing judgments about the superiority of the alternatives i and j, then their common but indirect support can also be taken into account.

Let experts e_1 and e_2 express hypotheses $h_1(i \succ j)$ and $h_2(i \prec j)$. Let μ_1 and μ_2 be their weight coefficients as well. Then, the hypothesis h_1 can be considered true with probability μ_1, and the hypothesis h_2 can be considered true with probability $1-\mu_1$, and vice versa. From this, the confidence coefficient $CF(h_2, e_1)$ in the truth of the hypothesis h_2 with support of the expert e_1 of the hypothesis h_1 can be calculated by (3) where $MB[h_2, e_1] = 1-\mu_1$ and $MD[h_2, e_1] = \mu_1$. The confidence coefficient $CF[h_1, e_2]$ is calculated similarly. Then, if there are more than two experts, their joint support for the hypothesis h can be taken into account by successive application of (4) in order to combine the total support of the experts already taken into account with the support of the next expert who is not taken into account yet.

Let $CF[h, e_1 \& e_2 \& \ldots \& e_n] = CF_n[h]$ be the coefficient of confidence in the validity of the hypothesis h supported by n experts and let $CF_i[h, e_i]$ be the coefficient of confidence in the hypothesis h supported by the ith expert. Then, the coefficient of confidence in the hypothesis h supported by all experts is calculated using a recurrent formula

$$CF_2[h] = \begin{cases} CF_1[h, e_1] + CF_2[h, e_2](1 - CF_1[h, e_1]), & CF_1[h, e_1] > 0,\ CF_2[h, e_2] > 0, \\ CF_1[h, e_1] + CF_2[h, e_2](1 + CF_1[h, e_1]), & CF_1[h, e_1] < 0,\ CF_2[h, e_2] < 0, \\ \dfrac{CF_1[h, e_1] + CF_2[h, e_2]}{1 - \min(|CF_1[h, e_1]|, |CF_2[h, e_2]|)}, & CF_1[h, e_1] \cdot CF_2[h, e_2] < 0, \end{cases}$$

$$i = \overline{3, n}, \tag{5}$$

$$CF_i[h] = \begin{cases} CF_{i-1}[h] + CF_i[h, e_i](1 - CF_{i-1}[h]), & CF_{i-1}[h] > 0,\ CF_i[h, e_i] > 0, \\ CF_{i-1}[h] + CF_i[h, e_i](1 + CF_{i-1}[h]), & CF_{i-1}[h] < 0,\ CF_i[h, e_i] < 0, \\ \dfrac{CF_{i-1}[h] + CF_i[h, e_i]}{1 - \min(|CF_{i-1}[h]|, |CF_i[h, e_i]|)}, & CF_{i-1}[h] \cdot CF_i[h, e_i] < 0. \end{cases}$$

This formula is used as the function $f(h)$.

PRACTICAL APPLICATION

To reduce the amount of calculations, let us consider the single-criterion problem of collective decision-making. Let $A = \{a_i \mid i = \overline{1,4}\}$ be a set of alternatives and let $E = (e_1/_{0.6}, e_2/_{0.8}, e_3/_{0.7}, e_4/_{0.9}, e_5/_{0.8})$ be the group of experts. Let the experts construct the following matrices of pairwise comparisons as well:

$$M_1 = \begin{vmatrix} 1 & 2 & 6 & 9 \\ 1/2 & 1 & 4 & 7 \\ 1/6 & 1/4 & 1 & 2 \\ 1/9 & 1/7 & 1/2 & 1 \end{vmatrix}, \quad M_2 = \begin{vmatrix} 1 & 1/3 & 1/5 & 2 \\ 3 & 1 & 3 & 2 \\ 5 & 1/3 & 1 & 3 \\ 1/2 & 1/2 & 1/3 & 1 \end{vmatrix}, \quad M_3 = \begin{vmatrix} 1 & 1/4 & 6 & 9 \\ 4 & 1 & 2 & 5 \\ 1/6 & 1/2 & 1 & 3 \\ 1/9 & 1/5 & 1/3 & 1 \end{vmatrix},$$

$$M_4 = \begin{vmatrix} 1 & 2 & 1/7 & 8 \\ 1/2 & 1 & 4 & 7 \\ 7 & 1/4 & 1 & 2 \\ 1/8 & 1/7 & 1/2 & 1 \end{vmatrix}, \quad M_5 = \begin{vmatrix} 1 & 2 & 1/6 & 7 \\ 1/2 & 1 & 3 & 7 \\ 6 & 1/3 & 1 & 2 \\ 1/7 & 1/7 & 1/2 & 1 \end{vmatrix}.$$

Since a single-criterion problem is considered, a single aggregated matrix M_A is constructed. When it comes to the pairs of alternatives (a_1, a_4), (a_2, a_3), (a_2, a_4), and (a_3, a_4), the judgments of experts about their advantages coincide, so the corresponding elements of the matrix M_A are calculated according to (1) as follows:

$$\overline{m}_{14} = 9^{0.16} \cdot 2^{0.21} \cdot 9^{0.18} \cdot 8^{0.24} \cdot 7^{0.21} = 6.05,$$

$$\overline{m}_{23} = 4^{0.16} \cdot 3^{0.21} \cdot 2^{0.18} \cdot 4^{0.24} \cdot 3^{0.21} = 3.13,$$

$$\overline{m}_{24} = 7^{0.16} \cdot 2^{0.21} \cdot 5^{0.18} \cdot 7^{0.24} \cdot 7^{0.21} = 5.06,$$

$$\overline{m}_{34} = 2^{0.16} \cdot 3^{0.21} \cdot 3^{0.18} \cdot 2^{0.24} \cdot 2^{0.21} = 2.34.$$

When it comes to alternatives a_1, a_2 and a_1, a_3, experts have polar opinions about their merits. For example, in the case of the pair a_1, a_2, experts e_1, e_4, e_5 believe that $h_1 (a_1 \succ a_2)$ and experts e_2, e_3 believe that $h_2 (a_1 \prec a_2)$. When it comes to the pair a_1, a_3 , experts e_1, e_3 believe that $h_3 (a_1 \succ a_2)$ and experts e_2, e_4, e_5 believe that $h_4 (a_1 \prec a_3)$. Note that since in the case of hypotheses h_1 and h_2 the condition of "if $CF_5 [h_1] > 0$, then $CF_5 [h_2] < 0$" is satisfied, it is enough to calculate the confidence coefficient in one of the hypotheses.

In accordance with (3), let us calculate the confidence coefficients in the hypothesis h_1 with it being supported by each expert as follows:

$$CF_1 [h_1, e_1] = \frac{MB [h_1, e_1] - MD [h_1, e_1]}{1 - \min (MB [h_1, e_1], MD [h_1, e_1])}$$

$$= \frac{\mu_1 - (1 - \mu_1)}{1 - \min (\mu_1, (1 - \mu_1))} = \frac{0.6 - 0.4}{1 - 0.4} = \frac{0.2}{0.6} = 0.33,$$

$$CF_2 [h_1, e_2] = \frac{MB [h_1, e_2] - MD [h_1, e_2]}{1 - \min (MB [h_1, e_2], MD [h_1, e_2])}$$

$$= \frac{(1 - \mu_2) - \mu_2}{1 - \min ((1 - \mu_2) - \mu_2)} = \frac{0.2 - 0.8}{1 - 0.2} = -\frac{0.6}{0.8} = -0.75$$

similarly, $CF_3 [h_1, e_3] = -0.57$, $CF_4 [h_1, e_4] = 0.89$, $CF_5 [h_1, e_5] = 0.75$.

Then, let us calculate the confidence coefficient in the hypothesis h_1 taking into account the judgments of all experts in accordance with (5) as follows:

$$CF_2 [h_1] = \frac{CF_1 [h_1, e_1] + CF_2 [h_1, e_2]}{1 - \min (| CF_1 [h_1, e_1] |, | CF_2 [h_1, e_2] |)} = -0.63,$$

$$CF_3 [h_1] = CF_2 [h_1] + CF_3 [h_1, e_3] + CF_2 [h_1] \cdot CF_3 [h_1, e_3] = -0.84,$$

$$CF_4 [h_1] = \frac{CF_3 [h_1] + CF_4 [h_1, e_4]}{1 - \min (| CF_3 [h_1] |, | CF_4 [h_1, e_4] |)} = 0.31,$$

$$CF_5 [h_1] = CF_4 [h_1] + CF_5 [h_1, e_5] - CF_4 [h_1] \cdot CF_5 [h_1, e_5] = 0.83.$$

Thus, the hypothesis h_1 is a group judgment about the dominance of alternatives a_1 and a_2, i.e., $a_1 \succ a_2$. Carrying out similar calculations for the hypothesis h_3, we obtain $CF_5 [h_3] = -0.97$, which testifies to the dominance of the alternative a_3 over the alternative a_1.

In this regard, the elements $\overline{m}_{12}$ and $\overline{m}_{31}$ of the matrix M_A are calculated by the following formulas in accordance with (2): $\overline{m}_{12} = (2^{0.26} \cdot 2^{0.39} \cdot 2^{0.35}) = 2$ and $\overline{m}_{31} = (5^{0.32} \cdot 7^{0.36} \cdot 6^{0.32}) = 5.98$. As a result, we obtain the aggregated matrix

$$M_1 = \begin{vmatrix} 1 & 2.00 & 0.17 & 6.05 \\ 0.50 & 1 & 3.13 & 5.06 \\ 5.98 & 0.32 & 1 & 2.34 \\ 0.16 & 0.2 & 0.43 & 1 \end{vmatrix},$$

whose eigenvector $V = (1.2, 1.7, 1.5, 0.3)$ sets the resulting metric ranking of the alternatives $a_2 \succ a_3 \succ a_1 \succ a_4$ in this case.

CONCLUSIONS

An approach to collective decision-making based on the analytic hierarchy process is proposed. This approach is based on the mechanism of constructing aggregated matrices of pairwise comparisons. The key point of this mechanism is the harmonization of opposite judgments of experts regarding the preference of alternatives. It is realized due to the selection of the most correct hypothesis. The basis for such a choice is the degree of confidence that the hypothesis can be considered reliable. At the same time, the degree of confidence is calculated using the Shortliff combination function. Such harmonization represents a computational model of group choice that is an independent component and can be used as a basis for developing collective decision-making procedures. In general, the proposed approach is quite natural and easy to use, and it harmoniously constitutes unity in the analytic hierarchy process.

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