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METHOD OF RESOLVING FUNCTIONS FOR GAME PROBLEMS OF APPROACH OF CONTROLLED OBJECTS WITH DIFFERENT INERTIA

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Abstract. *The authors consider the problem of approach of controlled objects with different inertia in dynamic game problems on the basis of the modern version of the method of resolving functions. For such objects, it is characteristic that the Pontryagin condition is not satisfied on a certain time interval, which significantly complicates the application of the method of resolving functions to this class of dynamic game problems. A method for solving such problems is proposed, which is associated with the construction of some scalar (resolving) functions, which qualitatively characterize the course of approach of controlled objects with different inertia and the efficiency of the decisions made. The method of resolving functions allows efficient use of the modern technique of multi-valued mappings in substantiating game constructions and obtaining meaningful results on their basis. The guaranteed times of game termintion are compared for different schemes of approaching of controlled objects. An illustrative example is given.*

Keywords: *controlled objects with different inertia, quasilinear differential game, multi-valued mapping, measurable selector, stroboscopic strategy, resolving function.*

INTRODUCTION

In the paper, we will analyze the problem of approach of controlled objects with different inertia in dynamic game problems on the basis of the method of resolving functions [1] and its modern version [2]. For such objects, it is characteristic that the Pontryagin condition is not satisfied on a certain time interval, which substantially complicates the application of the method of resolving functions to this class of dynamic game problems. The "boy and crocodile" problem [1] can be an example. The study [1] provides two methods of using terminal set to extend the class of dynamic problems to which the method of resolving functions is applicable.

In the paper, we will propose a solution technique for this problem that differs from the procedures considered in [1]. We will analyze special multi-valued mappings that generate upper and lower resolving functions of two types, which were first introduced in [3]. By means of these functions, we will obtain some sufficient solvability conditions for the problem of approach of controlled objects of different inertia in a guaranteed time. To illustrate the results, we will use the "boy and crocodile" example.

The present study continues the studies [1–4], is related to the publications [5–28], and expands the class of game problems of approach of controlled objects having a solution.

PROBLEM STATEMENT. THE GENERAL SCHEME OF THE METHOD

Let us consider the conflict-controlled process whose evolution can be described by the equality

$$z(t) = g(t) + \int_0^t \Omega(t, \tau)\varphi(u(\tau), \upsilon(\tau))\, d\tau, \ \ t \geq 0\,. \tag{1}$$

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Here, $z(t) \in R^n$, function $g(t)$, $g: R_+ \to R^n$, is Lebesgue measurable [9] and is bounded for $t > 0$, the matrix function $\Omega(t, \tau)$, $t \ge \tau \ge 0$, is measurable with respect to t and is summable with respect to τ for each $t \in R_+$. The control unit is defined by the function $\varphi(u, v)$, $\varphi: U \times V \to R^n$, continuous with respect to the set of variables on the direct product of nonempty compact sets U and V, and m, l, and n are natural numbers.

Controls of players $u(\tau)$, $u: R_+ \to U$, and $v(\tau)$, $v: R_+ \to V$, are measurable functions of time.

Along with the process (1), the terminal set M^* of cylindrical form is given,

$$M^* = M_0 + M, \tag{2}$$

where M_0 is a linear subspace from R^n and M is a convex compact set from the orthogonal complement L to the subspace M_0 in R^n.

The objectives of the first (u) and of the second (v) players are opposite. The first player (pursuer) tries to deduce the trajectory of the process (1) onto the terminal set (2) in the shortest time, and the second one (evader) tries to postpone the time when the trajectory hits the set M^* or to avoid the encounter.

Let us take the side with the first player and assume that if the game (1), (2) proceeds on the interval $[0, T]$, then we will choose first player's control at the time t based on the information about $g(T)$ and $v_t(\cdot)$, i.e., in the form of the measurable function

$$u(t) = u(g(T), v_t(\cdot)),\ t \in [0, T],\ u(t) \in U, \tag{3}$$

where $v_t(\cdot) = \{v(s): s \in [0, t]\}$ is the previous history of control of the second player by the time t, or in the form of countercontrol

$$u(t) = u(g(T), v(t)),\ t \in [0, T],\ u(t) \in U. \tag{4}$$

If, in particular, $g(t) = e^{At} z_0$, $\Omega(t, \tau) = e^{A(t-\tau)}$, $z(0) = z_0$ and e^{At} is a matrix exponential curve, then the control $u(t) = u(z_0, v_t(\cdot))$ is said to implement the quasistrategy [7], and countercontrol [5] $u(t) = u(z_0, v(t))$ is a manifestation of the Hajek stroboscopic strategy [8].

Let us formulate the necessary facts from the convex analysis [1, 10] in the form of a lemma.

LEMMA 1. Let $X \in R^n$ be a convex compact set and $\omega(\tau)$ be a nonnegative bounded measurable numerical function. Then $\int_0^T \omega(\tau) X d\tau = \int_0^T \omega(\tau)\, d\tau X$, $T > 0$. If $0 \in X$, $f(\tau) \in \omega(\tau) X$ and $\int_0^T \omega(\tau)\, d\tau \le 1$, then $\int_0^T f(\tau)\, d\tau \in X$, $f(\tau)$ is a measurable function, $\tau \in [0, T]$.

Denote by π the operator of orthogonal projection from R^n into L. Suppose $\varphi(U, v) = \{\varphi(u, v): u \in U\}$ and consider the multi-valued mappings

$$W(t, \tau, v) = \pi\Omega(t, \tau)\varphi(U, v),\ W(t, \tau) = \bigcap_{v \in V} W(t, \tau, v)$$

on the sets $\Delta \times V$ and Δ, respectively, where $\Delta = \{(t, \tau): 0 \le \tau \le t < \infty\}$. Assume that the multi-valued mapping $W(t, \tau, v)$ has closed values on the set $\Delta \times V$.

Pontryagin Condition. The multi-valued mapping $W(t, \tau)$ takes nonempty values on the set Δ.

Taking into account the assumptions regarding the matrix function $\Omega(t, \tau)$, we may conclude that for any fixed $t > 0$ the vector function $\pi\Omega(t, \tau)\varphi(u, v)$ is $\mathfrak{L} \otimes \mathfrak{B}$-measurable with respect to $(\tau, v) \in [0, t] \times V$ and is continuous with respect to $u \in U$. Therefore, on the basis of the direct image theorem [9], for any fixed $t > 0$, the multi-valued mapping $W(t, \tau, v)$ is $\mathfrak{L} \otimes \mathfrak{B}$-measurable with respect to $(\tau, v) \in [0, t] \times V$. If the Pontryagin condition is satisfied, then on the set Δ there is at least one selector $\gamma_0(t, \tau)$ of the mapping $W(t, \tau)$, $\gamma_0(t, \tau) \in W(t, \tau)$. Such selector is called the Pontryagin selector.

Let us formulate the Pontryagin condition in an equivalent form. On set Δ, there exists a Pontryagin selector $\gamma_0(t, \tau)$, for which the inclusion is true:

$$0 \in \bigcap_{v \in V} [W(t, \tau, v) - \gamma_0(t, \tau)].$$

Let $\gamma(t,\tau)$, $\gamma:\Delta \to L$, $\Delta = \{(t,\tau): 0 \le \tau \le t < \infty\}$ be some function, almost everywhere bounded, measurable with respect to t and summable with respect to τ, $\tau \in [0, t]$, for each $t > 0$, which we call shift function. Let M_1 be a convex compact set from the orthogonal complement L to the subspace M_0 in R^n such that if $m \in M_1$, then $-m \in M_1$ and $M_2 = M \overset{*}{-} M_1 = \{m \in L: m + M_1 \subset M\} = \bigcap_{m \in M_1} (M - m) \ne \varnothing$, where $\overset{*}{-}$ is the Minkowski geometrical difference [1]. We will call function $\gamma(t,\tau)$ and sets M_1 and M_2 feasible if the conditions and properties specified above are valid for them.

Let us consider some continuous matrix function $B(t,\tau)$ on set Δ. Its values are matrices of order k, k is dimension of vector υ. Denote $W_B(t,\tau,\upsilon) = \pi\Omega(t,\tau)\varphi(U, B(t,\tau)\upsilon)$, $\varphi_B(t,u,\upsilon) = \varphi(u, B(t,\tau)\upsilon) - \varphi(u,\upsilon)$, $t \ge \tau \ge 0$, $u \in U$, $\upsilon \in V$, and consider at $\tau \in [0,t]$, $t > 0$, $\upsilon \in V$ the multi-valued mapping

$$\Lambda_B(t,\tau,\upsilon) = \{\lambda \ge 0: \pi\Omega(t,\tau)\varphi_B(t,U,\upsilon) \subset \lambda M_1\}.$$

If the condition $\Lambda(t,\tau,\upsilon) \ne \varnothing$ is satisfied on set $\Delta \times V$, let us consider a scalar function $\lambda_B(t,\tau,\upsilon) = \inf\{\lambda: \lambda \in \Lambda_B(t,\tau,\upsilon)\}$, $\tau \in [0,t]$, $\upsilon \in V$. We can show [12] that the multi-valued mapping $\Lambda_B(t,\tau,\upsilon)$ is closed-valued, $\mathfrak{L} \otimes \mathfrak{B}$-measurable with respect to the set of (τ,υ), $\tau \in [0,t]$, $\upsilon \in V$, and function $\lambda_B(t,\tau,\upsilon)$ is $\mathfrak{L} \otimes \mathfrak{B}$-measurable with respect to the set (τ,υ), $\tau \in [0,t]$, $\upsilon \in V$, and therefore it is superpositionally measurable [12], i.e., $\lambda_B(t,\tau,\upsilon(\tau))$ is measurable with respect to τ, $\tau \in [0,t]$, for any measurable function $\upsilon(\cdot) \in V(\cdot)$, where $V(\cdot)$ is the set of measurable functions $\upsilon(\tau)$, $\tau \in [0,+\infty]$, with values from V. Note also that function $\lambda_B(t,\tau,\upsilon)$ is lower semicontinuous with respect to the variable υ and function $\sup_{\upsilon \in V} \lambda_B(t,\tau,\upsilon)$ is measurable with respect to τ, $\tau \in [0,t]$.

Condition 1. On set Δ there exists a matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$, and feasible set M_1, for which $\Lambda(t,\tau,\upsilon) \ne \varnothing$ and the inclusions are true:

$$0 \in \bigcap_{\upsilon \in V} [W_B(t,\tau,\upsilon) - \gamma(t,\tau)], \; \varphi_B(t,U,V) \subset \sup_{\upsilon \in V} \lambda_B(t,\tau,\upsilon) M_1.$$

Let us denote $\xi(t) = \xi(t, g(t), \gamma(t,\cdot)) = \pi g(t) + \int_0^t \gamma(t,\tau)\,d\tau$ and consider the set $\mathrm{P}(g(\cdot),\gamma(\cdot,\cdot)) = \left\{ t \ge 0: \xi(t) \in M_2, \int_0^t \sup_{\upsilon \in V} \lambda_B(t,\tau,\upsilon)dt \le 1 \right\}$. If the relations in curly brackets do not hold for any $t \ge 0$, then we put $\mathrm{P}(g(\cdot),\gamma(\cdot,\cdot)) = \varnothing$.

THEOREM 1. Let for the conflict-controlled process (1), (2) Condition 1 be satisfied, for some matrix $B(t,\tau)$, feasible functions $\gamma(t,\tau)$, and sets M_1 and M_2 the set $\mathrm{P}(g(\cdot),\gamma(\cdot,\cdot))$ be non-empty, and $\mathrm{P} \in \mathrm{P}(g(\cdot),\gamma(\cdot,\cdot))$. Then the game can be terminated at the time P with the use of control (4).

Proof. Let $\upsilon(\tau)$ be an arbitrary measurable selector of the compact set V, $\tau \in [0, \mathrm{P}]$. Let us specify the choice of control by the pursuer.

For $\upsilon \in V$ and $\tau \in [0, \mathrm{P}]$, let us consider the multi-valued compact-valued mapping

$$U(\tau,\upsilon) = \{u \in U: \pi\Omega(\mathrm{P},\tau)\varphi(u, B(\mathrm{P},\tau)\upsilon) - \gamma(\mathrm{P},\tau) = 0\}.$$

By virtue of the properties of parameters of process (1), the compact-valued mapping $U(\tau,\upsilon)$ is $\mathfrak{L} \otimes \mathfrak{B}$-measurable [12] for $\upsilon \in V$, $\tau \in [0, \mathrm{P}]$. Therefore, by the theorem on the measurable choice of selector [9], the multi-valued mapping $U(\tau,\upsilon)$ contains $\mathfrak{L} \otimes \mathfrak{B}$-measurable selector $u(\tau,\upsilon)$, which is a superpositionally measurable function [12].

Suppose that the control of the first player $u(\tau) = u(\tau, \upsilon(\tau))$, $\tau \in [0, \mathrm{P}]$. In view of the formula (1), we obtain

$$\pi z(\mathrm{P}) = -\int_0^{\mathrm{P}} \pi\Omega(\mathrm{P},\tau)\varphi_B(\mathrm{P}, u(\tau), \upsilon(\tau))\,d\tau$$

$$+\xi(\mathrm{P}) + \int_0^{\mathrm{P}} (\pi\Omega(\mathrm{P},\tau)\varphi(u(\tau), B(\mathrm{P},\tau)\upsilon(\tau)) - \gamma(\mathrm{P},\tau))d\tau.$$

Then by virtue of Condition 1, by the definition of the time P, we get

$$\pi\Omega(\mathrm{P},\tau)\varphi_B(\mathrm{P},u(\tau),\upsilon(\tau)) \in \sup_{\upsilon\in V} \lambda_B(\mathrm{P},\tau,\upsilon)M_1,$$

$$\int_0^{\mathrm{P}} \sup_{\upsilon\in V} \lambda_B(\mathrm{P},\tau,\upsilon)\,d\tau \le 1.$$

Therefore, by virtue of Lemma 1, the inclusion

$$\int_0^{\mathrm{P}} \pi\Omega(\mathrm{P},\tau)\varphi_B(\mathrm{P},u(\tau),\upsilon(\tau))d\tau \in M_1$$

is true; hence, $-\int_0^{\mathrm{P}} \pi\Omega(\mathrm{P},\tau)\varphi_B(\mathrm{P},u(\tau),\upsilon(\tau))d\tau \in M_1$. Taking into account the method of choosing the control by the first player, we obtain $\pi z(\mathrm{P}) \in M_1 + \xi(\mathrm{P}) \in M_1 + M_2 \subset M$ and $z(\mathrm{P}) \in M^*$, which completes the proof of the theorem.

Remark 1. Theorem 1 is an analog of the first direct Pontryagin method [1] for controlled objects with different inertia. If the Pontryagin condition is satisfied, let us suppose that $B(t,\tau)=E$, where E is a unit matrix, and transform Condition 1 to the Pontryagin condition.

For $\tau\in[0,t]$, $t>0$, $\upsilon\in V$, let us consider the multi-valued mapping

$$\mathfrak{A}(t,\tau,\upsilon)=\{\alpha\ge 0: [W_B(t,\tau,\upsilon)-\gamma(t,\tau)]\cap\alpha[M_2-\xi(t)]\ne\varnothing\}. \tag{5}$$

Condition 2. On set Δ, there exist matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$, and feasible sets M_1 and M_2, for which $\Lambda(t,\tau,\upsilon)\ne\varnothing$ and the inclusions hold:

$$0\in\bigcap_{\upsilon\in V}\{[W_B(t,\tau,\upsilon)-\gamma(t,\tau)]-\mathfrak{A}(t,\tau,\upsilon)[M_2-\xi(t)]\},$$

$$\varphi_B(t,U,V)\subset \sup_{\upsilon\in V}\lambda_B(t,\tau,\upsilon)M_1.$$

If Condition 2 is satisfied, let us consider the scalar function $\lambda_B(t,\tau,\upsilon)=\inf\{\lambda:\lambda\in\Lambda_B(t,\tau,\upsilon)\}$, $\tau\in[0,t]$, $\upsilon\in V$, the upper and lower resolving scalar functions [3]

$$\alpha^*(t,\tau,\upsilon)=\sup\{\alpha:\alpha\in\mathfrak{A}(t,\tau,\upsilon)\},\ \alpha_*(t,\tau,\upsilon)=\inf\{\alpha:\alpha\in\mathfrak{A}(t,\tau,\upsilon)\},$$

$$\tau\in[0,t],\ \upsilon\in V.$$

Let us show [12] that the multi-valued mapping $\mathfrak{A}(t,\tau,\upsilon)$ is closed-valued, $\mathfrak{L}\otimes\mathfrak{B}$-measurable with respect to the set (τ,υ), $\tau\in[0,t]$, $\upsilon\in V$, and the upper and lower resolving functions are $\mathfrak{L}\otimes\mathfrak{B}$-measurable with respect to the set (τ,υ), $\tau\in[0,t]$, $\upsilon\in V$; therefore, they are superpositionally measurable [12], i.e., $\alpha^*(t,\tau,\upsilon(\tau))$ and $\alpha_*(t,\tau,\upsilon(\tau))$ are measurable with respect to τ, $\tau\in[0,t]$, for any measurable function $\upsilon(\cdot)\in V(\cdot)$. Note also that the upper resolving function is upper semicontinuous, the lower one is lower semicontinuous with respect to the variable υ, and functions $\inf_{\upsilon\in V}\alpha^*(t,\tau,\upsilon)$ and $\sup_{\upsilon\in V}\alpha_*(t,\tau,\upsilon)$ are measurable with respect to τ, $\tau\in[0,t]$.

Let us considern the set

$$\mathrm{P}_*^1(g(\cdot),\gamma(\cdot,\cdot))=\left\{t\ge 0:\xi(t)\in M_2,\int_0^t \sup_{\upsilon\in V}\lambda_B(t,\tau,\upsilon)\,d\tau\le 1,\ \int_0^t \sup_{\upsilon\in V}\alpha_*(t,\tau,\upsilon)\,d\tau\le 1\right\}. \tag{6}$$

If the relations in curly brackets in Eq. (6) do not hold for any $t\ge 0$, then we put $\mathrm{P}_*^1(g(\cdot),\gamma(\cdot,\cdot))=\varnothing$.

THEOREM 2. Let for the conflict-controlled process (1), (2) Condition 2 be satisfied, for some matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$ and feasible sets M_1 and M_2 the set $\mathrm{P}_*^1(g(\cdot),\gamma(\cdot,\cdot))$ is not empty and $\mathrm{P}_*^1\in\mathrm{P}_*^1(g(\cdot),\gamma(\cdot,\cdot))$. Then the game can be terminated at time P_*^1 with the use of control (4).

Proof. Let $\upsilon(\tau)$ be an arbitrary measurable selector of the compact set V, $\tau \in [0, \mathrm{P}_*^1]$. Let us specify the choice of control by the pursuer.

For $\upsilon \in V$ and $\tau \in [0, \mathrm{P}_*^1]$, consider the compact-valued multi-valued mapping

$$U_*^1(\tau, \upsilon) = \{u \in U : \pi\Omega(\mathrm{P}_*^1, \tau)\varphi(u, B(\mathrm{P}_*^1, \tau)\upsilon) - \gamma(\mathrm{P}_*^1, \tau) \in \alpha_*(\mathrm{P}_*^1, \tau, \upsilon)[M_2 - \xi(\mathrm{P}_*^1)]\}.$$

By virtue of the properties of the parameters of process (1) and of the lower resolving function $\alpha_*(\mathrm{P}_*^1, \tau, \upsilon)$, the compact-valued mapping $U_*^1(\tau, \upsilon)$ is $\mathfrak{L} \otimes \mathfrak{B}$-measurable [12] for $\upsilon \in V$, $\tau \in [0, \mathrm{P}_*^1]$. Therefore, by the theorem on measurable choice of selector [9], multi-valued mapping $U_*^1(\tau, \upsilon)$ contains $\mathfrak{L} \otimes \mathfrak{B}$-measurable selector $u_*^1(\tau, \upsilon)$, which is a superpositionally measurable function [12].

Let the control of the first player be $u_*^1(\tau) = u_*^1(\tau, \upsilon(\tau))$, $\tau \in [0, \mathrm{P}_*^1]$. In view of the formula (1), we obtain

$$\pi z(\mathrm{P}_*^1) = -\int_0^{\mathrm{P}_*^1} \pi\Omega(\mathrm{P}_*^1, \tau)\varphi_B(\mathrm{P}_*^1, u_*^1(\tau), \upsilon(\tau))d\tau$$
$$+\xi(\mathrm{P}_*^1) + \int_0^{\mathrm{P}_*^1} (\pi\Omega(\mathrm{P}_*^1, \tau)\varphi(B(\mathrm{P}_*^1, \tau)u_*^1(\tau), \upsilon(\tau)) - \gamma(\mathrm{P}_*^1, \tau))\, d\tau. \tag{7}$$

By virtue of Condition 2, by the definition of the time P_*^1, we get

$$0 \in M_1,\ \pi\Omega(\mathrm{P}_*^1, \tau)\varphi_B(\mathrm{P}_*^1, u_*^1(\tau), \upsilon(\tau)) \in \sup_{\upsilon \in V} \lambda_B(\mathrm{P}_*^1, \tau, \upsilon) M_1,$$
$$\int_0^{\mathrm{P}_*^1} \sup_{\upsilon \in V} \lambda_B(\mathrm{P}_*^1, \tau, \upsilon)\, d\tau \le 1.$$

Then with regard for Lemma 1, the inclusion is true:

$$\int_0^{\mathrm{P}_*^1} \pi\Omega(\mathrm{P}_*^1, \tau)\varphi_B(\mathrm{P}_*^1, u_*^1(\tau), \upsilon(\tau))d\tau \in M_1$$

and by the assumption

$$-\int_0^{\mathrm{P}_*^1} \pi\Omega(\mathrm{P}_*^1, \tau)\varphi_B(\mathrm{P}_*^1, u_*^1(\tau), \upsilon(\tau))d\tau \in M_1.$$

Due to the choice of the control and by the definition of the time P_*^1, we get

$$0 \in M_2 - \xi(\mathrm{P}_*^1),$$

$$\pi\Omega(\mathrm{P}_*^1, \tau)\varphi(B(\mathrm{P}_*^1, \tau)u_*^1(\tau), \upsilon(\tau)) - \gamma(\mathrm{P}_*^1, \tau) \in \alpha_*(\mathrm{P}_*^1, \tau, \upsilon)[M_2 - \xi(\mathrm{P}_*^1)],$$

$$\int_0^{\mathrm{P}_*^1} \sup_{\upsilon \in V} \alpha_*(\mathrm{P}_*^1, \tau, \upsilon)\, d\tau \le 1.$$

Then with regard for Lemma 1, the inclusion is true:

$$\int_0^{\mathrm{P}_*^1} (\pi\Omega(\mathrm{P}_*^1, \tau)\varphi(B(\mathrm{P}_*^1, \tau)u_*^1(\tau), \upsilon(\tau)) - \gamma(\mathrm{P}_*^1, \tau))\, d\tau \in M_2 - \xi(\mathrm{P}_*^1).$$

Thus, the relation (7) yields

$$\pi z(\mathrm{P}_*^1) \in M_1 + \xi(\mathrm{P}_*^1) + M_2 - \xi(\mathrm{P}_*^1) = M_1 + M_2 \subset M$$

and hence, $z(\mathrm{P}_*^1) \in M^*$, which completes the proof of the theorem.

LEMMA 2. For the conflict-controlled process (1), (2), Condition 1 is true if and only if there exist matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$, and feasible sets M_1 and M_2, for which Condition 2 is true and $0 \in \mathfrak{A}(t,\tau,\upsilon)$ on the set $\Delta \times V$.

Proof. Let there exist matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$, and feasible sets M_1 and M_2, for which Condition 2 is true and $0 \in \mathfrak{A}(t,\tau,\upsilon)$ on the set $\Delta \times V$. Then zero value of α ensures nonempty intersection in expression (5); therefore, taking into account the condition $\Lambda(t,\tau,\upsilon) \neq \varnothing$, $\upsilon \in V$, we get $0 \in W_B(t,\tau,\upsilon) - \gamma(t,\tau)$, $(t,\tau) \in \Delta$, $\upsilon \in V$. From here, it follows that for $(t,\tau) \in \Delta$ we get $0 \in \bigcap_{\upsilon \in V} [W_B(t,\tau,\upsilon) - \gamma(t,\tau)]$, i.e., Condition 1 is true. Reverse reasoning yields the necessary conclusion.

Remark 2. If there exist matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$, and feasible sets M_1 and M_2, for which Condition 1 is satisfied, then by virtue of Lemma 2 $\alpha_*(t,\tau,\upsilon) = \inf\{\alpha : \alpha \in \mathfrak{A}(t,\tau,\upsilon)\} = 0$ on the set $\Delta \times V$.

Condition 3. On the set Δ, Condition 2 is satisfied and the inclusion is true:

$$0 \in \bigcap_{\upsilon \in V} \{[W_B(t,\tau,\upsilon) - \gamma(t,\tau)] - \sup_{\upsilon \in V} \alpha_*(t,\tau,\upsilon)[M_2 - \xi(t)]\}.$$

Remark 3. If there exist matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$, and feasible sets M_1 and M_2 for which Condition 1 is satisfied, then by analogy with Lemma 2, Condition 3 is satisfied and $\sup_{\upsilon \in V} \alpha_*(t,\tau,\upsilon) = 0$.

Let us consider the set

$$T(g(t),\gamma(\cdot,\cdot)) = \left\{ t \geq 0 : \int_0^t \sup_{\upsilon \in V} \lambda_B(t,\tau,\upsilon)\,d\tau \leq 1,\ \int_0^t \inf_{\upsilon \in V} \alpha^*(t,\tau,\upsilon)\,d\tau \geq 1,\ \int_0^t \sup_{\upsilon \in V} \alpha_*(t,\tau,\upsilon)\,d\tau < 1 \right\}. \tag{8}$$

If for some $t > 0$ we get $\alpha^*(t,\tau,\upsilon) \equiv +\infty$ for $\tau \in [0,t]$, $\upsilon \in V$, then it is natural to suppose the value of the respective integral in curly brackets in (8) to be equal to $+\infty$ and $t \in T(g(t),\gamma(\cdot,\cdot))$ if the other inequalities in curly brackets of this relation are true for this t. If the inequalities in (8) do not hold for all $t > 0$, put $T(g(t),\gamma(\cdot,\cdot)) = \varnothing$.

THEOREM 3. Let for the conflict-controlled process (1), (2) Condition 3 be satisfied, for some matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$ and feasible sets M_1 and M_2 set $T(g(\cdot),\gamma(\cdot,\cdot))$ be non-empty and $T \in T(g(\cdot),\gamma(\cdot,\cdot))$. Then the game can be terminated at time T with the use of control (3).

Proof. Let $\upsilon(\tau)$ be an arbitrary measurable selector of the compact set V, $\tau \in [0,T]$. Let us specify the choice of control by the pursuer.

First, let us consider the case $\xi(T, g(T), \gamma(\cdot,\cdot)) \notin M_2$ and introduce the control function

$$h(t) = 1 - \int_0^t \alpha^*(T,\tau,\upsilon(\tau))d\tau - \int_t^T \sup_{\upsilon \in V} \alpha_*(T,\tau,\upsilon)\,d\tau,\ \ t \in [0,T].$$

By the definition of T, we get

$$h(0) = 1 - \int_0^T \sup_{\upsilon \in V} \alpha_*(T,\tau,\upsilon)\,d\tau > 0,$$

$$h(T) = 1 - \int_0^T \alpha^*(T,\tau,\upsilon(\tau))\,d\tau \leq 1 - \int_0^T \inf_{\upsilon \in V} \alpha^*(T,\tau,\upsilon)\,d\tau \leq 0.$$

Since function $h(t)$ is continuous, there exists an instant of time t_*, $t_* \in (0, T]$ such that $h(t_*) = 0$. Note that switching time t_* depends on the previous history of control of the second player $\upsilon_{t_*}(\cdot) = \{\upsilon(s) : s \in [0, t_*]\}$.

We will call the time intervals $[0, t_*)$ and $[t_*, T]$ "active" and "passive," respectively. Let us describe the control of the first player on each of them. Consider the compact-valued mappings

$$U_1^*(\tau, \upsilon) = \{u \in U : \pi\Omega(T, \tau)\varphi(u, B(T, \tau)\upsilon) - \gamma(T, \tau)$$
$$\in \alpha^*(T, \tau, \upsilon)[M_2 - \xi(T)]\}, \ \tau \in [0, t_*), \tag{9}$$

$$U_*^1(\tau, \upsilon) = \{u \in U : \pi\Omega(T, \tau)\varphi(u, B(T, \tau)\upsilon) - \gamma(T, \tau)$$
$$\in \sup_{\upsilon \in V} \alpha_*(T, \tau, \upsilon)[M_2 - \xi(T)]\}, \ \tau \in [t_*, T]. \tag{10}$$

The multi-valued mappings $U_1^*(\tau, \upsilon)$ and $U_*^1(\tau, \upsilon)$ have nonempty images. By virtue of the properties of the parameters of process (1), functions $\alpha^*(T, \tau, \upsilon)$ and $\sup_{\upsilon \in V} \alpha_*(T, \tau, \upsilon)$, the compact-valued mappings $U_1^*(\tau, \upsilon)$, $\tau \in [0, t_*)$ and $U_*^1(\tau, \upsilon)$, $\tau \in [t_*, T]$ for $\upsilon \in V$ are $\mathfrak{L} \otimes \mathfrak{B}$-measurable [4]. Therefore, by the theorem on measurable choice of selector [9], in each of them there exists at least one $\mathfrak{L} \otimes \mathfrak{B}$-measurable selector $u_1^*(\tau, \upsilon)$ and $u_*^1(\tau, \upsilon)$, which are superpositionally measurable functions [12]. Suppose that control of the first player on the "active" interval is $u_1^*(\tau) = u_1^*(\tau, \upsilon(\tau))$, $\tau \in [0, t_*)$, and on the "passive" one it is $u_*^1(\tau) = u_*^1(\tau, \upsilon(\tau))$, $\tau \in [t_*, T]$.

In view of formula (1), for the selected controls we obtain

$$\pi z(T) = -\left[\int_0^{t_*} \pi\Omega(T, \tau)\varphi_B(T, u_1^*(\tau), \upsilon(\tau))d\tau + \int_{t_*}^{T} \pi\Omega(T, \tau)\varphi_B(T, u_*^1(\tau), \upsilon(\tau))d\tau\right]$$
$$+\xi(T) + \int_0^{t_*} (\pi\Omega(T, \tau)\varphi(u_1^*(\tau), B(T, \tau)\upsilon(\tau)) - \gamma(T, \tau))d\tau$$
$$+ \int_{t_*}^{T} (\pi\Omega(T, \tau)\varphi(u_*^1(\tau), B(T, \tau)\upsilon(\tau)) - \gamma(T, \tau))d\tau. \tag{11}$$

By virtue of Condition 2, by the definition of time T, we get

$$0 \in M_1, \ \int_0^T \sup_{\upsilon \in V} \lambda_B(T, \tau, \upsilon)\, d\tau \le 1,$$

$$\pi\Omega(T, \tau)\varphi_B(T, u_1^*(\tau), \upsilon(\tau)) \in \sup_{\upsilon \in V} \lambda_B(T, \tau, \upsilon)M_1, \ \tau \in [0, t_*),$$

$$\pi\Omega(T, \tau)\varphi_B(T, u_*^1(\tau), \upsilon(\tau)) \in \sup_{\upsilon \in V} \lambda_B(T, \tau, \upsilon)M_1, \ \tau \in [t_*, T].$$

Then with regard for Lemma 1, we obtain

$$\int_0^{t_*} \pi\Omega(T, \tau)\varphi_B(T, u_1^*(\tau), \upsilon(\tau))d\tau + \int_{t_*}^{T} \pi\Omega(T, \tau)\varphi_B(T, u_*^1(\tau), \upsilon(\tau))d\tau \in M_1$$

and by the assumtion

$$-\int_0^{t_*} \pi\Omega(T,\tau)\varphi_B(T, u_1^*(\tau), \upsilon(\tau))d\tau + \int_{t_*}^{T} \pi\Omega(T,\tau)\varphi_B(T, u_*^1(\tau), \upsilon(\tau))d\tau \in M_1.$$

Using the last inclusion and relations (9)–(11), we obtain

$$\pi z(T) \in M_1 + \xi(T) + \int_0^{t_*} \alpha^*(T,\tau,\upsilon(\tau))[M_2 - \xi(T)]d\tau + \int_{t_*}^{T} \sup_{\upsilon\in V} \alpha_*(T,\tau,\upsilon)[M_2 - \xi(T)]d\tau$$

$$= M_1 + \xi(T) + \int_0^{t_*} \alpha^*(T,\tau,\upsilon(\tau))d\tau[M_2 - \xi(T)] + \int_{t_*}^{T} \sup_{\upsilon\in V} \alpha_*(T,\tau,\upsilon)d\tau[M_2 - \xi(T)]$$

$$= M_1 + \xi(T)\left[1 - \int_0^{t_*} \alpha^*(T,\tau,\upsilon(\tau))d\tau - \int_{t_*}^{T} \sup_{\upsilon\in V} \alpha_*(T,\tau,\upsilon)d\tau\right]$$

$$+\left[\int_0^{t_*} \alpha^*(T,\tau,\upsilon(\tau))d\tau + \int_{t_*}^{T} \sup_{\upsilon\in V} \alpha_*(T,\tau,\upsilon)d\tau\right] M_2 = M_1 + M_2 \subset M.$$

Here, we took into account the equality $h(t_*) = 0$ and the inclusion $M_1 + M_2 \subset M$ and the passage in the integration of multi-valued mappings with set M_2 can be confirmed by applying the apparatus of support function [10].

For the case $\xi(T, g(T), \gamma(\cdot,\cdot)) \in M_2$, it will suffice to use Theorem 2.

Condition 4. On the set Δ, Condition 2 is satisfied and the inclusion is true:

$$0 \in \bigcap_{\upsilon\in V} \{[W_B(t,\tau,\upsilon) - \gamma(t,\tau)] - \inf_{\upsilon\in V} \alpha^*(t,\tau,\upsilon)[M_2 - \xi(t)]\}.$$

THEOREM 4. Let for the conflict-controlled process (1), (2), Conditions 3 and 4 be satisfied; for some matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$, and feasible sets M_1 and M_2, set $T(g(\cdot), \gamma(\cdot,\cdot))$ be non-empty, and $T \in T(g(\cdot), \gamma(\cdot,\cdot))$. Then the game can be terminated at time T with the use of control (4).

Proof. Let $\upsilon(\tau)$ be an arbitrary measurable selector of the compact set V, $\tau \in [0, T]$. Let us specify the choice of control of the pursuer.

First, let us consider the case $\xi(T, g(T), \gamma(\cdot,\cdot) \notin M_2$ and introduce the control function

$$h(t) = 1 - \int_0^{t} \inf_{\upsilon\in V} \alpha^*(T,\tau,\upsilon)d\tau - \int_{t}^{T} \sup_{\upsilon\in V} \alpha_*(T,\tau,\upsilon)d\tau, \quad t \in [0, T].$$

By the definition of T, we get

$$h(0) = 1 - \int_0^{T} \sup_{\upsilon\in V} \alpha_*(T,\tau,\upsilon)\,d\tau > 0, \quad h(T) = 1 - \int_0^{T} \inf_{\upsilon\in V} \alpha^*(T,\tau,\upsilon)\,d\tau \le 0.$$

Since the function $h(t)$ is continuous, there exists an instant of time t_*, $t_* \in (0, T]$, such that $h(t_*) = 0$. Note that the switching moment t_* does not depend on the previous history of control of the second player $\upsilon_{t_*}(\cdot) = \{\upsilon(s) : s \in [0, t_*]\}$.

We will call the time intervals $[0, t_*)$ and $[t_*, T]$ "active" and "passive," respectively. Let us describe the control of the first player on each of them. To this end, we will consider the compact-valued mappings

$$\tilde{U}_1^*(\tau,\upsilon) = \{u \in U : \pi\Omega(T,\tau)\varphi(u, B(T,\tau)\upsilon) - \gamma(T,\tau)$$

$$\in \inf_{\upsilon\in V} \alpha^*(T,\tau,\upsilon)[M_2 - \xi(T)]\}, \quad \tau \in [0, t_*), \tag{12}$$

$$\widetilde{U}_*^1(\tau, \upsilon) = \{u \in U : \pi\Omega(T,\tau)\varphi(u, B(T,\tau)\upsilon) - \gamma(T,\tau)$$

$$\in \sup_{\upsilon \in V} \alpha_*(T,\tau,\upsilon)[M_2 - \xi(T)]\}, \ \tau \in [t_*, T]. \tag{13}$$

The multi-valued mappings $\widetilde{U}_1^*(\tau, \upsilon)$ and $\widetilde{U}_*^1(\tau, \upsilon)$ have nonempty images. Due to the properties of the parameters of process (1), of functions $\inf_{\upsilon \in V} \alpha^*(T,\tau,\upsilon)$ and $\sup_{\upsilon \in V} \alpha_*(T,\tau,\upsilon)$, the compact-valued mappings $\widetilde{U}_1^*(\tau,\upsilon)$, $\tau \in [0, t_*)$, and $\widetilde{U}_*^1(\tau,\upsilon)$, $\tau \in [t_*, T]$, for $\upsilon \in V$ are $\mathfrak{L} \otimes \mathfrak{B}$-measurable [12]. Therefore, by the theorem on measurable choice of selector [9], in each of them there exists at least one $\mathfrak{L} \otimes \mathfrak{B}$-measurable selector $\widetilde{u}_1^*(\tau,\upsilon)$ and $\widetilde{u}_*^1(\tau,\upsilon)$, which are superpositionally measurable functions [12]. Suppose that control of the first player on the “active” interval is $\widetilde{u}_1^*(\tau) = \widetilde{u}_1^*(\tau, \upsilon(\tau))$, $\tau \in [0, t_*)$, and on the “passive” one it is $\widetilde{u}_*^1(\tau) = \widetilde{u}_*^1(\tau, \upsilon(\tau))$, $\tau \in [t_*, T]$.

Considering formula (1), for the selected controls we obtain

$$\pi z(T) = -\left[\int_0^{t_*} \pi\Omega(T,\tau)\varphi_B(T, \widetilde{u}_1^*(\tau), \upsilon(\tau))d\tau + \int_{t_*}^{T} \pi\Omega(T,\tau)\varphi_B(T, \widetilde{u}_*^1(\tau), \upsilon(\tau))d\tau\right]$$

$$+\xi(T) + \int_0^{t_*} (\pi\Omega(T,\tau)\varphi(\widetilde{u}_1^*(\tau), B(T,\tau)\upsilon(\tau)) - \gamma(T,\tau))d\tau$$

$$+\int_{t_*}^{T} (\pi\Omega(T,\tau)\varphi(\widetilde{u}_*^1(\tau), B(T,\tau)\upsilon(\tau)) - \gamma(T,\tau))d\tau. \tag{14}$$

By virtue of Condition 2, by the definition of time T, we get

$$0 \in M_1, \ \int_0^T \sup_{\upsilon \in V} \lambda_B(T,\tau,\upsilon)\, d\tau \le 1,$$

$$\pi\Omega(T,\tau)\varphi_B(T, \widetilde{u}_1^*(\tau), \upsilon(\tau)) \in \sup_{\upsilon \in V} \lambda_B(T,\tau,\upsilon) M_1, \ \tau \in [0, t_*),$$

$$\pi\Omega(T,\tau)\varphi_B(T, \widetilde{u}_*^1(\tau), \upsilon(\tau)) \in \sup_{\upsilon \in V} \lambda_B(T,\tau,\upsilon) M_1, \ \tau \in [t_*, T].$$

Then taking into account Lemma 1, we obtain

$$\int_0^{t_*} \pi\Omega(T,\tau)\varphi_B(T, \widetilde{u}_1^*(\tau), \upsilon(\tau))d\tau + \int_{t_*}^{T} \pi\Omega(T,\tau)\varphi_B(T, \widetilde{u}_*^1(\tau), \upsilon(\tau))d\tau \in M_1;$$

therefore, we get

$$-\left[\int_0^{t_*} \pi\Omega(T,\tau)\varphi_B(T, \widetilde{u}_1^*(\tau), \upsilon(\tau))d\tau + \int_{t_*}^{T} \pi\Omega(T,\tau)\varphi_B(T, \widetilde{u}_*^1(\tau), \upsilon(\tau))d\tau\right] \in M_1.$$

Taking into account the last inclusion and using (12)–(14), we obtain

$$\pi z(T) \in M_1 + \xi(T) + \int_0^{t_*} \inf_{\upsilon \in V} \alpha^*(T,\tau,\upsilon)[M_2 - \xi(T)]\, d\tau + \int_{t_*}^{T} \sup_{\upsilon \in V} \alpha_*(T,\tau,\upsilon)[M_2 - \xi(T)]\, d\tau$$

$$= M_1 + \xi(T) + \int_0^{t_*} \inf_{\upsilon \in V} \alpha^*(t, \tau, \upsilon) d\tau [M_2 - \xi(T)] + \int_{t_*}^{T} \sup_{\upsilon \in V} \alpha_*(T, \tau, \upsilon) d\tau [M_2 - \xi(T)]$$

$$= M_1 + \xi(T) \left[1 - \int_0^{t_*} \inf_{\upsilon \in V} \alpha^*(t, \tau, \upsilon) d\tau - \int_{t_*}^{T} \sup_{\upsilon \in V} \alpha_*(T, \tau, \upsilon) d\tau \right]$$

$$+ \left[\int_0^{t_*} \inf_{\upsilon \in V} \alpha^*(t, \tau, \upsilon) d\tau + \int_{t_*}^{T} \sup_{\upsilon \in V} \alpha_*(T, \tau, \upsilon) d\tau \right] M_2 = M_1 + M_2 \subset M.$$

Here, we took into account the equality $h(t_*) = 0$ and the inclusion $M_1 + M_2 \subset M$, and we can confirm the passage in the integration of multi-valued mappings with the set M_2 by applying the apparatus of support functions [10].

For the case $\xi(T, g(T), \gamma(T, \cdot)) \in M_2$, it will suffice to use Theorem 2.

MODIFICATION OF THE METHOD. RESOLVING FUNCTIONS OF SECOND KIND

Let us consider the multi-valued mapping

$$\mathfrak{A}(t, \tau) = \bigcap_{\upsilon \in V} \mathfrak{A}(t, \tau, \upsilon), \quad (t, \tau) \in \Delta. \tag{15}$$

Condition 5. On set Δ, Condition 2 is satisfied and the inclusion is true:

$$0 \in \bigcap_{\upsilon \in V} \{[W_B(t, \tau, \upsilon) - \gamma(t, \tau)] - \mathfrak{A}(t, \tau)[M_2 - \xi(t)]\}.$$

If Condition 5 is satisfied, then the multi-valued mapping $\mathfrak{A}(t, \tau)$ is not empty on set Δ and generates the upper and lower scalar resolving functions

$$\alpha^*(t, \tau) = \sup\{\alpha : \alpha \in \mathfrak{A}(t, \tau)\}, \ \alpha_*(t, \tau) = \inf\{\alpha : \alpha \in \mathrm{A}(t, \tau)\}, \ \tau \in [0, t].$$

We can show [12] that the multi-valued mapping $\mathfrak{A}(t, \tau)$ is closed-valued and $\mathfrak{L}$-measurable with respect to τ, $\tau \in [0, t]$, and the upper $\alpha^*(t, \tau)$ and lower $\alpha_*(t, \tau)$ resolving functions are $\mathfrak{L}$-measurable with respect to variable τ for fixed t.

Note 4. If for some feasible matrix $B(t, \tau)$, function $\gamma(t, \tau)$, and sets M_1 and M_2 Condition 3 is satisfied on set Δ, then $\sup_{\upsilon \in V} \alpha_*(t, \tau, \upsilon) \in \mathfrak{A}(t, \tau)$, $\tau \in [0, t]$. Then Condition 5 is satisfied and the equality holds: $\sup_{\upsilon \in V} \alpha_*(t, \tau, \upsilon) = \alpha_*(t, \tau)$, $\tau \in [0, t]$.
If for some matrix $B(t, \tau)$, feasible function $\gamma(t, \tau)$, and feasible sets M_1 and M_2 Condition 4 is satisfied on set Δ, then $\inf_{\upsilon \in V} \alpha^*(t, \tau, \upsilon) \in \mathfrak{A}(t, \tau)$, $\tau \in [0, t]$. Then Condition 5 is satisfied and the equality holds: $\inf_{\upsilon \in V} \alpha^*(t, \tau, \upsilon) = \alpha^*(t, \tau)$, $\tau \in [0, t]$.

Let us consider the set

$$\mathrm{P}_*^2(g(\cdot), \gamma(\cdot, \cdot)) = \left\{ t \geq 0 : \xi(t) \in M_2, \int_0^t \sup_{\upsilon \in V} \lambda_B(t, \tau, \upsilon) d\tau \leq 1, \int_0^t \alpha_*(t, \tau) d\tau < 1 \right\}. \tag{16}$$

If the inclusion and inequalities in curly brackets of (16) do not hold for any $t \geq 0$, we suppose $\mathrm{P}_*^2(g(\cdot), \gamma(\cdot, \cdot)) = \varnothing$.

THEOREM 5. Let Condition 5 be satisfied for the conflict-controlled process (1), (2), for some matrix $B(t, \tau)$, feasible function $\gamma(t, \tau)$, and feasible sets M_1 and M_2 the set $\mathrm{P}_*^2(g(\cdot), \gamma(\cdot, \cdot))$ be non-empty, and $\mathrm{P}_*^2 \in \mathrm{P}_*^2(g(\cdot), \gamma(\cdot, \cdot))$. Then the game can be terminated at time P_*^2 with the use of control (4).

Proof. Let $\upsilon(\tau)$ be an arbitrary measurable selector of the compact set V, $\tau \in [0, \mathrm{P}_*^2]$. Let us specify the choice of control of the pursuer.

For $\upsilon \in V$, $\tau \in [0, \mathrm{P}_*^2]$, consider the compact-valued multi-valued mapping

$$U_*^2(\tau, \upsilon) = \{u \in U : \pi\Omega(\mathrm{P}_*^2, \tau)\varphi(u, B(\mathrm{P}_*^2, \tau)\upsilon) - \gamma(\mathrm{P}_*^2, \tau) \in \alpha_*(\mathrm{P}_*^2, \tau)[M_2 - \xi(\mathrm{P}_*^2)]\} .$$

By virtue of the properties of the parameters of process (1) and of the lower resolving function $\alpha_*(\mathrm{P}_*^2, \tau)$, the compact-valued mapping $U_*^2(\tau, \upsilon)$ is $\mathfrak{L} \otimes \mathfrak{B}$-measurable [12] for $\upsilon \in V$, $\tau \in [0, \mathrm{P}_*^2]$. Therefore, by the theorem on measurable choice of selector [9], the multi-valued mapping $U_*^2(\tau, \upsilon)$ contains $\mathfrak{L} \otimes \mathfrak{B}$-measurable selector $u_*^2(\tau, \upsilon)$, which is a superpositionally measurable function [12].

Let the control of the first player be $u_*^2(\tau) = u_*^2(\tau, \upsilon(\tau))$, $\tau \in [0, \mathrm{P}_*^2]$. In view of formula (1), we obtain

$$\pi z(\mathrm{P}_*^2) = -\int_0^{\mathrm{P}_*^2} \pi\Omega(\mathrm{P}_*^2, \tau)\varphi_B(\mathrm{P}_*^2, u_*^2(\tau), \upsilon(\tau))d\tau$$
$$+\xi(\mathrm{P}_*^2) + \int_0^{\mathrm{P}_*^2} (\pi\Omega(\mathrm{P}_*^2, \tau)\varphi(u_*^2(\tau), B(\mathrm{P}_*^2, \tau)\upsilon(\tau)) - \gamma(\mathrm{P}_*^2, \tau))\, d\tau . \tag{17}$$

By virtue of Condition 2, by the definition of time P_*^2, we get

$$0 \in M_1,\ \pi\Omega(\mathrm{P}_*^2, \tau)\varphi_B(\mathrm{P}_*^2, u_*^2(\tau), \upsilon(\tau)) \in \sup_{\upsilon \in V} \lambda_B(\mathrm{P}_*^2, \tau, \upsilon) M_1,$$

$$\int_0^{\mathrm{P}_*^2} \sup_{\upsilon \in V} \lambda_B(\mathrm{P}_*^2, \tau, \upsilon)\, d\tau \le 1 .$$

Then with regard for Lemma 1, the inclusion is true:

$$\int_0^{\mathrm{P}_*^2} \pi\Omega(\mathrm{P}_*^2, \tau)\varphi_B(\mathrm{P}_*^2, u_*^2(\tau), \upsilon(\tau))\, d\tau \in M_1;$$

therefore, we get

$$-\int_0^{\mathrm{P}_*^2} \pi\Omega(\mathrm{P}_*^2, \tau)\varphi_B(\mathrm{P}_*^2, u_*^2(\tau), \upsilon(\tau))d\tau \in M_1.$$

Due to the choice of control and by the definition of time P_*^2, we get

$$0 \in M_2 - \xi(\mathrm{P}_*^2),\ \pi\Omega(\mathrm{P}_*^2, \tau)\varphi(B(\mathrm{P}_*^2, \tau)u_*^2(\tau), \upsilon(\tau)) - \gamma(\mathrm{P}_*^2, \tau) \in \alpha_*(\mathrm{P}_*^2, \tau)[M_2 - \xi(\mathrm{P}_*^2)],$$

$$\int_0^{\mathrm{P}_*^2} \sup_{\upsilon \in V} \alpha_*(\mathrm{P}_*^2, \tau)\, d\tau \le 1 .$$

Then with regard for Lemma 1, the inclusion holds:

$$\int_0^{\mathrm{P}_*^2} (\pi\Omega(\mathrm{P}_*^2, \tau)\varphi(B(\mathrm{P}_*^2, \tau)u_*^2(\tau), \upsilon(\tau)) - \gamma(\mathrm{P}_*^2, \tau))d\tau \in M_2 - \xi(\mathrm{P}_*^2) .$$

Thus, taking into account (17), we obtain

$$\pi z(\mathrm{P}_*^2) \in M_1 + \xi(\mathrm{P}_*^2) + M_2 - \xi(\mathrm{P}_*^2) = M_1 + M_2 \subset M$$

and hence, $z(\mathrm{P}_*^2) \in M^*$, which completes the proof of the theorem.

Remark 5. If for some matrix $B(t,\tau)$, feasible functions $\gamma(t,\tau)$, and feasible sets M_1 and M_2 Condition 1 is satisfied on set Δ, then $0 \in \mathfrak{A}(t,\tau)$, $\tau \in [0, t]$. Then Conditions 3 and 5 are satisfied and the equality holds

$$\sup_{v \in V} \alpha_*(t,\tau,v) = \alpha_*(t,\tau) = 0, \ \tau \in [0, t].$$

Let us consider the set

$$\Theta(g(t),\gamma(\cdot,\cdot)) = \left\{ t \ge 0 : \int_0^t \sup_{v \in V} \lambda_B(t,\tau,v)\, d\tau \le 1, \int_0^t \alpha^*(t,\tau)\, d\tau \ge 1, \int_0^t \alpha_*(t,\tau)\, d\tau < 1 \right\}. \tag{18}$$

If for some $t > 0$ $\alpha^*(t,\tau) \equiv +\infty$ for $\tau \in [0, t]$, then in this case it is natural to suppose the value of the respective integral in curly brackets of (18) to be equal to $+\infty$ and $t \in \Theta(g(t),\gamma(\cdot,\cdot))$ if other inequalities in curly brackets of this relation hold for this t. If the inequalities in (18) do not hold for all $t > 0$, suppose $\Theta(g(t),\gamma(\cdot,\cdot)) = \varnothing$.

THEOREM 6. Let Condition 5 be satisfied for the conflict-controlled process (1), (2), for some matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$, and feasible sets M_1 and M_2 set $\Theta(g(\cdot),\gamma(\cdot,\cdot))$ be non-empty, and $\Theta \in \Theta(g(\cdot),\gamma(\cdot,\cdot))$. Then the game can be terminated at time Θ with the use of control (4).

Proof. Let $v(\tau)$ be an arbitrary measurable selector of the compact set V, $\tau \in [0, \Theta]$. Let us specify the choice of control of the pursuer.

First, let us consider the case $\xi(\Theta, g(\Theta), \gamma(\cdot,\cdot) \notin M$ and introduce the control function

$$h(t) = 1 - \int_0^t \alpha^*(\Theta,\tau) d\tau - \int_t^\Theta \alpha_*(\Theta,\tau) d\tau, \ t \in [0, \Theta].$$

By the definition of Θ, we get

$$h(0) = 1 - \int_0^\Theta \alpha_*(\Theta,\tau)\, d\tau > 0, \ h(\Theta) = 1 - \int_0^\Theta \alpha^*(\Theta,\tau)\, d\tau \le 0.$$

Since the function $h(t)$ is continuous, there exists an instant of time t_*, $t_* \in (0, \Theta]$, such that $h(t_*) = 0$. Note that switching moment t_* does not depend on the previous history of control of the second player $v_{t_*}(\cdot) = \{v(s) : s \in [0, t_*]\}$.

We will call the time intervals $[0, t_*)$ and $[t_*, \Theta]$ "active" and "passive," respectively. Let us describe the control of the first player on each of them. To this end, consider the compact-valued mappings

$$\widetilde{U}_2^*(\tau, v) = \{u \in U : \pi\Omega(\Theta,\tau)\varphi(u, B(\Theta,\tau)v) - \gamma(\Theta,\tau) \in \alpha^*(\Theta,\tau)[M_2 - \xi(\Theta)]\}, \ \tau \in [0, t_*), \tag{19}$$

$$\widetilde{U}_*^2(\tau, v) = \{u \in U : \pi\Omega(\Theta,\tau)\varphi(u, B(\Theta,\tau)v) - \gamma(\Theta,\tau) \in \alpha_*(\Theta,\tau)[M_2 - \xi(\Theta)]\}, \ \tau \in [t_*, \Theta]. \tag{20}$$

The multi-valued mappings $\widetilde{U}_2^*(\tau, v)$ and $\widetilde{U}_*^2(\tau, v)$ have non-empty images. By virtue of the properties of the parameters of process (1), of functions $\alpha^*(\Theta,\tau)$ and $\alpha_*(\Theta,\tau)$, the compact-valued mappings $\widetilde{U}_2^*(\tau, v)$, $\tau \in [0, t_*)$, and $\widetilde{U}_*^2(\tau, v)$, $\tau \in [t_*, \Theta]$, for $v \in V$ are $\mathfrak{L} \otimes \mathfrak{B}$-measurable [12]. Therefore, by the theorem on measurable choice of selector [9], in each of them there exists at least one $\mathfrak{L} \otimes \mathfrak{B}$-measurable selector $\widetilde{u}_2^*(\tau, v)$ and $\widetilde{u}_*^2(\tau, v)$, which are superpositionally measurable functions [12]. Suppose that the control of the first player on the "active" interval is equal to $\widetilde{u}_2^*(\tau) = \widetilde{u}_2^*(\tau, v(\tau))$, $\tau \in [0, t_*)$, and on the "passive" one to $\widetilde{u}_*^2(\tau) = \widetilde{u}_*^2(\tau, v(\tau))$, $\tau \in [t_*, \Theta]$.

In view of formula (1), for the selected controls we obtain

$$\pi z(\Theta) = -\left[\int_0^{t_*} \pi\Omega(\Theta,\tau)\varphi_B(\Theta,\tilde{u}_2^*(\tau),v(\tau))d\tau + \int_{t_*}^{\Theta} \pi\Omega(\Theta,\tau)\varphi_B(\Theta,\tilde{u}_*^2(\tau),v(\tau))d\tau\right]$$

$$+\xi(\Theta) + \int_0^{t_*} (\pi\Omega(\Theta,\tau)\varphi(\tilde{u}_2^*(\tau), B(\Theta,\tau)v(\tau)) - \gamma(\Theta,\tau))d\tau$$

$$+\int_{t_*}^{\Theta} (\pi\Omega(\Theta,\tau)\varphi(\tilde{u}_*^2(\tau), B(\Theta,\tau)v(\tau)) - \gamma(\Theta,\tau))d\tau. \qquad (21)$$

By virtue of Condition 2, by the definition of time Θ, we get

$$0 \in M_1, \int_0^{\Theta} \sup_{v\in V} \lambda_B(\Theta,\tau,v)\, d\tau \le 1,$$

$$\pi\Omega(\Theta,\tau)\varphi_B(\Theta,\tilde{u}_2^*(\tau),v(\tau)) \in \sup_{v\in V} \lambda_B(\Theta,\tau,v)M_1,\ \tau\in[0,t_*),$$

$$\pi\Omega(\Theta,\tau)\varphi_B(\Theta,\tilde{u}_*^2(\tau),v(\tau)) \in \sup_{v\in V} \lambda_B(\Theta,\tau,v)M_1,\ \tau\in[t_*,\Theta].$$

Then with regard for Lemma 1, we obtain

$$\int_0^{t_*} \pi\Omega(\Theta,\tau)\varphi_B(\Theta,\tilde{u}_2^*(\tau),v(\tau))d\tau + \int_{t_*}^{\Theta} \pi\Omega(\Theta,\tau)\varphi_B(\Theta,\tilde{u}_*^2(\tau),v(\tau))d\tau \in M_1$$

and by the assumption we get

$$-\left[\int_0^{t_*} \pi\Omega(\Theta,\tau)\varphi_B(\Theta,\tilde{u}_2^*(\tau),v(\tau))d\tau + \int_{t_*}^{\Theta} \pi\Omega(\Theta,\tau)\varphi_B(\Theta,\tilde{u}_*^2(\tau),v(\tau))d\tau\right] \in M_1.$$

Taking into account the last inclusion and using (19)–(21), we obtain

$$\pi z(\Theta) \in M_1 + \xi(\Theta) + \int_0^{t_*} \alpha^*(\Theta,\tau)[M_2 - \xi(\Theta)]\,d\tau + \int_{t_*}^{\Theta} \alpha_*(\Theta,\tau)[M_2 - \xi(\Theta)]\,d\tau$$

$$= M_1 + \xi(\Theta) + \int_0^{t_*} \alpha^*(\Theta,\tau)\,d\tau[M_2 - \xi(\Theta)] + \int_{t_*}^{\Theta} \alpha_*(\Theta,\tau)d\tau[M_2 - \xi(\Theta)]$$

$$= M_1 + \xi(\Theta)\left[1 - \int_0^{t_*} \alpha^*(\Theta,\tau)d\tau - \int_{t_*}^{\Theta} \alpha_*(\Theta,\tau)d\tau\right] + \left[\int_0^{t_*} \alpha^*(\Theta,\tau)d\tau + \int_{t_*}^{\Theta} \alpha_*(\Theta,\tau)d\tau\right]M_2$$

$$= M_1 + M_2 \subset M.$$

Here, we took into account the equality $h(t_*) = 0$ and inclusion $M_1 + M_2 \subset M$, and we can confirm the passage in integration of multi-valued mappings with set M_2 by applying the apparatus of support functions [10].

For the case $\xi(\Theta, g(\Theta), \gamma(\cdot,\cdot)) \in M_2$, it will suffice to use Theorem 5.

COMPARING THE GUARANTEED TIMES

LEMMA 3. Let for the conflict-controlled process (1), (2) and some matrix $B(t,\tau)$, feasible functions $\gamma(t,\tau)$, and feasible sets M_1 and M_2 Condition 5 be satisfied and $\xi(t, g(t), \gamma(t,\cdot)) \notin M_2$. Then the inequalities hold:

$$\sup_{v \in V} \alpha_*(t,\tau,v) \le \alpha_*(t,\tau), \ (t,\tau) \in \Delta, \tag{22}$$

$$\inf_{v \in V} \alpha^*(t,\tau,v) \ge \alpha^*(t,\tau), \ (t,\tau) \in \Delta. \tag{23}$$

If, moreover, Condition 3 is satisfied, the inequality (22) turns to equality. If Condition 4 is true, then the inequality (23) is transformed into equality. If the multi-valued mapping $\mathfrak{A}(t,\tau,v)$ takes convex values on set $\Delta \times V$, then Conditions 3 and 4 are true and the equality takes place in relations (22), (23).

THEOREM 7. Let Condition 5 be satisfied for the conflict-controlled process (1), (2) and some matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$, and feasible sets M_1 and M_2. Then the inclusions takes place:

$$T(g(\cdot),\gamma(\cdot,\cdot)) \supset \Theta(g(\cdot),\gamma(\cdot,\cdot)) \supset \mathrm{P}^2_*(g(\cdot),\gamma(\cdot,\cdot)) \supset \mathrm{P}^1_*(g(\cdot),\gamma(\cdot,\cdot)) \supset \mathrm{P}(g(\cdot),\gamma(\cdot,\cdot)).$$

If, moreover, Conditions 3 and 4 are satisfied or if the multi-valued mapping $\mathfrak{A}(t,\tau,v)$ takes convex values on set $\Delta \times V$, then the equalities are true:

$$T(g(\cdot),\gamma(\cdot,\cdot)) = \Theta(g(\cdot),\gamma(\cdot,\cdot)), \ \mathrm{P}^2_*(g(\cdot),\gamma(\cdot,\cdot)) = \mathrm{P}^1_*(g(\cdot),\gamma(\cdot,\cdot)) = \mathrm{P}(g(\cdot),\gamma(\cdot,\cdot)).$$

The proofs of Lemma 3 and Theorem 7 follow from the constructions of the respective definitions, remarks, and theorems.

THEOREM 8. Let for the conflict-controlled process (1), (2), Condition 2 be satisfied, for some matrix $B(t,\tau)$, feasible function $\gamma(t,\tau)$, and feasible sets M_1 and M_2, the set $T(g(\cdot),\gamma(\cdot,\cdot))$ be non-empty, $T \in T(g(\cdot),\gamma(\cdot,\cdot))$, and the multi-valued mapping $\mathfrak{A}(t,\tau,v)$ take convex values for all (τ, v), $\tau \in [0,T]$, $v \in V$. Then the game can be terminated at time T with the use of control (4).

The proof immediately folows from Lemma 3 and Theorems 6 and 7.

A "BOY AND CROCODILE" EXAMPLE

The dynamics of the pursuer and evader is defined by the equations

$$\ddot{x} = u, \ x \in R^s, \ s \ge 2, \ \|u\| \le \rho, \ \rho > 0,$$

$$\dot{y} = v, \ y \in R^s, \ s \ge 2, \ \|v\| \le \sigma, \ \sigma > 0, \tag{24}$$

respectively. The pursuit is considered completed if $\|x - y\| \le l$.

The original problem (25) can be reduced to a conflict-controlled process as follows. Let us introduce new variables $(z_1, z_2) = z$,

$$z_1 = x - y, \ \dot{z}_2 = \dot{x} \tag{25}$$

and differentiate relations (25) with respect to time. Taking into account the original equations (24), we obtain

$$\dot{z}_1 = z_2 - v, \ \dot{z}_2 = u. \tag{26}$$

The terminal set takes the form $M^* = \{z : \|z_1\| \le l\}$, $M_0 = \{z : z_1 = 0\}$, $M = \{z : \|z_1\| \le l, \ z_2 = 0\}$. Denote

$$M_1 = l_1 S, \ M_2 = l_2 S = M \overset{*}{-} M_1 = lS \overset{*}{-} l_1 S = (l - l_1) S, \ l_2 = l - l_1, \ l > l_1.$$

Then

$$L = \{z : z_2 = 0\}, \ \pi = \begin{pmatrix} E & 0 \\ 0 & 0 \end{pmatrix}, \ \pi z = z_1, \ A = \begin{pmatrix} 0 & E \\ 0 & 0 \end{pmatrix}.$$

The domains of controls have the form

$$U=\left\{\begin{pmatrix}0\\ u\end{pmatrix}:\|u\|\le\rho\right\},\ V=\left\{\begin{pmatrix}v\\ 0\end{pmatrix}:\|v\|\le\sigma\right\}.$$

The fundamental matrix of the homogeneous system (26) is given by $e^{At}=\begin{pmatrix}E & tE\\ 0 & 0\end{pmatrix}$. Then we obtain $\pi e^{At}U=\rho tS$, $\pi e^{At}V=\sigma S$, $M=lS$, where S is a unit full-sphere of the subspace L centred at zero.

The Pontryagin condition is not satisfied on the interval $[0,\sigma/\rho)$:

$$\pi e^{At}U \overset{*}{-} \pi e^{At}V=\rho tS \overset{*}{-} \sigma S=(\rho t-\sigma)S=\varnothing,\ t\in[0,\sigma/\rho).$$

Let us choose the shift function $\gamma(t)\equiv 0$ and suppose

$$B(t)=\begin{cases}(\rho/\sigma)tE, & 0\le t\le\sigma/\rho,\\ E, & t>\sigma/\rho.\end{cases}$$

Then we get

$$0\in\bigcap_{v\in V}[W_B(t,\tau,v)-\gamma(t,\tau)]=\pi e^{At}U \overset{*}{-} \pi e^{At}\,B(t)V=\begin{cases}\{0\}, & t\in[0,\sigma/\rho],\\ (\rho t-\sigma)S, & t>\sigma/\rho.\end{cases}$$

By means of simple computation we obtain

$$\Lambda_B(t,\tau,v)=\begin{cases}[\lambda\ge 0:\ (1-(\rho/\sigma)t)v\subset\lambda l_1 S],\ 0\le t\le\sigma/\rho,\\ \{0\},\quad t>\sigma/\rho,\end{cases}$$

$$\lambda(t,v)=\begin{cases}\dfrac{(1-(\rho/\sigma)t)\|v\|}{l_1}, & t\in[0,\sigma/\rho],\\ 0, & t>\sigma/\rho,\end{cases}\qquad \max_{v\in\sigma S}\lambda(t,v)=\begin{cases}\dfrac{\sigma-\rho t}{l_1}, & t\in[0,\sigma/\rho],\\ 0, & t>\sigma/\rho.\end{cases}$$

Since

$$\varphi_B(t,U,V)=\pi e^{At}(E-B(t))V=\begin{cases}(\sigma-\rho t)S, & t\in[0,\sigma/\rho],\\ \{0\}, & t>\sigma/\rho,\end{cases}$$

we get $\varphi_B(t,U,V)=\max\limits_{v\in\sigma S}\lambda(t,v)l_1S$ and Condition 1 is satisfied.

If $l_1\ge\sigma^2/2\rho$, then the inequality holds for all $t\ge 0$:

$$(\rho t^2/2)-\sigma t+l_1\ge 0. \tag{27}$$

Therefore, for $l_1\ge\sigma^2/2\rho$ for all $t\ge 0$ the inequality holds:

$$\int_0^t\max_{v\in\sigma S}\lambda(\tau,v)\,d\tau\le\frac{\sigma t-(\rho t^2/2)}{l_1}\le 1.$$

Suppose $\xi(t)=\pi e^{At}z=z_1+tz_2$. Since Condition 1 is satisfied, Conditions 2 and 3 are true and $\alpha_*(t,\tau,v)=\sup\limits_{v\in V}\alpha_*(t,\tau,v)=0$. If $\xi(t)\in l_2S$, then by virtue of Theorem 1 or Theorem 2 the game can be terminated at time t with the use of control (4). With regard for inequality (27), the least such instant of time satisfies the equation

$$\|z_1+tz_2\|=(\rho t^2/2)-\sigma t+l,\ t\le\sigma/\rho.$$

Let $\xi(t) \notin l_2 S$. Then for $t-\tau \le \sigma/\rho$ the upper resolving function $\alpha^*(t,\tau,v)$ is defined from the relation $\|\rho(t-\tau)v-\alpha\xi(t)\| = \alpha l_2 + \rho(t-\tau)$, $v \in S$, and function $\alpha^*(t,\tau,v)$ is the larger positive root of the quadratic equation

$$(\|\xi(t)\|^2-(l_2)^2)\alpha^2 - 2[(v,\xi(t))+\rho(t-\tau)l_2]\alpha - [\rho^2(t-\tau)^2(1-\|v\|^2)] = 0$$

with respect to α if $\xi(t) \notin l_2 S$, $v \in S$.

After the evaluations we obtain

$$\min_{v\in S} \alpha^*(t,\tau,v) = 0, \quad t-\tau \le \sigma/\rho, \tag{28}$$

and the minimum is attained on the vector $v = -(\xi(t)/\|\xi(t)\|)$.

Let $\xi(t) \notin l_2 S$ and $t-\tau > \sigma/\rho$. Then the upper resolving function $\alpha^*(t,\tau,v)$ is defined from the relation $\|\sigma v - \alpha\xi(t)\| = \alpha l_2 + \rho(t-\tau)$, $v \in S$, and function $\alpha^*(t,\tau,v)$ is the larger positive root of the quadratic equation

$$(\|\xi(t)\|^2-(l_2)^2)\alpha^2 - 2[\sigma(v,\xi(t))+\rho(t-\tau)l_2]\alpha - [\rho^2(t-\tau)^2 - \sigma^2\|v\|^2] = 0$$

with respect to α if $\xi(t) \notin l_2 S$, $v \in S$.

After the calculations, we obtain

$$\min_{v\in S} \alpha^*(t,\tau,v) = \frac{\rho(t-\tau)-\sigma}{\|\xi(t)\|-l_2}, \quad t-\tau > \sigma/\rho. \tag{29}$$

Let us determine the time of game termination for $t > \sigma/\rho$. To this end, taking into account equalities (28) and (29), we can write

$$\int_0^t \min_{v\in S} \alpha^*(t,\tau,v)\,d\tau = \int_0^{t-(\sigma/\rho)} \min_{v\in S} \alpha^*(t,\tau,v)\,d\tau + \int_{t-(\sigma/\rho)}^{t} \min_{v\in S} \alpha^*(t,\tau,v)\,d\tau = \int_0^{t-(\sigma/\rho)} \frac{\rho(t-\tau)-\sigma}{\|\xi(t)\|-l_2}\,d\tau = 1.$$

From the last equality, we obtain the equation

$$\|z_1 + tz_2\| = (\rho t^2/2) - \sigma t + l - l_1. \tag{30}$$

The least positive root of Eq. (30) is the time of the game termination by virtue of Theorem 3 with the use of control (3). It can be easily verified that Condition 4 is true and the least positive root of Eq. (30) is the time of game termination by virtue of Theorem 4 with the use of control (4).

For $t = 0$, the left-hand side of Eq. (30) is equal to $\|z_1\|$ as t grows, it grows linearly, and the right-hand side is equal to $l - l_1$ and grows quadratically. Since $\|z_1\| > l - l_1$, for any z_1 and z_2 the time of game termination is finite.

The equality $z_1 + tz_2 = 0$ can only hold when Eq. (30) holds; therefore, this case is not considered.

CONCLUSIONS

We have considered the problem of approach of controlled objects with different inertia in dynamic game problems. We have formulated sufficient conditions of game termination in a finite guaranteed time in the case where the Pontryagin condition is not satisfied. We have introduced the upper and lower resolving functions of special type. On their basis, we have proposed two schemes of the method of resolving functions, which ensure termnation of the conflict-controlled process in the class of quasi-strategies and countercontrols. We have compared the guaranteed times of game termination for different schemes of approach of controlled objects with different inertia and have provided an illustrative example.

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