DOI 10.1007/s10559-021-00353-x
Cybernetics and Systems Analysis, Vol. 57, No. 2, March, 2021

$GI / G / 1$ LAKATOS-TYPE QUEUEING SYSTEM WITH T-RETRIALS

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UDC 519.872

Abstract. *We consider the Lakatos-type $GI / G / 1$ queueing system with T-retrials, i.e., the system with the FCFS service discipline and a constant orbit cycle time T. We have constructed a Markov chain for the system, proved its ergodicity condition, solved a system of equations for the stationary distributions of the system state probabilities, and derived formulas for the average number of requests and the average number of orbit cycles at specific relations of service and orbit time. We have developed an algorithm for statistical modeling of the considered system as well. Results of analytical and statistical modeling show consistency between them. We have indicated an essential property of the Lakatos-type system, namely, the fact that it can be used to evaluate a system in which the FCFS service discipline is not necessary.*

Keywords: *retrial queues, Lakatos-type queueing system, cyclic queueing systems, queueing systems with T-retrials, orbit, orbit cycle, Markov chain, queueing system ergodicity.*

INTRODUCTION

The retrial queueing theory has gained significant development over the last three decades. This process has stipulated a significant spread of telecommunication systems. Indeed, according to the classical Erlang system, a denied call attempt vanishes. However, if a call line is engaged, the call attempt can be repeated after a certain or random amount of time [1–3]. The other factors are computation systems and networks, where in the case of the retrial buffer being full, the retrials that are intended for the processor are blocked and repeated in a determined amount of time [4].

A substantial reason to develop the retrial queueing theory are modern optical information systems [5, 6] and control systems of aircraft landing approach [7, 8]. These systems necessitate the study of new queueing system models that are sometimes called Lakatos-type systems, retrial systems, and the *FCFS* (first come, first served) service discipline. Note that Laszlo Lakatos is a Hungarian mathematician, who was the first one to study a similar system to model the process of aircraft landing approach [9]. In this paper, a single-channel queueing system with Poisson arrivals with a parameter λ and a demonstrative service time with a parameter μ are considered. If a service channel is busy at the moment of an arrival entering the system, it will return in a time that is a factor of a certain coefficient T; however, this will occur not earlier than after all the preceding arrivals have been serviced. Using the embedded Markov chain method, a generating function of the queue length has been found, as well as a system ergodicity condition that has the following form:

$$\frac{\lambda}{\mu} < \frac{e^{-\lambda T}(1-e^{-\mu T})}{1-e^{-\lambda T}}. \tag{1}$$

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Note that in the classical queueing system with retrials that enter the system from outside or from the orbit, they are admitted to the service right away if at least one of the service channels is free. If all the channels are busy, then the arrival is directed (possibly, once again) to the orbit. Therefore, there is no possibility of queue-based service in the case of the classical queueing system.

By contrast with the classical queueing system, in the case of Lakatos-type systems, the retrials returning from the orbit are redirected to it again if there is even a single retrial that has entered the system before and has remained unserviced. A primary arrival is also directed to the orbit if even a single retrial exists on the service channel or in the orbit. Therefore, a Lakatos-type system combines the following two principles: the principles of retrials and of the *FCFS* service discipline (see [10–13]).

Let us denote a queueing system with retrials in a certain amount of time T by a system with T-retrials.

The aim of this paper is to study a more general than that of the discussed in [9] $GI/G/1$ system with the fixed time T and retrials, as well as the *FCFS* discipline service, i.e., a $GI/G/1$ Lakatos-type system with T-retrials.

$GI/G/1$ LAKATOS-TYPE SYSTEM WITH T-RETRIALS

Let us consider a single-channel queueing system with recurring arrivals and a continuous function of distribution $A(x)$ of the time between arrivals entering the system; with a general function of distribution $B(x)$ of service time, a fixed time T of an arrival staying in the orbit, and the *FCFS* service discipline. Therefore, the $M/M/1$ Lakatos model is generalized based on its arrivals and service time.

Let us determine the embedded Markov chain and find its ergodicity condition.

Let t_n be the moment of the nth arrival entering the system and $t_n + Tk_n$ be its service beginning moment. Note that k_n is always a nonnegative integer that is equal to the number of cycles of the nth arrival in the orbit. Therefore, let $\xi_n = t_{n+1} - t_n$ and Y_n be a service time of the nth arrival as well.

Let us find a relation between k_n and k_{n+1}. Let $k_n = i$. If $(k-1)T < Ti + Y_n - \xi_n < kT$, where $k \ge 1$ is an integer, then $k_{n+1} = k$; if $Ti + Y_n - \xi_n < 0$, then $k_{n+1} = 0$. Therefore, k_n is a homogenous Markov chain with transition probabilities $p_{ik} = P\{(k-i-1)T < Y_n - \xi_n < (k-i)T\}$, $k \ge 1$, and $p_{i0} = P\{Y_n - \xi_n < -Ti\}$.

Let us define $f_j = P\{(j-1)T < Y_n - \xi_n < jT\}$. We obtain

$$f_j = \int_0^{\infty} [B(x + jT) - B(x + (j-1)T)]\, dA(x). \tag{2}$$

In this case, the transition probabilities can be expressed in the following way:

$$p_{ik} = f_{k-i} \quad \text{if } 1 \le k \le i+1; \tag{3}$$

$$p_{i0} = \sum_{j=-\infty}^{-i} f_j. \tag{4}$$

Let us formulate and prove the ergodicity theorem.

THEOREM 1. If a series $\sum_{j=-\infty}^{\infty} jf_j$ fully coincides and $\sum_{j=-\infty}^{\infty} jf_j < 0$, then the Markov chain (k_n) is ergodic.

Proof. Under the condition of $k_n = i$,

$$\mathrm{E}\{k_{n+1}\} = \sum_{k=1}^{\infty} kp_{ik} = \sum_{k=1}^{\infty} kf_{k-i} = \sum_{j=1-i}^{\infty} (i+j)f_j = i\sum_{j=1-i}^{\infty} f_j + \sum_{j=1-i}^{\infty} jf_j \le i + \sum_{j=1-i}^{\infty} jf_j.$$

From Theorem 1, it follows that this expression is always finite and less than $i - \varepsilon$, $i > N$. According to the Mostafa indicator [14], the Markov chain k_n is additional, i.e., it has a stationary distribution.

In order to determine its ergodicity, it is enough to prove that it is aperiodic. Since $\sum jf_j < 0$, we will be able to find such a $k > 0$ that $f_{-k} > 0$. Let us fix the value $k_n = i$. If $i \ge k$, then we will obtain $k_{n+1} = i - k$ with the probability of f_{-k}; if $i< k$, then we will obtain $k_{n+1}=0$ with the probability not lower than f_{-k}. Therefore, it is possible to transition from the state (i) to the state ($k_{n+l}=0$) in a finite number of steps; after that, we will also obtain $k_{n+l+1}=0$ with the probability not lower than f_{-k}. From here, it follows that the Markov chain k_n is aperiodic. Therefore, it is ergodic.

Example 1. By substituting $dA(x)=\lambda e^{-\lambda x}dx$ and $B(x)=1-e^{-\mu x}$, $x\ge 0$, into (2), we obtain

$$f_j=\frac{\lambda}{\lambda+\mu}e^{-\mu(j-1)T}(1-e^{-\mu T}),\ j\ge 1,$$

$$f_j=\frac{\mu}{\lambda+\mu}e^{\lambda jT}(1-e^{-\lambda T}),\ j\le 0.$$

Performing a summation over j, we obtain

$$\sum_{j=-\infty}^{\infty} f_j=\frac{1}{\lambda+\mu}\left(\frac{\lambda}{1-e^{-\mu T}}-\frac{\mu e^{-\lambda T}}{1-e^{-\lambda T}}\right).$$

From here, it follows that the Lakatos condition from (1) is satisfied if and if the condition of theorem 1 is satisfied.

Note that a Lakatos-type system can be applied to evaluate a more complex system, where the *FCFS* service discipline is not necessary. Often, models with the *FCFS* service discipline are not comparable with real systems. This applies to the airplane landing system, when during the time in which one plane is directed to circle to land, the landing of another airplane can be performed. If we consider, however, the time W_n from the moment t_n until the last moment of the landing beginning of planes that have arrived earlier, then the following will always hold true: $W_n\le Tk_n$; therefore, the condition of Theorem 1 insures the ergodicity of W_n, i.e., the order of the more complex structure.

Let us derive the stationary distribution equation. Let us assume that the condition of Theorem 1 is satisfied and let us define $\pi_k=\lim_{n\to\infty} P\{k_n=k\}$, $k\ge 0$. We obtain the system of equations

$$\pi_k=\sum_{i=0}^{\infty}\pi_i p_{ik},\ k\ge 0; \tag{5}$$

$$\sum_{k=0}^{\infty}\pi_k=1. \tag{6}$$

From (3)–(5), we obtain

$$\pi_k=\sum_{i=0}^{\infty}\pi_i f_{k-i},\ k\ge 1; \tag{7}$$

$$\pi_0=\sum_{i=0}^{\infty}\pi_i\sum_{j=-\infty}^{-i} f_j. \tag{8}$$

Let us add the normalization condition from (6) to (7) and (8). The system of (7) and (8) can be solved in a recurring way. Note that the retrial service time is always lower or equal to T; therefore,

$$B(T+)=1. \tag{9}$$

Let us assume as well that $f_{-1}>0$. In this case, it follows from (2) that $f_j=0$ if all $j\ge 2$. Thus, we obtain

$$\pi_k=\sum_{j=0}^{k+1}\pi_i f_{k-i},\ k\ge 1.$$

From this equation system, all the unknown $\pi_2,\pi_3,\ldots$ can be expressed through π_0 and π_1; for example, $\pi_2=\frac{1}{f_{-1}}(\pi_1(1-f_0)-\pi_0 f_1)$. Therefore, we obtain

$$\pi_k=a_k\pi_0+b_k\pi_1,\ k\ge 0, \tag{10}$$

where a_k and b_k are the known constants ($a_0 = 1$, $b_0 = 0$, $a_1 = 0$, and $b_1 = 1$). By substituting (10) into (8), we obtain a linear relation between π_0 and π_1, which will allow us to express all the π_k through π_0. In order to determine π_0, the use of the normalization condition from (6) suffices.

Let us consider (7) and (8). Taking (9) into account, let us write the ergodicity condition of the Markov chain (k_n) as follows:

$$\sum_{k=-\infty}^{1} k f_k < 0. \tag{11}$$

Using a direct substitution, we can prove that under the condition of (11), system (7) and (8) have the following probabilistic solution:

$$\pi_j = (1-z)z^j, \; j \ge 1,$$

where z is a root of the equation

$$\sum_{k=-\infty}^{1} f_k z^{-k} = 1, \tag{12}$$

from the interval (0, 1).

The left-hand side of (12) is a convex function on the half-interval (0, 1], which approaches infinity as $z \to 0$ since $f_1 > 0$ and is equal to one if $z = 1$. Moreover, the left derivative of this function at the point $z = 1$ is positive due to (11). Therefore, there exists a unique root of Eq. (12) on the interval (0, 1).

This fact follows from the continuous right-side random walk theory as well [15].

Let us consider an important case, where the service time $Y_n = \tau$, $t_n - t_{n-1}$ is distributed exponentially with the parameter λ. Then,

$$f_1 = 1 - e^{-\lambda\tau};$$

$$f_k = e^{-\lambda\tau + \lambda kT}(1 - e^{-\lambda T}), \; k \le 0.$$

Then, the ergodicity condition from (11) takes the form of $z < 1$, where $z = e^{\lambda T}(1 - e^{-\lambda\tau})$.

Let the arrivals of the system be bulk Poisson arrivals, where the arrival number ζ in one group is a geometrically distributed random value

$$\mathrm{P}\{\zeta = k\} = (1-\theta)\theta^{k-1}, \; k \ge 1,$$

and intervals between groups are exponentially distributed random values with the parameter λ. Then, we obtain the following formula:

$$f_k = \theta 1_{\{k=1]} + (1-\theta) f_k^0,$$

where f_k^0 is the value of f_k in ordinary (non-bulk) Poisson arrivals with the parameter λ and $1_{\{k=1]}$ is a value that is equal to 1 if $k = 1$ and to 0 in the other case.

For f_k^0, the following formula is true:

$$f_k^0 = e^{-\lambda(\tau - kT)}(1 - e^{-\lambda T}) \text{ if } k \le 0;$$

$$f_1^0 = 1 - e^{-\lambda\tau}.$$

For the parameter z, we obtain either an equation

$$\frac{\theta}{z} + (1-\theta)\left(\frac{1-a}{z} + \frac{a(1-b)}{1-bz}\right) = 1$$

or an equation

$$\theta + (1-\theta)\left(1 - a + az\frac{1-b}{1-bz}\right) = z,$$

where we have defined $a = e^{-\lambda\tau}$ and $b = e^{-\lambda T}$ for the sake of writing simplification. Having solved this equation, we obtain the following formula for z under the condition that $z < 1$:

$$z = \frac{1 - a + a\theta}{b} = \frac{1 - e^{-\lambda\tau} + e^{-\lambda\tau}\theta}{e^{-\lambda T}}.$$

If $\theta \to 0$, then $z \to \frac{1-a}{b}$, which corresponds to the case of Poisson arrivals.

Let us derive the average value number of retrials in the orbit and the number of cycles of a retrial. Let $N(t)$ be the arrival number in the orbit at the moment in time t. Then, the integral $\int_0^T N(t)dt$ is the summed in-orbit time of the arrivals that entered the system on the interval $(0, T)$, excluding the remaining waiting time of the ones that were not admitted to the service until the moment in time T. From the ergodic point of view [14], in the case of large values of s we obtain

$$\int_0^s \mathrm{E}[N(t)]dt \sim \lambda s \mathrm{E}[KT],$$

$$\lambda = 1 / \int_0^\infty x dA(x),$$

where K is a stationary version of k_n. From here it follows that the ergodic average value of the retrial number in the orbit is as follows:

$$\lim_{s\to\infty} \frac{1}{s} \int_0^s \mathrm{E}[N(t)] = \lambda T \sum_{k=0}^{\infty} k(1-z)z^k = \frac{\lambda T z}{1-z}.$$

The stationary average value $\overline{K}$ of a retrial cycle number is determined by the formula $\overline{K} = \frac{z}{1-z}$.

STATISTICAL MODELING OF A $GI/G/1$ LAKATOS-TYPE SYSTEM WITH T-RETRIALS

Let us use the statistical modeling (Monte Carlo method) to obtain the numerical features of queueing systems and to examine their coherence to the analytical results obtained above. Let us apply the direct modeling method to estimate the service system features. Let us model a service system as a random process with a discrete time $\{X_n, n \geq 1\}$, where X_n is a vector of dimension k that includes information about the sought-after service system features and n are certain moments in time chosen depending on the modeling problem. The AMRandom module produced by ESB Consultancy [16] and created by Alan Miller has been applied in order to generate pseudorandom numbers.

Let us examine the ergodic condition of the $M/D/1$ Lakatos-type service system with T-retrials.

Let the demands (retrials) enter the system according to the Poisson law with the intensity λ with the constant service time that is equal to τ. Let the required waiting time in the orbit be constant as well and be equal to T ($T \geq \tau$). First of all, let us find an analytical expression of the ergodic condition for this service system. We obtain

$$f_j = e^{j\lambda T} e^{-\lambda\tau}[1 - e^{-\lambda T}], \; j \leq 0;$$

$$f_j = 1 - e^{-\lambda\tau}, \; j = 1;$$

$$f_j = 0, \; j > 1.$$

Let us consider the series

$$\sum_{j=-\infty}^{\infty} j f_j = 1 - \frac{e^{-\lambda\tau}}{1 - e^{-\lambda T}}.$$

Then, defining $\rho = \lambda\tau$, we obtain the following ergodic condition for this system:

$$\rho < \ln\left(\frac{1}{1-e^{-\lambda T}}\right). \tag{13}$$

Let us model this service system by a random process with the discrete time $\{X_n, n \geq 1\}$, where X_n is the number of demands in the system and n is the moment in time when the demands entered the system.

Let us provide a modeling algorithm of the demand number dependence on time in the system.

Step 1. Introduce the parameters (assume that the arrival intensity is $\lambda = 1$): $\rho = \lambda\tau = 1 \cdot \tau = \tau$ is the system utilization/service time, T is orbit time, and N is the number of demands in the system.

Step 2. Initialize the variables: $N_L \leftarrow 0$, because the initial number of demands in the system is equal to zero.

Every demand in the system is determined by the time interval between the current time and the moment when the demand is leaving the system (its service end moment) γ_i, $i = 1, \ldots, N$.

The first demand enters the system. Then, $\gamma_1 \leftarrow \tau$, $N \leftarrow 1$, and $C \leftarrow 1$ is a counter of the number of demands that have entered the system.

Step 3. The main part.

Generate an exponentially distributed time interval with the parameter $\lambda = 1$ between demand arrivals $\xi \leftarrow -\ln(1-\omega)$, where ω is a random value that is evenly distributed on the interval [0,1] and where $C \leftarrow C+1$, as the next demand enters the system.

Recalculate the values γ_i and N as follows:

$$\gamma_i \leftarrow \gamma_i - \xi, \; i = 1, \ldots, N_L,$$

$$N_L \leftarrow \sum_{i=1}^{N_L} I\{\gamma_i > 0\},$$

where $I\{\cdot\}$ is an indicator function. Extract all the zero and negative elements from the array γ_i^L.

Since the demand entering the system will be serviced last, find the minimum integer $j > 0$ for which the inequality $\gamma_i < jT$, $i = 1, \ldots, N_L$, is true. Then,

$$\gamma_{N_L+1} \leftarrow jT + \tau,$$

$$N_L \leftarrow N_L + 1.$$

Step 4. Examination of the fact weather all the demands entered the system. If $C < N$, then return to Step 3.

The algorithm stops.

As a result of the algorithm, we obtain the dependence of the number of demands in the system on time.

Let the arrivals parameter be $\lambda = 1$ and the time of a demand spent in the orbit be $T = 1.5$. Then, from (13) it follows that the inequality $\rho < -\ln(1-e^{-1.5}) \approx 0.2525$ is the ergodic condition for this system. Let us prove this statement by modeling. Let us present the form of possible dependencies of the number of demands in the system on time for the cases of $\rho = 0.2$, $\rho = 0.252$, $\rho = 0.253$, and $\rho = 0.4$ (Figs. 1–4). From Fig. 1, in particular, we can reach an empirical conclusion that the system is ergodic since it is emptied and the average number of demands does not increase with time. From Fig. 2, we can reach an empirical conclusion that the system remains ergodic; however, its emptying becomes sparser. From Fig. 3 it follows that the average number of demands in the system increases with time; therefore, we reach an empirical conclusion that the system will not be ergodic any longer. It is evident from Fig. 4 that the demand number speed in the system increases; therefore, we can reach an empirical conclusion that the system is not ergodic.

Thus, the modeling results have proved that (13) is true.

Let us construct a graph of the average ergodic value of the demand number in the orbit depending on the different parameter sets of the system (Figs. 5–7).

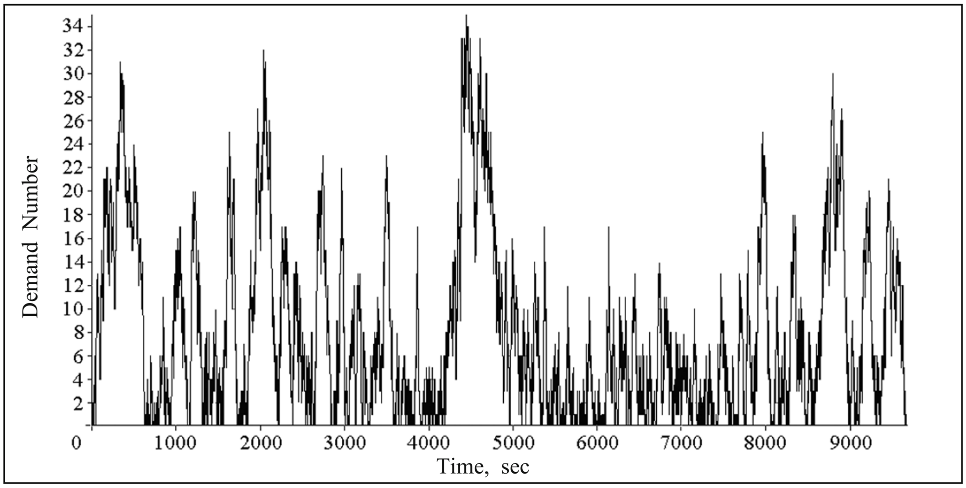

Fig. 1. Dependence of the number of demands in the system on the time, $\rho = 0.2$.

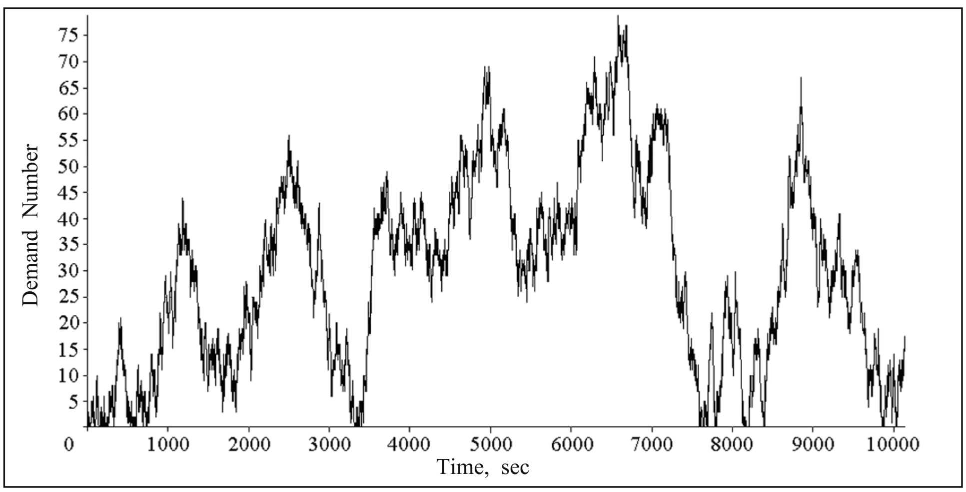

Fig. 2. Dependence of the number of demands in the system on the time, $\rho = 0.252$.

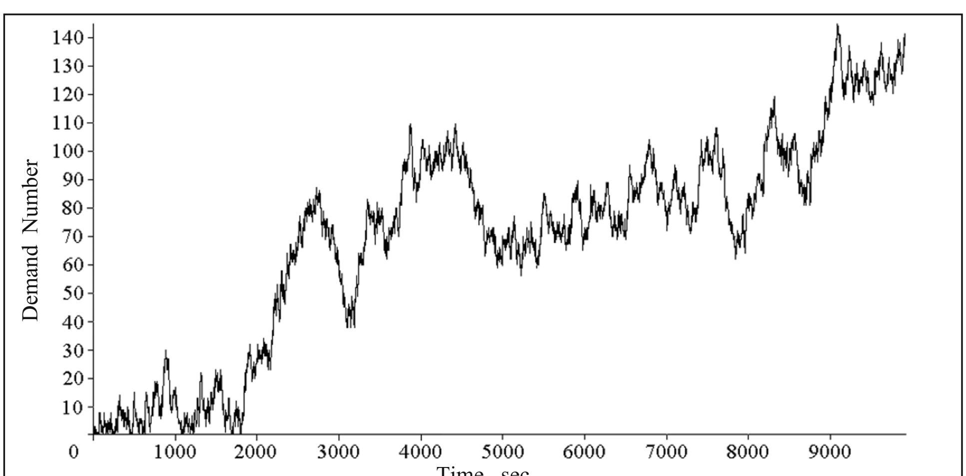

Fig. 3. Dependence of the number of demands in the system on the time, $\rho = 0.253$.

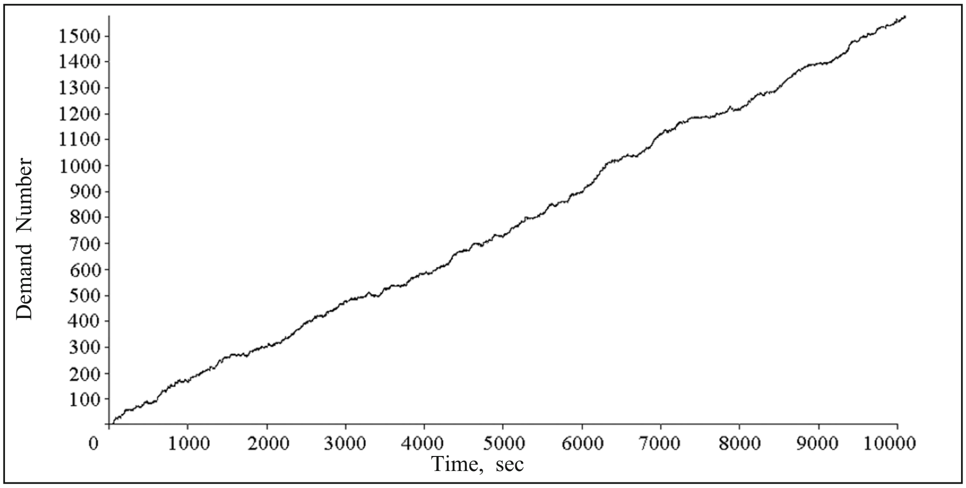

Fig. 4. Dependence of the number of demands in the system on the time, $\rho = 0.4$.

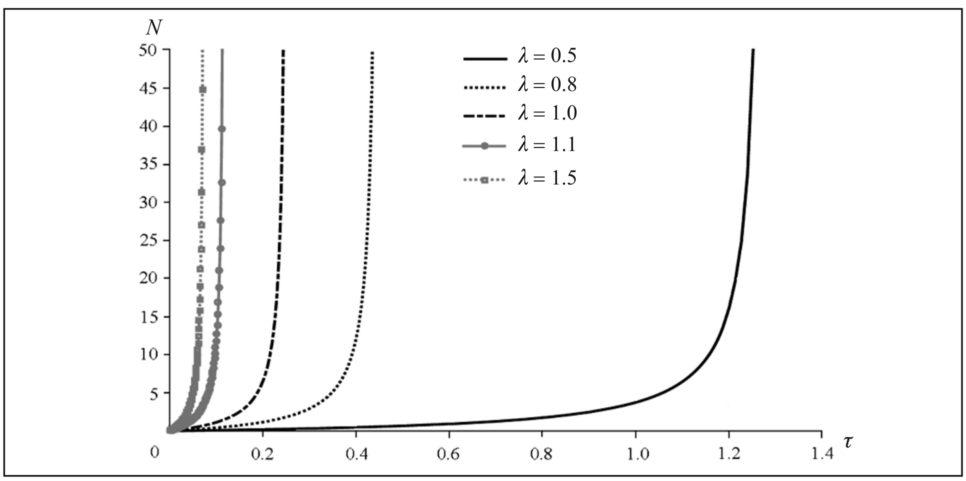

Fig. 5. Dependence of the average ergodic value of the number of retrials in the orbit N on τ under the conditions that $\lambda = \{0.5, 0.8, 1.0, 1.1, 1.5\}$, $T = 1.5$.

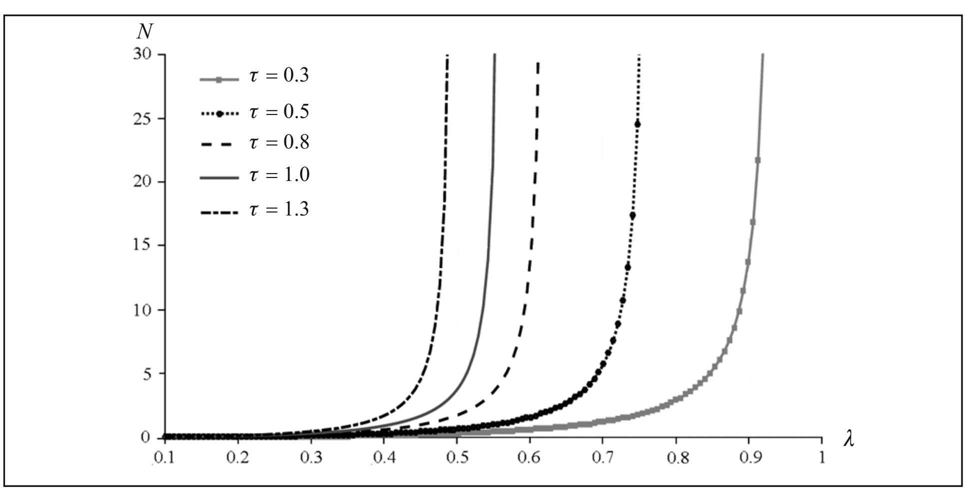

Fig. 6. Dependence of the average ergodic value of the number of retrials in the orbit N on λ under the condition that $\tau = \{0.3, 0.5, 0.8, 1.0, 1.3\}$, $T = 1.5$.

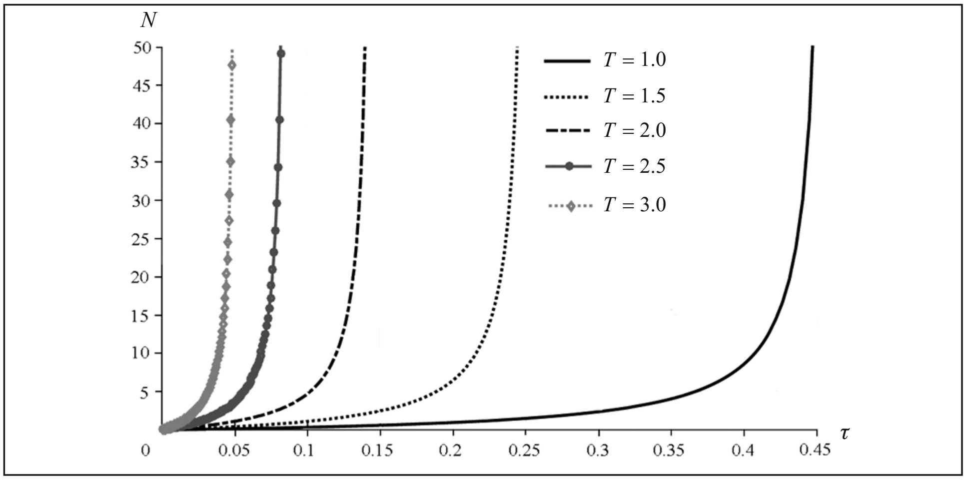

Fig. 7. Dependence of the average ergodic value of the number of retrials in the orbit N on τ under the condition that $\lambda = 1.0$, $T = \{1.0, 1.5, 2.0, 2.5, 3.0\}$.

The graphs in Figs. 5–7 visually demonstrate the fact that in the case of a certain parameter relation of the system, the retrial number in the orbit starts to sharply increase. This indicates the fact that in the case of the increase of the non-fixed parameter of the system, inequality (13) will not hold.

CONCLUSIONS

A single-channel $GI/G/1$ Lakatos-type service system with T-arrivals has been considered in this paper. Application examples of such a system are different telecommunication network nodes, airplane control systems of landing approach, microring resonators, optical delay lines, etc. For the system under study, we have found the ergodic condition, as well as the stationary distribution of the corresponding Markov chain under a specific relation between the service time and the time in the orbit. The average numbers of retrials in the orbit and orbit cycles have been considered as the system performance efficiency indicators. The obtained analytical and statistical modeling results correlate well.

Despite its peculiar queueing, the Lakatos system holds a major significance, as it can be applied to evaluate more complex systems, where there exist no restrictions of the *FCFS* service discipline.

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