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ON BAXTER TYPE THEOREMS FOR GENERALIZED RANDOM GAUSSIAN PROCESSES WITH INDEPENDENT VALUES

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Abstract. *We construct suitable families of basic functions and prove theorems of Baxter type for generalized Gaussian random processes with independent values. These theorems are used to divide families of such processes into classes. The singularity of probability measures corresponding to representatives of different classes is proved.*

Keywords: *generalized random process, theorems of Baxter type, singularity of probability measures.*

INTRODUCTION

The use of generalized functions [1] and their stochastic analogs (generalized random processes and fields) are known to substantially expand the possibilities of adequate description of various systems, for example, when the modern theory of differential equations is used for such description [2].

In the paper, we will consider one class of stochastic generalized functions: generalized Gaussian random processes with independent values. In some sense, such processes are analogs of regular processes with independent increments and are important, for example, for the theory of stochastic differential equations [3]. Some properties of these processes are described in [3, 4]. In what follows, we will analyze this class of functions for the so-called Baxter property.

For regular random processes, theorems of Baxter type (the Levi–Baxter theorems) are statements about convergence, to the deterministic constant, of normalized sums of nonlinear functions of increments of the process on the intervals that form partition of the interval $[0, T]$. Such sums are called Baxter ones. Squared increments are often considered as the above-mentioned functions . Statements of this type pertain to stochastic analysis. On the other hand, they admit natural statistical interpretations, which allow using respective results to estimate (identify) parameters of the processes or fields being observed [5]. Many mathematicians (noteworthy are the pioneer studies [6, 7]) have been analyzed sufficient convergence conditions of Baxter sums for normal random processes and fields for half a century. The Levi–Baxter theorems for generalized random functions are less studied [8–10]. In the present paper, we will analyze convergence of Baxter sums for generalized Gaussian random processes with independent values and constant coefficients in the representation of a covariance functional [4, p. 355]. The results will be applied to classify families of such processes and measure singularity conditions.

BAXTER SUMS FOR GENERALIZED PROCESSES

Let K be the space of real infinitely differentiable functions with compact carriers, defined on a real straight line R. A continuous random linear functional $\xi = (\xi, \varphi)$, $\varphi \in K$, in space K is called a generalized random process [4]. Below,

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we will consider real generalized random processes. To generate Baxter sums for generalized random processes, two-parameter sets of functions from space K of the form

$$\{\chi_{t,h}\} = \{\chi_{t,h} \in K \mid t \in R,\ h \in (0,1),\ \operatorname{supp}\chi_{t,h} \subset [t, t+h]\}, \tag{1}$$

nondecreasing unbounded sequence $(b(n))$ of natural numbers, and sequence of series of functions

$$\chi_{k,n} = \chi_{t,h}|_{t=k/b(n),\, h=1/b(n)},\quad k = 0, 1, 2, \ldots, b(n)-1,\quad n \ge 1, \tag{2}$$

are used.

Baxter sum $S_n(\xi)$ for the generalized random process ξ and family of functions $\{\chi_{k,n}\}$ is a random variable

$$S_n(\xi) = \sum_{k=0}^{b(n)-1} (\xi, \chi_{k,n})^2. \tag{3}$$

A generalized random process ξ is called a generalized Baxter (Baxter type) random process if for some family of functions of the form (2) sequence of random variables $S_n(\xi)$, $n \ge 1$, defined by Eq. (3) converges in one sense or another to a deterministic positive constant: $S_n(\xi) \to c > 0$ as $n \to \infty$. We will call respective family of functions $\{\chi_{t,h}\}$ suitable for the generalized random process ξ.

Definition 1. The family of functions (1) is called a family of type O_2 (o_2) if for functions of this family

$$\int_t^{t+h} \chi_{t,h}^2(x)dx = h + o(h),\ h \to 0+ \left(\int_t^{t+h} \chi_{t,h}^2(x)dx = o(h),\ h \to 0+ \right)$$

uniformly with respect to $t \in R$.

Example 1. A family of functions $\left\{\rho_{t,h}: R \to [0,1] \mid t \in R,\ h \in \left(0, \frac{1}{2}\right)\right\} \subset K$ such that for arbitrary $t \in R$, $h \in \left(0, \frac{1}{2}\right)$ the carrier $\operatorname{supp}\rho_{t,h} \subset [t, t+h]$ and $\rho_{t,h}(x) = 1$ on the interval $(t+h^2,\ t+h-h^2)$ is of type O_2.

Example 2. Let function $\varphi \in K$ with the carrier on the interval $[0,1]$ be such that $\int_0^1 \varphi^2(y)dy = 1$. For $t \in R$, $h \in (0,1)$, we suppose $\chi_{t,h}(x) = \varphi\left(\frac{x-t}{h}\right)$, $x \in R$. This family of functions is of type O_2. Indeed, for arbitrary $t \in R$, $h \in (0,1)$, we get

$$\int_t^{t+h} \chi_{t,h}^2(x)dx = \int_t^{t+h} \varphi^2\left(\frac{x-t}{h}\right)dx = h\int_0^1 \varphi^2(y)dy = h.$$

GENERALIZED RANDOM PROCESSES WITH INDEPENDENT VALUES

Definition 2 [4]. A generalized random process $\xi = (\xi, \varphi)$, $\varphi \in K$, is called a generalized random process with independent values if for arbitrary functions $\varphi, \psi \in K$ with carriers without comon interior points, random variables (ξ, φ) and (ξ, ψ) are independent.

THEOREM 1 [8]. A covariance functional $B(\varphi, \psi)$, $\varphi, \psi \in K$, for generalized random processes with the values independent at each point can be defined by the formula

$$B(\varphi, \psi) = \int_{-\infty}^{+\infty} \sum_{j,k \ge 0} R_{jk}(x)\varphi^{(j)}(x)\psi^{(k)}(x)dx, \tag{4}$$

where only a finite number of functions $R_{jk}(x)$ are nonzero on each finite interval.

Remark 1 [1, p. 356]. For each positive definite bilinear functional of the form (4), there exists a Gaussian generalized random process with independent values whose covariance functional is equal to the given one.

Let us consider the family of all Gaussian generalized random processes with independent values and zero mean values, whose contractions onto the subspace $K([0,1]) = \{\varphi \in K \mid \operatorname{supp}\varphi \subset [0,1]\}$ of space K have a covariance functional of the form

$$B(\varphi,\psi) = \sum_{j,k\geq 0} b_{jk} \int_{-\infty}^{+\infty} \varphi^{(j)}(x)\psi^{(k)}(x)dx,\ \varphi,\psi \in K([0,1]), \tag{5}$$

where $b_{jk} \in R$, and the number of nonzero terms is finite.

LEMMA 1. It is possible to present any covariance functional (5) as

$$B(\varphi,\psi) = \sum_{k=0}^{N} c_k \int_0^1 \varphi^{(k)}(x)\psi^{(k)}(x)dx,\ \varphi,\psi \in K([0,1]), \tag{6}$$

where N is a nonnegative integer, $c_k \in R$, $0 \leq k \leq N$, $c_N \neq 0$.

Proof. Let the covariance functional have the form (5), i.e., there exists a nonnegative integer M such that

$$B(\varphi,\psi) = \sum_{j,k=0}^{M} b_{kj} \int_0^1 \varphi^{(k)}(x)\psi^{(j)}(x)dx,\ \varphi,\psi \in K([0,1]).$$

Suppose

$$A = \{(k,j) \mid 0 \leq k,\ j \leq M,\ j-k \text{ is even}\},\ B = \{(k,j) \mid 0 \leq k,\ j \leq M,\ j-k \text{ is odd}\},$$

so that

$$B(\varphi,\psi) = \left(\sum_{(k,j)\in A} + \sum_{(k,j)\in B} \right) b_{kj} \int_0^1 \varphi^{(k)}(x)\psi^{(j)}(x)\,dx,\ \varphi,\psi \in K([0,1]). \tag{7}$$

Integrating by parts yields

$$\int_0^1 \psi^{(k)}(x)\varphi^{(j)}(x)\,dx = \begin{cases} \int_0^1 \varphi^{(k)}(x)\psi^{(j)}(x)dx, & (k,j)\in A, \\ -\int_0^1 \varphi^{(k)}(x)\psi^{(j)}(x)dx, & (k,j)\in B. \end{cases}$$

Thus,

$$B(\psi,\varphi) = \left(\sum_{(k,j)\in A} - \sum_{(k,j)\in B} \right) b_{kj} \int_0^1 \varphi^{(k)}(x)\psi^{(j)}(x)dx,\ \varphi,\psi \in K([0,1]). \tag{8}$$

Due to symmetry of the covariance functional, adding Eq. (7) and Eq. (8) yields

$$2B(\varphi,\psi) = 2\sum_{(k,j)\in A} b_{kj} \int_0^1 \varphi^{(k)}(x)\psi^{(j)}(x)dx.$$

By means of integration by parts, we can verify that for $(k,j)\in A$

$$\int_0^1 \varphi^{(k)}(x)\psi^{(j)}(x)dx = \begin{cases} \int_0^1 \varphi^{(l)}(x)\psi^{(l)}(x)dx, & (k-j)/2 \text{ even}, \\ -\int_0^1 \varphi^{(l)}(x)\psi^{(l)}(x)dx, & (k-j)/2 \text{ odd}, \end{cases}$$

where $l = \dfrac{k+j}{2}$.

Thus, $B(\varphi,\psi)=\sum_{l\ge 0} c_l \int_0^1 \varphi^{(l)}(x)\psi^{(l)}(x)dx$, where $c_l \in R$, and the number of terms is finite. For $N=\max\{l \mid l\ge 0,\, c_l \ne 0\}$, we obtain (6).

The lemma is proved.

Remark 2. The covariance functional of the generalized random process $P(d/dt)\eta$, where η is white noise and $P(d/dt)$ is a differential operator with constant coefficients, can also be reduced to the form (6).

CONVERGENCE OF THE BAXTER SUMS

THEOREM 2 [9, Corollary 2.2]. Let $\xi=(\xi,\varphi)$, $\varphi\in K$, be a generalized Gaussian random process with independent values and zero expectation, $\{\chi_{k,n}\}$ be a sequence of series of functions of the form (2), $S_n(\xi)=\sum_{k=0}^{b(n)-1}(\xi,\chi_{k,n})^2$, $n\ge 1$, be a sequence of Baxter sums. Then for mean square convergence

$$S_n(\xi)-ES_n(\xi)\to 0 \tag{9}$$

as $n\to\infty$, it is necessary and sufficient that

$$v_n^{(0)}(\xi)=\sum_{k=0}^{b(n)-1}(E(\xi,\chi_{k,n})^2)^2\to 0 \text{ as } n\to\infty. \tag{10}$$

If series $\sum_{n=1}^{\infty} v_n^{(0)}(\xi)$ converges, then convergence almost sure (a.s.) takes place in (9).

THEOREM 3. Let $\xi=(\xi,\varphi)$, $\varphi\in K$, be a generalized Gaussian random process with independent values with zero expectation and covariance functional (6), and the two-parameter family of functions $\{\alpha_{t,h}\}$ of the form (1) satisfy the following conditions:

(i) family of functions $\{\alpha_{t,h}^{(N)}\}$ is of type O_2;

(ii) for $N\ge 1$, for any $l\in\{0,1,\dots,N-1\}$, family of functions $\{\alpha_{t,h}^{(l)}\}$ is of type o_2.

Then

$$S_n(\xi)=\sum_{k=0}^{b(n)-1}(\xi,\alpha_{k,n})^2\to c_N \tag{11}$$

in the mean square as $n\to\infty$. If series $\sum_{n=1}^{\infty}\frac{1}{b(n)}$ converges, then convergence a.s. takes place in (11).

Proof. Since the covariance functional has the form (6),

$$E(\xi,\alpha_{k,n})^2=c_N\int_0^1(\alpha_{k,n}^{(N)}(x))^2dx+c_{N-1}\int_0^1(\alpha_{k,n}^{(N-1)}(x))^2dx$$

$$+\ldots+c_1\int_0^1(\alpha_{k,n}^{(1)}(x))^2dx+c_0\int_0^1(\alpha_{k,n}(x))^2dx=c_N\left(\frac{1}{b(n)}+o\left(\frac{1}{b(n)}\right)\right)$$

$$+c_{N-1}o\left(\frac{1}{b(n)}\right)+\ldots+c_1\,o\left(\frac{1}{b(n)}\right)+c_0\,o\left(\frac{1}{b(n)}\right)=\frac{c_N}{b(n)}+o\left(\frac{1}{b(n)}\right),\quad n\to\infty,$$

it follows that

$$ES_n(\xi)=\sum_{k=0}^{b(n)-1}E(\xi,\alpha_{k,n})^2=c_N(1+o(1))+o(1)\to c_N \text{ as } n\to\infty. \tag{12}$$

Further,

$$v_n^{(0)}(\xi) = \sum_{k=0}^{b(n)-1} (E(\xi, \alpha_{k,n})^2)^2 = \sum_{k=0}^{b(n)-1} \left(\frac{c_N}{b(n)} + o\left(\frac{1}{b(n)} \right) \right)^2$$

$$= b(n)\left(\frac{c_N}{b(n)} + o\left(\frac{1}{b(n)} \right) \right)^2 = \frac{c_N^2}{b(n)} + o\left(\frac{1}{b(n)} \right) \to 0 \text{ as } n \to \infty.$$

From relations (9), (10), and (12), the statement of the theorem follows.

Example 3. Let generalized Gaussian random process ξ satisfy the conditions of Theorem 3 and the family of functions $\{\chi_{t,h}\}$ be defined in Example 2. Let us prove that the family of functions $\alpha_{t,h} = \frac{h^N}{d_N} \chi_{t,h}$, $t \in R$, $h > 0$, where $d_N = \left(\int_0^1 (\varphi^{(N)}(t))^2 \, dt \right)^{1/2}$, satisfies Conditions 1 and 2 of Theorem 3. For $N = 0$, it follows from Example 2 that these conditions are satisfied. Let $N \geq 1$. We get

$$\alpha_{t,h}^{(l)}(x) = \frac{h^{N-l}}{d_N} \varphi^{(l)}\left(\frac{x-t}{h} \right), \; x \in R, \; 0 \leq l \leq N.$$

For $l = N$, we obtain

$$\int_t^{t+h} (\alpha_{t,h}^{(N)}(x))^2 \, dx = \frac{1}{d_N^2} \int_t^{t+h} \left(\varphi^{(N)}\left(\frac{x-t}{h} \right) \right)^2 dx = \frac{h}{d_N^2} \int_0^1 (\varphi^{(N)}(y))^2 \, dy = h,$$

and for $0 \leq l \leq N-1$, we get

$$\int_t^{t+h} (\alpha_{t,h}^{(l)}(x))^2 \, dx = \frac{h^{2(N-l)}}{d_N^2} \int_t^{t+h} \left(\varphi^{(l)}\left(\frac{x-t}{h} \right) \right)^2 dx = \frac{h^{2(N-l)+1}}{d_N^2} \int_0^1 (\varphi^{(l)}(y))^2 \, dy = o(h), \; h \to 0+.$$

By virtue of Theorem 3, the family of functions $\{\alpha_{t,h}\}$ is suitable for the generalized Gaussian random process ξ with zero expectation and covariance functional (6). Relation (11) holds in this case.

To generate a suitable family of functions for the generalized random process ξ, the specific form of functions of the original family $\{\chi_{t,h}\}$ in Example 2 is important. Let us show how it is possible to generate suitable families of functions for the generalized Gaussian process with covariance functional (6), based on the arbitrary family of functions $\{\chi_{t,h}\}$ of type O_2.

TRANSFORMATIONS *L*, *M*

Let us define transformations L, M of the main functions $\chi_{t,h}$ by the following equalities:

$$(L\chi_{t,h})(x) = \chi_{t,h}(2x - t) - \chi_{t,h}(2x - (t+h)), \; x \in R, \tag{13}$$

$$(M\chi_{t,h})(x) = \int_{-\infty}^{x} \chi_{t,h}(y) dy, \; x \in R. \tag{14}$$

Noteworthy are the following properties of transformations L, M.

(i) carrier $\operatorname{supp}(L\chi_{t,h}) \subset [t, t+h]$. The graph of function $L\chi_{t,h}$ is centrally symmetric with respect to point $\left(t + \frac{h}{2}, 0 \right)$.

(ii) carrier $\operatorname{supp}(ML\chi_{t,h}) \subset [t, t+h]$.

(iii) differentiation formulas:

$$\frac{dM\chi_{t,h}}{dx}(x) = \chi_{t,h}(x),\ x \in R; \tag{15}$$

$$\frac{dL\chi_{t,h}}{dx}(x) = 2L\frac{d\chi_{t,h}}{dx}(x),\ x \in R. \tag{16}$$

LEMMA 2. (i) If family of functions $\{\chi_{t,h}\}$ is of type O_2, then family of functions $\{L\chi_{t,h}\}$ is of type O_2 and family of functions $\{ML\chi_{t,h}\}$ is of type o_2.

(ii) If family of functions $\{\chi_{t,h}\}$ is of type o_2, then families of functions $\{L\chi_{t,h}\}$ and $\{ML\chi_{t,h}\}$ are of type o_2.

Proof. Let us prove the first statement, the second one can be proved similarly. Due to the first property of transformation L, we get

$$\int_t^{t+h} (L\chi_{t,h}(x))^2\,dx = 2\int_t^{t+\frac{h}{2}} \chi_{t,h}^2(2x-t)\,dx = \int_t^{t+h} \chi_{t,h}^2(y)\,dy = h + o(h),\ h \to 0+,$$

i.e., family of functions $\{L\chi_{t,h}\}$ is of type O_2. Then, using Eqs. (13)–(16), we get

$$\int_t^{t+h} (ML\chi_{t,h}(x))^2\,dx = \int_t^{t+h}\left(\int_{-\infty}^{x}(L\chi_{t,h})(y)\,dy\right)^2 dx = \int_t^{t+h}\left(2\int_t^{t+\frac{h}{2}}\chi_{t,h}(2y-t)\,dy\right)^2 dx$$

$$= \int_t^{t+h}\left(\int_t^{t+h}\chi_{t,h}(s)\,ds\right)^2 dx = h\left(\int_t^{t+h}\chi_{t,h}(s)\,ds\right)^2$$

$$\le h^2\int_t^{t+h}\chi_{t,h}^2(s)\,ds = h^2(h+o(h)) = o(h),\ h \to 0+,$$

whence it follows that family of functions $\{ML\chi_{t,h}\}$ is of type o_2.

The lemma is proved.

LEMMA 3. For all non-negative integers k and l such that $l \le k$, the equality holds

$$\frac{d^l}{dx^l}((ML)^k\chi_{t,h}) = 2^{\frac{l(l-1)}{2}} L^l (ML)^{k-l}\chi_{t,h}. \tag{17}$$

Proof. For $l = 0$ and any $k \ge 0$, the statement is trivial. In what follows, we assume $l \ge 1$. Let us apply mathematical induction with respect to k. For $k = 1$, we get the equality $\frac{d}{dx}(ML\chi_{t,h}) = L\chi_{t,h}$. Assume that Eq. (17) holds for k, $l \in \{0, 1, \dots, k\}$. Let us prove Eq. (17) for $k+1$ and all $l \in \{1, \dots, k, k+1\}$. We get

$$\frac{d^l}{dx^l}((ML)^{k+1}\chi_{t,h}) = \frac{d^l}{dx^l}(ML(ML)^k\chi_{t,h}) = \frac{d^{l-1}}{dx^{l-1}}(L(ML)^k\chi_{t,h})$$

$$= 2^{l-1}L\frac{d^{l-1}}{dx^{l-1}}((ML)^k\chi_{t,h}).$$

By the induction hypothesis, the last expression has the form

$$2^{l-1} L 2^{\frac{(l-1)(l-2)}{2}} L^{l-1} \frac{d^{l-1}}{dx^{l-1}} ((ML)^{k-(l-1)} \chi_{t,h}) = 2^{\frac{l(l-1)}{2}} L^l (ML)^{k+1-l} \chi_{t,h}.$$

The lemma is proved.

Let family of functions $\{\chi_{t,h}\}$ be of type O_2. For each $k \geq 0$, let us define family of functions

$$A_k = \left\{ \alpha_{t,h;k} = 2^{-\frac{k(k-1)}{2}} ((ML)^k \chi_{t,h}) \mid t \in R,\, h > 0 \right\}. \tag{18}$$

Note that by virtue of Lemma 3

$$\frac{d^k \alpha_{t,h;k}}{dx^k} = L^k \chi_{t,h},\ k \geq 0. \tag{19}$$

THEOREM 4. Let $\xi = (\xi, \varphi)$, $\varphi \in K$, be a generalized Gaussian random process with independent values with zero expectation and covariance functional (6) and two-parameter family of functions $A_N = \{\alpha_{t,h;N}\}$ be defined by Eq. (18). Then

$$S_n(\xi) = \sum_{k=0}^{b(n)-1} (\xi, \alpha_{k,n;N})^2 \to c_N \tag{20}$$

in the quadratic mean as $n \to \infty$. If series $\sum_{n=1}^{\infty} \frac{1}{b(n)}$ converges, then convergence with probability one takes place in (20).

Proof. Due to the first statement of Lemma 2 and Eq. (19) for non-negative integer N, the family of functions $\left\{\frac{d^N \alpha_{t,h;k}}{dx^N}\right\}$ is of type O_2, and for $N \geq 1$ for each $l \in \{0, \ldots, N-1\}$ the family of functions $\left\{\frac{d^l \alpha_{t,h;k}}{dx^l}\right\}$ is of type o_2. Thus, Theorem 4 follows from Theorem 3.

SINGULARITY OF MEASURES

Let N be a non-negative integer, $c_N > 0$. Denote by $G(N, c_N)$ the class of all generalized Gaussian random processes with zero mean value, whose contractions to the interval $[0, 1]$ have the covariance functional (6), where N and c_N are fixed. In what follows, we assume that the series $\sum_{n=1}^{\infty} \frac{1}{b(n)}$ converges. Note that for $(N, c_N) \neq (\widetilde{N}, c_{\widetilde{N}})$, classes $G(N, c_N)$ and $G(\widetilde{N}, c_{\widetilde{N}})$ are disjoint.

THEOREM 5. Let the statistical structure of $(\Omega, \sigma, P_1, P_2)$ be such that a generalized Gaussian random process with independent values ξ belongs to the class $G(N, c_N)$ with respect to the probability measure P_1 and belongs to the class $G(\widetilde{N}, c_{\widetilde{N}})$ with respect to the probability measure P_2. Then for $(N, c_N) \neq (\widetilde{N}, c_{\widetilde{N}})$, probability measures P_1 and P_2 are orthogonal.

Proof. Let $(N, c_N) \neq (\widetilde{N}, c_{\widetilde{N}})$. Assume that series $\sum_{n=1}^{\infty} \frac{1}{b(n)}$ converges. If $N \neq \widetilde{N}$, then assuming that $N < \widetilde{N}$, we get

$$X_1 = \left\{ \omega \in \Omega \,\middle|\, \sum_{k=0}^{b(n)-1} (\xi, \alpha_{t,h,\widetilde{N}})^2 \to 0, \quad n \to \infty \right\},$$

$$X_2 = \left\{ \omega \in \Omega \,\middle|\, \sum_{k=0}^{b(n)-1} (\xi, \alpha_{t,h,\widetilde{N}})^2 \to c_{\widetilde{N}},\ \ n \to \infty \right\}.$$

Then, as follows from Theorem 4 and from the proof of Theorem 3, $P_1(X_1) = P_2(X_2) = 1$. However, $X_1 \cap X_2 = \varnothing$. The singularity is proved. The case $N = \widetilde{N}$, $c_N \neq c_{\widetilde{N}}$ can be considered similarly.

Remark 3. If the equality $(N, c_N) = (\widetilde{N}, c_{\widetilde{N}})$ holds, measures P_1 and P_2 are equivalent on the σ-algebra generated by $\xi(\varphi)$, $\varphi \in K([0,1])$ [11].

CONCLUSIONS

We have proposed the families of suitable functions and proved theorems of Baxter type for generalized Gaussian random processes with independent values and zero expectation. This has allowed us to divide such family of functions into a continual set of pairwise disjoint classes. The probability measures corresponding to representatives of different classes are orthogonal, and those corresponding to representatives of one class are equivalent.

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