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CHEBYSHEV APPROXIMATION OF FUNCTIONS OF SEVERAL VARIABLES

P. S. Malachivskyy,[1] Ya. V. Pizyur,[2†] R. P. Malachivskyi,[3] and O. M. Ukhanska[2‡] UDC 519.65

Abstract. *The authors propose an algorithm to construct Chebyshev approximation for functions of several variables by a generalized polynomial as a limiting approximation in the norm of space L^p as $p \to \infty$. It is based on serial construction of power-average approximations using the least squares method with variable weight function. The convergence of the method provides an original way to consistently refine the values of the weight function, which takes into account the results of approximation at all previous iterations. The authors describe the methods of calculating the Chebyshev approximation with absolute and relative errors. The results of test examples confirm the efficiency of using the method to obtain Chebyshev approximation of tabular continuous functions of one, two, and three variables.*

Keywords: *functions of several variables, Chebyshev (uniform) approximation, power-mean approximation, least squares method, variable weight function.*

INTRODUCTION. PROBLEM STATEMENT

Let a continuous function of n variables $f(X)$, where vector $X = \{(x_1, x_2, \ldots, x_n)\}^{\mathrm{T}}$, be defined on a point set $\Omega = \{x_{i,j}\}_{i=0,\, j=0}^{n,\, s_i}$. It is required to approximate this function by an expression $F_m(a; X)$, where $F_m(a; X)$ is a generalized polynomial

$$F_m(a; X) = \sum_{i=0}^{m} a_i \varphi_i(X) \tag{1}$$

with respect to the system of linearly independent basis functions $\varphi_i(X)$, $i = \overline{0, m}$, where a_i, $i = \overline{0, m}$, are unknown parameters: $\{a_i\}_{i=0}^{m} \in A$, $A \subseteq R^{m+1}$, and R^m is an m-dimensional vector space. We will call expression $F_m(a^*; X)$ the Chebyshev approximation of function $f(X)$ on the point set Ω if it satisfies the condition

$$\max_{X \in \Omega} |f(X) - F_m(a^*; X)| = \min_{a \in A} \max_{X \in \Omega} |f(X) - F_m(a; X)|. \tag{2}$$

Chebyshev approximation of functions of several variables is used to solve various applied problems, in particular, to design hardware for measurement of physical quantities whose value depends on several information signals [1, 2].

To create Chebyshev approximation of functions of several variables, three techniques are mainly used: optimization methods, serial computing of Chebyshev approximation with respect to each variable, and Remez-type

[1]Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, National Academy of Sciences of Ukraine, Lviv, Ukraine, *Petro.Malachivskyy@gmail.com*. [2]National University "Lviv Polytechnic," Lviv, Ukraine, [†]*pizyur@yahoo.com*; [‡]*oksana.m.ukhanska@lpnu.ua*. [3]Lohika Systems, Lviv, Ukraine, *romanmalachivsky@gmail.com*.
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iteration schemes [3, 4]. In [5], an improved algorithm is proposed for generating the Chebyshev approximation of a function of several variables by a generalized polynomial; it solves the linear programming problem with regard for the features of uniform approximation. Algorithms for computing the Chebyshev approximation of functions of several variables and available program implementations are presented in [6, 7]. In the paper, we will propose a method for generating the Chebyshev approximation of functions of several variables as a limiting approximation in the norm of space L^p as $p \to \infty$, which sequentially generates power-mean approximations [8]. This method is a further development of the idea of the Remez α-algorithm [9]. Power-mean approximations are calculated by the least squares method with the use of a variable weight function whose value is specified sequentially with regard for all previous approximations.

POWER-MEAN APPROXIMATION OF FUNCTIONS

To estimate the error of power-mean approximation of functions, the norm in space L^p

$$\|\Delta\|_{L^p} = \left(\int_{\alpha_n}^{\beta_n} \ldots \int_{\alpha_2}^{\beta_2} \int_{\alpha_1}^{\beta_1} |\Delta(X)|^p \, \partial x_1 \partial x_2 \ldots \partial x_n \right)^{1/p}, \quad 1 \le p < \infty, \tag{3}$$

is used, where $x_i \in [\alpha_i, \beta_i]$, $i = \overline{1, n}$, and $\Delta(X) = f(X) - F_m(a; X)$. For $1 \le p < \infty$, $\|\Delta\|_{L^p}$ gets intermediate values between $\|\Delta\|_{L^1}$ and $\|\Delta\|_C$ [3, 9], where $\|\Delta\|_C$ is the norm in the space of continuous functions.

In the discrete case, the norm of the Euclidean space E^p is used for error estimation of the power-mean approximation. The error of power-mean approximation of function $f(X)$ defined on the point set Ω by expression (1) can be estimated in the norm

$$\|\Delta\|_{E^p} = \left(\sum_{X \in \Omega} |\Delta(X)|^p \right)^{1/p}, \tag{4}$$

where $\Delta(X) = f(X) - F_m(a; X)$, $1 \le p < \infty$. Similarly to the continuous case, the limiting value of the norm $\|\Delta\|_{E^p}$ as $p \to \infty$ corresponds to the norm in the space of continuous functions $\|\Delta\|_C$.

The possibility of obtaining the Chebyshev approximation as a limiting approximation in space L^p as $p \to \infty$ is analyzed in detail in [9], where E.Ya. Remez theoretically substantiated convergence of the computing schemes for creation of the Chebyshev approximation on the basis of power-mean approximation. The α-algorithm provides calculation of the Chebyshev approximation of a function of several variables, as an improved one, with the use of α-correction of power-mean approximation of some rather high power of p_s. This algorithm consists in iterative generation of power-mean approximations for the powers

$$p_0 < p_1 < p_2 < \ldots < p_s, \tag{5}$$

where $p_0 = 2$, $p_1 \ge 3$. Higher powers are recommended to be chosen from the relation $p_i / p_{i-1} = 4$, $i = \overline{2, s-1}$ [9]. The power-mean approximation can be found by the least squares method

$$\sum_{X \in \Omega} \rho_i(X)(f(X) - F_m(a; X))^2 \xrightarrow[a \in A]{} \min, \quad i = 0, 1, \ldots, s, \tag{6}$$

with the weight function

$$\rho_0(X) = 1, \quad \rho_i(X) = |\Delta_i(X)|^{p_i - 2}, \quad i = 1, 2, \ldots, s, \tag{7}$$

where $\Delta_i(X) = f(X) - F_{m,i-1}(a; X)$, $F_{m,i}(a; X)$ is power-mean approximation of the function $f(X)$ by the expression $F_m(a; X)$, which corresponds to the power exponent p_i.

Beginning with the power p_1 (5), the power-mean approximations $F_{m,i}(a; X)$, $i = \overline{1, s}$, obtained by the method (6), (7) are adjusted by the α-correction [9]. For the adjustment, the least squares method is used

$$\sum_{X\in\Omega}\rho_{i,j}(X)((f(X)-F_{m,i,j-1}(a;X))-F_m(a;X))^2 \xrightarrow[a\in A]{} \min, \tag{8}$$
$$i=1,2,\ \ldots,s,\ \ j=1,2,\ldots,$$

with variable weight function

$$\rho_{i,j}(X)=|\Delta_{i,j}(X)|^{p_i-2}, \tag{9}$$

where

$$\Delta_{i,j}(X)=f(X)-F_{m,i,j-1}(a;X), \tag{10}$$

$F_{m,i,0}(a;X)=F_{m,i}(a;X)$, $F_{m,i,j}(a;X)$ is power-mean approximation of the function $f(X)$ by the expression $F_m(a;X)$ obtained with the use of the weight function $\rho_{i,j}(X)$. The adjusted power-mean approximation $F_{m,i,j}(a;X)$ of power p_i of function $f(X)$ can be found by the formula

$$F_{m,i,j}(a;X)=F_{m,i,j-1}(a;X)+\alpha\bar{F}_{m,i,j}(a;X), \tag{11}$$

where $\bar{F}_{m,i,j}(a;X)$ is approximation obtained as a result of solution of problem (8), and the value of parameter α can be found from the condition

$$\max_{X\in\Omega}|f(X)-F_{m,i,j-1}(a;X)-\alpha\bar{F}_{m,i,j}(a;X)| \xrightarrow[\alpha,\alpha<0]{} \min. \tag{12}$$

Adjustment of the power-mean approximation (11) of power p_i is carried out until the required accuracy ε is attained:

$$|\max_{X\in\Omega}|\Delta_{i,j}(X)|-\max_{X\in\Omega}|\Delta_{i,j-1}(X)||\le\varepsilon\max_{X\in\Omega}|\Delta_{i,j}(X)|. \tag{13}$$

When this condition is satisfied, the power-mean approximation of power p_i of function $f(X)$ is set equal to $F_{m,i}(a;X)=F_{m,i,j}(a;X)$ and passage to calculating the power-mean approximation of power p_{i+1} (5) is performed.

The program implementation of the α-algorithm is described in [10]. In [4, 11], to obtain uniform approximation of functions of several variables $f(X)$ by the expression $F_m(a;X)$ (1), it is proposed to use the least squares method

$$\sum_{X\in\Omega}\rho_r(X)(f(X)-F_m(a;X))^2 \xrightarrow[a\in A]{} \min,\ \ r=0,1,\ \ldots, \tag{14}$$

with successive adjustment of the variable weight function

$$\rho_0(X)\equiv 1,\ \ \rho_r(X)=\prod_{i=1}^{r}|\Delta_i(X)|^2,\ \ r=1,2,\ \ldots, \tag{15}$$

where $\Delta_k(X)=f(X)-F_{m,k-1}(a;X)$, $k=\overline{1,r}$, $F_{m,k}(a;X)$ is approximation, by the least squares method, of the function $f(X)$ with the weight function $\rho_k(X)$.

The use of weight function (15) has ensured nearly uniform distribution of the approximation error at the points of representation of function $f(X)$ [4, 11]. In [11], it is proposed to adjust the approximation obtained by the method (14), (15) with the use of an additive (symmetrizable) correction.

METHOD TO DETERMINE PARAMETERS OF CHEBYSHEV APPROXIMATION OF FUNCTION OF SEVERAL VARIABLES

Generation of the Chebyshev approximation of tabular functions of several variables is based on the idea of successive obtaining of approximations in space E^p for $p=2,3,4,\ldots$ [8]. To generate an approximation in the space E^p, we will use the least squares method (14) with the weight function

$$\rho_0(X) \equiv 1,\ \rho_r(X) = \prod_{i=1}^{r} |\Delta_i(X)|,\ r = 1, \ldots, p-2,\ p = 3, 4, \ldots, \tag{16}$$

where $\Delta_k(X) = f(X) - F_{m,k-1}(a; X)$, $k = \overline{1, r}$, and $F_{m,k}(a; X)$ is approximation of the function $f(X)$ with the weight function $\rho_k(X)$ by the least squares method.

The least squares method (14) with variable weight function (16) ensures successive obtaining of power-mean approximations $F_{m,r}(a; X)$, $r = 0, 1, \ldots$, of the function $f(X)$ in the space E^{r+2}. According to (16), the value of the weight function increases at each iteration (14) proportionally to the absolute value of the approximation error

$$\mu_r(X) = |f(X) - F_{m,r}(a; X)| \tag{17}$$

of the function $f(X)$ by the expression $F_{m,r}(a; X)$ obtained at the previous iteration.

Since the greatest proportional increase of the weight function (16) corresponds to point X $(X \in \Omega)$ with the greatest deviation (17), application of such adjustment of the value of the weight function for iterations (14) successively reduces a decrease in the approximation error of function $f(X)$ on the point set X $(X \in \Omega)$

$$\hat{\mu}_0 > \hat{\mu}_1 > \ldots > \hat{\mu}_r, \tag{18}$$

where

$$\hat{\mu}_r = \max_{X \in \Omega} \mu_r(X). \tag{19}$$

Thus, application of the weight function (16), which proportionally increases at each iteration of (14) by the absolute value of the error (17) of modeling of the value of function $f(X)$ causes successive reduction of the error of its modeling (19) by the approximation $F_{m,r}(a; X)$. Successive reduction of the error of modeling of the values of function $f(X)$ as a result of each subsequent iteration (14) with the weight function (16) substantiates convergence of the iterations (14), (16).

End of iterations (14) can be controlled by attaining some prescribed accuracy ε:

$$|\hat{\mu}_{r-1} - \hat{\mu}_r| \le \varepsilon \hat{\mu}_r. \tag{20}$$

Using the absolute value on the left-hand side of condition (20) is caused by possible round-off errors in calculation of the approximation errors $\mu_r(X)$. Noteworthy is that accumulation of round-off errors is not typical for the method (14), (16). The round-off errors obtained in the solution of problem (14) are only typical for it.

Then the obtained approximation is corrected with the use of a symmetrizing correction

$$\bar{a}_0 = (\mu_{\max} + \mu_{\min}) / 2, \tag{21}$$

where $\mu_{\max} = \max_{X \in \Omega} (f(X) - F_{m,r}(a; X))$ and $\mu_{\min} = \min_{X \in \Omega} (f(X) - F_{m,r}(a; X))$. As a result, the required approximation of the continuous function $f(X)$ defined on the point set $X \in \Omega$ by the generalized polynomial (1) will be defined as follows:

$$F_m(a; X) = F_{m,r}(a; X) + \bar{a}_0. \tag{22}$$

This method reminds the Remez scheme [12] for obtaining the Chebyshev approximation of a function of one variable according to which the approximation error after each iteration decreases due to introduction of a point with the greatest deviation to the alternance.

Thus, successive adjustment of the values of weight function (16) with regard for the errors of modeling of the values of function $f(X)$ by the results of all previous approximations by the least squares method ensures convergence of the iterative scheme (14), (16) and respectively convergence of the method of calculation of the Chebyshev approximation. By specifying the value of ε in (20), we can attain the required calculation accuracy for the parameters of the Chebyshev approximation of function $f(X)$.

DETERMINING THE PARAMETERS OF CHEBYSHEV APPROXIMATION OF THE FUNCTION OF SEVERAL VARIABLES WITH A RELATIVE ERROR

If a continuous function $f(X)$ on a point set Ω does not take zero values, then a similar method allows obtaining the Chebyshev approximation of $f(X)$ with a relative error. To generate the Chebyshev approximation of the function $f(X)$ with a relative error, we will use the least squares method (14) with the weight function

$$\rho_0(X) = \frac{1}{f^2(X)}, \quad \rho_r(X) = \prod_{i=1}^{r} |\Theta_i(X)|, \quad r = 1, \ldots, p-2, \quad p = 3, 4, \ldots, \tag{23}$$

where

$$\Theta_k(X) = \frac{f(X) - F_{m,k-1}(a; X)}{f(X)}, \quad k = \overline{1, r}, \tag{24}$$

and $F_{m,k}(a; X)$ is approximation of function $f(X)$ by the least squares method with the weight function $\rho_k(X)$.

When generating an approximation with a relative error, we can control the end of iterations (14) with the weight function (23) by attaining some prescribed accuracy ε according to the condition (20), where

$$\hat{\mu}_r(X) = \max_{X \in \Omega} |\Theta_{r+1}(X)|, \tag{25}$$

where $\Theta_{r+1}(X)$ is the error of modeling the function $f(X)$ by the expression $F_{m,r}(a; X)$ obtained by the least squares method (14) at the rth iteration.

Then we adjust the obtained approximation $F_{m,r}(a; X)$ with the relative error with the use of the symmetrizing correction

$$b = \frac{2 f(X_{\max}) f(X_{\min})}{F_{m,r}(a; X_{\min}) f(X_{\max}) + F_{m,r}(a; X_{\max}) f(X_{\min})}, \tag{26}$$

where $X_{\max}$ is a point at which relative approximation error $\Theta_{r+1}(X)$ (24) attains the greatest value on the point set $X \in \Omega$ and $X_{\min}$ is the point at which the relative error is the least. As a result, the required approximation of the continuous function $f(X)$ defined on the point set $X \in \Omega$ by the generalized polynomial (1) with the relative error can be found by the formula

$$F_m(a; X) = b F_{m,r}(a; X). \tag{27}$$

The value of the correction b (26) is defined as a solution of the one-parameter problem of the Chebyshev approximation of function $f(X)$ by the expression $bF_{m,r}(a; X)$ on the point set $X \in \Omega$ with the relative error

$$\max_{X \in \Omega} \left| \frac{f(X) - bF_{m,r}(a; X)}{f(X)} \right| \xrightarrow[b]{} \min. \tag{28}$$

The results of computation of the parameters of the Chebyshev approximation for test examples confirm good convergence of the iteration process (14) with the weight functions (16) and (23) in case of approximation of functions of one, two, and three variables. In particular, in the solution of test examples for the functions defined on a set of 121 points, coincidence of two to three significant digits in the approximation error was attained for $\varepsilon = 0.003$ in six to nine iterations (14) for both weight function (16) for absolute error and weight function (23) for relative error.

Example 1. Let us find the Chebyshev approximation by a quadratic polynomial of the function of one variable $y(x) = \sqrt{1 + 2x + 3x^2}$ defined at points x_i, $i = \overline{0, 20}$, where $x_i = 0.1i$.

With the use of the proposed method for $\varepsilon = 0.003$ in the condition (20) in eight iterations (14) with the weight function (16) for the function $y(x)$ the polynomial

$$P_2(x) = 1.006720776 + 0.7076428791x + 0.7445228708x^2, \tag{29}$$

was obtained; with regard for the correction $\bar{a}_0 = 0.0000619405$, it provides the absolute approximation error 0.015613905. In the calculation of the Chebyshev approximation of function $y(x)$, the approximation error at iterations (14) attained the following values:

0.021899282, 0.0163219491, 0.016111283, 0.016045417, 0.015902523, 0.015792476, 0.015720242, 0.015675845.

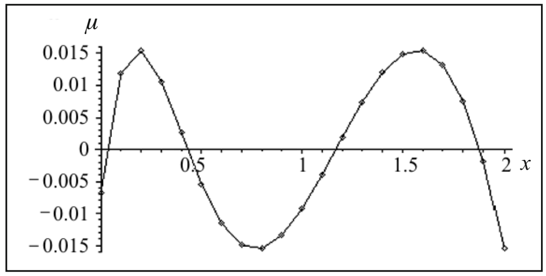

Fig. 1. The curve of the error of approximation of function $y(x)$ by polynomial (29).

Chebyshev approximation of functions $y(x)$ by the quadratic polynomial obtained by the Remez iteration scheme [12] with adjustment of alternance points by the Vallee–Poussin algorithm ensures the approximation error 0.01544. The error of approximation by polynomial (29) is by 0.0001739 greater than the error of the Chebyshev approximation obtained by the Remez scheme. The error of approximation by the polynomial (29) 1.12% exceeds the error of the Chebyshev approximation obtained by the Remez scheme.

The curve of approximation error (29) is presented in Fig. 1.

The approximation error curve presented in the figure corresponds to the characteristic property of the Chebyshev approximation: it has four extremum points where the absolute value of the deviation is the largest (within the given accuracy) and the deviation sign at these points alternates. These extremum points coincide with the points of alternance obtained for the approximation by the Remez scheme [12].

The least value of the error of approximation of function $y(x)$ by a quadratic polynomial with the use of the proposed method was attained at the 117th iteration (14) with the weight function (16) and was equal to 0.01544.

Approximation of function $y(x)$ by a quadratic polynomial with a relative error with the use of the iteration method (14), (23) for $\varepsilon = 0.003$ was obtained in seven iterations. Polynomial

$$P_2(x) = 1.008795347 + 0.7207929316x + 0.7265246023x^2 \tag{30}$$

ensures relative approximation error 0.94999% with the correction $b = 0.9999505612$.

Chebyshev approximation of function $y(x)$ by a quadratic polynomial with relative error obtained by the Remez iteration scheme [12] with adjustment of alternation points by the Vallee–Poussin algorithm provides the 0.9337% approximation error. The approximation error (30) is 0.0163% greater than the Chebyshev approximation error obtained by the Remez scheme, which is 1.75% of the Chebyshev approximation error.

Example 2. Find the Chebyshev approximation of the function $z_1(x, y) = \sqrt{x^2 + y^2}$ specified at points (x_i, y_j), $i = \overline{0,10}$, $j = \overline{0,10}$, where $x_i = 0.1i$, $y_j = 0.1j$, by a quadratic polynomial with respect for variables x and y.

With the use of the proposed method for the function $z_1(x, y)$ in seven iterations (14) with the weighting function (16), condition (20) was satisfied for $\varepsilon = 0.003$. The obtained polynomial

$$\begin{aligned} P_{2,2}(x, y) = {} & 0.03161824134 + 0.7318695249x + 0.7318695249y \\ & - 0.6459607105xy + 0.2640058033x^2 + 0.2640058033y^2 \end{aligned} \tag{31}$$

provides the absolute approximation error of function $z_1(x, y)$ equal to 0.036805375, with the correction $\overline{a}_0 = -0.00041013265$.

Figure 2 shows the surface view of the approximation error (31).

The surface shown in Fig. 2 confirms that the characteristic property of the Chebyshev approximation is satisfied, i.e., the sign of the deviations with the largest absolute value alternate. It also follows from this figure that there are five points at which the approximation error takes the largest value. This number of extremum points corresponds to the solved problem. Since the function $z_1(x, y)$ is symmetric with respect to the arguments x and y, its approximation by a quadratic polynomial must also be symmetric, i.e., this polynomial can be represented as

$$P_{2,2}(x, y) = a + b(x + y) + c(x^2 + y^2) + dxy. \tag{32}$$

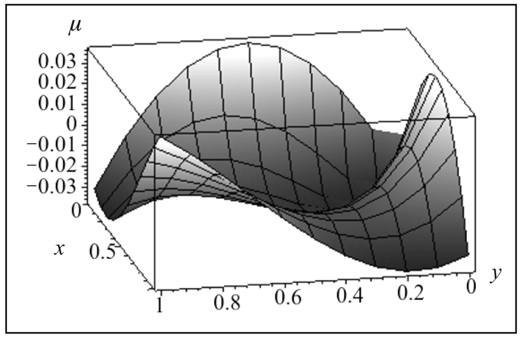

Fig. 2. The surface of the error of approximation of function $z_1(x, y)$ by polynomial (31).

According to the characteristic property of the Chebyshev approximation, approximation by the polynomial (32) with four unknown parameters should be characterized by the presence of five points with the largest absolute value of the deviation. In approximation (31), the values of the coefficients of the same powers of variables x and y almost coincide. The value of the error of approximation of function $z_1(x, y)$ by the method (14), (16) in the form of polynomial (32) coincided with the value of the error of approximation by polynomial (31).

The values of positive and negative deviations of approximation (31) with the largest absolute values from the values of function $z_1(x, y)$ coincide within the given accuracy. This is proved, in particular, by the relatively small value of the correction $\bar{a}_0 = -0.00041013265$, which is 1.09% of the obtained absolute approximation error (31).

Example 2 is taken from [9], where to obtain the Chebyshev approximation of the function $z_1(x, y)$ by a quadratic polynomial with respect to the variables x and y, the α-algorithm is used with regard for the possibilities of effective error reduction. The absolute error of approximation of function $z_1(x, y)$ by a quadratic polynomial obtained in [9] was 0.036351. The theoretical value (obtained in [9]) of the absolute error of the Chebyshev approximation of the function $z_1(x, y)$ by a quadratic polynomial is 0.036310. Therefore, the approximation error (31) exceeds by 0.000495375 the theoretically obtained error value, which is 1.36% of the theoretically obtained value of the error of approximation of the function $z_1(x, y)$ by a quadratic polynomial.

The least value of the absolute error of approximation of the function $z_1(x, y)$ by a quadratic polynomial by the method (14), (16) was obtained at the 55th iteration. The obtained polynomial

$$P^*_{2,2}(x, y) = 0.03393618707 + 0.7230553918x + 0.7230553918y$$
$$- 0.6483527459xy + 0.2731844281x^2 + 0.2731844281y^2 \tag{33}$$

provides the absolute error of approximation 0.03624001. The correction is $\bar{a}_0 = -0.000026703$. Obtaining the approximation error (33) less than the theoretically calculated error value in [9] can be explained by the fact that the calculated value of the Chebyshev approximation error in [9] corresponds to the approximation of the analytically given function $z_1(x, y)$.

Example 3. Find the Chebyshev approximation of function $z_2(x, y, t) = e^{-xyt}$ defined at points (x_i, y_j, t_r), $i = \overline{0, 10}$, $j = \overline{0, 10}$, $r = \overline{0, 10}$, where $x_i = 0.1i$, $y_j = 0.1j$, and $t_r = 0.1r$, by a first-order polynomial for each variable x, y, and t.

Using the proposed method (14), (16) for $\varepsilon = 0.003$, in seven iterations we have obtained approximation of the function $z_2(x, y, t)$ by the polynomial

$$P_{3,1}(x, y, t) = 0.9787294020 - 0.01152161174x - 0.01152135336y + 0.005123721063xy - 0.01152121170t$$
$$+ 0.005123542576tx + 0.005123241453yt - 0.6312148958xyt, \tag{34}$$

which provides absolute approximation error 0.0395586 with the correction $\bar{a}_0 = -0.00200142015$.

The Chebyshev approximation of function $z_2(x, y, t)$ by the first-degree polynomial with respect to each variable x, y, and t with the relative error with the use of iterations (14) with weight function (23) for $\varepsilon = 0.003$ was obtained in eight iterations. The polynomial

$$P_{3,1}(x, y, t) = 0.9974817898 - 0.002695606042x - 0.002696040932y + 0.002240020262xy - 0.002694776433y$$
$$+ 0.002238596308xt + 0.002239105684yt - 0.2594474214xyt, \qquad (35)$$

provides the relative approximation error 0.567% with the correction $b = 0.9998333224$.

CONCLUSIONS

The method proposed for constructing the Chebyshev approximation of continuous tabular functions of several variables makes it possible to calculate the approximation by the generalized polynomial (1) with the required accuracy. The method is reliable, efficient, and simple to implement. The results of solution of the test examples confirm rather fast convergence of the proposed method when constructing the Chebyshev approximation with absolute and relative errors for functions of one, two, and three variables.

It is expedient to use this method for the high-accuracy calibration of measuring devices for physical values that depend on several information signals, in particular, pressure gauges, thermoanemometric fluid flow rate meters, etc.

REFERENCES

1. V. A. Yatsuk and P. S. Malachivskyy, Methods to Improve Measurement Accuracy [in Ukrainian], Beskid Bit, Lviv (2008).
2. T. Bubela, P. Malachivskyy, Y. Pokhodylo, M. Mykyychuk, and O. Vorobets, "Mathematical modeling of soil acidity by the admittance parameters," Eastern-European J. of Enterprise Technologies, Vol. 6, No. 10 (84), 4–9 (2016).
3. L. Collatz and W. Krabs, Approximation Theory. Chebyshev Approximations and their Applications [Russian translation], Nauka, Moscow (1978).
4. P. S. Malachivskyy, Y. N. Matviychuk, Ya. V. Pizyur, and R. P. Malachivskyi, "Uniform approximation of functions of two variables," Cybern. Syst. Analysis, Vol. 53, No. 3, 426–431 (2017).
5. A. O. Kalenchuk-Porkhanova and L. P. Vakal, "Constructing the best uniform approximations of functions of several variables," Comp. Zasoby, Merezhi ta Systemy, No. 6, 141–148 (2007).
6. A. A Kalenchuk-Porkhanova, "Best Chebyshev approximation of functions of one and many variables," Cybern. Syst. Analysis, Vol. 45, No. 6, 988–996 (2009).
7. A. A. Kalenchuk-Porkhanova and L. P. Vakal, "A software package for function approximation," Comp. Zasoby, Merezhi ta Systemy, No. 7, 32–38 (2008).
8. P. S. Malachivskyy, Ya. V. Pizyur, and R. P. Malachivskyi, "Calculating the Chebyshev approximation of functions of several variables," in: Computational Methods and Systems of Information Transformation, Proc. of the 5th Sci.-Tech. Conf., Lviv, October 4–5, 2018, Lviv, Institute of Physics and Mathematics of NASU, No. 5, 35–38 (2018).
9. E. Ya. Remez, Fundamentals of the Numerical Methods of Chebyshev Approximation [in Russian], Naukova Dumka, Kyiv (1969).
10. L. V. Petrak, "A program to construct an approximating polynomial for a function of several variables," Optimization Programs (Approximation of Functions), UNTs Acad. Sci. USSR, Sverdlovsk, Iss. 6, 145–157 (1975).
11. P. S. Malachivskyy, B. R. Montsibovich, Ya. V. Pizyur, and R. P. Malachivskyi, "An algorithm for uniform approximation of functions of several variables," Mathem. and Computer Modeling, Ser. Phys. Math. Sci., Vol. 15, 106–112 (2017).
12. P. S. Malachivskyy and V. V. Skopetsky, Continuous and Smooth Minimax Spline Approximation [in Russian], Naukova Dumka, Kyiv (2013).