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MULTICRITERIA DECISION MAKING BASED ON A SET OF OPTIMIZATION METHODS

M. M. Potiomkin[1†] **and I. Y. Svida**[1‡]

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Abstract. *The authors provide an approach to multicriteria decision making in the analysis of complex systems. It is based on the results obtained from a number of methods (ranking as well as forming of the kernel), which increases the substantiation of the results due to increasing the completeness of alternatives comparison.*

Keywords: *alternative, multicriteria decision making, multicriteria method, completeness of comparison, preferred range, complex system, kernel of alternatives.*

INTRODUCTION

Methods of multicriteria decision making based on comparing of alternatives are widely used in the analysis of complex systems.

A typical scheme of multicriteria decision-making has the following form: determining input data → generating variants (alternatives) → for each variant, determining the values of all parameters used to assess the system under study → deleting the variants for which criterion constraints (if any) are vioalted → deleting Pareto-nonoptimal variants (if any) → comparing the remaining variants by the selected multicriteria method → choosing the best variant according to the method being applied (ranking of alternatives, etc., depending on the type of problem being solved).

At the same time, according to [1], the tendency of complex use of multicriteria methods is observed, as well as development of new methods, which combine the features of several methods that were applied separately earlier. This tendency is implemented, for example, in the WASPAS [2] and MOORA [3] methods.

However, if several multicriteria methods are applied, there occurs an issue related to comparing the results obtained by different methods for one decision-making problem. This is demonstrated in [4] on the example of comparing the results obtained for one problem by six methods. In the comparison of 11 variants, the alternative that was ranked the first by the PROMETHEE II method became the third by the ELECTRE II method and the fifth by the MAI method.

To assess the consistency of the results obtained by two multicriteria methods, the study [5] recommends to use a correlation coefficient, and [6] proposes two additional approaches: to estimate correlation by the Kendall's coefficient of concordance or to assess the consistency of the first (the best) three terms of the series of alternatives arranged according to the results obtained by different methods. It is clear that the results are consistent the best when the first three alternatives in all the series are identical and their ranks in all the series are identical as well.

Applying these recommendations allows us to establish the fact of convergence (or divergence) of the results obtained by different methods; however, recommendations as to the method of choosing the best alternative in case of their divergence are almost absent in special literature.

Thus, we may conclude that within the typical scheme of multicriteria decision-making it is impossible to clarify what one should do when the results obtained by different methods do not coincide.

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[1]Central Scientific and Research Institute of the Armed Forces of Ukraine, Kyiv, Ukraine, [†]*favorite_p@ukr.net*; [‡]*mou.ivan.svyda@gmail.com*. Translated from Kibernetika i Sistemnyi Analiz, No. 4, July–August, 2018, pp. 91–97. Original article submitted June 15, 2017.

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Considering the aforesaid, the task was formulated to develop an approach to multicriteria decision-making on the basis of the set of optimization methods that would generalize the results obtained by all the used methods.

THE APPROACH TO MULTICRITERIA DECISION-MAKING

Initial data for the development of this approach are as follows: the set of *m* parameters used to estimate the system under study and the values of coefficients of their relative importance w_j $(j=1,\ldots,m)$; criterion constraints on the ranges of the parameters; a set of *n* alternatives $(i=1,\ldots,n)$, each being characterized by the values of each parameter (E_{ij} is the value of the *i*th parameter for the *j*th alternative).

The tasks of the approach are to generate the kernel that contains either one best alternative or several incomparable alternatives (if it is impossible to determine the best one) as well as to rank the alternatives.

The proposed approach provides carrying out calculations in several stages.

First Stage. For each alternative, correspondence of the values of all parameters that characterize it to the criterion constraints is checked. Alternatives for which the values of at least one of the parameters do not correspond to these constraints are removed from further consideration.

Second Stage. A Pareto-optimal set of alternatives is generated by their pairwise comparison. Alternatives that are not Pareto-optimal are removed from further consideration.

These stages are traditional. Once they have been implemented, the initial set of alternatives can decrease due to excluding the alternatives that are unpromising for the further analysis.

Third Stage. The list of methods to be used for comparing of alternatives are determined.

This stage can be implemented in two variants, depending on whether consultations with the decision-maker (DM) are possible.

In the first variant, the list the multicriteria optimization methods to be used further is generated based on the results of consultations with the DM.

In the second variant, when consultations with the DM are impossible, all the known (available) methods should be used.

The necessity in such an implementation of the variant of choosing multicriteria optimization methods can be explained by the idea that underlies the principles of problem solution under risk, where the game between active (player) and passive (nature) sides is considered.

Within the framework of the considered approach, a multicriteria optimization expert plays active role, and DM plays passive role.

The variant of carrying out consultations with the DM corresponds to the case where the quantity and list of strategies of the passive side are determined explicitly. If consultations with the DM are impossible, the active side has to consider the whole range of strategies of the nature, each corresponding to one multicriteria optimization method.

Actually, in such approach, the passive side can be considered a certain hypothetical DM who can ensure the necessary completeness of the comparison of alternatives at the expense of using the set of methods defined by it. This allows determining all the possible variants of decision-making. In turn, the DM (who is authorized to choose the best alternative for the system), out of the whole set of possible variants determined by a hypothetical DM, should select the one (as to their judgement) that corresponds the best to the features of the specific situation.

Let us consider a hypothetical situation, where it is necessary to compare two alternatives (*A* and *B*), which are characterized by two parameters (*x* and *y*), which should attain maximum values and are expressed, for example, by utility functions. Figure 1 shows the coordinates of points that characterize the alternatives (*A* and *B*), as well as typical points with respect to which they can be compared. For example, the Pareto principle and ELECTRE I method provide comparing the alternatives with each other (*A* with respect to *B* and vice versa).

Moreover, from Fig. 1 one can see that alternatives can be comparable: to the absolute best standard $P_1(1,1)$; to a relative best standard $P_2(x_B, y_A)$, generated by the best values of available alternatives; to the absolute worst standard $P_4(0,0)$; to a relative worst standard $P_3(x_A, y_B)$; to two standards $P_2(x_B, y_A)$ and $P_3(x_A, y_B)$.

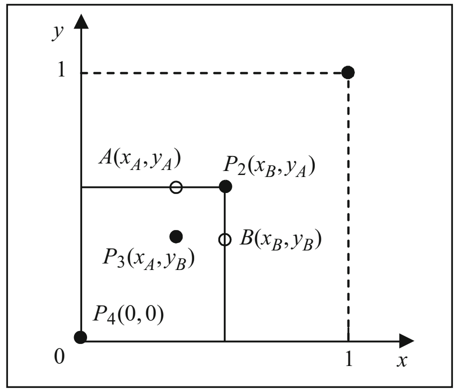

Fig. 1. Layout of typical points with respect to which alternatives can be compared.

Formally, it is also possible to propose other approaches to determining the standards to be used to compare the alternatives.

Thus, the list of decision-making methods being used should be representative enough to ensure the completeness of the comparison of alternatives to reveal the features of their differences.

Fourth Stage. A series of optimization problems are being solved according to certain list of methods, and the result for each method is as follows: for ranking methods these are ranks of alternatives; for kernel generation methods this is composition of kernels.

Fifth Stage. The results obtained by the methods of ranking of alternatives are generalized with the use of the approaches that underlie the principles of data generation for problem solution under risk and the Pareto principle. To this end, a payoff matrix is generated, in which alternatives are considered as strategies of the active side and methods as strategies of the passive side (DM), and the payoff matrix contains the ranks of alternatives.

The Pareto principle can be applied to the obtained matrix; and it is carried out at this stage. The alternative with a smaller rank is assumed to be better. The result of this stage is the kernel of Pareto-optimal alternatives, which are characterized by the best ranks.

Sixth Stage. All the obtained kernels are united: those obtained by the kernel generating methods (at the fourth stage) with ones that were generated based on the Pareto optimization of payoff matrix. If the final kernel includes several alternatives, this testifies about their incomparability with respect to the advantage criteria used in the applied methods.

Seventh Stage. Ranking of alternatives is performed. To this end, the initial payoff matrix (i.e., the matrix before Pareto optimization) is expanded by including strategies that correspond to the kernel generating methods. Since alternatives in the kernel are incomparable, the unit rank is assigned to all the alternatives that appear at the kernel. The other elements are filled with zero value.

For ranking of alternatives, their rank averaged over all the applied methods is used; it can be calculated by the formula

$$r_{ci} = \frac{\sum\limits_{j=1}^{N_p+N_k} r_j}{\sum\limits_{\substack{j=1 \\ r_j \neq 0}}^{N_p+N_k} 1}, \quad i = 1, \ldots, n,$$

where N_p and N_k are the number of the applied ranking and kernel generating methods, respectively; n is the total number of alternatives being compared; r_j is the rank of alternative obtained by the jth method, i.e., average is carried out only by the methods for which the rank of alternative has no zero value.

Eighth Stage. For averaged ranks of alternatives, clusterization procedure is executed, which reveals almost indistinguishable alternatives and excludes the situations where an alternative has an unreasonable advantage over another one.

Ninth Stage. Clustered ranks are used to generate a preferred range of alternatives where better alternative corresponds to a smaller value of the rank.

Tenth (the Last) Stage. Recommendations for the DM are generated, with detailed description of differences between the alternatives revealed as a result of application of formal methods. This stage is completely creative and provides a generalized analysis of differences between alternatives with regard for the results of all the calculations.

If the objective of the study was to find the alternatives that are expedient to be submitted to DM for consideration, then alternatives are only considered that belong to the kernel generated at the sixth stage. In this case, DM's recommendations focus on their separate features, which determined their place in the preferred range generated at the ninth stage.

If the objective of the study was ranking of alternatives, then preferred range of alternatives is submitted to the DM for consideration, and the attention in the recommendations is focused on the alternatives that appeared in the kernel generated at the sixth stage.

EXAMPLE OF APPLYING THE PROPOSED APPROACH

To show the possibility of practical application of the proposed approach, we will use input data from the example from [7], where a rational variant of some system should be chosen based on the known characteristics of $n = 10$ variants of the system. The characteristics of the variants are presented in Table 1, from which we can see that each variant is assessed with respect to $m = 6$ parameters ($E_1 - E_6$), out of which $E_1 - E_3$ need maximization, and the other ones need minimization. Here, all the parameters are assumed to have identical importance.

In addition, note that approaches to determining the best variant are illustrated in [7] with the use of two methods (taxonomy [8] and additive convolution), which have shown contradictory ranking results as to determining the leaders (Table 1). Such results of using different methods are a practical proof of the need for application of the proposed approach.

Since borrowed data are used, we will consider that the variants under consideration correspond to criterion constraints on the values of parameters, i.e., the composition of alternatives will not vary based on the results of the first stage.

According to the second stage, we will exclude from further consideration the alternatives that are not Pareto-optimal, namely, variants #4 and #9, i.e., eight alternatives are subject to further comparison.

In implementing the third stage, we will consider that consultations with the DM are impossible and calculations should be carried out with the use of all known (available) methods.

Since the calculations are illustrative, in addition to the two methods used in [7], we will use only two methods: modified ELECTRE II method [9], which allows generating the kernel of alternatives without involving experts, and the TOPSIS method [10].

At the fourth stage, we will solve optimization problems by means of the ELECTRE II and TOPSIS methods, since the results have been already received in [7] for other methods.

As a result of the fourth stage, we obtain the following: kernel composition for the kernel generating method ELECTRE II (the kernel consists of one alternative #5); a preferred range of alternatives for the TOPSIS method (the preferred range is as follows: ## 5, 3, 1, 7, 6, 2, 10, 8).

According to the fifth stage, we form the payoff matrix of the ranks of alternatives (Table 2).

Applying the Pareto principle to the ranks of alternatives presented in Table 2 testifies that alternatives ## 1, 2, 6, 7, 8 are not Pareto optimal since by all the used methods they have the ranks larger than the ranks of alternative #3. Alternative #10 is not Pareto optimal either: by all the methods it has the ranks that are larger than the ranks of alternative #5.

Thus, based on the set of the applied methods of ranking by ranks, the kernel of Pareto optimal alternatives consists of alternatives #3 and #5.

At the sixth stage, we combine the kernels obtained at the fourth and fifth stages. Respectively, the combined kernel contains two alternatives: #3 and #5.

TABLE 1. Values of Parameters Used to Estimate Variants of the System [7] and Results of the Ranking of Variants

Variant, i	Values of Parameters						Ranks of Variants with Respect to	
	$E_{i1}\uparrow$	$E_{i2}\uparrow$	$E_{i3}\uparrow$	$E_{i4}\downarrow$	$E_{i5}\downarrow$	$E_{i6}\downarrow$	taxonomy	additive convolution
1	0.852	0.903	0.724	0.085	0.216	0.102	3	5
2	0.741	0.935	0.827	0.064	0.177	0.245	6	6
3	0.815	0.839	0.896	0.106	0.118	0.143	1	3
4	0.778	0.806	0.689	0.128	0.255	0.163	9	9
5	0.926	0.742	0.862	0.043	0.098	0.082	2	1
6	0.741	0.871	0.827	0.085	0.137	0.225	4	7
7	0.667	0.903	0.793	0.064	0.235	0.123	8	8
8	0.852	0.839	1.000	0.128	0.275	0.143	5	4
9	0.667	0.806	0.896	0.106	0.294	0.266	10	10
10	0.778	0.903	0.965	0.177	0.059	0.184	7	2

TABLE 2. Payoff Matrix of the Ranks of Alternatives Obtained by the Ranking Methods

No.	Method	Alternative Number							
		1	2	3	5	6	7	8	10
1	Taxonomy	3	6	1	2	4	8	5	7
2	Additive convolution	5	6	3	1	7	8	4	2
3	TOPSIS	3	6	2	1	5	4	8	7

TABLE 3. Augmented Matrix of the Ranks of Alternatives

No.	Method	Alternative Number							
		1	2	3	5	6	7	8	10
1	Taxonomy	3	6	1	2	4	8	5	7
2	Additive convolution	5	6	3	1	7	8	4	2
3	TOPSIS	3	6	2	1	5	4	8	7
4	ELECTRE II	0	0	0	1	0	0	0	0
Sum of the ranks		11	18	6	5	16	20	17	16
The number of methods based on which the alternative has received a nonzero rank		3	3	3	4	3	3	3	3
Average rank		3.67	6	2	1.25	5.33	6.67	5.67	5.33
Ranks of clustered alternatives		3	6	2	1	4	7	5	4

At the seventh stage, in order to perform further ranking of alternatives, an augmented matrix is generated, which will contain the results obtained by all the applied methods (Table 3). Handling these data yields the ranks of alternatives averaged over all the applied methods.

According to the eighth stage, average ranks of alternatives are analyzed as to the possibility of their clusterization. The value of average ranks presented in Table 3 testify that alternatives #6 and #10 belong to one cluster.

At the ninth stage, we generate a preferred range of alternatives with regard for the results of their clusterization. It has the following form: ## 5, 3, 1, (6, 10), 8, 2, 7.

At the tenth stage, we formulate recommendations for the DM. With regard for the composition of the combined kernel obtained at the sixth stage, it is expedient to give only two alternatives, #3 and #5, for DM's consideration. Note that alternative #5 is much better, which follows from its substantial advantage over alternative #3 with respect to average rank due to the advantage given to it by three methods out of the four used ones.

However, more often the DM needs not the analysis of the advantages of alternatives with respect to the parameters obtained in the calculations by optimization methods (membership in the kernel, average ranks, etc.) but describing their differences with respect to the initial parameters. Then the analysis of data presented in Table 1 testifies that alternative #3 dominates over alternative #5 with respect to only two parameters (E_2 and E_3) and can compete with it only if parameters E_2 and E_3 are much more important for the DM as compared with the other parameters. Otherwise, it is expedient to consider alternative #5 the best.

CONCLUSIONS

Unlike the well-known approaches, the proposed approach is oriented to multicriteria decision-making with the use of the set of optimization methods, which increases the completeness of the comparison of alternatives and validity of the obtained results.

The further studies will determine the set of methods that are expedient to be applied in multicriteria decision-making within the framework of the proposed approach.

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