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PROBLEMS OF GROUP PURSUIT WITH INTEGRAL CONSTRAINTS ON CONTROLS OF THE PLAYERS. I

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Abstract. *The paper studies problems of group pursuit for linear differential games with integral constraints. The problems are analyzed on the basis of Chikrii's method of resolving functions. The proposed method substantiates the parallel approach strategy, i.e., the Π-strategy. The new sufficient solvability conditions are obtained for problems of group pursuit. As an example, two classes of problems are considered, namely, the Pontryagin control example and a group pursuit with a simple motion for the case of "l-catch."*

Keywords: *problem of group pursuit, integral constraint, resolving function, strategy, guaranteed time of pursuit.*

1. PROBLEM STATEMENT

Consider a linear differential game in a finite-dimensional Euclidean space, described by the system of equations

$$\dot{z}_i = A_i z_i + B_i u_i - C_i v, \ z_i(0) = z_i^0, \tag{1}$$

where $z_i \in R^{n_i}, u_i \in R^{p_i}, v \in R^q$; $n_i \ge 1$, $p_i \ge 1$, $q \ge 1$, $i = \overline{1, m}$ is the set of integer numbers from 1 to m; A_i, B_i, and C_i are constant rectangular $n_i \times n_i, n_i \times p_i$, and $n_i \times q$ matrices, respectively; z_i^0 is the initial state of the ith object; u_i is the control parameter of the ith pursuer; v is the control parameter of the evader. The realizations of the parameters $u_i, i = \overline{1, m}$, and v at the end of the game should be measurable functions from the class $L_p[0, T]$, $p > 1$, and satisfy the constraints

$$\int_0^T |u_i(\tau)|^p \, d\tau \le \rho_i, \ \rho_i > 0, \ i = \overline{1, m}, \tag{2}$$

$$\int_0^T |v(\tau)|^p \, d\tau \le \sigma, \ \sigma \ge 0, \tag{3}$$

respectively, where $T > 0$ (the case $T = +\infty$ is not excluded). In what follows, we will call such controls admissible and will denote their sets by U_T^i, $i = \overline{1, m}$, and V_T, respectively.

The terminal set consists of the union of sets $M_1, M_2, \ldots, M_m$, each having the form $M_i = M_i^0 + M_i^0$, where M_i^0 is a linear subspace from R^{n_i} and M_i^1 is a convex compact subset of the orthogonal complement L_i to the subspace M_i^0 in R^{n_i}.

Definition 1. In game (1)–(3) the set of mappings $u_i\colon [0, T] \times V_T \Rightarrow U_T^i$, $i = \overline{1, m}$, is called a strategy of the group of pursuers if the following conditions are satisfied:

(i) admissibility: for each $v(\cdot) \in V_T$ the inclusion $u_i(\cdot) = u_i(\cdot, v(\cdot)) \in U_T^i$, $i = \overline{1, m}$, holds;

(ii) the property of being Volterrian: if for any $t \in [0, T], v^1(\cdot), v^2(\cdot) \in V_T$ the equality $v^1(\tau) = v^2(\tau)$ holds for almost all $\tau \in [0, t]$, then $u_i^1(\tau) = u_i^2(\tau)$ almost everywhere on $[0, t]$, where $u_i^1(\cdot) = u_i(\cdot, v^1(\cdot))$, $u_i^2(\cdot) = u_i(\cdot, v^2(\cdot))$.

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Definition 2. In game (1)–(3) from the initial position $z^0 = \{z_1^0, z_2^0, \ldots, z_m^0\}$ it is possible to complete the pursuit in time $T = T(z^0)$ if at least for one value of i, $i = \overline{1, m}$, the absolute continuous solution $z_i(t)$ of the Cauchy problem $\dot{z}_i = A_i z_i + B_i u_i(t, \upsilon(t)) - C_i \upsilon(t)$, $z_i(0) = z_i^0$, belongs to the terminal set M_i in time not exceeding $T = T(z^0)$, i.e., $z_i(t^*) \in M_i$ for some $t^* \in [0, T]$. The number $T(z^0)$ is called guaranteed time of pursuit.

Finding the initial positions from which the pursuit can be ended in a finite time is one of the pursuit problems. The pursuit problem has been sufficiently studied. First of all, noteworthy are the studies [1–11], whose methods and results are generalized and developed in [12–19], etc.

Differential games with integral and multi-type constraints on controls of the players have been intensively investigated [20–36], etc.

In the paper, we will analyze the problem of group pursuit in linear differential games with integral constraints. We will be based on Pontryagin's formalization [1, 2] and use Chikrii's method of resolving functions [8]. Note that the class of strategies introduced above for the group of pursuers is the widest from the point of view of awareness level. Actually, in the paper we will develop a specific strategy, which uses a much smaller amount of current information. Namely, the time interval $[0, T]$ consists of active and passive parts for the pursuers. On each active section, the appropriate pursuer applies actually the stroboscopic strategy $u_i(t, \upsilon)$, where the so-called resolving function plays the dominant role; in the passive part it is equated to zero. To determine the time of passage from the active section to the passive one, the pursuer needs the information about the control of the evader on the time interval $[0, t]$, i.e., about the function $\upsilon_t(\cdot) = \{\upsilon(\tau) : 0 \le \tau \le t\}$ for each current value of time t.

The present part of the study consists of five sections. Section 2 proposes a general pursuit scheme where the problem is divided into two substantially different cases depending on the parameters of system (1) and on the parameters of constraints (2) and (3). In Section 3, the resolving function is defined and its properties are analyzed for each case. These functions are used in Secs. 4 and 5 to prove Theorems 1 and 2 about the possibility of terminating the pursuit. The proof of these theorems employs the ideas from [8, 22–24, 34]. In the second part of the study we will apply the proposed solution technique to Pontryagin's control example [1, 2] and the problem of group pursuit in simple motion of players for the case of *l*-catch.

2. MAIN ASSUMPTIONS

In what follows, index i runs from 1 to m by default. Moreover, the statement of the problems includes a fixed parameter p, $p > 1$, which is omitted in the notation.

Let π_i be the operator of orthogonal projection from R^{n_i} onto the subspace L_i. Consider the linear mappings $\pi_i e^{A_i t} B_i R^{p_i} \to L_i$ and $\pi_i e^{A_i t} C_i R^q \to L_i$ for $t \ge 0$.

Assumption 1. The equation $\pi_i e^{A_i t} B_i F = \pi_i e^{A_i t} C_i$ has a solution $F = F_i(t)$, which is a continuous and nonsingular matrix for all t, $t \ge 0$.

Let us use the matrix $F_i(\cdot)$ to construct the function

$$\chi_i(t, s) = \sup_{\upsilon(\cdot) \in V_1[s, t]} \int_s^t |F_i(t - \tau)\upsilon(\tau)|^p \, d\tau, \ 0 \le s \le t,$$

where $V_1[s, t] = \left\{ \upsilon(\cdot) : \int_s^t |\upsilon(\tau)|^p \, d\tau \le 1 \right\}$. The quantity $\mu_i = \sup_{0 \le s \le t < \infty} \chi_i(t, s)$ is called the Nikol'skii coefficient [23]. It is easy to verify that $\mu_i > 0$; however, the case $\mu_i = +\infty$ is not excluded.

Assumption 2. The inequality is true

$$\frac{\rho_1}{\mu_1} + \frac{\rho_2}{\mu_2} + \ldots + \frac{\rho_m}{\mu_m} > \sigma. \tag{4}$$

If inequality (4) holds, it is clear that all the coefficients μ_i cannot be equal to $+\infty$ simultaneously.

Two cases are possible:

(a) $\rho_i / \mu_i \le \sigma$ for any i;

(b) $\rho_i / \mu_i > \sigma$ for some i.

The pursuit problem is analyzed differently for these cases. In case (a), σ is divided as follows:

$$\sigma = \sigma_1 + \sigma_2 + \ldots + \sigma_m, \tag{5}$$

where $\sigma_i = \frac{\sigma \rho_i}{\mu_i}\left(\frac{\rho_1}{\mu_1} + \frac{\rho_2}{\mu_2} + \ldots + \frac{\rho_m}{\mu_m}\right)^{-1}$. It can be easily seen that $\rho_i > \sigma_i \mu_i$ for all i. In this case, the pursuit strategy is constructed in Sec. 4 (Theorem 1). In case (b), to complete the pursuit it will suffice that only the pursuers for which $\rho_i \ge \sigma \mu_i$ actively participate (Sec. 5, Theorem 2).

3. CONSTRUCTION OF THE RESOLVING FUNCTION

3.1. Resolving Function for Case (a). Let us introduce the multi-valued mapping

$$U_i(t-\tau, v, \lambda) = (|F_i(t-\tau)v|^p + \lambda \delta_i)^{1/p} \pi_i e^{(t-\tau)A_i} B_i S^{p_i} - \pi_i e^{(t-\tau)A_i} C_i v, \tag{6}$$

where $0 \le \tau \le t$, $v \in R^q$, $\lambda \ge 0, \delta_i = \rho_i - \sigma_i \mu_i$, S^{p_i} is a ball of radius 1 centered at zero of the space R^{p_i}.

LEMMA 1. The inclusion $0 \in U_i(t-\tau, v, \lambda)$ holds for all i, t, τ, v, λ such that $0 \le \tau \le t, v \in R^q$ and $\lambda \ge 0$.

Proof. Consider the multi-valued mapping (6) for $\lambda = 0$. Then

$$U_i(t-\tau, v, 0) = |F_i(t-\tau)v| \pi_i e^{(t-\tau)A_i} B_i S^{p_i} - \pi_i e^{(t-\tau)A_i} C_i v.$$

In view of Assumption 1 we get $\pi_i e^{(t-\tau)A_i} C_i v = \pi_i e^{(t-\tau)A_i} B_i F_i(t-\tau)v$, whence

$$U_i(t-\tau, v, 0) = |F_i(t-\tau)v| \pi_i e^{(t-\tau)A_i} B_i S^{p_i} - \pi_i e^{(t-\tau)A_i} B_i F_i(t-\tau)v$$

$$= \begin{cases} |F_i(t-\tau)v| \pi_i e^{(t-\tau)A_i} B_i \left(S^{p_i} - \dfrac{F_i(t-\tau)v}{|F_i(t-\tau)v|} \right) & \text{if } F_i(t-\tau)v \ne 0, \\ \{0\} & \text{if } F_i(t-\tau)v = 0. \end{cases}$$

From $F_i(t-\tau)v \in R^{p_i}$ and $\dfrac{F_i(t-\tau)v}{|F_i(t-\tau)v|} \in S^{p_i}$ it is seen that $0 \in U_i(t-\tau, v, 0)$ for all $0 \le \tau \le t$ and $v \in R^q$.

It is easy to verify that for $\delta_i > 0$ the multi-valued mapping (6) monotonically increases in the parameter $\lambda \ge 0$, i.e., from $\lambda_1 > \lambda_2$ the inclusion $U_i(t-\tau, v, \lambda_2) \subset U_i(t-\tau, v, \lambda_1)$ follows. As a result, we obtain that $0 \in U_i(t-\tau, v, 0) \subset U_i(t-\tau, v, \lambda)$ for all $\lambda \ge 0$, $0 \le \tau \le t$ and $v \in R^q$.

The lemma is proved.

LEMMA 2. If $\pi_i e^{(t-t_i)A_i} z_i \notin M_i^1$, where $z_i \in R^{n_i}, 0 \le t_i \le t$, then the function

$$\lambda_i(t-t_i, t-\tau, v, z_i) = \max\{\lambda \ge 0 : \lambda(M_i^1 - \pi_i e^{(t-t_i)A_i} z_i) \cap U_i(t-\tau, v, \lambda) \ne \varnothing\}, \tag{7}$$

defined as a resolving one for the *i*th pursuer, is upper semicontinuous with respect to the variables τ and v, where $t_i \le \tau \le t$ and $v \in R^q$.

Proof. Let us show the correctness of the definition of function (7). To this end, we will consider the multi-valued mappings $\lambda(M_i^1 - \pi_i e^{(t-t_i)A_i} z_i)$ and $U_i(t-\tau, v, \lambda)$ as the mappings dependent only on λ by fixing the other variables. For brevity, we introduce the notation $K_i(\lambda) = \lambda(M_i^1 - \pi_i e^{(t-t_i)A_i} z_i)$, $U_i(\lambda) = U_i(t-\tau, v, \lambda)$ and $Q_i(\lambda) = K_i(\lambda) \cap U_i(\lambda)$. Let us show that the domain of definition of the multi-valued mapping $Q_i(\lambda)$, i.e., $\operatorname{dom} Q_i = \{\lambda : Q_i(\lambda) \ne \varnothing\}$, is a nonempty compact set.

First, let us show the boundedness of $\mathrm{dom}\,Q_i$. To this end, suppose by contradiction that there exists a sequence $\lambda_n \in \mathrm{dom}\,Q_i$ such that $\lambda_n \to +\infty$ as $n \to \infty$. By Theorem 1.1 from [7] we obtain that $Q_i(\lambda) \neq \varnothing$ if and only if the inequality

$$\min_{|\psi|=1} (W_{U_i(\lambda)}(\psi) + W_{K_i(\lambda)}(-\psi)) \geq 0$$

holds, where $\psi \in L_i$. Hence, from the properties of support function [37] and the specific form of multi-valued mappings $K_i(\lambda)$ and $U_i(\lambda)$ we get

$$(|F_i(t-\tau)v|^p + \lambda\delta_i)^{1/p} W_{\pi_i e^{(t-\tau)A_i} B_i S^{p_i}}(\psi)$$
$$-(\pi_i e^{(t-\tau)A_i} C_i v, \psi) + \lambda W_{-\pi_i e^{(t-t_i)A_i} z_i + M_i^1}(-\psi) \geq 0 \tag{8}$$

for all $\psi, |\psi|=1$. According to Lemma 1, for all ψ, $|\psi|=1$, the inequality

$$(|F_i(t-\tau)v|^p + \lambda\delta_i)^{1/p} W_{\pi_i e^{(t-\tau)A_i} B_i S^{p_i}}(\psi) - (\pi_i e^{(t-\tau)A_i} C_i v, \psi) \geq 0$$

holds. Therefore, if $W_{-\pi_i e^{(t-t_i)A_i} z_i + M_i^1}(-\psi) \geq 0$, then inequality (8) holds for all $\lambda \geq 0$. It remains to consider the case where $W_{-\pi_i e^{(t-t_i)A_i} z_i + M_i^1}(-\psi) < 0$. From the fact that $\pi_i e^{(t-t_i)A_i} z_i \notin M_i^1$ and M_i^1 is a convex compact set, we obtain that the set

$$\Gamma_i = \{\psi \in L_i : |\psi|=1,\ W_{-\pi_i e^{(t-t_i)A_i} z_i + M_i^1}(-\psi) < 0\}$$

is nonempty. Then there exists ψ from Γ_i such that inequality (8) does not hold beginning with some $\lambda > 0$, which contradicts the assumption. Hence, the set $\mathrm{dom}\,Q_i$ is bounded.

It remains to show that $\mathrm{dom}\,Q_i$ is closed. Since the multi-valued mappings $U_i(\lambda)$ and $K_i(\lambda)$ for all $\lambda \geq 0$ are compact-valued and continuous, their support functions $W_{U_i(\lambda)}(\psi)$ and $W_{K_i(\lambda)}(-\psi)$ are also continuous for all $\lambda \geq 0$ and $\psi \in \Gamma_i$ [37]. Therefore, the function $\gamma_i(\lambda) = \min_{|\psi|=1} [W_{U_i(\lambda)}(\psi) + W_{K_i(\lambda)}(-\psi)]$ is also continuous in λ, $\lambda \geq 0$. Hence $\mathrm{dom}\,Q_i = \{\lambda : \gamma_i(\lambda) \geq 0\}$ is closed, which completes the proof of its compactness.

If $\mathrm{dom}\,Q_i$ is a compact set from $[0, +\infty)$, then there exists its greatest element; we will take it as the function $\lambda_i(t-t_i, t-\tau, v, z_i)$ and will show that this function is upper semicontinuous in the variables τ and v, where $t_i \leq \tau \leq t$, $v \in R^q$.

As is known from [8, 17], if a function $g(x, y)$ is continuous on the product of compact sets X and Y, which are subsets of some finite-dimensional Euclidean spaces, then the nonempty multi-valued mapping $N(x) = \{y \in Y : g(x, y) \geq 0\}$ for all $x \in X$ is upper semicontinuous on X.

Hence, the continuity of the function

$$\gamma_i(\lambda, t-t_i, t-\tau, v, z_i) = \min_{|\psi|=1} [W_{U_i(t-\tau, v, \lambda)}(\psi) + W_{\lambda(M_i^1 - \pi_i e^{(t-t_i)A_i} z_i)}(-\psi)]$$

in the variables λ, τ, and v, where $\lambda \geq 0$, $t_i \leq \tau \leq t$, $v \in R^q$, yields that the multi-valued mapping

$$Q_i(t-t_i, t-\tau, v, z_i) = \mathrm{dom}\,Q_i = \{\lambda : \gamma_i(\lambda, t-t_i, t-\tau, v, z_i) \geq 0\}$$

is upper semicontinuous in the variables τ and v. From here and from the fact that $Q_i(t-t_i, t-\tau, v, z_i)$ is a compact-valued mapping, we can easily obtain that the function $\lambda_i(t-t_i, t-\tau, v, z_i) = \max Q_i(t-t_i, t-\tau, v, z_i)$ is also upper semicontinuous in τ and v, where $t_i \leq \tau \leq t$ and $v \in R^q$.

The lemma is proved.

3.2. Resolving Function for Case (b). Let $\rho_i > \mu_i\sigma$ for $i = \overline{1, k_1}$, $\rho_i = \mu_i\sigma$ for $i = \overline{1+k_1, k_2}$, and $\rho_i < \mu_i\sigma$ for $i = \overline{k_2+1, m}$. In this case, as is told in Sec. 2, to solve the pursuit problem, it will suffice that pursuers with the indices from 1 to k_2 participate.

As in case (a), a multi-valued mapping (6) is introduced, but with a modification where the constant $\delta_i = \rho_i - \mu_i\sigma$ for $i = \overline{1, k_1}$ and $\delta_i = 0$ for $i = \overline{k_1+1, k_2}$.

It can be easily verified that Lemma 1 is also true in this case. Therefore, similarly to Lemma 2 we can prove the following statement.

LEMMA 3. If $\pi_i e^{tA_i} z_i^0 \notin M_i^1$, where $z_i^0 \in R^{n_i}, t \ge 0$, then the function

$$\lambda_i(t, t-\tau, \upsilon, z_i^0) = \max\{\lambda \ge 0 : \lambda(M_i^1 - \pi_i e^{tA_i} z_i^0) \cap U_i(t-\tau, \upsilon, \lambda) \ne \varnothing\}$$

defined as resolving one for the ith pursuer, where $i = \overline{1, k_2}$, is upper semicontinuous in the variables τ and υ when $0 \le \tau \le t, \upsilon \in R^q$.

For $i = \overline{1 + k_2, m}$ we assume that $\lambda_i(t, t-\tau, \upsilon, z_i^0) \equiv 0$.

4. THEOREM ON THE POSSIBILITY OF TERMINATING THE PURSUIT FOR CASE (a)

Let Assumptions 1 and 2 be satisfied. For case (a) σ is represented as (5) and for each ith pursuer the appropriate resolving function $\lambda_i(t - t_i, t-\tau, \upsilon, z_i)$, upper semicontinuous in τ and υ, is constructed. Now we use this resolving function to introduce the function

$$\Lambda_i(t, t_i, z_i) = 1 - \inf_{\upsilon(\cdot) \in \upsilon_{\sigma_i}[t_i, t]} \int_{t_i}^{t} \lambda_i(t - t_i, t - \tau, \upsilon(\tau), z_i) d\tau, \ 0 \le t_i \le t,$$

where $V_{\sigma_i}[t_i, t] = \left\{ \upsilon(\cdot) : \int_{t_i}^{t} |\upsilon(\tau)|^p \, d\tau \le \sigma_i \right\}$, t_i is a fixed time, $z_i \in R^{n_i}$. Let $T_i = T_i(t_i, z_i)$ be the first positive root of the equation $\Lambda_i(t, t_i, z_i) = 0$ with respect to t. If there is no such root, we assume that $T_i = \infty$.

Assumption 3 (for case (a)). For each i there exists a continuous mapping $T_i^* : [0, +\infty) \times R^{n_i} \to R^1$ such that $T_i(t_i, z_i) \le T_i^*(t_i, z_i)$ and $T_i^*(t_i, z_i) < \infty$.

THEOREM 1. If Assumptions 1, 2, and 3 are satisfied for the initial position z^0, then in game (1) with constraints (2) and (3) in case (a) the pursuit can be terminated in a finite time $T = T(z^0)$.

We will prove Theorem 1 in four steps.

4.1. The Structure of the Strategy. For fixed t_i and z_i let us introduce the set

$$M_i^1(T_i - \tau, \upsilon) = \{m_i^1 \in M_i^1 : \lambda_i(T_i - t_i, T_i - \tau, \upsilon, z_i)(m_i^1 - \pi_i e^{(T_i - t_i)A_i} z_i)$$
$$\in (|F_i(T_i - \tau)\upsilon|^p + \lambda_i(T_i - t_i, T_i - \tau, \upsilon, z_i)\delta_i)^{1/p} \pi_i e^{(T_i - \tau)} B_i S^{p_i} - \pi_i e^{(T_i - \tau)A_i} C_i \upsilon\},$$

where $t_i \le \tau \le T_i$, $T_i = T_i(z_i, t_i)$ and $i = \overline{1, m}$. From the upper semicontinuity of the function $\lambda_i(T_i - t_i, T_i - \tau, \upsilon, z_i)$ with respect to (τ, υ) it follows that the multi-valued mappings

$$\lambda_i(T_i - t_i, T_i - \tau, \upsilon, z_i)(M_i^1 - \pi_i e^{(T_i - t_i)A_i} z_i),$$

$$(|F_i(T_i - \tau)\upsilon|^p + \lambda_i(T_i - t_i, T_i - \tau, \upsilon, z_i)\delta_i)^{1/p} \pi_i e^{(T_i - \tau)} B_i S^{p_i} - \pi_i e^{(T_i - \tau)A_i} C_i \upsilon$$

are upper semicontinuous with respect to (τ, υ). By Lemma 1.7.5 from [38], the intersection of these sets $M_i^1(T_i - \tau, \upsilon)$ is Borel measurable from (τ, υ). Hence, there exists a single-valued Borel measurable branch $m_i^1(T_i - \tau, \upsilon) \in M_i^1(T_i - \tau, \upsilon)$ (Lemma 1.7.7. [38]). Then the inclusion follows from the definition of resolving function (7)

$$\lambda_i(T_i - t_i, T_i - \tau, \upsilon, z_i)(m_i^1(T_i - \tau, \upsilon) - \pi_i e^{(T_i - t_i)A_i} z_i) + \pi_i e^{(T_i - \tau)A_i} C_i \upsilon$$
$$\in (|F_i(T_i - \tau)\upsilon|^p + \lambda_i(T_i - t_i, T_i - \tau, \upsilon, z_i)\delta_i)^{1/p} \pi_i e^{(T_i - \tau)} B_i S^{p_i}.$$

Since all the conditions of Filippov–Casten Theorem 1.7.10 [38, 39] are satisfied, there exists a Borel measurable single-valued branch $\hat{u}_i(T_i-\tau,v)$ from S^{p_i} such that

$$\lambda_i(T_i-t_i,T_i-\tau,v,z_i)(m_i^1(T_i-\tau,v)-\pi_i e^{(T_i-t_i)A_i}z_i)+\pi_i e^{(T_i-\tau)A_i}C_i v$$
$$=(|F_i(T_i-\tau)v|^p+\lambda_i(T_i-t_i,T_i-\tau,v,z_i)\delta_i)^{1/p}\pi_i e^{(T_i-\tau)}B_i\hat{u}_i(T_i-\tau,v), \tag{9}$$

where $t_i\le\tau\le T_i$, $v\in R^q$. In view of the last equality in (9), it is possible to determine the strategy for the ith pursuer in the form

$$u_i(T_i-\tau,v)=(|F_i(T_i-\tau)v|^p+\lambda_i(T_i-t_i,T_i-\tau,v,z_i)\delta_i)^{1/p}\hat{u}_i(T_i-\tau,v), \tag{10}$$

which is Borel measurable for $t_i\le\tau\le T_i, v\in R^q$.

4.2. An Auxiliary Lemma. Lemma 4. If $\pi_i e^{(T_i-t_i)A_i}z_i^*\notin M_i^1$, $z_i^*\in R^{n_i}$, $0\le t_i\le T_i$, and the inequality $\int_{t_i}^{T_i}|v(\tau)|^p d\tau\le\sigma_i$ holds for control $v=v(\tau)$, $t_i\le\tau\le T_i$, then the pursuit can be terminated from the point z_i^* for the ith pursuer in time T_i-t_i, where $T_i=T_i(t_i,z_i^*)$ is the first positive root of the equation $\Lambda_i(t,t_i,z_i^*)=0$ with respect to t.

Proof. Let us introduce the control function

$$\Lambda_i^*(t,t_i,v(\cdot),z_i^*)=1-\int_{t_i}^{t}\lambda_i(T_i-t_i,T_i-\tau,v(\tau),z_i^*)d\tau,$$

where $v=v(\tau)$, $t_i\le\tau\le t$, is the control of the evader for which the inequality $\int_{t_i}^{t}|v(\tau)|^p d\tau\le\sigma_i$ holds. It is obvious that $\Lambda_i^*(t_i,t_i,v(\cdot),z_i^*)=1$ and the function $\Lambda_i^*(t,t_i,v(\cdot),z_i^*)$ in the variable t, $t_i\le t\le T_i$ is uniformly continuous and monotonically nonincreasing. From here and from Assumption 3 it follows that there exists time $t_i^*, t_i<t_i^*\le T_i$, such that

$$\Lambda_i^*(t_i^*,t_i,v(\cdot),z_i^*)=0 \tag{11}$$

and $\Lambda_i^*(t,t_i,v(\cdot),z_i^*)>0$ for all t, where $t_i\le t<t_i^*$.

Then we prescribe to the ith pursuer in the time interval $[t_i,T_i]$ to implement strategy (10) in the form

$$u_i(T_i-\tau,v(\tau))=(|F_i(T_i-\tau)v(\tau)|^p+\lambda_i^*(T_i-t_i,T_i-\tau,v(\tau),z_i^*)\delta_i)^{1/p}\hat{u}_i(T_i-\tau,v(\tau)), \tag{12}$$

where

$$\lambda_i^*(T_i-t_i,T_i-\tau,v(\tau),z_i^*)=\begin{cases}\lambda_i(T_i-t_i,T_i-\tau,v(\tau),z_i^*) & \text{when } t_i\le t\le t_i^*,\\ 0 & \text{when } t_i^*<t\le T_i.\end{cases}$$

The superposition of a Borel and a Lebesgue measurable functions is known to be Lebesgue measurable [40]. Therefore, the control of the ith pursuer (12) is also Lebesgue measurable for the arbitrary admissible control of the evader $v(\tau)$, $t_i\le\tau\le T_i$.

Let now the evader choose control $v(\cdot)\in V_{\sigma_i}[t_i,T_i]$, and the pursuer implement the strategy corresponding to control (12). Then for the equation

$$\dot{z}_i=A_i z_i+B_i u_i(T_i-\tau,v(\tau))-C_i v(\tau), z_i^*=z_i(t_i)$$

the Cauchy formula

$$z_i(t)=e^{(t-t_i)A_i}z_i^*+\int_{t_i}^{t}e^{(t-\tau)A_i}(B_i u_i(T_i-\tau,v(\tau))-C_i v(\tau))d\tau$$

is true. From here, for the time $T_i=T_i(t_i,z_i^*)$ in view of Eq. (9) and the form of the control of the pursuer (12) we find

$$\pi_i z_i(T_i) = \pi_i e^{(T_i - t_i)A_i} z_i^* + \int_{t_i}^{t_i^*} \lambda_i(T_i - t_i, T_i - \tau, v(\tau), z_i^*)(m_i^1(T_i - \tau, v(\tau)) - \pi_i e^{(T_i - t_i)A_i} z_i^*) d\tau$$

$$= \pi_i e^{(T_i - t_i)A_i} z_i^* \left(1 - \int_{t_i}^{t_i^*} \lambda_i(T_i - t_i, T_i - \tau, v(\tau), z_i^*) d\tau \right) + \int_{t_i}^{t_i^*} \lambda_i(T_i - t_i, T_i - \tau, v(\tau), z_i^*) m_i^1(T_i - \tau, v(\tau)) d\tau.$$

Since $\Lambda_i^*(t_i^*, t_i, v(\cdot), z_i^*) = 0$, from the Satimov lemma [15] we obtain

$$\pi_i z_i(T_i) = \int_{t_i}^{t_i^*} \lambda_i(T_i - t_i, T_i - \tau, v(\tau), z_i^*) m_i^1(T_i - \tau, v(\tau)) d\tau \in M_i^1 \int_{t_i}^{t_i^*} \lambda_i(T_i - t_i, T_i - \tau, v(\tau), z_i^*) d\tau = M_i^1$$

or $z_i(T_i) \in M_i$.

It now remains to show that the selected control $u_i(T_i - \tau, v(\tau))$, $t_i \le \tau \le T_i$, is admissible. By the definition of μ_i we have

$$\int_{t_i}^{T_i} |F_i(T_i - \tau) v(\tau)|^p \, d\tau \le \sigma_i \sup_{v(\cdot) \in V_1[t_i, T_i]} \int_{t_i}^{T_i} |F_i(T_i - \tau) v(\tau)|^p \, d\tau = \sigma_i \chi_i(T_i, t_i) \le \sigma_i \mu_i.$$

Then from (12) by choosing the control of the pursuer and from Eq. (11) we find that

$$\int_{t_i}^{T_i} |u_i(T_i - \tau, v(\tau))|^p \, d\tau \le \int_{t_i}^{T_i} |F_i(T_i - \tau) v(\tau)|^p \, d\tau$$

$$+ \delta_i \int_{t_i}^{t_i^*} \lambda_i(T_i - t_i, T_i - \tau, v(\tau), z_i^*) d\tau \le \sigma_i \mu_i + \delta_i = \rho_i,$$

as was to be shown.

Let $T_i = T_i(t_i, z_i^*)$ be the first instant of time when $\pi_i e^{(T_i - t_i)A_i} z_i^* \in M_i^1$ and $\int_{t_i}^{T_i} |v(\tau)|^p \, d\tau \le \sigma_i$. In this case, for the ith pursuer in the time interval $t_i \le \tau \le T_i$ it will suffice to implement control (12) on the assumption that $\lambda_i^*(T_i - t_i, T_i - \tau, v(\tau), z_i^*) \equiv 0$ on $[t_i, T_i]$. Then it is easy to verify that game (1) will be completed in time $T_i - t_i$ from the point z_i^* by the ith pursuer.

The lemma is proved.

4.3. The Main Part of the Proof. Consider the differential game (1) for $i = 1$:

$$\dot{z}_1 = A_1 z_1 + B_1 u_1 - C_1 v, \ z_1(0) = z_1^0. \tag{13}$$

By Assumption 3, there exists a finite solution $T_1 = T_1(z_1^0)$ of the equation $\Lambda_1(t, t_1, z_1^0) = 0$, where $t_1 = 0$. It is assumed that $\pi_1 e^{tA_1} z_1^0 \notin M_1^1$ for $t \in [0, T_1]$.

Let $v = v(\cdot)$ be an arbitrary admissible function of the evader. Then two cases are possible for the control $v(\tau)$, $0 \le \tau \le T_1$:

(i) $\int_0^{T_1} |v(\tau)|^p \, d\tau \le \sigma_1$; (ii) $\int_0^{T_1} |v(\tau)|^p \, d\tau > \sigma_1$.

In case (i) by Lemma 4 the first pursuer in differential game (13) completes the pursuit in time $T_1 = T_1(z_1^0)$ from the point z_1^0 by applying control (12) for $i = 1$. Hence, game (1) from the point z^0 will also be completed in time $T(z^0) = T_1(z_1^0)$.

In case (ii) there exists time $t_2 < T_1$ such that $\int_0^{t_2} |\upsilon(\tau)|^p \, d\tau = \sigma_1$. Till the moment t_2, the first pursuer can use control (12) for $i = 1$. However, from this point t_2 from the position

$$z_2^* = z_2(t_2) = e^{t_2 A_2} z_2^0 - \int_0^{t_2} e^{(t_2-\tau)A_2} C_2 \upsilon(\tau) d\tau$$

the second pursuer starts constructing the control, for which we assumed till the time t_2 that $u_2(\cdot) \equiv 0$. Then all the procedure of pursuit repeats for the second pursuer, like for the first pursuer. For the control $\upsilon = \upsilon(\tau)$, when $t_2 \le \tau \le T_2$, where $T_2 = T_2(t_2, z_2^*)$ is the first positive root of the equation $\Lambda_2(t, t_2, z_2^*) = 0$, the following two cases are possible:

(i′) $\int_{t_2}^{T_2} |\upsilon(\tau)|^p \, d\tau \le \sigma_2$; (ii′) $\int_{t_2}^{T_2} |\upsilon(\tau)|^p \, d\tau > \sigma_2$.

In case (i′) by Lemma 4 the second pursuer from the point z_2^* completes the pursuit in time $T_2 - t_2$. Then game (1) from the point z^0 is completed in time $T(z^0) = T_2(t_2, z_2^*)$ by the second pursuer.

In case (ii′), there exists a time $t_3 < T_2$ such that $\int_{t_2}^{t_3} |\upsilon(\tau)|^p \, d\tau = \sigma_2$. As above, from the time t_3 from the position $z_3^* = z_3(t_3) = e^{t_3 A_3} z_3^0 - \int_0^{t_3} e^{(t_3-\tau)A_3} C_3 \upsilon(\tau) d\tau$ similarly to the first two pursuers, the third pursuer, for which we assume till the time t_3 that $u_3(\cdot) \equiv 0$, starts to implement the strategy, etc.

Thus, implementing in turn their controls $u_1, u_2, \ldots,_1 u_{m-1}$, the pursuers can complete game (1) from the initial position z^0 till the $m-1$th step in time $T(z^0) = T_i(t_i, z_i^*)$, where $1 \le i \le m-1$ and $z_1^* = z_1^0$. The last statement for $i \ge 3$ can be proved in the same way as for $i = 1, 2$.

If the pursuit is not terminated from the initial position z^0 till the $(m-1)$th step, there will become a time $t_m < T_{m-1}$ such that $\int_{t_{m-1}}^{t_m} |\upsilon(\tau)|^p \, d\tau = \sigma_{m-1}$, and the mth pursuer starts to implement control (12) from the point $z_m^* = z_m(t_m) = e^{t_m A_m} z_m^0 - \int_0^{t_m} e^{(t_m-\tau)A_m} C_m \upsilon(\tau) d\tau$ for $i = m$, and control $u_m(T_m - \tau, \upsilon(\tau))$, $t_m \le \tau \le T_m$, is applied until $\int_{t_m}^{T_m} |\upsilon(\tau)|^p \, d\tau \le \sigma_m$, where $T_m = T_m(t_m, z_m^*)$ is the first positive root of the equation $\Lambda_m(t, t_m, z_m^*) = 0$. Since $\sigma_m = \sigma - (\sigma_1 + \ldots + \sigma_{m-1})$ and it is the last part of the evader's resource, we obtain for arbitrary $\upsilon(\tau), t_m \le \tau \le T_m$, that

$$\int_{t_m}^{T_m} |\upsilon(\tau)|^p \, d\tau \le \sigma - \left(\int_0^{t_2} |\upsilon(\tau)|^p \, d\tau + \int_{t_2}^{t_3} |\upsilon(\tau)|^p \, d\tau + \ldots + \int_{t_{m-1}}^{t_m} |\upsilon(\tau)|^p \, d\tau \right) = \sigma_m .$$

Therefore, in view of Lemma 4, it takes time $T_m - t_m$ for the mth pursuer from the position z_m^* till the time of the pursuit termination. Hence, game (1) with constraints (2), (3) from the point z^0 is also completed in time $T(z^0) = T_m(t_m, z_m^*)$.

Note that the point z_m^* depends on the choice of the admissible control $\upsilon = \upsilon(\tau)$, $0 \le \tau \le t_m$, and thus the function $T_m = T_m(t_m, z_m^*)$ also depends on this control.

4.4. End of the Proof or Boundedness of the Function $T_m(t_m, z_m^*)$. The boundedness of the functions $T_2(t_2, z_2^*), \ldots, T_{m-1}(t_{m-1}, z_{m-1}^*)$ yields the boundedness of $T_m(t_m, z_m^*)$. First, let us show the boundedness of the function $T_2(t_2, z_2^*)$.

By the Cauchy–Bunyakovskii inequality and the matrix inequality $\|e^{At}\| \le e^{\|A\|t}$ for $z_2^* = z_2(t_2, v_{t_2}(\cdot))$, where $v_{t_2}(\cdot) = \{v(\tau): 0 \le \tau \le t_2\}$, we get

$$|z_2^*| \le e^{\|A_2\| t_2}[\,|z_2^0| + \|C_2\| \sqrt[p]{t_2\sigma_2}\,] < e^{\|A_2\|T_1}(|z_2^0| + \|C_2\| \sqrt[p]{T_1\sigma_2}),$$

which shows that the position of z_2^* is bounded. The last inequality is obtained from the condition $t_2 < T_1$, where $T_1 = T_1(z_1^0)$. In view of Assumption 3 there exists function $T_2^*(t_2, z_2^*)$, continuous in (t_2, z_2^*) such that $T_2(t_2, z_2^*) \le T_2^*(t_2, z_2^*)$. For the function $T_2^*(t_2, z_2^*)$ on $[0, T_1] \times Q_2$, where $Q_2 = \{z_2^*: |z_2^*| \le e^{\|A_2\|T_1}(|z_2^0| + \|C_2\| \sqrt[p]{T_1\sigma_2})\}$, there exists the greatest value T_2^*. From here the boundedness of the function $T_2(t_2, z_2^*)$ follows.

The boundedness of $T_3(t_3, z_3^*)$ follows from the boundedness of T_2, etc. Finally, we obtain the boundedness of the function $T_m(t_m, z_m^*)$. Thus, the time of the completion of the differential game (1) $T(z^0)$ is bounded from above.

Theorem 1 is proved.

Remark 1. For each i we assumed that $\pi_i e^{(t-t_i)A_i} z_i^* \notin M_i^1$ for $t \ge t_i$. If now for some $i, i = \overline{1, m}$, there exists the first instant of time $T_i = T_i(t_i, z_i^*)$ such that $\pi_i e^{(T_i - t_i)A_i} z_i^* \in M_i^1$ and $\Lambda_i(t, t_i, z_i^*) \ne 0$ for all t, $t_i \le t < T_i$, then, as is mentioned at the end of the proof of Lemma 4, in this case it is also possible in game (1) to terminate the group pursuit from the position z^0 in time $T(z^0) = T_i(t_i, z_i^*)$ by the ith pursuer.

5. THEOREM ON THE POSSIBILITY OF TERMINATING THE PURSUIT IN CASE (b)

In Sec. 3.2 we have defined the resolving function $\lambda_i(t, t-\tau, v, z_i^0)$, upper semicontinuous in τ, $0 \le \tau \le t$, and $v \in R^q$. Now we will use it to introduce the function

$$\Lambda(t, z^0) = 1 - \inf_{v(\cdot)\in V_\sigma[0,t]} \max_{i=\overline{1,k_2}} \int_0^t \lambda_i(t, t-\tau, v(\tau), z_i^0)\,d\tau,$$

where $V_\sigma[0,t] = \left\{ v(\cdot): \int_0^t |v(\tau)|^p\,d\tau \le \sigma \right\}$, and for $i = \overline{k_2+1, m}$ assume $\lambda_i \equiv 0$. Let $T' = T'(z^0)$ be the first positive root of the equation $\Lambda(t, z^0) = 0$; if there is no such root, we assume $T'(z^0) = +\infty$.

Assumption 4 (for case (b)). Let for the position z^0 there exist a finite time $T' = T'(z^0)$.

THEOREM 2. If Assumptions 1, 2, and 4 are satisfied, then from the position z^0 in game (1) with constraints (2), (3) in case (b) the pursuit can be ended in time $T' = T'(z^0)$.

Proof. Let Assumptions 1, 2, and 4 be satisfied for some position z^0. Let us consider the control function

$$\Lambda^*(T', t, z^0, v(\cdot)) = 1 - \max_{i=\overline{1,k_2}} \int_0^t \lambda_i(T', T'-\tau, v(\tau), z_i^0)\,d\tau,$$

where $v = v(\cdot)$ is an arbitrary admissible control of the evader. It is obvious that $\Lambda^*(T', 0, z^0, v(\cdot)) = 1$ and the function $\Lambda^*(T', t, z^0, v(\cdot))$ is continuous in t, $0 \le t \le T'$. By Assumption 4, there exists an instant of time t^* such that $t^* \le T'$ and $\Lambda^*(T', t^*, z^0, v(\cdot)) = 0$. In this case, $\Lambda^*(T', t, z^0, v(\cdot)) > 0$ for all t, $0 \le t < t^*$.

Now consider the multi-valued mapping

$$M_i^1(T'-\tau, v) = \{m_i^1 \in M_i^1 : \lambda_i(T', T'-\tau, v, z_i^0)(m_i^1 - \pi_i e^{T'A_i} z_i^0)$$
$$\in (|F_i(T'-\tau)v|^p + \lambda_i(T', T'-\tau, v, z_i^0)\delta_i)^{1/p} \pi_i e^{(T'-\tau)A_i} B_i S^{p_i} - \pi_i e^{(T'-\tau)A_i} C_i v\},$$

for fixed z_i^0, $i=\overline{1,k_2}$, and $T'=T'(z^0)$. Like in the proof of Theorem 1, the multi-valued mapping $M_i^1(T'-\tau,\upsilon)$ is measurable in (τ,υ), where $0\le\tau\le T', \upsilon\in R^q$. Hence, there exists a single-valued Borel measurable branch $m_i^1(T'-\tau,\upsilon)\in M_i^1(T'-\tau,\upsilon)$ (Lemma 1.7.7 [38]). By Lemma 3, the inclusion holds

$$\lambda_i(T',T'-\tau,\upsilon,z_i^0)(m_i^1(T'-\tau,\upsilon)-\pi_i e^{T'A_i}z_i^0)+\pi_i e^{(T'-\tau)A_i}C_i\upsilon$$
$$\in(|F_i(T'-\tau)\upsilon|^p+\lambda_i(T',T'-\tau,\upsilon,z_i^0)\delta_i)^{1/p}\pi_i e^{(T'-\tau)A_i}B_i S^{p_i}.$$

Since the functions $\lambda_i(T',T'-\tau,\upsilon,z_i^0)$, $i=\overline{1,k_2}$, and $m_i(\tau,\upsilon)$ are Borel measurable in (τ,υ), by the Filippov–Casten Theorem 1.7.10 [38, 39] the equation

$$\lambda_i(T',T'-\tau,\upsilon,z_i^0)(m_i^1(T'-\tau,\upsilon)-\pi_i e^{T'A_i}z_i^0)+\pi_i e^{(T'-\tau)A_i}C_i\upsilon$$
$$=(|F_i(T'-\tau)\upsilon|^p+\lambda_i(T',T'-\tau,\upsilon,z_i^0)\delta_i)^{1/p}\pi_i e^{(T'-\tau)A_i}B_i\hat{u}_i \tag{14}$$

is uniquely solvable in the class of Borel measurable functions. Denote this solution by $\hat{u}_i(T'-\tau,\upsilon)$, where $\hat{u}_i(T'-\tau,\upsilon)\in S^{p_i}$ for $0\le\tau\le T'$, $\upsilon\in R^q$.

During the game, for the admissible control of the evader $\upsilon(\tau)$, $0\le\tau\le T'$, the pursuers should implement the strategies $u_i(T'-\tau,\upsilon)$, $0\le\tau\le T'$, $i=\overline{1,k_2}$, in the form of Lebesgue measurable controls

$$u_i(T'-\tau,\upsilon(\tau))=(|F_i(T'-\tau)\upsilon(\tau)|^p$$
$$+\lambda_i^*(T',T'-\tau,\upsilon(\tau),z_i^0)\delta_i)^{1/p}\hat{u}_i(T'-\tau,\upsilon), \tag{15}$$

where

$$\lambda_i^*(T',T'-\tau,\upsilon(\tau),z_i^0)=\begin{cases}\lambda_i(T',T'-\tau,\upsilon(\tau),z_i^0) & \text{for } 0\le t\le t^*,\\ 0 & \text{for } t^*<t\le T'.\end{cases}$$

Let us show that the proposed control $u_i(\tau,\upsilon(\tau))$, $0\le\tau\le T'$, allows completing the pursuit for the arbitrary admissible control $\upsilon=\upsilon(\tau)$, $0\le\tau\le T'$, in time $T'=T'(z^0)$. To this end, we consider the Cauchy problem

$$\dot{z}_i=A_i z_i+B_i u_i(T'-\tau,\upsilon(\tau))-C_i\upsilon(\tau),\ z_i(0)=z_i^0,$$

for each $i=\overline{1,k_2}$. Then based on the Cauchy formula we get

$$z_i(T')=e^{T'A_i}z_i^0+\int_0^{T'}e^{(T'-\tau)A_i}[B_i u_i(T'-\tau,\upsilon(\tau))-C_i\upsilon(\tau)]d\tau.$$

From (14) and (15) we find

$$\pi_i z_i(T')=\pi_i e^{T'A_i}z_i^0+\int_0^{T'}\pi_i e^{(T'-\tau)A_i}[B_i u_i(T'-\tau,\upsilon(\tau))-C_i\upsilon(\tau)]d\tau$$
$$=\pi_i e^{T'A_i}z_i^0+\int_0^{t^*}\lambda_i(T',T'-\tau,\upsilon(\tau),z_i^0)(m_i^1(T'-\tau,\upsilon(\tau))-\pi_i e^{T'A_i}z_i^0)d\tau$$
$$=\pi_i e^{T'A_i}z_i^0\left(1-\int_0^{t^*}\lambda_i(T',T'-\tau,\upsilon(\tau),z_i^0)d\tau\right)+\int_0^{t^*}\lambda_i(T',T'-\tau,\upsilon(\tau),z_i^0)m_i^1(T'-\tau,\upsilon(\tau))d\tau.$$

But for t^* there exists i^0 from $\overline{1,k_2}$ such that $\int_0^{t^*}\lambda_{i^0}(T',T'-\tau,\upsilon(\tau),z_{i^0}^0)d\tau=1$. Indeed, if we suppose the opposite, then the inequality $1-\int_0^{t^*}\lambda_i(T',t'-\tau,\upsilon(\tau),z_i^0)d\tau>0$ should hold for all $i=\overline{1,k_2}$, which yields

$$\min_{i=\overline{1,k_2}}\left(1-\int_0^{t^*}\lambda_i(T',T'-\tau,\upsilon(\tau),z_i^0)d\tau\right)=1-\max_{i=\overline{1,k_2}}\int_0^{t^*}\lambda_i(T',T'-\tau,\upsilon(\tau),z_i^0)d\tau>0.$$

However, this contradicts the fact that $\Lambda^*(T', t^*, \upsilon(\cdot), z_i^0) = 0$; therefore, for i^0 we obtain

$$\pi_{i^0} z_{i^0}(T') = \pi_{i^0} e^{TA_{i^0}} z_{i^0}^0 \left(1 - \int_0^{t^*} \lambda_{i^0}(T', T' - \tau, \upsilon(\tau), z_{i^0}^0) d\tau \right)$$

$$+ \int_0^{t^*} \lambda_{i^0}(T', T' - \tau, \upsilon(\tau), z_{i^0}^0) m_{i^0}^1 (T' - \tau, \upsilon(\tau)) d\tau = \int_0^{t^*} \lambda_{i^0}(T', T' - \tau, \upsilon(\tau), z_{i^0}^0) m_{i^0}^1 (T' - \tau, \upsilon(\tau)) d\tau$$

$$\in \int_0^{t^*} \lambda_{i^0}(T', T' - \tau, \upsilon(\tau), z_{i^0}^0) M_{i^0}^1 d\tau = M_{i^0}^1 \int_0^{t^*} \lambda_{i^0}(T', T' - \tau, \upsilon(\tau), z_{i^0}^0) d\tau = M_{i^0}^1$$

or $z_{i^0}(T') \in M_{i^0}$. Thus, game (1) from the initial position z^0 in case (b) is completed in time $T' = T'(z^0)$.

It remains to show the admissibility of the control $u_i = u_i(T' - \tau, \upsilon(\tau))$, $0 \le \tau \le T'$. Since by construction for all $i = \overline{1, k_2}$

$$\int_0^{T'} |u_i(T' - \tau, \upsilon(\tau))|^p d\tau = \int_0^{T'} |F_i(T' - \tau)\upsilon(\tau)|^p d\tau + \delta_i \int_0^{t^*} \lambda_i(T', T' - \tau, \upsilon(\tau), z_i^0) d\tau,$$

the easily checked inequalities

$$\int_0^{T'} |F_i(T' - \tau)\upsilon(\tau)|^p d\tau \le \sigma\mu_i, \quad \int_0^{t^*} \lambda_i(T', T' - \tau, \upsilon(\tau), z_i^0) d\tau \le 1$$

yield the inequality $\int_0^{T'} |u_i(T' - \tau, \upsilon(\tau))|^p d\tau \le \sigma\mu_i + \delta_i$. Since the constant $\delta_i = \rho_i - \sigma\mu_i$ for $i = \overline{1, k_1}$ and $\delta_i = \rho_i - \sigma\mu_i = 0$ for $i = \overline{k_1 + 1, k_2}$, we arrive at the inequality $\int_0^{T'} |u_i(\tau, \upsilon(\tau))|^p \le \rho_i$ for all $i = \overline{1, k_2}$.

Theorem 2 is proved completely.

Remark 2. If for some i^0 and $z_{i^0}^0$ there exists a time $T'_{i^0} = T'(z_{i^0}^0)$ such that $\pi_{i^0} e^{T'_{i^0} A_{i^0}} z_{i^0}^0 \in M_{i^0}^1$ and $\Lambda(t, z_{i^0}^0) \ne 0$ for all $0 \le t \le T'_{i^0}$, the i^0th pursuer, constructing the control $u_{i^0} = u_{i^0}(T'_{i^0} - \tau, \upsilon(\tau))$, $0 \le \tau \le T'(z_{i^0}^0)$, like in (15), assuming that $\lambda_{i^0}^*(T', T'_{i^0} - \tau, \upsilon, z_i^0) \equiv 0$ on $[0, T'(z_{i^0}^0)]$, completes game (1) from the point z^0 in time $T' = T'(z_{i^0}^0) = T'(z^0)$.

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