

## CATASTROPHE RISK MANAGEMENT FOR SUSTAINABLE DEVELOPMENT OF REGIONS UNDER RISKS OF NATURAL DISASTERS

T. Yu. Ermolieva<sup>a</sup> and I. V. Sergienko<sup>b</sup>

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*The paper considers models and approaches to the analysis and decision making under catastrophic risks. It is shown that the design of optimal robust strategies for the flood risk management can be approached as a stochastic spatially explicit optimization problem combining the goals and constraints of various agents such as producers, farmers, individuals, governments, insurers, reinsurers, and investors. The approach is illustrated with a case study on catastrophic flood risks, which shows the importance of an appropriate combination of ex-ante and ex-post structural and financial measures.*

**Keywords:** *catastrophe risk insurance, stochastic optimization, adaptive Monte Carlo method, probabilistic constraints, sustainable development.*

### INTRODUCTION

In the paper, we review the models and the approaches developed for the analysis and decision making on management under catastrophic risks. It is important to take into account the risk of catastrophes because of fast growing losses caused by natural disasters and man-caused catastrophes, which inevitably results in serious social, economic, and environmental problems.

As leading insurance companies evaluated [7], direct losses from natural catastrophes have increased three-fold in the last three decades. The frequency and power of recent hurricanes, earthquakes, storms, wars, and disease outbreaks have revealed a tendency to escalating economic and human losses. The major factor promoting such growth is the lack of specific knowledge and neglect of risks. In particular, economic planning strategies often disregard the direct dependence of losses on land-use practice and on industry, people, and capital arrangement in catastrophe-prone areas. This indicates that the state policy is oriented mainly to the mitigation rather than prevention of natural disasters [23]. The necessity of saving public funds requires reappraising the preventive measures for mitigation and risk reduction. The expediency of protective measures should be justified in view of the economic and social factors.

Solving a versatile challenge such as planning under natural risks requires a strict risk-specific methodology that would account for major factors influencing public safety and security of economic, land-use, and life-activity objects. It is also necessary to develop methods and mathematical models for quantitative assessment and prediction of risk management strategies under incomplete information.

Traditional deterministic models do not consider a 500-year flood (repeating once every 500 years on the average) as an event that may affect the lives of the current and many subsequent generations. However, this event may occur today, next week, or next year. For example, the floods that took place in 2002 in Central Europe were evaluated as 100-, 250-, 500-, and 1000-year events. The most destructive catastrophe caused by a nuclear power plant accident is classed as an event with the risk level  $10^7$  (i.e., may occur once every  $10^7$  years on the average).

Models with truncated uncertainties represented by a finite set of possible outcomes, well known to the whole society, dominate in the classical economic theory. Therefore, their consequences can be assessed and reimbursed by the whole

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<sup>a</sup>International Institute for Applied Systems Analysis, Laxenburg, Austria, [ermol@iiasa.ac.at](mailto:ermol@iiasa.ac.at). <sup>b</sup>V. M. Glushkov Institute of Cybernetics, National Academy of Sciences of Ukraine, Kyiv, Ukraine, [aik@public.icyb.kiev.ua](mailto:aik@public.icyb.kiev.ua). Translated from *Kibernetika i Sistemnyi Analiz*, No. 3, pp. 112–128, May–June 2008. Original article submitted June 11, 2007.

society via financial markets. Catastrophe management under such assumptions does not involve difficulties [3]. The analysis of catastrophic risks and management of related losses require new models to be developed.

Note that the risk insurance theory developed irrespective of fundamental economic ideas [3, 23, 25]. The central problem of this theory deals with modeling the probability distribution of future losses [25] to be used to evaluate the probabilities of ruin, assigning insurance premiums, concluding reinsurance contracts, etc. The fundamental principle of this theory is the assumption that risks occur frequently, are equally distributed, and independent. Road accidents satisfy such assumptions since decisions on the premium rate, insurance compensation, and ruin probability of insurance companies are made based on extensive statistics. Insurance companies can manage frequent independent daily risks by the simple strategy: “the more risks in the company’s portfolio, the better.”

In contrast to traditional risks, catastrophes have complex space-time characteristics. They result in mutually dependent destructions, which, in turn, causes a cascade of insurance claims and compensations whose amount depends on the region of accident, arrangement and volume of property, and population density in this region. Damages and compensations obviously depend also on loss-reducing measures, agreements with investors, reinsurance contracts, etc. All this discredits the traditional principle of the incorporation of risks and the standard theory of extreme values [9, 25].

In each specific region, catastrophes are scarce, i.e., a new catastrophe may be unlike those occurred earlier. Since catastrophes have no similarity, there are no necessary real observations; therefore, control strategies cannot be evaluated using traditional statistical methods based on the law of averages and on the assumption of the independence of claims and losses [9, 25]. In case of catastrophes, the probability of bankruptcy of a company or inefficiency of local government can be reduced not due to the direct incorporation of risks but by accounting correctly for the space-time distributions of risks and their interdependences.

The present paper reviews the approaches to studying catastrophic risks based on the models and methods developed at the International Institute for Applied Systems Analysis (Laxenburg, Austria) together with the V. M. Glushkov Institute of Cybernetics, National Academy of Sciences of Ukraine. In Sec. 1, we discuss the shortcomings of the traditional approaches and deterministic models, and in Sec. 2, we consider the key features of the model developed to analyze catastrophic flood of the Tisza river in Hungary and Ukraine.

New approaches and models are rather general and can be applied to study and manage various risks. Methods of the traditional insurance and finance theories evaluate extreme events in terms of monetary units [9]. The approaches proposed allow studying the events not evaluated in this sense but described by multidimensional spatial distribution laws, i.e., the cases for which the standard theory of extreme events has no adequate solutions.

The basic problem of the models is to adequately account for specific features of catastrophic risks, i.e., their space-time characteristics, probability of serious mutually dependent losses, lack of sufficient historical data on events occurred in a specific place, need for long-term planning and a balanced combination of preventive and mitigation measures, robustness of decisions, allowance for the variety of goals and constraints of agents such as farmers, producers, individuals, land-use planning agencies, central and local authorities, insurance companies, investors, and catastrophe funds.

In Sec. 3, we show that decisions may be made under catastrophic risks using stochastic optimization models that account for the goals and constraints of the agents participating in catastrophe management. The section summarizes general ideas of the Adaptive Monte Carlo (AMC) optimization [10, 12, 16, 19, 20].

Section 4 illustrates that catastrophe-managing strategies maximize the well-being of regions and provide their sustainable development. The importance of financial measures such as catastrophe funds, catastrophic bonds (securities, shares) and credits is emphasized. Accounting for financial measures in catastrophe management allows evaluating optimal strategies of insurance compensations, possibilities of loss reallocation through catastrophe funds and financial markets, and the role of financial instruments. In particular, it is shown how the model was applied to evaluate a multipillar program on compensating and allocating losses as a result of the flood of the Tisza river in Hungary. The stages of the program consisted in partial compensation of losses by the central government, property insurance via a catastrophe fund based on individual risks, and external credit for reinsurance of obligations of the fund. The development of integrated programs on catastrophic safety implies evaluating the efficiency and expediency of structural measures such as dams, weeper channels, tanks, etc. Many studies emphasized the importance of integrated programs for the regions where insurance market is insufficient or absent. In Sec. 5, we briefly discuss such a program for a flood-prone region).

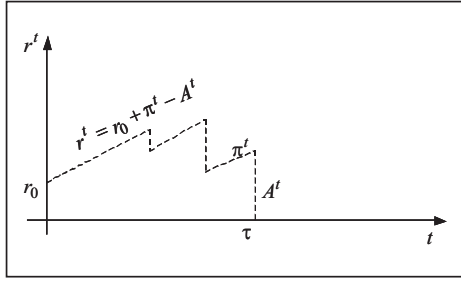


Fig. 1. Random risk-reserve  $r^t$  trajectory.

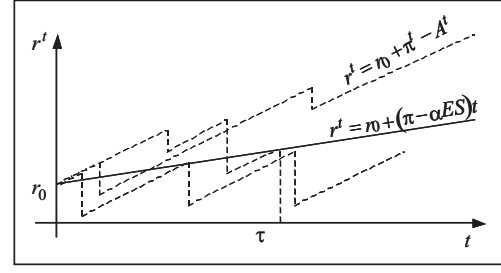


Fig. 2. Trajectories of the anticipated (solid line) and actual (dashed line) increase in the risk-reserve  $r^t$ .

## 1. SIMPLIFIED MODEL OF CATASTROPHE RISK MANAGEMENT

Catastrophes happen unexpectedly in time and space and cause sudden heavy losses that cannot be described (modeled) in terms of average quantities. Let us consider a stylized model of economic development of a region functioning under a sudden catastrophe. This model can also describe the process of insurance or accumulation of monetary reserve of a catastrophe fund. The basic variable, risk-reserve  $r^t$  of the fund at the time of  $t$  can be presented by the formula  $r^t = r_0 + \pi^t - A^t$ , where  $\pi^t$  are aggregated premiums,  $A^t$  are insurance compensations,  $r_0$  is the initial level of the reserve. The process  $A^t = \sum_{k=1}^{N(t)} S_k$ , where  $N(t)$ ,  $t \geq 0$ , is a random number of insurance claims on the interval  $[0, t]$ ,  $N(0) = 0$ , and  $\{S_k\}_1^\infty$  is a random sequence of compensation levels. Figure 1 shows that the inflow of premiums  $\pi^t$  increases the level of reserve  $r^t$ , while payments of compensations  $A^t$  reduce its level.

The main management problem in this case is to avoid the situations where  $r^t$  goes below an admissible level (default or bankruptcy), say below zero, which is possible with the probability  $\Psi = P\{r^t \leq 0, t > 0\}$ .

Traditional actuarial approaches employ the following simplified principles. Assume that  $S_k$  are independent random variables, a random process  $N(t)$  is characterized by the intensity  $\alpha$ , i.e.,  $E\{N(t)\} = \alpha t$ ,  $EA^t = ES\alpha t$ , where  $ES$  represents an average level of insurance compensations, and  $\pi^t = \pi t$ ,  $\pi > 0$ . The expected reserve  $r^t$  grows in time if  $\pi - ES\alpha > 0$ , which, however, ignores complex time dependences between arrivals of insurance claims  $S_k$  (by possible claim clusters) and their amounts, which may be significant despite a low level of average losses  $ES$  and the subsequent bankruptcy probability,  $r^t \leq 0$ . Thus, a real random process  $r^t$  is replaced in actuarial approaches by a function  $r^t = r_0 + (\pi - \alpha ES)t$  linear in time  $t$ . The difference  $\pi - \alpha ES$  is called safety load. As follows from the strong law of large numbers,  $[\pi^t - A^t] / t \rightarrow [\pi - \alpha ES]$  with probability 1. Therefore, in case of positive safety load,  $\pi > \alpha ES$ , one may expect that a real random profit  $\pi^t - A^t$  for a sufficiently large  $t$  will also be positive with correct choice of premiums  $\pi = (1 + \rho)\alpha ES$ , where  $\rho = (\pi - \alpha ES) / \alpha ES$ . However, this will be so only if bankruptcy does not occur till the time  $t$ .

As follows from Fig. 2, despite a guaranteed average growth of the reserve (the trajectory  $r^t = r_0 + (\pi - \alpha ES)t$ ), the actual reserve,  $r^t = r_0 + \pi^t - A^t$ , may go below an admissible level. In other words, replacing a complex jump process of growth by a simple deterministic model yields false conclusions. Various risk management measures may loosen the stiffness of claim distribution and reduce the default probability. One of such measures is constructing dams, which, on the one hand, can reduce the frequency of shallow flood and, on the other hand, can increase the probability of catastrophic flood. The claim value  $S$  depends on the compensations fixed by insurance contracts for different catastrophe-prone areas. The important variables in decision making are  $r_0$ ,  $\pi$ , reinsurance contracts, and the level of credit.

## 2. MODELING OF CATASTROPHES

In Sec. 1, we briefly outlined the methodological difficulties of decision making under catastrophes. Damage and reimbursement of losses depend on the geography of catastrophes, arrangement of property and infrastructure, risk-reducing preventive measures, and on insurance contracts and state reimbursements. As a rule, state reimbursements are paid from the

funds accumulated due to direct taxes; thus, losses and reimbursements are reallocated in the country. It happens that poor men subsidize rich men who are living in landscape-attractive regions that are however subject to serious risks. Catastrophes cause mutually dependent losses, reimbursements are directed to different regions affected by the same natural disaster. To account for the variety of spatial characteristics of catastrophes, the model should have a sufficient geographical detailing [11-14, 45]. Spatial variables of the model include characteristics of the agents participating in risk management such as the real estate value appraisal, preventive measures in each specific place, insurance coverage, and land-use practice.

**2.1. Analysis of Catastrophic Flood.** To illustrate the approaches and models under discussion, let us consider the results of studying catastrophic flood of the Tisza river (Hungary) [12, 13, 29]. To model the flood, its consequences, and management strategies, the information collected by employees of various organizations in Hungary [13, 29] was used. Note that integrated approaches are of special significance for the countries such as Hungary. In Hungary, as well as in many other countries and regions of the world, losses caused by flood and other natural disasters are reimbursed mainly from the state budget and, to a lesser degree, from the funds gathered by the people living in the zones subject to catastrophes [29]. Insurance companies cover only a part of losses, which is very small even in developed countries. The government of Hungary is concerned about the frequency and amount of compensations and cost of preventive measures. As compensations grow, central authorities and ministries, including many local authorities [29], consider necessary to transfer the liability for flood losses to local authorities, individual builders, and farmers and thus to raise their consciousness in arranging housing and agricultural holdings and constructing infrastructures. However, such a decision increases the efficiency and speeds up the implementation of preventive and loss-reducing measures only with a serious analysis and thought-out assessment of alternative on-site strategies.

In countries such as Hungary and Ukraine [37], flood problems are mainly due to the poverty and nonmobility of the population. In this case, planning and implementing preventive measures can be one of the most efficient solutions. For example, a correctly planned multistage program to help people suffering from natural disaster can provide guaranteed compensations. Developing such a program involves experts in various scientific fields, politicians, individuals, business executives, etc. In particular, planning such a program requires the analysis of the frequency and power of probable catastrophic events, capital vulnerability assessment in catastrophe-prone areas, which, in turn, needs so-called models of catastrophes. A three-dimensional model of catastrophic flood developed directly for the region of the Tisza river in Hungary is proposed in [13]. The model was intended to generate nonexistent and incomplete data and information on past and future floods, possible damage, and their dependence on different management strategies. For efficient control of catastrophic risks, the model assumes and emphasizes the necessity of tight cooperation of all concerned parties.

In regions with insufficiently developed economy, decisions under catastrophic risks should assume the possibility of incorporating and reallocating these risks [35, 2, 8]. The model under consideration proposes to do this via a catastrophe fund that fixes premium and compensation levels in view of individual on-site risks. Such a fund stimulates the capital accumulation in the region to take measures in case of catastrophes. To provide a sustainable operation of the fund, indicators such as expected overpayments by fund participants and expected fund deficiency are used to compute its parameters. These and other indicators, together with the so-called stopping time, allow the assessment of decisions stable against the most destructive catastrophic events. The papers [15, 16] show how the discontinuous probabilistic constraints specifying a stability level can be approximated using a convex stochastic optimization problem, which leads to CVaR-type risk measures.

**2.2. Modeling Catastrophic Events.** The absence and inaccuracy of the historical data available on catastrophes in a specific region often hamper the analysis and choice of risk management strategies. Spatial (geographically detailed) simulation models of catastrophes render a significant help in this case. They allow supplementing the data with scenarios of possible catastrophes and losses at the level of individual households, farmland groups, city, and region under threat of natural disaster such as flood, drought, earthquake, hurricane, and epidemic.

Many contributors (in particular, [45]) emphasize the need for such models, which become an indispensable tool in land use planning, arrangement of infrastructure and industrial enterprises, development of emergency systems, estimation of possible losses. Models of catastrophes use the information on physical processes, knowledge of experts, judgements of participants in risk management (stakeholders).

However, the data necessary for model development exist, as a rule, only in an aggregated form, inapplicable to analyze and simulate local processes. For example, extensive statistics on the origin of catastrophes may exist at the level of a country, which does not provide sufficient information on possible space-time catastrophes directly in the places of their origin. Disaggregation methods can be used to achieve a required data detailing level. Data disaggregation problems arise, as a rule, in analyzing the level and frequency of precipitation, studying the weather catastrophes caused by climate changes,

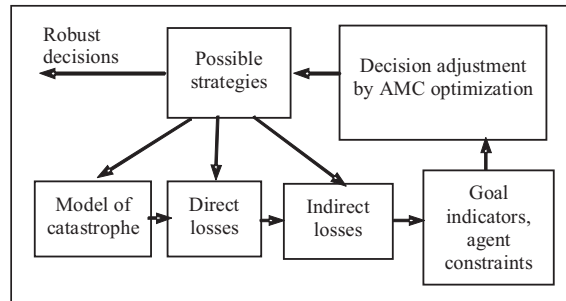


Fig. 3. Search for robust decisions using the AMC optimization method.

modeling cattle epidemics, and analyzing socio-economic processes. These situations require adequate disaggregation procedures [21] compatible with models of catastrophes and decision-making under catastrophic risks.

The integrated risk control model for decision making under the flood of the Tisza river (Hungary) consists of three main moduli: flood model, vulnerability model, and a multiagent economic model of economic growth. The flood model (consisting of the river model and inundation model) assesses the possible increase in the river level and inundation due to a possible precipitation scenario. A digitized relief map of the region is used, and the flood level and duration is calculated for each disaggregated cell. For each flood scenario, the model calculates losses using vulnerability curves, which associate each type of building with a potential damage level. The curves also take into account codes of buildings, their age, number of floors, type of property, etc.

The multiagent economic model is represented by a stochastic model of economic growth. The model transforms the evaluated space-time property damage into losses and incomes of agents participating in risk management. The main agents in the case under consideration are the central government, catastrophe fund, individual households (cells or regions), producers, farmers, etc. It is obvious that the choice of agents, allowance for their goals and constraints, and involving them in risk management depends on a set of damage preventing and reducing measures being evaluated.

Catastrophic models can generate damage (loss) and income scenarios in various places under various precipitation scenarios and various measures on preventing catastrophes and reducing and reallocating losses. There are uncertainties and substantial variations in losses and incomes. A 50-year flood may occur in five days or in 70 years. Different agents, or stakeholders, are especially concerned about such variations since the funds accumulated by the time of a catastrophe may appear insufficient to cope with heavy losses. To avoid bankruptcy, the catastrophe fund may increase premiums, which however may cause serious overpayments of premiums and reduce the demand for insurance packages. In the case of constructing dams as protection measures, premiums may decrease, which will increase the demand for insurance. A dam-protected area can stimulate the allocation of capital. However, the amount of losses can be incommensurable in case of sudden breakout of the dam (see Sec. 4).

### 3. ADAPTIVE MONTE CARLO METHOD

The conclusions drawn as a result of traditional modeling of “if-then” scenarios are of limited utility. In general, the scenario analysis allows gaining an insight into isolated observations of direct damages, losses, and incomes of various agents (Sec. 4) for different strategies. However, the number of alternate scenarios assessed by the integrated model of catastrophes can be considerable. For example, for a region divided into ten sections and with only ten possible levels of insurance compensations (0%, 10%..., 100%), the number of alternate compensations equals  $10^{10}$ . Sequential analysis of all the alternatives may be long-term and expensive. If it takes one second on the average to analyze one scenario, computing all the alternatives will require more than 100 years. Moreover, the loss-reducing measures in the case of one flood scenario cannot guarantee positive results in the case of other scenarios.

An important methodological question in the analysis of catastrophic risks is how to avoid the shortcomings of the “if-then” scenario approach, how to assess the set of measures that guarantees long-term sustainable activity of the region.

Practical implementation of each specific decision influences the general losses from catastrophes and thus increases the well-being of the region. As the studies [11–18] show, robust decisions can be searched for using the method of Adaptive Monte Carlo (AMC) optimization (the procedure is detailed and analyzed in [10, 15, 16, 20]). Figure 3 gives a schematic of



the catastrophic model and optimization procedure. Beginning with the initial arbitrary approximate decision (strategy), the model assesses the influence of this decision on possible losses. The efficiency of the required strategy is evaluated using indicators (the “Indicators, goals, constraints of agents” unit) specifying the goals and constraints of agents. Based on this, the current decision is adjusted in the “Decision adjustment by the AMC optimization” unit. The computations stop when the current decisions or strategies satisfy the goals and constraints of agents.

Developing an integrated catastrophic model is a key point in catastrophe risk management. As is generally known, the traditional Monte-Carlo method is applied to evaluate the integral  $G(x) = \int g(x) d\mu$ , where  $\mu$  is a probabilistic measure, and  $g$  is a measurable function. The measure  $\mu$  is often given implicitly using other known measures. In the case of AMC optimization, the function  $G(x)$  is an example where  $g$  and  $\mu$  depend (in contrast to the standard Monte Carlo) on decisions, which are sequentially adjusted as a result of estimating  $g$  for different values of unknown variables.

As a rule, searching for optimal strategies is complicated with the fact that the measure  $\mu$  is not given explicitly, and the analytic computation of  $G(x)$  is virtually impossible. The standard Monte Carlo approach can be considered as the unbiased estimation of the function  $G(x)$ . The less the standard deviation of the estimate for given observations, the better. By the method of AMC optimization is meant a technique that sequentially changes the measure  $\mu$ , adapting it to newly arrived observations to increase the sampling efficiency.

In the present paper, the AMC method is used in a more general sense, where sampling efficiency is only a criterion of a more general optimization problem in view of feasible solutions, objective functions, and constraints. The function  $G(x)$  depends on the unknown solutions  $x$ , and the task is to estimate the optimal value of  $G(x)$  by sampling the values of the random function  $g(x)$  for different  $x$ . AMC optimization methods apply the general ideas of stochastic programming, namely, the stochastic quasigradient method (see, for example, [10, 20]).

The general idea of the adaptive gradient approach used to improve sampling results was proposed first by E. Pugh [38]. However, this procedure requires integrals to be estimated. Stochastic optimization methods allow the method of sequential decrease in standard deviation to be applied in combination with the optimization with respect to feasible solutions without special computational difficulties.

Let us consider a probabilistic measure  $\nu$  with the same carrier as that of the measure  $\mu$  has, i.e., taking the zero value everywhere where  $\mu$  is zero. In this case, the derivative  $d\mu/d\nu$  exists,

$$G(x) = \int g(x, \omega) d\mu(\omega) = \int g(x, \omega) \frac{d\mu}{d\nu} d\nu(\omega) := \int \tilde{g}(x, \omega) d\nu(\omega),$$

where  $\tilde{g}(x, \omega) = g(x, \omega) \frac{d\mu}{d\nu}$ . The variance has the form

$$V \arg \tilde{g}(x, \omega) = \int g^2(x, \omega) \left( \frac{d\mu}{d\nu} \right)^2 d\nu - G^2(x).$$

The purpose of the adaptive sampling method is to find the distribution of  $\nu$  minimizing  $\Psi = E_{\omega} g^2(x, \omega) \left( \frac{d\mu(\omega)}{d\nu(\omega)} \right)^2 = \int g^2 \left( \frac{d\mu}{d\nu} \right)^2 d\nu$ .

Assume that the set of distributions  $\nu$  is indexed by the components of the vector  $y = (y_1, y_2, \dots, y_k)$ . The function  $\Psi$  is a function  $\Psi(y)$  of  $y$ , and derivatives of this function with respect to  $y$  (if the regularity conditions are satisfied) are defined as follows:

$$\frac{\partial \Psi}{\partial y_l} = \left( \int g^2 \left( \frac{d\mu}{d\nu} \right)^2 d\nu \right)_{y_l} = \left( \int g^2 \frac{d\mu}{d\nu} d\mu \right)_{y_l} = \int g^2 \left( \frac{d\mu}{d\nu} \right)_{y_l} \frac{d\mu}{d\nu} d\nu.$$

Assume that the sequence of measures  $\nu_k$  is specified by the sequence of vectors  $\{y^k\}$ . Assuming  $\nu_k$  to be known, it is necessary to find the value of  $\nu_{k+1}$ , such that reduces  $\Psi$  for the current  $\nu = \nu_k$ , i.e., we take  $y^{k+1}$  defined by

$$y_l^{k+1} = y_l^{k+1} - \sigma g^2(x^k, \omega^k) \frac{\partial}{\partial y_l} \left( \frac{d\mu}{d\nu} \right)_{y=y^k} \left( \frac{d\mu}{d\nu} \right), \quad l = \overline{1, k},$$

where  $\omega^k$  is a sampling from  $\nu_k$ ,  $\sigma_k > 0$  is a positive  $(q^0, y^0, q^1, y^1, \dots, q^k, y^k)$ -measurable random variable satisfying

some natural requirements of the stochastic quasigradient method [10, 20]. This procedure requires the exact values of  $\frac{d}{dy_l} \left( \frac{d\mu}{dv} \right)_{y=y^k}$ ,  $\left( \frac{d\mu}{dv} \right)_{y=y^k}$ , which are not specified explicitly since the measure  $\mu$  is not given explicitly. These quantities can be replaced with statistical estimates similarly to standard approaches. The convergence of the resultant processes easily follows from the general convergence results for the stochastic gradient method [10].

The adaptive sampling procedure can be combined with the sequential search for the optimal solution of the function  $G(x)$ . In the general case, the AMC optimization procedure has the following scheme. Assume that the vector  $x$  includes not only risk control variables but also the components of the vector  $y$  that influence the sampling efficiency, as considered above. The AMC optimization procedure starts with an arbitrary initial value of  $x^0$  and sequentially improves the value of the solution according to the rule  $x^{k+1} = x^k - \rho_k \xi^k$ ,  $k=0,1,\dots$ , where the step size  $\rho_k > 0$  satisfies the conditions  $\sum_{k=0}^{\infty} \rho_k = \infty$ ,  $\sum_{k=0}^{\infty} \rho_k^2 < \infty$ . For example, we may put  $\rho_k = 1/k+1$ . A random vector  $\xi^k$  is the estimate of the gradient  $G_x(x)$  (or its analog) of the nonsmooth function  $G(x)$ . The value of this vector is estimated from random observations of the function  $G(x)$ . Let  $G^k$  be a random observation of the function  $G(x)$  for  $x=x^k$  and  $\tilde{G}^k$  be a random observation of the function  $G(x)$  for  $x=x^k + \delta_k h^k$ . The quantity  $\delta_k$  is positive,  $\delta_k \rightarrow 0$ ,  $k \rightarrow \infty$ , and  $h^k$  is an independent observation of the vector  $h$  with independent components uniformly distributed on  $[-1,1]$ . Then  $\xi^k$  can be chosen from the condition  $\xi^k = [(\tilde{G}^k - G^k)/\delta_k]h^k$ . A formal analysis of this method, in particular, for discontinuous objective functions, is based on the general ideas of the stochastic quasigradient method (see [10, 20] and further references in [12, 15, 16, 19]).

#### 4. MODEL OF CATASTROPHE RISK MANAGEMENT

As already mentioned, catastrophe risk management requires integrated approaches to decision making. This implies, in particular, the assessment of a set of structural and financial measures to prevent, reduce, and reallocate losses, which would ensure a long-term sustainable development of the region in view of possible catastrophes. A risk management measure is immediate involving numerous agents and administrative structures (stakeholders) in decision making. Allowing for the variety of these agents, their goals, and constraints is a key problem. Many agents may go bankrupt not having coped with the consequences of catastrophes if their risk vulnerability is high or the risk portfolio is improper.

A general model of decision making under conditions of catastrophic risks is proposed in [11, 12, 14, 16, 17]. Its applications are discussed in [1, 2, 5, 12, 13, 18]. The studies [1, 2] consider seismic risks. The papers [21, 22] analyze the cases of cattle diseases.

In the present section, we consider the main features of the model applied for the analysis and decision making in the case of catastrophic flood of the Tisza river [12, 13]. A multistage program has been proposed for compensations and cost reduction as a result of natural disaster. At the first stage, partial compensation of losses by the central government was planned. The remaining losses were proposed to be reimbursed via a local catastrophe fund functioning on the principles of mandatory retention and payments by all people in the region. In the case of the lack of financial assets, the fund plans an external credit. The funds for maintaining and restoration of structural measures such as dams, bridges, tanks, and channels are assigned and paid via the catastrophe fund.

To reflect the space characteristics of catastrophes, the region under study is divided into sections (cells). The sections can unite several private households, a group of agricultural lands or farms, a section of a transport turnpike or a gas pipeline, an administrative area. In the general case, the resolution selected and the subdivision into sections determine and provide the required degree of detailing the losses and decisions being modeled. In our example, a comparatively small territory is divided into  $1500 \times 1500$  disaggregated sections (cells). The flood scenarios were generated by a catastrophe model and used to determine the necessity of robust feasible decisions on reducing potential losses. The so-called structural decisions on the construction or modernization of dams can reduce the losses. Financial decisions allocate losses at the region or country level or reimburse the losses via financial markets using available financial mechanisms such as credits, shares, reinsurance, and other securities.

Let a vector  $x = (x_1, x_2, \dots, x_n)$  specify the decision vector, then the losses  $L_j^t$  at the cell  $j$  at the time  $t$  are a function of  $x$ , i.e.,  $L_j^t(x)$ . For example the function  $L_j^t(x) = \max\{x_j^1, \min(L_j^t, x_j^2)\} - x_j^1$  describes losses at the section  $j$ , which has an

insurance contract with the minimum reimbursement level  $x_j^1$  and the maximum level not exceeding  $x_j^2$ .

In the general case, the vector combines decisions of different agents, including governmental decisions as to the dimensions of dams or the compensation level (determined by a part of general losses  $\sum_{j=1}^m L_j^t$ ). Decisions of insurance companies concern premiums and volumes of compensations in the case of catastrophes.

The time interval of the simulation model is determined by the instant of catastrophe, which is called stopping time. For the region of the Tisza river under consideration, the instant of the catastrophe was associated with the dam collapse and prompt flooding. The failure may occur due to a flood scenario with an average periodicity of 100, 150 or 1000 years. The periodicity depends on the frequency and volume of headwater flows due to snow melting in spring, intensive rains, sometimes snow melting during winter thawing weather.

Assume that  $\tau$  is a random instant of the first catastrophe within the time interval  $[0, T]$ , where  $T$  is a planning interval, say 50 years. Since  $\tau$  is associated with dam collapse, the probability distribution generally depends on the solutions  $x$ , for example, on the dimensions and reliability of dams, construction of tanks, land-use practice, etc.

Let  $L_j^\tau$  correspond to random losses at the section  $j$  at the instant  $t = \tau$ . Assume also that the reserve of the catastrophe fund is estimated only relative to financial decisions on loss allocation and reimbursement. If  $\pi_j$  is a payment from the section  $j$  to the catastrophe fund, the fund reserve at the time  $\tau$ , including the state compensation  $\nu \sum_j L_j^\tau$ , is determined by the relationship  $\tau \sum_j \pi_j + \nu \sum_j L_j^\tau - \sum_j \varphi_j L_j^\tau$ , where  $0 \leq \varphi_j \leq 1$  is the ratio of losses reimbursed to the section  $j$ .

In a more general case, this indicator may include variables corresponding to state grants, deductions for reconstruction and restoration of structural measures such as dams, tanks, etc. For example, a pair  $(\pi_j, \varphi_j)$  determines an insurance contract for the section  $j$ . Loss reimbursement to victims is assumed to be paid via the catastrophe fund.

The program stability depends on the assets accumulated in the fund for compensations, i.e., on the probability of the event

$$e_1 = \tau \sum_j \pi_j + \nu \sum_j L_j^\tau - \sum_j \varphi_j L_j^\tau \leq 0. \quad (1)$$

Moreover, the stability depends on the desire of clients to pay fixed premiums, i.e., on the premium overpayment probability

$$e_2 = \tau \pi_j - \varphi_j L_j^\tau \geq 0, \quad j = 1, \dots, m. \quad (2)$$

To increase the program stability, along with the state grants in the amount of  $\nu \sum_j L_j^\tau(x)$ , the catastrophe fund may provide a credit  $y$  at the price of  $q$ , which transforms (1) into the relationship

$$e_3 = \tau \sum_j \pi_j + \nu \sum_j L_j^\tau - \sum_j \varphi_j L_j^\tau + y - \tau q y \leq 0. \quad (3)$$

It is assumed that the catastrophe fund pays  $qy$  up to the moment of collapse of one of the dams caused by a flood flow scenario. In the case of dam damage, the fund immediately obtains a credit in the amount of  $y$  monetary units.

The difference between payments of compensations and the credit is rather significant: the credit is paid systematically while the payment of compensations occurs suddenly at the time of  $\tau$  and significantly affects the fund reserve. In the absence of credit, the state may need more resources to provide quickly a timely recovery of the region affected by the catastrophe. The credit cost abruptly increases after a catastrophe, and it is often impossible to obtain a credit.

In the general case, budgetary constraints require developing wider approaches to dynamic control of reserves, which would provide a level of stability and efficiency of an insurance company or a fund. For example, apart from a credit (or in addition to it), it is possible to invest in liquid assets. The problems discussed in the present paper are more specific: to illustrate the role and capabilities of a catastrophe fund in the catastrophic flood management.

Inequalities (2) and (3) specify the important events that limit the choice of decisions determining the flood abatement program, i.e., the state compensation level  $\nu$ , payments from the fund  $\varphi_j$ , fees  $\pi_j$ , the level and cost of the credit  $y$  and  $q$ . Note that the investigations using the model described are based on computing the so-called fair premiums  $\pi_j$ ,  $(\pi_j, \varphi_j)$  satisfying the equilibrium conditions (2), (3), but not the premiums according to traditional actuarial principles presented in Sec. 1. As shown in Sec. 4, actuarial premiums cannot guarantee the stability. The event probabilities (2), (3) and the values of indicators  $e_2, e_3$  determine a stable operation of the program. More generally, the program stability is guaranteed by the



probabilistic constraint

$$P[e_2 \geq 0, e_3 \leq 0] \leq p, \quad (4)$$

where  $p$  specifies the admissible fund bankruptcy probability (for example, the bankruptcy can take place once in a 100 years). Constraint (4) is equivalent to the so-called stability constraint for insurance companies, which is applied in insurance business. It is commonly supposed in stochastic optimization that inequality (4) specifies the probabilistic constraint [10, 16, 20, 36].

Assume that the vector  $x$  consists of the components  $\pi_j$ ,  $\varphi_j$ , and  $y$ . The purpose of the program is formulated as the minimization of the expected losses  $F(x) = c + \gamma \left( \nu E \sum_j L_j^\tau + y \right)$  under constraint (4). The definition of expected losses  $F(x)$  requires additional interpretation. It reflects the state interest to reduce uncovered losses,  $c = E \sum_j (1 - \varphi_j) L_j^\tau$ , and at the same time to reduce state payments  $\nu E \sum_j L_j^\tau + y$ . The estimate of the optimal values of  $\nu$  and  $y$  requires an explicit introduction and allowance for constraints on state funds allocated for loss payments and probabilistic constraints similar to (4). Determining the optimal level of the state compensation is a complicated optimization problem whose solution would need a significant modification of the model, which is beyond this paper.

Constraint (4) introduces methodological difficulties even in the case where  $\tau(x)$  does not depend on  $x$ , and events (2), (3) are determined by linear functions of solution variable (see [20, p. 8], [15, 16]). Minimizing the expected losses under constraints (4) is a complicated stochastic optimization problem since catastrophes and sudden losses can result in the discontinuity of constraints (4) [11, 17, 20]. Moreover, probabilistic constraints make the optimization of the function  $F(x)$ , in contrast to the optimization of a standard utility function, highly nonlinear with respect to the probabilistic measure  $P$ . The fees  $\pi_j$  are usually described by the expectation, which leads to high nonlinearity of the probabilistic constraints with respect to the probabilistic measure. The approaches to solving a similar optimization problem in view of the stopping time  $\tau(x)$  were considered in [15].

There is a relationship between the minimization of the function  $F(x)$  under nonlinear and, probably, discontinuous probabilistic constraints (4) and the minimization of convex functions, which has an important economic interpretation (see, for example, [12, 16]). Let us consider the function

$$G(x) = F(x) + \alpha E \max \left\{ 0, \sum_j \varphi_j L_j^\tau - \nu \sum_j L_j^\tau - \tau \sum_j \pi_j - y + \tau q y \right\} + \beta E \sum_j \max \{ 0, \tau \pi_j - \varphi_j L_j^\tau \}, \quad (5)$$

where  $\alpha$  and  $\beta$  are some positive parameters. This function highly depends on the loss probability  $L_j$  and on the stopping moment  $\tau$ , which may vary under the influence of  $\pi_j$ ,  $\varphi_j$ ,  $\nu$ , and  $y$ . It is possible to show [15, 16]) that for sufficiently large  $\alpha$  and  $\beta$  the minimization of the function  $G(x)$  yields a solution  $x$  such that  $F(x)$  tends to the minimum of the function  $F(x)$  under constraints (4) for any level of  $p$ .

The minimization of  $G(x)$  has a simple economic interpretation. The function  $F(x)$  estimates the expected direct losses under given decisions (strategies). The second term corresponds to the expected (budgetary) deficiency of the program in case the fund will pay all the obligations. It can also be considered as an additional capital necessary to cover the losses, which can be obtained as a loan after a catastrophe with the loan cost  $\alpha$ . The third term can similarly be interpreted as the expected loan after the catastrophe in order to cover overpayments of premiums. Obviously, sufficiently big prices  $\alpha$  and  $\beta$  promote satisfying constraints (2) and (3). For example, a loan with high prices allows monitoring bankruptcy constraints (4). It is easy to see that the second term in  $G(x)$  together with the optimum credit level  $y$  monitors the CVaR risk measure considered in [4, 27, 39, 46].

Indeed, the minimization of  $G(x)$  is an example of a stochastic minimax problem [20, Sec. 22]. The optimality condition for these problems yields the optimality condition for the credit  $y$ . For example, assuming that  $G(x)$  is continuously differentiable, which follows directly from the continuity of the cumulative distribution function of losses  $L_j^t$  (despite the nonsmoothness of random functions under the sign of expectation), it is easy to see that the optimal credit level  $y > 0$  satisfies the following equation:

$$\frac{\partial G}{\partial y} = \gamma - \alpha P \left[ \sum_j \varphi_j L_j^\tau - \nu \sum_j L_j^\tau - \tau \sum_j \pi_j > y \right] = 0. \quad (6)$$

Thus, the optimal credit level is determined by the inverse distribution of the random variable  $\sum_j \varphi_j L_j^\tau - \nu \sum_j L_j^\tau - \tau \sum_j \pi_j$ , equal to the relationship  $\gamma / \alpha \leq 1$ . Thus, the expectation in the second expression for the optimal  $y$  can be calculated

provided that  $y$  is the inverse distribution  $\sum_j \varphi_j L_j^\tau - \nu \sum_j L_j^\tau - \tau \sum_j \pi_j$ , which corresponds to the CVaR definition presented in [4, 39].

More general risk measures are determined by the optimality condition for  $G(x)$  with respect to the premiums  $\pi_j, \varphi_j$ .

The significance of an economic indicator such as deficiency was discussed in [4, 9, 16, 27]. The relation of CVaR with linear programming problems was considered in [4, 26]. Note that  $G(x)$  is a convex function when  $\tau$  and  $L_j^\tau$  do not depend on  $x$ . In this case, the stochastic minimax problem can be approximated by a linear programming problem [14]. Of interest is the case where  $\tau$  and  $L_j^\tau$  explicitly depend on  $x$ . In such a situation, a solution can be found only with the AMC optimization method. This method is described in detail in [15, 16].

## 5. NUMERICAL EXPERIMENTS

Let us illustrate the application of the above approaches and models to develop management strategies for catastrophic flood of the Tisza river in Hungary. As already mentioned, the region under study has been divided into 1500×1500 disaggregated cells, which could be grouped up into 40–300 significant sections, according to the purposes of study and the type of decisions. Sections united private households, groups of agricultural lands or farms, a section of a transport turnpike or a gas pipeline, administrative area. Each disaggregated cell or section was characterized by a monetary estimate of property and the degree of flood vulnerability. The resolution and division into sections provided a necessary level of detailing of losses and decisions. The planning horizon of strategies enveloped 50 years. The model required up to 10,000 simulations to derive the required convergence of solutions by the AMC optimization method.

Flood scenarios were classed as events with a return period of 100, 150, and 1000 years. They were modeled according to the probability estimates of intensive precipitation that lead to dam collapse. There are three dams in the region under study. A collapse of one dam could entail flood whose power was determined by the frequency and level of precipitation and dam reliability. The consequences of various proposals on improving the region stability were analyzed. These proposals included, among others, increasing investments in the dam renewal, evacuation of property, and production relocation followed by flooding.

In each case, it was required to estimate the catastrophe fund fees  $\pi_j$  satisfying the robustness condition discussed in Sec. 3. The fees were estimated according to the condition (3), which differs from the traditional actuarial principles. For example, if actuarial premiums are often computed based on average aggregated losses regardless constraints (4), they do not take into account possible additional constraints, for example, on incomes of the participants of the program, their desire to pay fixed premiums, governmental means available to pay allowances to demanders. In the model proposed, the allowance for these and similar constraints guarantees the estimate of so-called robust fair premiums minimizing the function (5). The premiums are calculated based on the optimization for each disintegrated cell or section, take into account the loss distribution on-site (at the level of individual households, farms), and satisfy the stochastic equilibrium conditions (4).

The degree of dependence of the catastrophe fund on the external assistance (credits, shares, securities, government aid) is determined by the negative values of the indicator  $e_1$  calculated for the optimal solution. Figures 4 and 5 illustrate the model evaluation results for actuarial and robust fair fees. The axis  $x$  is the fund reserve (negative values of  $e_1$  indicate that the reserve is insufficient. The number of Monte Carlo simulations and the probability distribution function is plotted on the  $y$  axis.

In practical problems [1, 2, 11–13], histograms of random indicators and constraints (1)–(4) are evaluated simultaneously with the minimization of function (5). Negative values of indicators specify the necessity to increase or reduce the penalty coefficients (risk factors)  $\alpha, \beta$ , and  $\gamma$  that ensure the required fund survival level (its reliability)  $p$ .

As is seen from Fig. 4, the inflow of actuarial premiums is insufficient in many cases to compensate losses since  $e_1$  is often negative. In more than 2000 scenarios (out of 10,000) of catastrophes being modeled, the fund turned out to be unable to satisfy damage compensation claims. Such a situation required from the government to participate more actively in loss reimbursement, for example, by rendering a direct financial assistance or external loan. As is seen from Fig. 6, actuarial premiums lead to frequent and substantial overpayments (a high probability of the outcomes where the level of the premiums arrived at the fund substantially exceeds the claims paid).

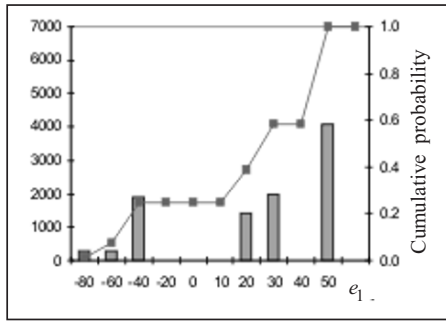


Fig. 4. Model evaluation for actuarial premiums.

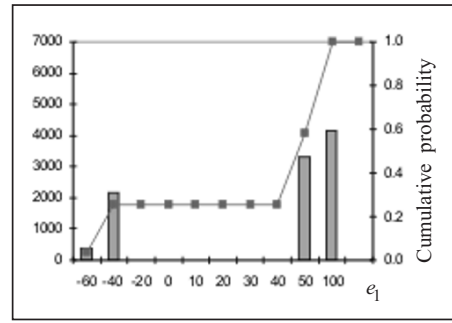


Fig. 5. Model evaluation for robust fair premiums.

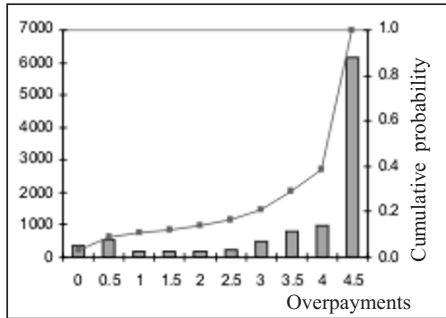


Fig. 6. Model evaluation for actuarial premiums.

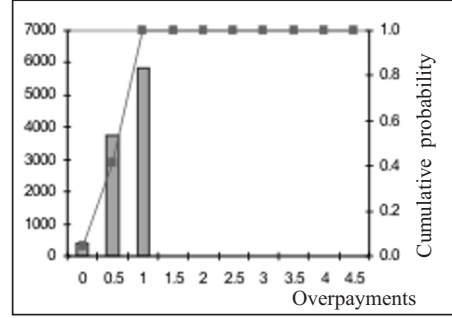


Fig. 7. Model evaluation for robust fair premiums.

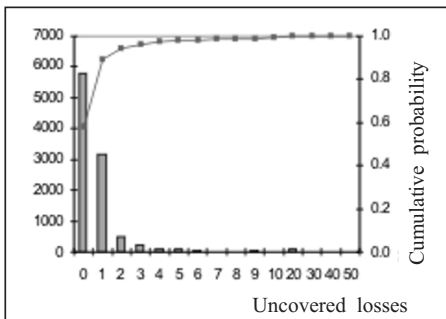


Fig. 8. Distribution of uncovered losses for actuarial premiums.

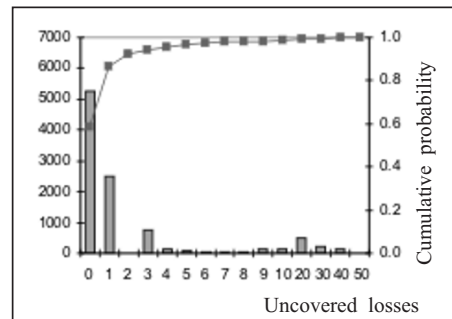


Fig. 9. Distribution of uncovered losses for robust fair premiums.

Robust fair premiums estimated according to the minimization condition (5) improve the fund operation. Figure 5 shows that the needs of the fund in an external credit for such premiums have been reduced substantially (fewer negative values on the horizontal axis). Optimal premiums (5) decrease also premium overpayments (the distribution of the indicator  $\sum_j \max\{0, \tau\pi_j - \varphi_j L_j^r\} / \tau$ ). Figures 8 and 9 show the distribution of uncovered losses. Obviously, simultaneously with estimating fair premiums (Fig. 9), optimal insurance coverages are estimated. The probabilities of bankruptcy (negative  $e_1$  in Fig. 4) and overpayments (Fig. 6) decrease up to feasible values (Figs. 5 and 7).

## CONCLUSIONS

We have discussed models and approaches for the analysis and decision makings under catastrophic risks. A peculiarity of the models proposed is that they allow for special features of catastrophes (serious, mutually dependent losses, lack of sufficient historical information on possible catastrophes, impossibility of their exact prediction, necessity of

long-term planning and adequate space-time resolution of models and decisions). The models are of current importance due to fast growing losses from natural disasters that cause heavy socio-economical and environmental consequences.

The stochastic optimization models proposed here allow developing robust strategies under catastrophic risks in view of the goals and constraints of different agents participating in the planning, namely, producers, farmers, individuals, the government (central and local authorities), insurance companies, investors, etc. In particular, these methods were applied to estimate optimal strategies for catastrophic flood of the Tisza river in Hungary and Ukraine. The main conclusion of the studies on catastrophic flood management is the need for integrated approaches including balanced application of preventive measures that reduce catastrophe probabilities, and measures on catastrophe mitigation and cost reallocation at the regional, national, and international levels.

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