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CALCULATING THE PROCESS OF LOOSE MATERIAL MIXING

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On the basis of scientific provisions of statistical mechanics and the entropy method arising from these provisions, the article substantiates the physical and mathematical modeling of the kinetics of the process of loose material mixing in a mixing installation, while the blade of the machine is simulated by an absorbing wall. Using the resulting solution to the task on the counting density of weight particle distribution near the blade (as the probability of this event), an equation for calculating the mixing factor (a parameter of mixture quality) is derived. A method for calculating the basic control parameter of the mixing process (the macro-diffusion factor) independent of the process parameters of the blade installation is proposed. On the basis of experiments on mixing portions of loose materials, the adequacy of the results of calculating the kinetics of the process of mixing portions of the product according to the developed algorithm has been confirmed.

Keywords: powder-like substance, blade, diffusion mixing, physical and mathematical modeling, heterogeneity factor.

In the technological processes of the chemical, food and other industries, crushed materials (powder-like or granular) are one of the most common forms of using mixtures in the processing and production of various substances. The quality of the products obtained in this case largely depends on the degree of dispersion of the material and its preliminary processing [1–4].

Blending installations (intended, conditionally, for processing mixtures of a composition that is homogeneous by all appearances) and mixing installations (for processing mixtures with various geometric and physical and mechanical parameters of components) used in practice are very diverse. In this regard, it is obvious that it is necessary to develop applied calculation models and algorithms that are independent of the parameters of substances and the design features of processing installations, based on advanced information technologies and theoretical provisions.

Since the main working body of industrial blending installations (BI) in many cases is a blade device, then when setting and solving such a problem, it is advisable to use the absorbing wall as a geometric model of the BI blade. In this case, the factor of mixing adjacent particles of the loose mixture is identified with the absorption factor of particles by an absorbing wall moving in a specified manner.

Based on the solution of the physical and mathematical model formulated in this way (using the provisions of the theory of probability), an algorithm for the quantitative analysis of the mixing process of a powder-like substance was developed [5, 6].

From the analysis of publications devoted to the issue of mixing loose materials, it was revealed that the mechanical mixing of powder-like substances is investigated in various aspects. However, taking into account the complexity of modeling the nature of the process, the publications often consider separate issues of an applied

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nature and only in some cases investigations are carried out from a more general, theoretically substantiated point of view.

For example, it has been revealed that during mixing, the distribution of mixture particles and the mixture homogeneity in the cross section of the mixer change predominantly in accordance with the diffusion mechanism of the process (not only in drum-type apparatuses, but also in blade mixers) [2].

To assess the influence of the investigated parameters on the mixture homogeneity in the cross section of the BI [4], the process was modeled as a random motion of the key component particle moving along the mixer axis. Based on this concept, the isotropic approximation of the kinetic ratio such as the Fokker–Planck equation (FP) [1] is used to describe the mixing process.

A qualitative analysis of the images of the mixture in the cross sections of a batch mixer [3] revealed a zone of active mixing and a zone of transporting loose material, separated by a break line.

In [4] it was shown that the calculated ratios obtained on the basis of theoretical modeling are more preferable (in comparison with the regression ratios based on experimental data); therefore, the calculated model ratios can be the basis of the engineering method for calculating the mixer.

However, in studies in the field of mixing loose materials, there is no clear justification and formalization of the physical model of the mixing process, there are no calculation dependencies for assessing the mixture quality and the process efficiency that are convenient for practical use.

On the basis of scientific provisions and software products of the Mathcad information environment, using the example of a numerical analysis of a homogeneous (only by particle size) mixing process of a loose material, a calculation algorithm was proposed and further tested by mathematical modeling and experimental research of the mixing process using a blade installation [6].

This article presents a solution to a more general (in comparison with the problem in [6]) problem of analytical and numerical study of the process of mixing portions of loose materials in a mixing installation (MI). The basis for calculating the process of processing powder-like substances by mixing is a scientific hypothesis about simulating a blade by an absorbing wall and a developed analytical apparatus with a numerical algorithm for solving the set problem.

For mixtures of particles of the equal density, the mixing processes were theoretically and experimentally investigated: in one experiment, mixing of various mass portions of particles of monodisperse composition ("multi-colored" particles) was investigated; in the other experiment, mixing of equal mass portions of particles of various sizes was investigated.

Formulation of the Problem

The main objective of mixing the mixture particles is to ensure a homogeneous distribution of the components in the mixture over the entire volume of the medium being processed; but in real conditions, this is practically impossible to achieve due to the fact that the process of mixing particles develops as a random phenomenon. Therefore, advanced methods of quantitative assessment of the mixture quality, when mixing particles of loose material, are based on the methods of statistical analysis [5–9].

Taking into account the provisions of this scientific branch and the significant uncertainty of the kinetics of the product being processed, the mathematical description of the mixing process can be obtained on the basis of an information scientific approach and, in particular, the entropy method.

In practice, the powders being processed are a multiphase continuum; however, in this study (as in many other cases), the mixture quality was assessed on the basis of the distribution of one of the mixture components, provided that the loose medium under study is a two-component heterogeneous system.

Solution of the Problem

In industry, an approximately homogeneous distribution of the components of the particles clusters throughout the entire mixture volume is ensured using batch and continuous mixers, where in stages of processing the following processes develop in the working volume of the machine: convective mixing by redistribution of particles through the interface between layers; diffusion mixing; grouping (under the action of inertial, gravitational or other forces) and segregation of particles with similar geometric or physicochemical parameters.

In the mixing process, which proceeds quite intensively, convective mixing invariant to the physicochemical properties of particles predominates at first. The convective mixing is determined mainly by the dynamics of motion of the MI working bodies. Further, when the components are distributed over the mixer volume, the diffusion mixing has the main influence. Then, the segregation of particles begins to influence the kinetics of the process.

The duration of diffusion mixing depends both on the mixer parameters and on the physical and mechanical properties of the mixture components: particle size distribution; density; particle shapes; the state of the particles surface, their moisture content; flowability. Mixing is more efficient when the properties of particles differ insignificantly, since with large differences in the density and particle size of the mixture components, the segregation process intensifies.

With an increase in the number of mixture components, the efficiency of the mixing process decreases. It was revealed that if the mixture contains components in small quantities, then the process duration increases significantly; in this case, the components with higher dispersion are better distributed.

The subject of quantitative research of this study is the diffusion mixing of a two-component powder-like substance in a blade batch mixer.

The solution of the problem of the distribution of particles chaotically moving near the absorbing wall has the form [7]

$$n/n_0 = \operatorname{erf}\left[x/(4Dt)^{1/2}\right] = \operatorname{erf}(\lambda x), \tag{1}$$

where n/n_0 is the counting concentration of particles, $(4Dt)^{1/2} = \left(\sigma\sqrt{2}\right)$, D is the diffusion factor, t is the time, σ is the standard deviation, $\operatorname{erf}(\lambda x)$ is the integral of probabilities (Crump function), $\lambda = 1/\left(\sigma\sqrt{2}\right)$.

If, in real conditions, the blade with the particles moves as a mechanical system translationally with a constant velocity U, then in a moving frame of reference (at $x' = x - Ut$), Eq. (1) can be transformed to the form

$$n/n_0 = \operatorname{erf}\left[\lambda(x - Ut)\right]. \tag{2}$$

According to Eq. (2), the particles concentration gradient on the wall (at $x = 0$) is equal to

$$q(t) = D\left.\frac{\partial n}{\partial x}\right|_{x=0} = \sqrt{\frac{D}{\pi t}}\exp\left[-U^2t/(4D)\right],$$

then the specific (in terms of area) number of particles deposited on the wall surface over a time period dt is

$$dN(t) = q(t)\,dt = n_0\sqrt{D/(\pi t)}\exp\left[-U^2t/(4D)\right]dt\ .$$

Number of particles deposited over time t

$$N'(t) = n_0\sqrt{\frac{D}{\pi}}\int_0^t t^{-1/2}\exp\left[-U^2 t/(4D)\right]dt\,. \tag{3}$$

In this case, the ratio of this number of particles to the volume of the deposited particles $1^2 \times l$ $(l = Ut)$

$$N'(t)/(n_0 \cdot l) = J(t)$$

and then

$$J(t, D, U) = \sqrt{\frac{D}{\pi l^2}}\int_0^t t^{-1/2}\exp\left[-U^2 t/(4D)\right]dt\,. \tag{4}$$

In the framework of the adopted model of the mixing process kinetics, the ratio of Eq. (4) is the basis for calculating the dependence of the specific number of mixed particles (in relation to the initial number of particles of the material) on the process duration.

In its turn, from the point of view of the provisions of statistical mechanics, the occurrence of a single random event — the specific number of particles absorbed by the blade as an absorbing wall (hence, by agreement, mixed) for a fixed period $t = T$ of the time of the blade movement — in its meaning is the probability (Eq. (4)) of this event.

Therefore, the probability p of the occurrence of at least one independent random event specified by Eq. (4) during the i tests can be determined from the dependence [10]

$$p = 1 - \left[1 - J(t, D, U)\cdot j\right]^i, \tag{5}$$

where J is calculated by Eq. (4), j is the number of MI blades, i is the number of movements of the MI blade for the time period $\tau = iT$ of the experimental study.

At that, the value p is taken as the heterogeneity factor (HF) of the mixture:

$$p = n_{\mathrm{m}}/n,$$

where n_{m} is the number of mixed particles.

As follows from the analysis of Eq. (5), the relative frequency of mixed particles (probability p) depends on the process parameters t, U, i, j, as well as on the macro-diffusion factor D, calculated based only on the physical and geometrical parameters of powder portions.

In order to do that, it is necessary to solve the equation transcendental in terms of parameter D, which follows from Eq. (5):

$$F(t, D, U, i, j, k) = 1 - \left[1 - J(t, D, U)\cdot j\right]^i - k = 0\,, \tag{6}$$

i.e. to find (in symbolic form) the value of the macro-diffusion factor

$$D = F^{-1}(t, D, U, i, j, k). \tag{7}$$

Fig. 1. Exposure of the dispersion composition by the color of the particles being mixed at the time moments of 0, 18, 38, 58 s from the start of the MI operation (initial exposure: (a) colorless particles above, (b) colorless particles below).

The value of the parameter k is determined as a maximum permissible (regulatory) for the process conditions of the MI value of the heterogeneity factor. The parameter k is calculated, in particular, when two portions of the phases being mixed have one or several common quantitative indicators (for example, particle size, portion mass, etc.), since it provides the possibility of calculating the ratio between the counting concentrations of particles in portions of the mixed clusters of particles.

On the basis of the calculated ratio between the counting concentrations of particles (in accordance with the requirement to ensure an approximately homogenous distribution of the heterogeneity factor of the phases being mixed), obviously, as the calculated value of the parameter k, the value of the parameter corresponding to the minimum specific value of the counting concentration of particles in the mixture as a whole is chosen.

After determining the macro-diffusion factor D, its value is substituted in Eq. (5), and then the dependence of the HF on the duration of the mixing process, which is determined as a number of revolutions of the MI blade, is determined by calculation.

Numerical and Full-Scale Experiments

As objects of trial mixing of loose materials, clusters of caryopses of agricultural crops were used (Fig. 1) in portions weighing 125 g and 250 g (in the initial position, respectively, colorless particles above, colored ones below) of particles with a diameter of 1.6 mm (*first option*), as well as portions of particles with diameters of 1.6 mm and 3.2 mm (in the initial position, respectively, colored particles above, colorless ones below) weighing 250 g (*second option*).

To substantiate the calculation of the efficiency of the particles mixing process, experiments were carried out on the equipment of HURAKAN NKM-KS5 type (BI power 0.5 kW, blade rotation frequency $n = 60\ \text{min}^{-1}$). Since the visualization of the experimental data was recorded at the blade radius $R = 0.05$ m, in the calculations $U = 0.314$ m/s was taken.

The number of particles (caryopses) of each color in the upper layer of the mixture was measured visually every 2 s of the BI operation, i.e. every two revolutions of the blade (which was corrected by the factor $j = 2$ in Eq. (5)), in rectangular 10×20 mm windows for smaller particles and 20×30 mm windows for two particle sizes, while providing the ability to observe up to 40 particles.

Photographing of the entire surface of the mixture in the apparatus was carried out at the initial (zero), 9th, 19th and 29th experimental measurements, i.e. at the time moments of 0, 18, 38, 58 s from the start of the BI operation (see Fig. 1).

The experiments were carried out until an approximately uniform distribution of particles in the working volume of the MI, including the stages of convective and diffusion processing (Fig. 2, curves *1*, *2*), was provided.

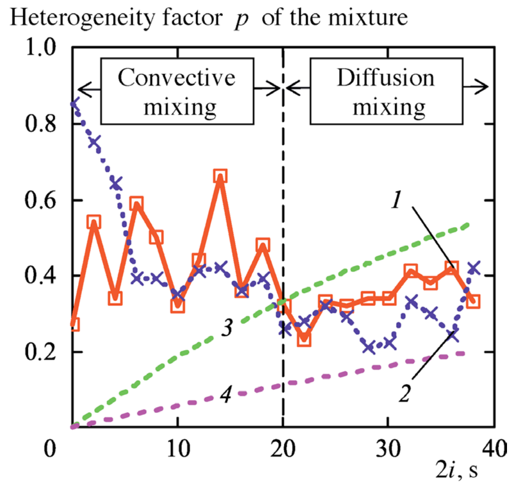

Fig. 2. Dependences of the heterogeneity factor p of the mixture for particles of different colors on the duration t of the mixture processing: *(1)* in the experiment, colored particles are below (250 g), colorless particles are above (125 g), particle size 1.6 mm, *(2)* in the experiment, colorless particles with a diameter of 1.6 mm are below, colored particles with a diameter of 3.2 mm are above, the mass of particles portions is 250 g, *(3)* regulatory heterogeneity factor of the mixture, calculated for the experimental conditions (see Fig. 1a, macro-diffusion coefficient $D = 9.8 \cdot 10^{-4}$ m^2/s), *(4)* regulatory heterogeneity factor of the mixture, calculated for the experimental conditions (see Fig. 1b, $D = 2.9 \cdot 10^{-4}$ m^2/s).

When calculating the mixing process (for two options), the parameter k was determined as follows.

First Option (Fig. 1a): in the experiments, portions of particles differing only in color with a portion mass ratio of 2:1 (hence, with the same ratio in terms of the number of particles in portions) were processed.

Since the HF of mixing particles of the same size is determined by the mass of the smallest of the portions, the authors took $k = 1/3$, and in the calculations the following initial data on the coefficients of Eq. (7) were used: $U = 0.314$ m/s, $i = 10$, $j = 2$, $k = 1/3$.

On the basis of the value of the macro-diffusion factor D calculated by Eq. (6), a curve of the regulatory HF of mixing was constructed (see Fig. 2, curve *3*).

Second Option (Fig. 1b): since for the equal masses of portions, the particle size ratio was 2:1 (and therefore the calculated ratio of the number of particles was 1:8), in the calculations the authors took $k = 1/9$; on the basis of this value, curve *4* was constructed (see Fig. 2).

Processing the results of the experimental determining of HF p_i $(i = 10, 11, \ldots, 19)$ for two options of mixing the clusters of particles at the diffusion stage of the process (see Fig. 2), i.e. during the period from 20 to 38 s, was carried out according to the formulas

$$a = \sum_{i=10}^{19} P_i/10, \qquad \sigma = \sqrt{\sum_{i=10}^{19} (P_i - a)^2/10},$$

where a is the arithmetic mean value, σ is the standard deviation.

When processing the data, it was revealed that: for the *first option* $a_1 = 0.342$, $\sigma_1 = 0.051$; for the *second option* $a_2 = 0.287$, $\sigma_2 = 0.058$.

A qualitative analysis of the calculation results based on visualizations (see Fig. 1) and dependencies (see Fig. 2, curves *1*, *2*) revealed that: the process of mixing particles of equal sizes proceeds faster and with higher efficiency than the process of mixing particles of different sizes — curve *1*, constructed according to the experimental data for the *first option*, enters the area of diffusion mixing earlier and deviates from the HF mean value less $(\sigma_1 < \sigma_2)$ than curve *2* (see Fig. 2), which is confirmed by the results of experiments by other authors [4] and by the shape of theoretical regulatory curves of the heterogeneity factor (see Fig. 2, curves *3*, *4*).

CONCLUSIONS

An algorithm for quantitative analysis of the quality of a mixture of loose materials (in a wide range of particle fineness) during the mixing process, based on the diffusion model and the provisions of the theory of probabilities, has been developed.

On the basis of the study carried out (according to the diffusion model of the process of processing particles by mixing that was proposed in the article), the calculation of the macro-diffusion factor as a control parameter of the mixing process is substantiated.

On the basis of the developed algorithm, an experimental study and numerical simulation of the process of mixing portions of loose materials differing in physical and geometrical properties has been carried out.

The proposed physical and mathematical and numerical analysis of the mixing process can be useful in the development and simulation of innovative samples of mixing equipment.

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