CALCULATION OF THE PROCESS OF SLURRY SEPARATION IN THE ROTOR OF A CONTINUOUS SEDIMENTATION CENTRIFUGE

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UDC 66.011:66.067.5

The process of deposition of the solid phase of a suspension in the rotor of a continuous sedimentation centrifuge with a rigid wall and periodic discharge of the sediment was investigated based on the provisions of the hydrodynamics of slow movements. Results of quantitative analysis of the process of centrifugal separation of the suspension show that the calculation method of the efficiency of this process should be improved taking into account the layered structure of the flow based on the first model.

Keywords: *centrifuge, rotor, sedimentation, suspension, device performance.*

Continuous sedimentation centrifuges, or tubular centrifuges (TC), are devices for separating the liquid phase from the solid with the minimum content of liquid in the sediment. They are used for the treatment of wastewater (containing chrome, claydite and coal dust, clay, kaolin, shale, pumice, etc.), in the nuclear industry for enrichment of uranium feedstock, in the chemical and metallurgical production of plutonium for the separation and processing of difficult-to-filter radiochemical suspensions, and in other industries [1–5].

Sedimentation centrifuges, the distinguishing feature of which is the presence of a continuous rotor, have a number of advantages over sedimentation tanks: compact size; the possibility of placing them directly in production facilities and including them in the technological scheme of production; higher clarification efficiency.

Methods for performing sedimentation centrifugation [2]:

1) centrifugation of a separate volume of the suspension without replenishing the suspension and without removing the fugate and sediment during the process;

2) centrifugation with a continuous supply of the suspension and with the removal of the fugate, but without removing the sediment during the process; and

3) centrifugation with a continuous supply of the suspension and a continuous removal of the fugate and sediment.

The purpose of this work is to study the process of separation of a suspension in the working volume of a centrifuge with a continuous supply of the suspension and with the removal of the fugate without removal of the sediment during the process, i.e., in a continuous sedimentation centrifuge (method 2) based on the model of a layered fluid flow regime in the rotor of a centrifugal device.

Continuous sedimentation centrifuges for clarifying suspensions are used when the content of high-dispersion impurities is low; when it is necessary to classify solid particles by size and density; when separating resistant emulsions.

The problem of quantitative analysis of the kinetics of the process of separation of a suspension in the rotor of a TC was considered in [1, 2, 4, 6–10]. However, a number of factors that determine the efficiency of the separation process have not been accounted for yet, namely: the features of hydrodynamics of the intra-rotational flow; slurry kinetics during processing in the device; formation of a layer of sediment on the rotor wall; optimization of the centrifugation process. To solve these engineering problems, an accurate method for calculating and predicting the course of this process is necessary.
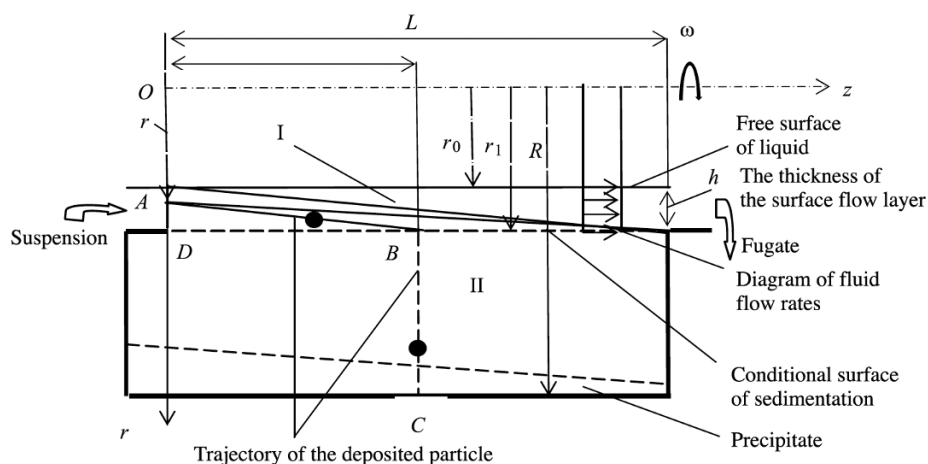
Fig. 1. Scheme for calculating the sedimentation process in the lower half of the meridional cross section of the centrifuge rotor.

In the second half of the twentieth century, the problems of the kinematics of the intra-rotor flow of TC were investigated by Sokolov, Shkoropad, Gol'din, Reuter, and other scientists [1, 2, 11]. The theories of a layered flow (assuming the existence of laminar flow in the rotor), surface flow, and flow lines were subjected to critical analysis.

One of the first experimental studies of the surface flow regime in augerless sedimentation centrifuges was carried out by Shkoropad, who visualized the behavior of a colored liquid in intra-rotary flow [2]. In turn, Reuter [11] found, as a result of an experimental study of the flow pattern in a relatively short rotor ($D_{rt} = 300$ mm, $L = 227$ mm), that there are three characteristic zones in the working volume of this device [11]: a thin surface layer of the flowing fluid (with a thickness on the order of several millimeters); zone of constant capacity (in which the fluid currents are weakly expressed); circulation zone near the overflow board. It was assumed that the thickness of the draining layer depends on the height of the support, which is formed when the fugate is poured over the annular side of the rotor.

Shkoropad suggested that in short rotors ($L/D_{rt} < 1$) the slurry flows in a layer of variable thickness and, consequently, with a speed that varies both along the length of the rotor and through the thickness of the layer.

Development of this model of flow in an augerless rotor of a sedimentation centrifuge led to the surface-vortex flow model [12], according to which liquid flow in TC rotors has characteristics of surface flow, but the liquid filling the rotor under the thin draining layer does not remain stationary. Vortices of different sizes form at the interface (this term is conventionally adopted). The intensity of vortex formation depends on the average velocity of the draining layer, vortex formation is accompanied by the mixing and exchange of the "surface" and "internal" volumes of the liquid at the interface.

Sokolov and his co-workers showed that in centrifuges with a long rotor ($L/D_{rt} > 5$), a solution of ink injected in a trickle into the liquid stream is washed out by the stream to a depth exceeding the thickness of the liquid layer above the edge of the overflow wall; the velocity of the streams at different depths of flow is different, and the highest velocity is in the surface layer.

Other studies have shown that, with a pronounced surface character of the flow, weak fluid currents also exist in the so-called constant-capacity zone. In the laminar flow regime, the jet stream starts from the supply site and, expanding, goes towards the overflow wall, where it again narrows, with a vortex zone forming a slowly rotating toroidal vortex. Other features of intra-rotary flow were also detected experimentally.

By reviewing the results of experimental data, it was concluded that in these studies (including the theoretical analysis of the Gol'din flow lines) the specific features of flow in a TC have been studied in detail. However, the problem of quantitative and qualitative analysis of the actual phenomenon of centrifugal sedimentation of a suspension in a heterogeneous liquid + solid system, which is essentially the main purpose of calculating the process of centrifugal separation of a suspension in a rotor machine, has remained virtually unaffected.

In this paper, a numerical simulation of the process of separation of a liquid system in the working volume of a continuous sedimentation centrifuge (taking into account the features of flow structure) was performed in terms of quantitative analysis of process efficiency of the accumulation of the solid phase of the processed suspension in the rotor of the machine.

With regard to the analysis of sedimentation of the solid phase in a low-concentration, fine-dispersed suspension, it is legitimate to use the concept of internal and external hydrodynamic problems. In this case, regardless of the mutual influence of the liquid and the solid, the results of calculating (according to the solution of the internal problem) the velocity and pressure fields along the main flow are then used in solving the external problem of particle kinetics in a liquid flow [1, 2, 7]. Since the process of separation of a suspension of this type is realized precisely in a sedimentation centrifuge, the use of this concept in solving the problem is justified.

Let us assume that the processed suspension is a low-concentration liquid mixture containing spherical particles of small size; the rotor of a centrifuge with periodic discharge of the sediment is in rotational motion with a constant angular velocity ω; the axisymmetric flow bounded by the region $r_0 \le r \le R$ (r_0 and R represent the radii of the free surface of the fluid and the rotor, respectively) and $0 \le z \le L$ (L is the length of the rotor) moves in the positive direction of the z axis (Fig. 1). In order to quantify the kinematic characteristics of the flow, we refer the volume under investigation to the cylindrical coordinate system $Or\varphi z$ rotating together with the rotor.

Then (assuming that the flow is axisymmetric), we start from the surface flow model in the projection onto the meridional section Orz. For convenience of quantitative analysis of the sedimentation slurry kinetics, it is advisable to divide the working volume of the centrifuge conditionally at the "interface" (according to Shkoropad) into two parts: I – region of the surface flow of the liquid (main flow) with dimensions $r_0 \le r \le r_1, 0 \le z \le L, r_1 = r_0 + h$ (h is the thickness of the surface current layer); II – region of constant capacitance with dimensions $r_1 \le r \le R, 0 \le z \le L$ (see Fig. 1).

Studies of the hydrodynamics of a number of TC showed that the relative motion of the fluid system in the rotor is slow. Therefore, it is possible to analyze the sedimentation of the solid phase in the moving reference frame using the Stokes formula when calculating the drag forces for particle motion.

Thus, in determining the particle precipitation velocity v_r in the rotor, the dependence [1], which is valid for fine powder, can be used,

$$v_r = kr\delta^2, \tag{1}$$

where

$$k = \omega^2 \Delta / 18\mu; \tag{2}$$

$\Delta = \rho_s - \rho_l > 0$; ρ_s, ρ_l represent the densities of the solid particles and the liquid, respectively, kg/m^3; μ is the dynamic viscosity of the liquid, Pa·sec; and δ is the particle diameter, m.

In turn, we assume that the particle in the axial direction Oz moves with the velocity v_z along with the flow, as a suspended particle.

In addition, without violating the generality of the formulation of the problem, the velocity u_z of the liquid in region I of the surface layer is calculated as its expenditure value. Then, as the axial component of the velocity of the particle, we take

$$v_z = w_0, \tag{3}$$

where

$$w_0 = \frac{V}{\pi(r_1^2 - r_0^2)}; \tag{4}$$

$V = Q/3600$ is the flow rate, m^3/sec; and Q is centrifuge productivity, m^3/h.

The selected geometric structure of the channel, which simulates the meridional cross section of the working volume of the centrifuge as the sum of zones I and II, determines the geometry of the ABC trajectory of the sedimenting particle in the intra-rotational flow (see Fig. 1). In zone I, where the actual shear surface flow of the slurry develops, the particle moves to the interface $r = r_1$ along the section AB of the path ABC. Further, in accordance with the adopted channel model, the particle in the zone of constant capacitance II under the action of external forces will move along the radius in the section BC of the trajectory ABC, then settling on the surface $r = R$.
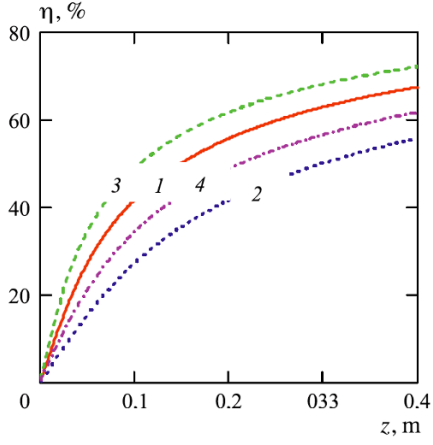
Fig. 2. Specific mass of the solid phase η settling on the rotor wall as a function of the axial coordinate z and the thickness h of the surface flow layer: $h = 0.001$ m: $1)$ $Q = 0.1$ m³/h, $2)$ $Q = 0.2$ m³/h; $h = 0.005$ m: $3)$ $Q = 0.1$ m³/h, $4)$ $Q = 0.2$ m³/h ($\delta_{av} = 4.77 \cdot 10^{-6}$ m, $\alpha = 10^{10}$, $\omega = 2000$ rad/sec, $\mu = 0.01$ Pa·sec).
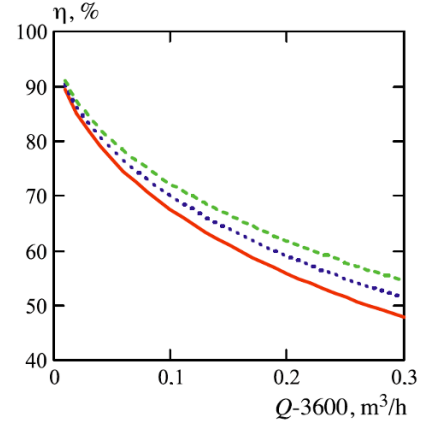

Fig. 3. Clarification coefficient η as a function of the flow rate of the suspension Q: $1)$ $h = 0.001$ m; $2)$ $h = 0.003$ m; $3)$ $h = 0.005$ m ($\delta_{av} = 4.77 \cdot 10^{-6}$ m, $\omega = 2000$ rad/sec, $L = 0.4$ m).
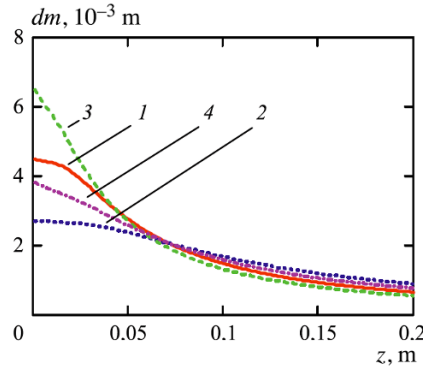

Fig. 4. The thickness of the sediment layer dm settling on the rotor wall as a function of the axial coordinate z and the thickness h of the surface flow layer: $h = 0.001$ m: $1)$ $Q = 0.1$ m³/h, $2)$ $Q = 0.2$ m³/h; $h = 0.005$ m: $3)$ $Q = 0.1$ m³/h, $4)$ $Q = 0.2$ m³/h ($L = 0.4$ m, $\delta_{av} = 4.77 \cdot 10^{-6}$ m, $\alpha = 10^{10}$, $\omega = 2000$ rad/sec, $\mu = 0.01$ Pa·sec).

Thus, it is possible that the instantaneous deposition surface is a cylindrical surface $r = r_1$ (interface) on which a sediment layer is conventionally formed, which is then projected on the rotor wall $r = R$.

To reveal the features of solid phase accumulation on the surface $r = r_1$, it is necessary to know the expression of the current critical diameter of the particle moving from point $A(r, 0)$ to point $B(r_1, z)$, where $r_0 \leq r \leq r_1, 0 \leq z \leq L$.

Based on the values of the projections of the velocities v_r and v_z given by Eqs. (1)–(4) for the particle moving along the trajectory AB, the calculated value of the current critical diameter is expressed in the form

$$\delta(r, z) = \sqrt{\frac{w_0 \ln(r_1 / r)}{kz}}, \quad z \neq 0, \tag{5}$$

where w_0 is determined by Eq. (3) and k – by Eq. (2).

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Let $F(\delta)$ be the mass distribution function of the particles,

$$F(\delta) = \frac{m(\delta < \delta')}{m_0}, \tag{6}$$

where m is the mass of particles of size $\delta < \delta'$; m_0 is the mass of particles in the initial suspension; and δ' is calculated by formula (5).

In this case, the clarification coefficient for the triangular region ABD (as a functional of the axial coordinate z) [7] is

$$\eta(z) = \frac{m_0 - m_1(z)}{m_0} = \frac{2}{r_1^2 - r_0^2} \int_{r_0}^{r_1} \Phi[\delta(r, z)] r \, dr, \tag{7}$$

where $m_1(z)$ is the mass of the solid in the fugate; and $\Phi(\delta)$ is the characteristic function complementing the function $F(\delta)$ defined by Eq. (6) to unity.

The value of η, calculated from Eq. (7), is the relative mass of the solid phase in the flow region located under the line AB (that is, the mass of the solid to be deposited on the surface $r = r_1$, and hence also on the rotor wall). In turn, the quantity $\varepsilon = 1 - \eta$ (entrainment coefficient) is the relative mass of the solid phase in the region located above the line AB (and diverted from the rotor together with the fugate) (see Fig. 1).

Since the mass of the solid settling at a distance z on the section of the rotor wall of length Δz per unit time is

$$\Delta m(z) = [\eta(z + \Delta z) - \eta(z)] m_0 \approx \eta'(z) \Delta z m_0, \tag{8}$$

(where $\eta'(z)$ is the derivative of the function $\eta(z)$ with respect to z), and the mass m_{set} of the solid settling along the length of the rotor L over the same period of time is

$$m_{\text{set}} = \eta(L) m_0, \tag{9}$$

the specific mass of the settling particles at a distance z (in the section Δz) per unit time according to equations (7)–(9) is approximately

$$\Delta m(z) / m_{\text{set}} = \frac{\eta'(z) \Delta z}{\eta(L)}, \tag{10}$$

where $m_0 = c_0 \rho_s V$; c_0 is the mass (or volume) concentration of the solid phase in the suspension.

The derivative $\eta'(z)$ in (10), in accordance with expression (7), has the form

$$\eta'(z) = \frac{-2}{R^2 - r_0^2} \int_{r_0}^{R} \varphi[\delta(r, z)] \frac{d\delta}{dz} r \, dr, \tag{11}$$

where the mass density distribution of the solid phase is

$$\varphi(\delta) = F'(\delta) = -\Phi'(\delta). \tag{12}$$

In addition, according to Eq. (5),

$$\frac{d\delta}{dz} = -\frac{1}{2} \sqrt{\frac{w_0 \ln(r_1 / r)}{k z^3}}. \tag{13}$$

Thus, the forming line of the surface, which limits the volume of the solid phase (by mass) on the rotor wall, is calculated based on dependences (7), (11)–(13).

The following values were used for the numerical experiment: $R = 0.02$ m; $L = 0.4$ m; $r_0 = 0.015$ m; $h = 0.001, 0.005$ mm; $Q = 0.1, 0.2, 0.3$ m³/h; $\omega = 2000$ rad/sec; $\rho_s = 1100$ kg/m³; $\rho_l = 1000$ kg/m³; $\mu = 0{,}01$ Pa·sec; $c_0 = 1\%$.

In the considered calculation example, two models of the liquid flow in the meridional section of the rotor were investigated:

1) the thickness of the surface current layer is relatively small – 1 mm, the thickness of the constant capacity layer is significant – 4 mm;

2) the thickness of the surface flow layer is 5 mm, there is no layer of constant capacity.

As an expression for the density distribution of the solid phase in the initial suspension, the Rayleigh law [1] was conditionally selected:

$$\varphi(\delta) = \frac{1}{2}\sqrt{\frac{\alpha}{\pi}}\exp(-\alpha\delta^2), \qquad (14)$$

and therefore the characteristic distribution function of the solid phase by mass takes the form

$$\Phi(\delta) = 1 - F(\delta), \qquad (15)$$

where

$$F(\delta) = 2\sqrt{\frac{\alpha}{\pi}}\int_0^\delta \exp(-\alpha\delta^2)d\delta. \qquad (16)$$

Assuming (conditionally) that the parameter $\alpha = 10^{10}$ m^{-2} in dependences (14), (16), we arrive at the value of the mean particle size $\delta_{av} = 4677 \cdot 10^{-6}$ m, characteristic of fine dispersions.

In the section $(0, z)$, taking into account the value of the clarification coefficient $\eta(z)$ (7), the relative mass of the solid phase $m_2(z)$ settling on the wall of the solid-state rotor is calculated (Fig. 2). In turn, based on dependencies (7), (14)–(16) for $z = L$ and different values of device productivity Q, the relative mass of the solid, settling on the rotor walls is determined (Fig. 3).

The relative change in the thickness of the sediment layer (and, hence, the slope of the guide surface of the sediment layer along the rotor axis) was determined according to Eq. (10), as reflected by the graphs (Fig. 4).

According to Fig. 2 (curve *1* corresponds to the calculation result for model 1 of the flow; and curve *3*, to model 2 of the flow), the dependence of the parameter $\eta(z)$, consistent with the physical meaning, on the parameters of the centrifugation process was found, namely (for given values of the regime parameters), the increase of $\eta(z)$ with increasing distance from the entrance to the rotor and an increase in the thickness h of the surface flow layer, as well as a decrease in η with an increase in the flow rate Q of the liquid and a decreasing thickness h.

From the analysis of the graphs (see Fig. 3), it follows that the clarification coefficient decreases with an increase in the flow rate Q of the liquid and a decrease in the thickness h of the surface flow layer.

In the case when the values of the axial coordinate are small ($0 < z < 0.04$ m), i.e., near the entrance to the rotor, an increase in the thickness of the sediment layer is observed at low liquid flow rates Q and high values of the thickness h of the surface flow layer (see Fig. 4). In turn, far from the entrance to the channel, the results of calculations are opposite to the previous case: the sediment layer increases at high values of the liquid flow Q and low values of the thickness h of the surface flow layer, which is revealed by analyzing the arrangement of the curves in the range of the axial coordinate $z > 0.07$ m.

Thus, if the surface flow layer is relatively small (model 1 of the flow), an insignificant increase in the sediment layer is observed near the entrance to the rotor; in the absence of a layer of constant capacity (model 2 of the flow), the sediment layer increases more. As a result, far from the entrance to the rotor (due to the balance condition of the precipitating solid), the tendency in accumulation of sediment on the rotor wall naturally develops in the opposite direction: the specific mass of the solid phase in the sediment according to model 1 of flow exceeds the corresponding specific mass value calculated based on model 2. In general, far from the entrance to the rotor, stabilization of sediment accumulation process is noted in calculations for both flow models.

Based on the results of the numerical experiment for a device with an elongated rotor ($L/D_{rt} = 10$), a strong dependence of the calculation result on the selected hydrodynamic model of the suspension flow in the centrifuge rotor was revealed. In the field of real values of the process parameters for different models, the difference in the results of numerical analysis for long rotors reaches 10% (see Fig. 3), for short rotors this difference is even greater.

Quantitative analysis of the kinetics of the process of separation of the suspension in the working volume of a tubular centrifuge, based on the commonly used flow model 2 (i.e., assuming that the entire fluid system moves in the rotor [1, 2, 7, etc.]) leads to overestimated data on the effectiveness of the process. Since model 1 is based on the refined flow structure, the calculation results should be carried out taking into account the data on the thickness of the surface flow layer.

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