

UNSTEADY PRESSURE DYNAMICS OF POLYMER FLOODING RESERVOIRS CONSIDERING CONCENTRATION CHANGES

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In this paper a model is established for unstable seepage flow with polymer concentration and pressure diffusion coupling, considering the effects of polymer molecular diffusion, adsorption, and viscoelasticity of polymer solution in the formation. The factors are close to the actual seepage parameters of the injected reservoir. For the nonlinear adsorption, the combined variable and the analytical iterative method are used to obtain the approximate analytical solution of the model. According to the concentration model, the relationship between concentration and pressure distribution is obtained. Using the model theory curve to fit the well test data, the seepage parameters of the formation are obtained, and the reflection characteristics of the unstable wellbore pressure derivative curve are analyzed.

Keywords: *molecular diffusion, nonlinear adsorption, viscoelasticity, combined variables, analytical solutions.*

Introduction

Savins [1] has conducted detailed research on the flow of non-Newtonian fluids such as polymers in porous media. Van Poolen et al. [2] published a theoretical analysis that discussed the effect of shear rate on viscosity and considered viscosity as a geometrically distributed function. The partial differential equations were solved by a finite difference numerical method. The results showed that the transient pressure response

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characteristics of non-Newtonian flow are very different from those of Newtonian flow. Bondor et al. [3] gave a numerical simulation of polymer flooding in composite reservoirs but did not take into account the transient flow of the fluid. Odeh and Yang [4] derived the wellbore pressure response solution of the Laplace transform space of a non-Newtonian power law fluid. Ikoku and Ramey [5] considered the influence of wellbore storage and the skin effect and obtained the Laplace transform space wellbore pressure response solution of a non-Newtonian power law fluid. Lund and Ikoku [6] studied the unstable pressure dynamics of non-Newton/Newtonian fluid reservoirs. Xu et al. [7] proposed the infinite reservoir Laplace space spherical transient pressure solution and discussed the characteristics of wellbore pressure at an early stage and later times of flooding. In the above studies, the unstable seepage of the polymer solution in the formation and establishment of the well test method model considered only the influence of shear rate on viscosity of the polymer solution, and the influence of viscoelasticity of the polymer and the concentration diffusion were rarely considered. The diffusion and adsorption of the solution in the formation can cause significant changes of concentration distribution of the polymer in the formation, resulting in a change in the viscosity of the polymer. Viscosity degradation and the viscoelastic effects of the polymer solution determine the pressure distribution in the reservoir and the complexity of the pressure diffusion equation. The concentration distribution and variation law are the basis for determining the pressure diffusion equation.

In the past, the analytical solution was used for the one-dimensional flow problem of distribution and variation of polymer concentration. Perkins and Johnston [8] discussed the diffusion and dispersion phenomena in a porous medium. Coats and Smith [9] studied the relationship between mechanical dispersion and velocity, and Brigham [10] proposed a correlation model for polymer concentration and diffusion of the constant diffusion coefficient, and obtained an analytical solution in one-dimensional space. Cheng et al. [11] established a well test analysis method for polymer flooding composite reservoir and a model considering the influence of shear rate and polymer concentration on polymer viscosity but did not take into account the effects of polymer viscoelasticity, adsorption, and formation permeability changes. There are few models for considering molecular diffusion and nonlinear adsorption in radial systems, usually solved by numerical methods. However, it was shown [12] that due to the influence of sharp concentration fronts, the numerical solution resulted in serious numerical divergence. Therefore, establishing a concentration model and accurately determining its distribution and variation is important for accurately determining the pressure conductivity model.

In this paper, by integrating the influence of various factors, the coupled seepage model of concentration and pressure is developed to explain the unstable well test data that the traditional model cannot match.

Concentration and pressure coupled seepage model

Assume that the fluid flow and adsorption in the formation are isothermal; fluid and formation pore media are microcompressible; adsorption is irreversible and complies with the Langmuir isotherm adsorption model; short-term injection does not cause thermal degradation; the polymer solution flow complies with the generalized Darcy law; the inaccessible pore volume could be neglected; the formation is isotropic and homogeneous; the fluid flow is radial flow; the density change caused by the polymer is neglected; and the effect of the capillary force is ignored.

The convection flux density of the polymer solution in the formation is proportional to the concentration of the polymer C_p . Polymer molecular diffusion flux is proportional to the polymer solution concentration gradient ∇C_p .

The adsorption of polymer in the formation medium is assumed to be a monolayer single layer adsorption on the solid surface, assuming q is the ratio of the adsorption area of the solid to the total surface area of the solid

$$q = \frac{a_c C_p}{1 + b_c C_p} \quad (1)$$

To establish the pressure-diffusion equation of the polymer flooding reservoir, as with the Newtonian fluid, it is necessary to consider the change in permeability due to adsorption, the change in fluid viscosity due to shear, polymer concentration changes, and polymerization, and also the effect of viscoelasticity of the solution.

The change in permeability caused by adsorption is expressed by the permeability reduction coefficient R_k derived as the ratio of the effective permeability of the formation before and after the injection of the polymer solution:

$$R_k = 1 + \frac{(R_{keq} - 1) b_k C_p}{1 + b_k C_p} \quad (2)$$

Studying the rheological model of different concentrations of polymer solution for practically applicable ranges of polymer concentration generally shows that the rheological curves in a double logarithmic scale demonstrate the typical characteristics: when the shear rate is very low, the polymer solution behavior is similar to that of a Newtonian fluid, and the apparent viscosity does not change with the shear rate. Then, when the shear rate increases up to a certain extent, the apparent viscosity decreases almost linearly with increase in the shear rate, which is typical for a pseudoplastic fluid. When the rate continues to rise to a larger value (generally over 10^4 s^{-1}), the Newtonian fluid characteristics are again displayed. The viscosity at a zero shear rate is derived as μ_0 , apparent viscosity as μ_p , and intrinsic viscosity μ_∞ , respectively, and $\mu_0 > \mu_p > \mu_\infty$. It can be seen that the polymer solution exhibits complex rheological properties, and within a certain range the rheological behavior can be reflected by the power law mode. However, the Meter model can better reflect the relationship between viscosity and shear rate over the entire shear rate range.

The apparent viscosity μ_p at a given shear rate can be determined from μ_0 using the Meter formula:

$$\mu_p = \mu_w + \frac{\mu_0 - \mu_w}{1 + \left(\frac{v}{V_{1/2}}\right)^{Pa-1}} \quad (3)$$

where

$$\mu_0 = \mu_w \left[1 + (A_1 C_p + A_2 C_p^2 + A_3 C_p^3) \right] \quad (4)$$

In the formula, $\nu_{1/2}$ and P_a are both dependent on the molecular weight, degree of hydrolysis, and concentration of the polymer and added salts in the solution. However, it can be seen from the experimental analysis that the parameters in the Meter formula are related to the zero shear viscosity of the polymer. The parameter P_a reflects the structural characteristics of the polymer. For a partially hydrolyzed polyacrylamide of the linear structure type of the same flexible chain, the parameter P_a is insensitive to changes in solution conditions (such as polymer concentration, solvent, and temperature). The parameter $\nu_{1/2}$ comprehensively reflects the pseudoplastic characteristic of the polymer solution.

Considering the viscoelastic properties of the polymer solution, Masuda [13] proposed the Darcy term to describe the viscosity of the polymer as

$$\mu_{aq} = \mu_p + \mu_{elas} = (1 + C^* \nu^m) \mu_p \quad (5)$$

where C^* is related to the relaxation time of the polymer solution, and m is a constant depending on the complexity of the pore geometry of the porous medium.

On this basis, based on the principle of material balance, the concentration diffusion equation is transformed into a dimensionless form:

$$\frac{d\left(\zeta' \frac{dC_{pD}}{d\zeta'}\right)}{d\zeta} = -\left(1 + \frac{1-\varphi}{\varphi} \frac{\rho_\gamma}{\rho_p}\right) \zeta' \frac{dC_{pD}}{d\zeta'} + F \frac{dC_{pD}}{d\zeta'} \quad (6)$$

where the dimensionless concentrations are

$$C_{pD} = \frac{C_p}{C_0}; \quad \zeta' = \frac{r^2}{4Dt} = \frac{r_D^2}{4t_D}; \quad a_c = aC_0; \quad b_c = bC_0; \quad q' = \frac{a_c}{(1 + b_c C_{pD})^2}$$

where C_0 is the initial concentration of the polymer solution, $F = \frac{Q}{4\pi h \varphi D}$; $r_D = r/r_w$, $t_D = Dt/r_w^2$ is the molecular diffusion coefficient, and r_w is the actual wellbore radius.

The initial and boundary conditions are

$$C_{pD} = 0(\zeta') \quad (7)$$

$$C_{pD} = 1(\zeta') \quad (8)$$

Since q' contains the function C_{pD} , it is first linearized, so that the concentration loss caused by the adsorption term is firstly ignored. The initial iteration concentration is obtained:

$$C_{pD} = \frac{\Gamma(F/2, \zeta')}{\Gamma(F/2)} \quad (9)$$

where $\Gamma(\frac{F}{2})$ and $\Gamma(\frac{F}{2}, \zeta')$ are Gamma functions and incomplete Gamma functions, respectively.

Substituting the initial iteration concentration equation (9) into q to solve Eq. (6), the concentration distribution and variation can be expressed as

$$C_{pD} = 1 - AA \int_0^{\zeta'} u^{\frac{F}{2}-1} \exp(-u) \exp\left(\int_0^u kq' ds\right) du \quad (10)$$

where $k = \frac{(1-\varphi)\rho_r}{\rho_p\varphi}$; $AA = \left[\int_0^\infty u^{\frac{F}{2}-1} \exp(-u) \exp\left(\int_0^u kq' ds\right) du \right]^{-1}$.

The above integral can be calculated by the Gauss numerical integration method.

The theoretical curve of the concentration diffusion model is shown in Fig.1 ($k=5, F=3.0$).

Based on the principle of material balance, the dimensionless pressure diffusion equation of the formation is obtained:

$$\frac{d\left[\zeta R_0(\zeta) \frac{dp_D}{d\zeta}\right]}{d\zeta} = \zeta \frac{dp_D}{d\zeta} \quad (11)$$

The corresponding boundary conditions and initial conditions are as follows:

$$\left(R\zeta \frac{\partial p_D}{\partial \zeta} \right)_{\zeta \rightarrow 0} = -\frac{1}{2} \quad (12)$$

$$p_D = 0 (\zeta \rightarrow \infty) \quad (13)$$

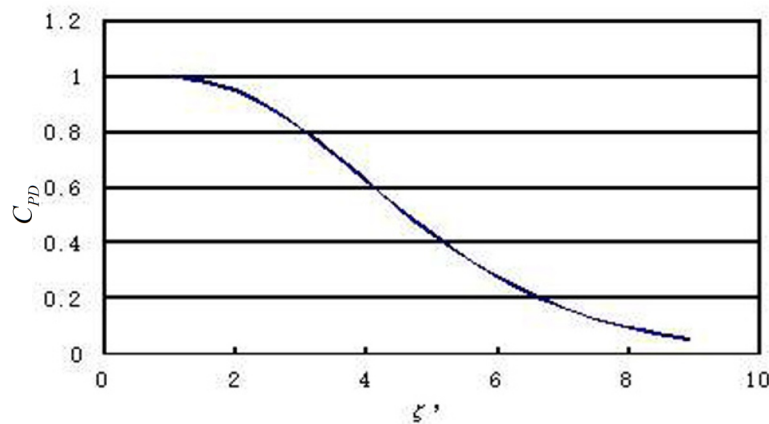


Fig.1. Theoretical curve of polymer concentration distribution change.

where the dimensionless quantities are defined as follows:

$$p_D = \frac{p - p_0}{\frac{Q \mu_w}{2 \pi K h}}; \quad t_D = \frac{\eta t}{r_w^2}; \quad \zeta = \frac{r_D^2}{4 t_D}$$

The pressure transmission coefficient $\eta = \frac{\phi \mu_w C_t}{K}$, the relationship between ω and ω^s in the concentration model is

$$\zeta = \frac{D}{\eta} \zeta', \quad R = \frac{R_1}{R_2}; \quad \frac{1}{R_1} = \left[1 + \frac{\frac{\mu_0 - 1}{\mu_w}}{1 + \left(\frac{v}{v_{1/2}}\right)^{pa-1}} \right] (1 + C^* v^m); \quad R_2 = R_K$$

The corresponding boundary conditions and initial conditions are as follows:

$$\left(R \zeta \frac{\partial p_D}{\partial \zeta} \right)_{\zeta \rightarrow 0} = -\frac{1}{2} \tag{14}$$

$$p_D = 0 (\zeta \rightarrow \infty) \tag{15}$$

Equation (11) is solved using an iterative method. The effect of shear viscosity is ignored, and the viscosity change caused by concentration is considered.

Assume $R_0 = \frac{1}{R_2}$. Since the concentration is only related to ζ , then R_0 is only related to ζ , and on both sides of Eq. (11) it can be seen that the pressure is only related to ζ . Then Eq. (11) is transformed into

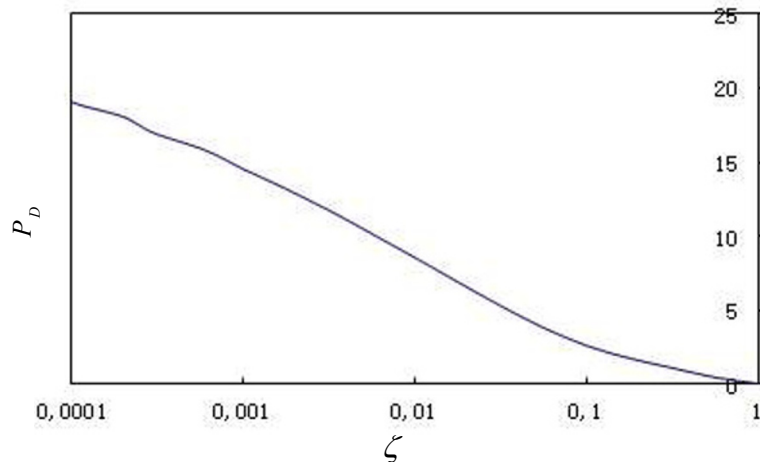


Fig.2. Semi-logarithmic curve between dimensionless pressure and ζ .

$$\frac{d(\zeta R_0(\zeta) \frac{dp_D}{d\zeta})}{d\zeta} = -\zeta \frac{dp_D}{d\zeta} \quad (16)$$

Setting , the general solution of the above homogeneous equation is

$$Y_0 = -\frac{1}{2} \exp \left[-\int_0^\zeta \frac{1}{R_0(u)} du \right] \quad (17)$$

The pressure versus ω derivative of Eq. (16) is approximated as the pressure derivative of Eq. (11), so Eq. (11) is written in the form

$$\frac{\partial \left(\frac{RY_0}{R_0} \right)}{\partial \zeta} + \frac{Y_0}{R_0} = t_D \frac{\partial p_D}{\partial t_D} = R_3(\zeta, t_D) = \frac{\partial R_1}{\partial \zeta} Y_0 + \frac{(1-R_1)Y_0}{R_0} \quad (18)$$

and

$$R_3(\zeta, t_D) = \left[(A'_1 + 2A'_2 C_{pD} + 3A'_3 C_{pD}^2) f - (1 + A'_1 + A'_3 C_{pD}^2 + A'_3 C_{pD}^3) \frac{\partial f}{\partial \zeta} \right] \frac{Y_0}{f^2} + (1-R_1) \frac{Y_0}{R_0} \quad (19)$$

where $b = b_k C_0$, $A'_1 = A_1 C_0$, $A'_2 = A_2 C_0$, $A'_3 = A_3 C_0$.

The equivalent shear rate of porous media is derived from certain simplification conditions:

$$v \approx \frac{D_1 V}{(K\phi)^{1/2}} = \frac{D_1 Q}{2\pi h r \phi (K\phi)^{1/2}} \approx \frac{D_2}{r} \quad (20)$$

$$\text{where } D_2 = \frac{D_1 Q}{2\pi h \phi (K\phi)^{1/2}}, \quad f = \left[1 + \frac{\frac{\mu_0}{\mu_w} - 1}{1 + \eta_c (\zeta t_D)^{\frac{1-p_a}{2}}} \right] \left(1 + C^{**} (\zeta t_D)^{\frac{-m}{2}} \right), \quad \text{and} \quad \eta_c = \left(\frac{2r_w D_2}{v_{1/2}} \right)^{p_a-1}, \quad C^{**} = C^* (2r_w D_2)^m$$

Then Eq. (11) is solved regarding pressure distribution and variation in the formation as

$$p_D = \int_\zeta^\infty \left\{ \frac{1}{uR} \exp \left(-\int_0^u \frac{1}{R} ds \right) \left[\frac{1}{2} - \int_0^u R_3 \exp \left(\int_0^s \frac{1}{R} dl \right) ds \right] \right\} du \quad (21)$$

To get the wellbore pressure change, simply bring $r = r_w$ into the variable ω of Eq. (11). The above integral can be calculated by high-precision Laguëll numerical integration.

Figures 2 and 3 present the dimensionless pressure and Boltzmann variables, the dimensionless pressure and dimensionless time under wellbore conditions, and the dimensionless pressure derivative and dimensionless time curve under wellbore conditions.

Figure 2 is a model curve of the dimensionless pressure corresponding to parameters $P_a = 1.5$, $b = 1.5$, $A_1 = 1.0$, $A_2 = 2 \text{ cm}^3/\text{g}$, $A_3 = 1.0 \text{ cm}^6/\text{g}^2$, $m = 1.3$, $v_{1/2} = 5.0 \text{ s}^{-1}$, $C^{**} = 0.2$, $\eta/D = 100$, $R_{keq} = 2.0$, and ζ in semi-logarithmic coordinates. Figure 3 is the model curve of the corresponding dimensionless wellbore pressure derivative and dimensionless time in double logarithmic coordinates.

It can be seen from Fig. 3 that the pressure increases with increase in ω . As shown in Fig. 3, there is a peak in the early stage of the pressure derivative curve, and then it upturns, but the increase becomes more and more slow. At the early stage of injection, the seepage resistance is smaller. As the polymer injection time increases, the seepage resistance becomes larger and larger, showing the radial seepage characteristics of a non-Newtonian flow. The slower amplitude is the viscosity shear reduction and viscoelastic effect. It is caused by the increase in distance.

Fitting analysis of well test data in injection wells

The GX15-13 polymer daily injection quantity is $Q = 1331 \text{ cm}^3/\text{s}$, pressure is recorded for more than 20 h using a small-diameter storage pressure gauge, initial pressure is $p_0 = 1.2 \times 10^6 \text{ Pa}$, and the main parameters of well and stratum are as follows: water viscosity $\mu_w = 1.2 \times 10^{-3} \text{ Pa}\cdot\text{s}$, effective thickness $h = 1500 \text{ cm}$, formation porosity $\phi = 0.21$, comprehensive compression coefficient $C_t = 1200 \text{ Pa}^{-1}$, wellbore radius $r_w = 10 \text{ cm}$, injected polymer concentration $C_0 = 0.85 \text{ g/cm}^3$, $A_1 = 1.2$, $A_2 = 2.3$, $A_3 = 0.8$, $b = 1.6$, and $R_{keq} = 1.9$.

By fitting the measured pressure data to the above theoretical curve, the double logarithmic fitting of the wellbore pressure derivative is shown in Fig.4, where square points represent actual test data and graphs represent model theoretical curves. The calculated formation permeability is $K = 0.665 \mu\text{m}^2$, the Meter viscosity model index is $P_a = 1.65$, and the elastic viscosity index is $m = 1.45$. It can be seen that the measured curve is basically consistent with the theoretical curve. From the logarithmic curve of the pressure derivative, the early

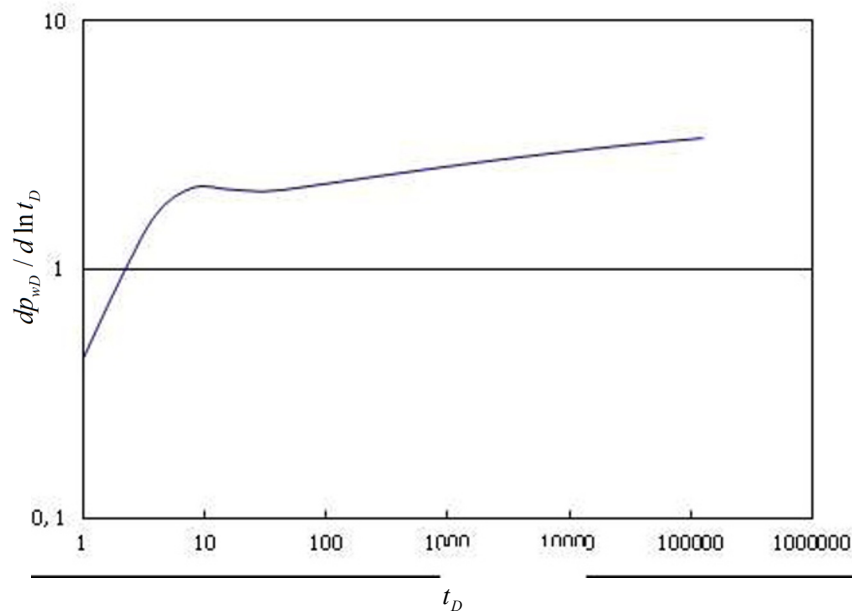


Fig.3. Double logarithmic curve between dimensionless wellbore pressure derivative and time.

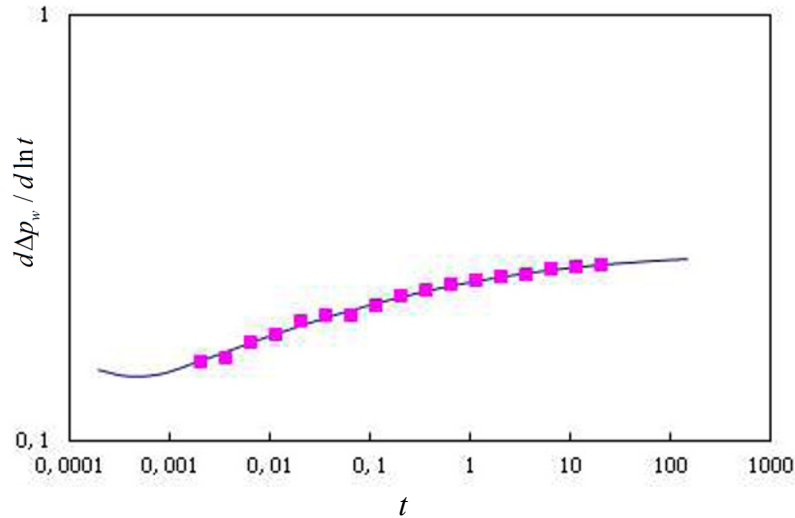


Fig. 4. Measured pressure derivative double logarithmic fitting.

stage is almost straight up, showing the reflection characteristics of a non-Newtonian fluid flow. The viscosity of the polymer solution is small due to shearing around the wellbore. As the distance increases, the shearing effect is weakened, the viscosity of the polymer solution increases, the flow capacity decreases, and the shear and viscoelasticity play a major role; but in the middle and late stages, the pressure derivative upset becomes slower, indicating that the farther away from the well, the smaller the local polymer concentration, the smaller the viscosity, and the higher the permeability. When the flow capacity increases, the non-Newtonian effect is weakened, and the pressure derivative curve is slowed down. From this, it can be seen that the pressure derivative characteristics are different from the power law fluid pressure derivative characteristics, and the pressure derivative of the power law fluid is always up-lined.

Conclusions

The coupling equation of concentration diffusion and pressure diffusion in polymer flooding reservoir is established, which overcomes the error caused by the assumption that the polymer concentration is constant in the unstable pressure diffusion model of a power law fluid.

Nonlinear adsorption of polymer solutions is considered in the polymer concentration diffusion. An iterative method is used to find an approximate analytical solution for the pressure diffusion model.

In the pressure diffusion model parameters, the influence of various factors, including the concentration change, the permeability change, and the variable model of the viscosity function affecting the viscoelasticity and the concentration function of the broad shear rate range are fully considered. The resulting model theoretical curve is closer to the reservoir pressure dynamics, and the well and formation parameters calculated by fitting the well test data are more accurate.

ACKNOWLEDGMENTS

This work is supported by National Natural Science Foundation (No. 51974247, No. 41502311, No. 51874241), Key Laboratory Research Project of Education Department of Shaanxi Province

(No. 15JS088), and Shaanxi Basic Research Projects in Natural Sciences (No. 2018JM5054, No. 2019JQ-807).

NOMENCLATURE

- D — molecular diffusion coefficient, cm^2/s ;
 φ — formation porosity, fraction;
 C_p — polymer solution concentration, g/cm^3 ;
 C_0 — initial concentration of polymer solution, g/cm^3 ;
 V — apparent flow rate, cm/s ;
 q — the adsorption rate, dimensionless;
 a_c, b_c — Langmuir isotherm adsorption coefficient, cm^3/g ;
 r — radial distance, cm ;
 r_w — wellbore radius, cm ;
 ρ_p — polymer solution density, g/cm^3 ;
 ρ_a — rock density, g/cm^3 ;
 Q — injection amount of polymer solution, cm^3/s ;
 h — effective thickness of the formation, cm ;
 R_k — permeability reduction coefficient, dimensionless;
 R_{keq} — maximum permeability reduction coefficient, dimensionless;
 b_k — parameters in the permeability reduction coefficient formula, cm^3/g ;
 i_p — apparent viscosity of the polymer solution, PaRs;
 μ_0 — zero shear viscosity of the polymer solution, PaRs;
 μ_w — water viscosity, PaRs;
 μ_{aq} — viscosity of the polymer solution as it flows through the pores considering the effect of viscoelasticity, PaRs;
 p — pressure, Pa;
 p_0 — initial pressure, Pa;
 t — injection time, s;
 ν — shear rate, s^{-1} ;
 $\nu_{1/2}$ — shear rate corresponding to the average of zero and infinite shear viscosity, s^{-1} ;
 P_a — Meter viscosity model index, dimensionless;
 μ_{elas} — viscoelastic viscosity of the polymer solution, PaRs;
 N_{deb} — Deborah number, dimensionless;
 C^* — coefficient in the viscoelastic expression, dimensionless;
 m — elastic viscosity index, dimensionless;
 t_{fr}, t_{pr} — respectively the relaxation time of the fluid and the characteristic time of the flow, s;
 K — effective permeability of the formation before injection of polymer solution, cm^2 ;
 η — pressure transmission coefficient, cm^2/s ;
 C_t — formation comprehensive compression coefficient, Pa^{-1} ;
 D_1 — coefficient of the relationship between the equivalent shear rate and the apparent velocity, dimensionless;
 $A_1, A_2, \text{ and } A_3$ — influence coefficients of concentration on viscosity, cm^3/g , cm^6/g^2 , cm^9/g^3 .

SUBSCRIPTS

- D — dimensionless;
 p — polymer solution.

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