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MATHEMATICAL DESCRIPTION OF GAS DRAINAGE RADIUS FOR UNDERGROUND GAS STORAGE

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Most underground gas storage in China encounters the problem of evaluating gas drainage. In order to evaluate the gas reservoir parameters, we propose a mathematical model for calculating the gas drainage radius. The model describes the gas seepage mechanism based on material balance, and lets us evaluate the gas drainage radius for any well pattern and also to calculate the optimal production parameters in a limited production time range. Experimental results have demonstrated the accuracy of the model. We have conducted studies to assess the nature of the dependence on the drainage radius on various reservoir parameters. The results have shown that the radius depends on multiple variables. For example, the radius increases as production time and the rock permeability increase. The field tests conducted have shown that the radius values calculated based on the mathematical model quite accurately match the actual parameters for existing underground gas storage at the Dagang field.
Key words: *underground gas storage, drainage radius, reservoir volume, well pattern, seepage mechanism, material balance.*

In the past 20 years, 25 underground gas storage (UGS) reservoirs have been constructed and put into operation in China with total volume 4.7 billion m^3, which made it possible to significantly improve the

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technology for safe gas storage and transport. Nevertheless, existing capacities cannot satisfy the growing demands for natural gas. With start of construction of the gas pipeline from Russia to China [2], the need arises for increasing gas storage capacity.

Traditionally, high-permeability reservoirs are used to construct underground gas storage, but formations with appropriate geological characteristics have already been used in existing projects [3]. It is becoming important to construct gas storage based on low-permeability reservoirs, but difficulties arise in ensuring optimal gas injection and production rates within a limited time period. For operation of underground gas storage in high-permeability formations [4-7] or in shallow formations [8, 9], a number of special wells are drilled, but for low-permeability reservoirs such a well pattern may prove to be ineffective. An obvious difference between gas production and storage reservoirs involves the different production times, pressure intervals, and well types, and so this is why an effective well pattern for producing reservoirs may be ineffective for underground gas storage. For example, the life of a producing reservoir is usually 20-30 years [10, 11], while the gas production period in underground gas storage ranges from 90 days to 150 days and the injection time ranges from 170 days to 230 days. In normal reservoir development, the operating pressure interval uniformly decreases from the initial reservoir pressure to the depletion pressure [12-14], but the operating pressure interval for underground gas storage varies between the maximum and minimum pressure. Wells in producing reservoirs are designed for gas production, while wells in underground gas storage are used both for injecting gas and for producing gas.

Existing methods for evaluating the gas drainage radius and well patterns do not take into account the intrinsic differences between gas-producing reservoirs and underground gas storage. The need arises for an accurate method for designing the optimal well pattern based on a quantitative evaluation of the gas drainage radius, within a limited time period and for different pressure intervals.

The drainage radius for a well in gas storage is defined as the distance from the center of the well to the flow boundary in the reservoir. The reservoir volume within the boundaries of the drainage radius corresponds to the quantity of gas flowing from the reservoir to the wellbore within a certain time interval. Pressure drawdown at the well boundary propagates within the reservoir volume, and the pressure distribution dynamically changes during the entire gas storage period. The average pressure within the boundaries of the drainage radius decreases from the maximum operating pressure to the minimum value. The ultimate bottom-hole pressure should be sufficient to ensure stable operation of the reservoir and also to uplift gas to the surface.

The drainage radius of an individual well should be equal to the theoretical drainage radius. If the actual radius is significantly larger than the theoretical drainage radius, this means that the gas volume within the boundaries of the actual drainage radius cannot be completely produced within a limited time interval, i.e., only some of the reservoir volume is used for gas storage and the working volume of the underground gas storage is significantly reduced. On the contrary, if the actual drainage radius is significantly smaller than the theoretical radius, the gas is produced until the well is depleted, the well is shut in, and the well capacity is not optimally utilized. Obviously for optimal operation, the actual drainage radius should be close to the theoretical radius. The aim of this work was to evaluate the theoretical gas drainage radius for an individual well in underground gas storage.

The model was developed for the gas production process, taking into account the following initial assumptions:

• we consider the gas storage reservoir as a volume of cylindrical shape, with a single well located at the center of the cylinder;

• the reservoir has sufficient thickness in the vertical direction;

• the formation within the drainage radius is isotropic and homogeneous;

• the reservoir has good connectivity;

• we consider gas flow as single-phase fluid flow;

• gas production in the well occurs at a constant rate.

Based on the definition of drainage radius and the initial assumptions, the problem is reduced to calculations for a single well producing gas at a constant rate from a cylindrical reservoir with a closed boundary.

The problem was solved as follows. First of all, we introduced an equation for unstable radial gas flow. Secondly, we determined the boundary conditions for the equation: the inner boundary condition is constant rate of gas production, and the outer boundary condition is a closed boundary for the reservoir. Thirdly, the maximum operating pressure is assumed to be the initial pressure. We obtain the partial differential equation (1) for gas percolation [15] for the boundary conditions:

$$\begin{cases} \dfrac{\partial^2 P_p}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial P_p}{\partial r} = \dfrac{\phi \mu c_i}{K}\dfrac{\partial P_p}{\partial t}, \quad r_w < r < R_e, \quad t > 0 \\ r\dfrac{\partial P_p}{\partial r}\bigg|_{r=r_w} = \dfrac{Q\phi}{2\pi Kh}, \quad t > 0 \\ \dfrac{\partial P_p}{\partial r}\bigg|_{r=R_e} = 0, \quad t > 0 \\ \varphi_{D_p}\left(r_D, t_D\right)_{t_D=0} = 0, \quad 1 \le r_D \le R_{e_D}, \quad t_D = 0 \end{cases} \tag{1}$$

where P_p is the pseudopressure, MPa^2/(mPa·s); r is the distance from down-hole, m; ϕ is the porosity of the formation, %; μ is the gas viscosity, mPa·s; c_t is the total compressibility of the formation, MPa^{-1}; K is the permeability of the formation, 10^{-3} μm^2; t is the production time, days; R_e is the drainage radius of the well, m; Q is the gas production rate under the reservoir conditions, m^3/day; $P_{P\max}$ is the maximum operating pseudopressure, MPa^2/(mPa·s); r_w is the wellbore radius, m.

In order to solve Eq. (1), we introduce four dimensionless parameters, as shown in Eq. (2). From Eq. (1), substituting the dimensionless quantities, we obtain Eq. (3):

$$\begin{cases} \varphi_{D_p} = \left(P_{p_{\max}} - P_p\right) / \left(P_{p_{\max}} Q_D\right) \\ Q_D = P_{sc} Q_{sc} T / \left(\pi K h P_{p_{\max}} T_{sc}\right) \\ t_D = Kt / \left(\phi \mu c_t r_w^2\right) \\ r_D = r / r_w \end{cases} \tag{2}$$

$$\begin{cases} \dfrac{\partial^2 \varphi_{D_p}}{\partial r_D^2} + \dfrac{1}{r_D}\dfrac{\partial \varphi_{D_p}}{\partial r_D} = \dfrac{\partial \varphi_{D_p}}{\partial t_D}, \quad 1 < r_D < R_{e_D}, \quad t_D > 0 \\ \left.\dfrac{\partial \varphi_{D_p}}{\partial r_D}\right|_{r_D=1} = -1, \quad t > 0 \\ \left.\dfrac{\partial \varphi_{D_p}}{\partial r_D}\right|_{r_D=R_{eD}} = 0, \quad t_D > 0 \\ \varphi_{D_p}\left(r_D, t_D\right)_{t_D=0} = 0, \quad 1 \le r_D \le R_{e_D}, \quad t_D = 0 \end{cases} \tag{3}$$

where φ_{D_p} is the dimensionless pressure; P_{sc} is the pressure under standard conditions, MPa; Q_{sc} is the gas rate under standard conditions, m^3/day; T_{sc} is the temperature under standard conditions, 293 K; Q_D is the dimensionless gas rate; t_D is the dimensionless time; r_D is the dimensionless distance from down-hole; R_{eD} is the dimensionless drainage radius.

In order to determine the boundary values, we use a Laplace transformation:

$$\begin{cases} \dfrac{d^2 \overline{\varphi}_{D_p}(s)}{dr_D^2} + \dfrac{1}{r_D}\dfrac{d\overline{\varphi}_{D_p}(s)}{dr_D} = s\varphi_{D_p}(s) \quad 1 < r_D < R_{e_D} \\ \left.\dfrac{d\overline{\varphi}_{D_p}}{dr_D}\right|_{r_D=1} = -\dfrac{1}{s} \\ \left.\dfrac{d\overline{\varphi}_{D_p}}{dr_D}\right|_{r_D=R_{eD}} = 0 \end{cases} \tag{4}$$

In Laplace coordinates, the analytical solution of Eq. (4) can be rewritten as an integral variable:

$$\overline{\varphi}_{D_p}\left(r_D, t_D\right) = \frac{\left[K_1\left(R_{e_D}\sqrt{s}\right)I_0\left(r_D\sqrt{s}\right) + I_1\left(R_{e_D}\sqrt{s}\right)K_0\left(r_D\sqrt{s}\right)\right]}{s^{3/2}\left[I_1\left(R_{e_D}\sqrt{s}\right)K_0\left(r_D\sqrt{s}\right) - K_1\left(R_{e_D}\sqrt{s}\right)I_0\left(r_D\sqrt{s}\right)\right]} \tag{5}$$

where I_0, I_1 are respectively the zero-th order and first-order modified Bessel functions of the first kind; K_0, K_1 are respectively the zero-th order and first-order modified Bessel functions of the second kind.

Using an inverse Laplace transformation, the expression for the dimensionless pressure variable as a function of space and time can be rewritten as

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$$\varphi_{D_p}\left(r_D,t_D\right)=\frac{2t_D}{R_{e_D}^2-1}+\frac{R_{e_D}^2}{R_{e_D}^2-1}\left(\ln\frac{R_{e_D}}{r_D}+\frac{r_D^2}{2R_{e_D}^2}\right)-\frac{3R_{e_D}^4-4R_{e_D}^2\ln R_{e_D}-2R_{e_D}^2-1}{4\left(2R_{e_D}^2-1\right)^2}-$$

$$-\pi\sum_{n=1}^{\infty}\frac{\mathrm{e}^{a_n^2 t_D}J_1^2\left(R_{e_D}a_n\right)\left[N_1\left(a_n\right)J_0\left(r_D a_n\right)-J_1\left(a_n\right)N_0\left(r_D a_n\right)\right]}{a_n\left[J_1^2\left(R_{e_D}a_n\right)-J_1^2\left(a_n\right)\right]} \tag{6}$$

where J_0, J_1 are respectively the zero-th order and first-order Bessel functions of the first kind; N_0, N_1 are respectively the zero-th order and first-order Bessel functions of the second kind.

We solve Eq. (6), substituting the expression for the bottom-hole pressure with r_D = 1; higher-order terms can be neglected since the value of R_D is much greater than 1. Thus for the dimensionless bottom-hole pressure, we obtain the equation

$$\varphi_{D_{P_w}}\left(t_D\right)=\frac{2t}{R_{e_D}^2}+\ln\frac{R_{e_D}}{r_D}-\frac{3}{4}+\frac{2\,\mathrm{e}^{a_1^2 t_D}J_1\left(R_{e_D}a_1\right)}{a_1^2\left[J_1^2\left(R_{e_D}a_1\right)-J_1^2\left(a_1\right)\right]} \tag{7}$$

In dimensional variables, Eq. (7) takes on the form:

$$\left(P_{p_{\max}}-P_{p_{wf}}\right)\frac{\pi KhT_{sc}}{P_{sc}Q_{sc}T}=\frac{2\eta t}{R_e^2}+\ln\frac{R_e}{r_w}-\frac{3}{4}-0.84\,\mathrm{e}^{14.682\frac{\eta t}{R_e^2}} \tag{8}$$

where T is the temperature, K; η is the pressure transmitting coefficient of the formation, $\eta = K/(\phi\mu c_t)$; P_{Pwf} is the dimensionless bottom-hole pseudopressure.

Expressing the average dimensionless reservoir pressure as an integral over the area, in Laplace coordinates we obtain the equation

$$\varphi_{D_{P_{averagew}}}\left(t_D\right)=\frac{1}{A\iint\varphi_{D_{P_w}}\left(r_D,t_D\right)dA} \tag{9}$$

where $\varphi_{D_{P_{averagew}}}$ is the dimensionless average pressure of the drainage area.

Applying an inverse Laplace transformation, the average reservoir pressure can be calculated by substituting Eq. (6) into Eq. (9):

$$P_{average}=P_{\max}-\frac{Q_{sc}tB_g}{\pi\phi c_t hR_e^2} \tag{10}$$

where $P_{\max}$, $P_{\min}$ are respectively the maximum and minimum operating pressure of the underground gas storage, MPa; B_g is the volume coefficient of the gas.

In the limiting case, the average pressure in Eq. (10) is equal to the minimum operating pressure. Substituting the minimum operating pressure into Eq. (10), we obtain:

$$P_{\max} - P_{\min} = \frac{Q_{sc} t B_g}{\pi \phi c_t h R_e^2} \tag{11}$$

If we simultaneously solve Eqs. (11) and (8), the rate parameter is eliminated. The mathematical expression for the theoretical drainage radius for gas stored in underground gas storage takes on the form:

$$\left[\frac{\left(P_{p_{\max}} - P_{p_{wfp}}\right) B_g T_{sc}}{P_{sc}\left(P_{\max} - P_{\min}\right) T} - 2\right] \frac{\eta t_p}{R_{e_p}^2} = \ln \frac{R_{e_p}}{r_w} - \frac{3}{4} - 0.84\,\mathrm{e}^{14.682 \frac{\eta t}{R_{e_p}^2}} \tag{12}$$

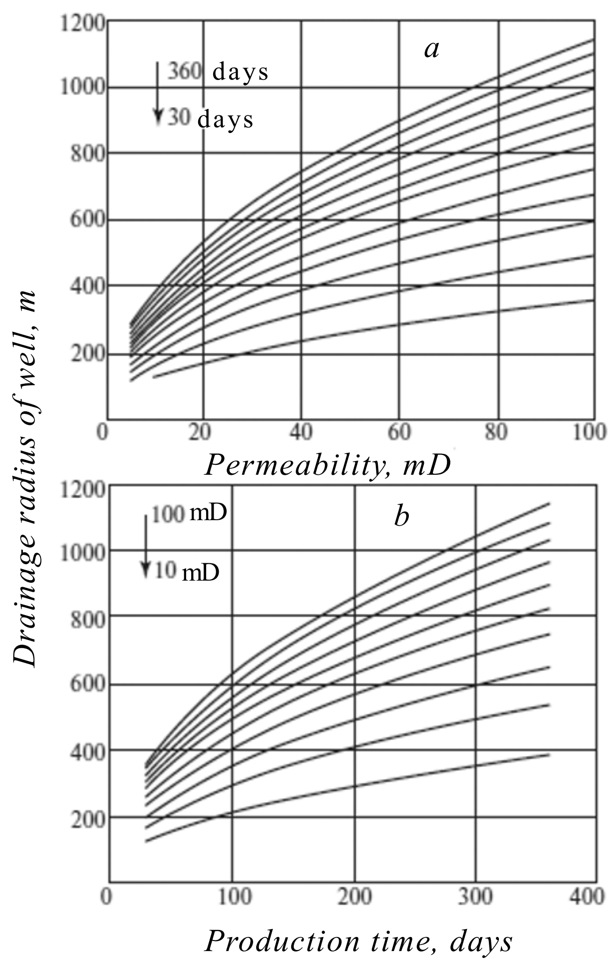

Fig. 1. Drainage radius vs. permeability (*a*) and production time (*b*).

Table 1

Well No.	Permeability, mD	Calculated drainage radius, m	Actual radius, m	Relative error, %
1	4	151	169	-11.9
2	9	220	197	10.5
3	13	245	266	-8.6
4	16	289	312	-8.0
5	25	356	328	7.9
6	30	365	402	-10.1
7	36	422	437	-3.6
8	42	427	447	-4.7
9	49	488	463	5.1
10	55	484	513	-6.0
11	64	553	583	-5.4
12	70	542	577	-6.5
13	81	618	634	-2.6
14	86	598	626	-4.7
15	92	617	669	-8.4
16	96	629	674	-7.2
17	100	683	672	2.6

where P_{Pwfp} is the dimensionless flowing bottom-hole pseudopressure in the gas withdrawal phase; R_{ep} is the drainage radius of the well in the withdrawal phase, m; t_p is the production time in the withdrawal phase, days.

Eq. (12) describes the relationship between the gas drainage radius and a number of reservoir parameters, such as the maximum and minimum operating pressure, the drawdown pressure, the gas production time, the permeability and temperature in the formation, etc. All these parameters together uniquely and quite accurately describe the reservoir. We need to determine the nature of the relationship between the drainage radius and each parameter. In this work, we have applied the numerical method of secant iteration [16] to calculate the drainage radius.

In order to evaluate the influence of the production time, we calculated the drainage radius for a number of values of the production time while keeping all the other parameters unchanged. Figure 1 shows the drainage radius vs. the permeability of the formation and vs. the production time.

In Fig. 1, we see that the drainage radius increases as the production time increases. When the production time is constant, the radius depends directly on the permeability of the formation. Therefore the production time is a critical parameter for controlling the drainage radius, and consequently determining the optimal gas production schedule will help control the drainage radius.

In the calculations given above, we also took into account the dependence of the drainage radius on the permeability of the formation. We see that as the permeability increases, the drainage radius also increases (see Fig. 1). Therefore, during operation of an underground gas storage in low-permeability reservoirs, in order to increase the gas depletion area we need to increase the number of wells.

In order to compare the calculated parameters with the actual characteristics of an underground gas storage, we selected the Dagang gas storage that was put into operation in 2000. Currently the Dagang underground gas storage is used to compensate for peak loads for the Shaan–Jing pipeline and to meet

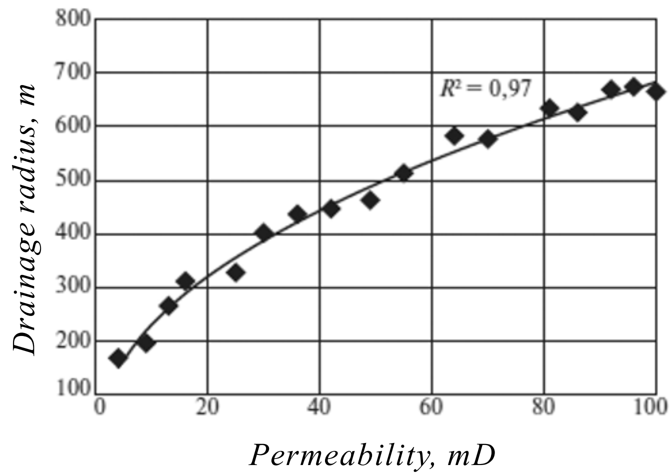

Fig. 2. Relationship between the calculated (line) and actual (points) drainage radius.

Beijing's demands. After 16 years of operation, the characteristic parameters for the underground gas storage, including the drainage area and the working gas volume, can be considered stable and constant, which makes the Dagang underground gas storage a good case study for verification of the model for calculating the drainage radius for actual wells.

Based on Eq. (12) and the parameters of the Dagang underground gas storage, we calculated and determined the calculated and actual values of the drainage radius for 17 wells. The actual values of the radius were determined by rate transient analysis [17] from the production data and the type of curves. The overall results are given in Table 1 and Fig. 2.

The results from comparison of calculated and experimentally determined values of the drainage radius let us conclude that the proposed model is sufficiently accurate. The average difference between the calculated and actual values is not greater than 12%, and for well 13 the difference between the values is less than 3%.

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