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A Simple Model for Spatially-averaged Wind Profiles Within and Above an Urban Canopy

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Abstract This paper deals with the modelling of the flow in the urban canopy layer. It critically reviews a well-known formula for the spatially-averaged wind profile, originally proposed by Cionco in 1965, and provides a new interpretation for it. This opens up a number of new applications for modelling mean wind flow over the neighbourhood scale. The model is based on a balance equation between the obstacle drag force and the local shear stress as proposed by Cionco for a vegetative canopy. The buildings within the canopy are represented as a canopy element drag formulated in terms of morphological parameters such as λ_f and λ_p (the ratios of plan area and frontal area of buildings to the lot area). These parameters can be obtained from the analysis of urban digital elevation models. The shear stress is parameterised using a mixing length approach. Spatially-averaged velocity profiles for different values of building packing density corresponding to different flow regimes are obtained and analysed. The computed solutions are compared with published data from wind-tunnel and water-tunnel experiments over arrays of cubes. The model is used to estimate the spatially-averaged velocity profile within and above neighbourhood areas of real cities by using vertical profiles of λ_f.

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1 Introduction

Modelling wind flow in urban areas has recently received much attention because of the increasing concern worldwide for the health effects of high pollution concentrations in cities. An important requisite of wind flow models is their ability to produce fast solutions as they need to be run many times for different meteorological conditions and building arrangements. Following Britter and Hanna (2003), an operational approach to the modelling of flow over complex geometry may be based on the identification of relevant spatial scales. The two obvious scales are the street (canyon) scale and the city scale. However there is an intermediate scale that is referred to as the neighbourhood scale. This is somewhat less precisely defined than the street scale and the urban scale as it is recognised that there are a number of scales between these two. In order to identify the neighbourhood scale we may need to identify some kind of homogeneity for the statistical properties of building characteristics. For instance, we may require that the standard deviation of mean building height is small. As a consequence, we can regard the neighbourhood area as an horizontally homogeneous urban canopy. For most cities, especially European, the neighbourhood scale is a spatial scale of the order of 1 km (Britter and Hanna 2003) over which a gross parameterisation of the flow can be achieved. By adopting this criterion, each neighbourhood is parameterised in terms of height-dependent morphological parameters which have a horizontal homogeneity. Such parameters are derived by means of statistical analysis of detailed building characteristics data. From this perspective each neighbourhood is the equivalent of a land use description of the city. Therefore, the urban area can be represented as a composition of horizontally homogeneous neighbourhoods, each with its own statistical characteristics. The neighbourhood scale is particularly important when seeking useful parameterisations for dispersion models. The estimation of dispersion of airborne pollutants and hazardous material on this scale is likely to require knowledge of the flow within and above the urban canopy (Britter and Hanna 2003). For practical purposes, computational fluid dynamics (CFD) models can be used for resolving the flow around every building but they are computationally expensive and site-specific. In order to calculate the drag effect of the flow field, it is necessary to use grid cells whose size is about 1 m or less. As a consequence a complete computation of the flow field for the entire city at the building resolving scale would require at least 10^{10}–10^{11} grid cells. For air quality applications, this calculation has to be repeated for each meteorological condition. However, full CFD simulations of flow over arrays of obstacles are increasingly carried out with a different aim; that of deriving simpler physical models e.g. Hamlyn and Britter (2005), suitable for incorporation into operational dispersion models. The present objective in an urban air quality context is to improve the prediction, using simple models, of wind velocity profiles at the neighbourhood scale within the urban canopy layer.

Some measurements of airflow from both field e.g. Louka (1998); Allwine et al. (2002, 2004); Arnold et al. (2004); Rotach et al. (2004); Mestayer and Coll (2005) and wind-tunnel and water-tunnel experiments (Kastner-Klein et al. 2000; Hall et al. 1998; Macdonald 2000; Cheng and Castro 2001) are available. Also, a review by Roth (2000) gives a good overview of available turbulence studies in urban areas. Due to advances in understanding flow and turbulence mechanisms, the number of modelling studies has recently increased. Simpler approaches to predict wind distribution within cities are also available (Bentham and Britter 2003; Coceal and Belcher 2004) but the application of these results to real cities still needs to be investigated. The objective of this paper is to derive a simple model for the flow within and above an urban canopy in which the canopy can be described in terms of building morphological parameters. The underlying idea of this work is similar in concept to that of Coceal and Belcher (2004) and in that it consists of representing the roughness elements within the urban canopy as a porous medium, and therefore permeable to the air flow. In our approach the wind is representative of a neighbourhood area and therefore, as in Coceal and Belcher (2004), the average wind velocity is equivalent to the spatially-averaged wind velocity over the area available for the flow. Each element within the urban canopy exerts a drag force on the local airflow. These effects on the spatially-averaged wind velocity are represented as a body force. This approach avoids the unnecessary detail and the large computational cost of accurately resolving the flow around each individual building.

The model investigated here is of a type proposed by Cionco (1965) for the vegetative canopy and later re-analysed by Macdonald (2000) for the urban canopy. Therefore, our model results in an extension of the above existing models for the prediction of spatially-averaged wind profiles within and above neighbourhood areas of real cities. A real city geometry differs greatly from cube arrays because of its building height variability that affects the flow and results in higher levels of turbulence at the top of the canopy than with cube arrays. In particular, this height variability will reduce the likelihood of skimming flow and enhance the fluid exchange between the canopy and the flow above. To capture this aspect of the flow is important when determining mixing and pollutant dispersion within the urban canopy.

Following the above objectives, the article is organised as follows: first, we briefly review Cionco's and Macdonald's models and second we discuss the changes made to those models to obtain our model. We derive some solutions suitable for arrays of cubes and discuss various mixing length formulations. Finally, we discuss some spatially-averaged wind profiles using morphological parameters of some neighbourhoods of European and North American cities. Morphological parameters are derived using Matlab© based image analysis of digital elevation models (DEMs) following Ratti et al. (2006).

2 Previous Models for Spatially-averaged Wind Profiles

The main objective of our model is the prediction of the spatially-averaged wind profile for real cities at the neighbourhood scale starting from a momentum balance equation in the urban canopy. The momentum balance equation was first used by Cionco (1965) for the vegetative canopy and then extended by Macdonald (2000) to an array of cubes as a first attempt to represent the urban canopy. We want to extend the approach further by including the variation of building heights typical of the urban canopy. The main observation made by Macdonald (2000) was that the drag depends on the obstacle packing density and therefore on the morphometric parameters λ_p and λ_f (the ratios of plan area and frontal area of buildings to the lot area, respectively) which for cube arrays are the same. In the model developed here we argue that the main difference between a cube array (typically studied in the wind-tunnel) and real cities is the variation of building heights within a neighbourhood. We therefore seek a morphometric parameter that reflects the building height variability and can be included within the same mathematical framework developed by Cionco and Macdonald. To better describe the main physical ideas of our model we review their work in the following two sections.

2.1 Cionco's Model

Cionco (1965) studied the properties of the turbulent momentum transfer in a vegetative canopy. He proposed a mathematical model for the effects of the drag of obstacles in terms of height, air density and shear stress. In Cionco's model the vegetative canopy is the region which extends from the ground to the top of the obstacles, e.g. the trees. Within this region it is assumed that there is a minimal vertical variation in the mixing length so that it can be taken as constant, while, above the canopy, it increases linearly with height. In Cionco's work it is assumed that the momentum loss inside the canopy is proportional to the square of the mean velocity U^2. This is equivalent to assuming that the expression for the drag coefficient C_D for flow around an obstacle is constant. The drag coefficient is a non-dimensional form of the drag force on the body i.e.:

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} \tag{1}$$

where F_D is the aerodynamic drag force, ρ is the air density, U is the magnitude of the flow velocity and A is the cross-sectional area of the obstacle. Values of C_D are usually obtained in wind-tunnels. Typically, the drag coefficient is a function of the Reynolds number. However, if the Reynolds number is large enough and separation of the flow at the sharp edges of the obstacle has already occurred, C_D is no longer a function of the Reynolds number. The drag coefficient is then constant. Equation 1 also indicates that for flow around sharp edged bodies at high Reynolds number the drag force on the obstacle is proportional to U^2. Thus it is equivalent to Cionco's assumption.

It should be noted here that while C_D is known for various isolated cuboid obstacles such as buildings, C_D for cities is generally not available. It might require specific measurements in wind-tunnels for each city. It is generally accepted that the overall non-dimensional drag force over a city can be expressed in terms of the surface roughness length z_0. The roughness length is another form of the drag coefficient in the sense that by rewriting Eq. 1 as:

$$C_D = \frac{F_D}{\frac{1}{2}\rho U_{\text{ref}}^2 A} \tag{2}$$

where A now represents a plan area of a part of the city and U_{ref} is a reference velocity measured at a reference height z_{ref} that is within the logarithmic region of the velocity profile. Therefore,

$$C_D = 2\frac{u_*^2}{U_{\text{ref}}^2} = 2\frac{1}{\kappa}\left(\ln\frac{z_{\text{ref}}}{z_0}\right)^{-2} \tag{3}$$

where u_* is the friction velocity. The right-hand side of Eq. 3 is valid for neutral atmospheric stability. For non-neutral flow a similar expression can be determined. Both C_D and z_0 quantify the momentum exchange between the urban surface and the atmosphere; an exchange that is influenced by the interference between individual buildings.

Returning to Cionco's arguments, as a result of the drag forces exerted on the obstacles there will be a momentum loss within the canopy. If we assume that there is no variation of the flow in the downstream direction, locally there will be a balance between the loss of momentum and the drag forces. Of course this momentum loss is replaced by a net momentum flux from above the urban canopy into the urban canopy. This is the basic fluid dynamic statement made by Cionco of which the model is a straightforward mathematical consequence.

From the assumption of a local equilibrium that is, a balance between the obstacle drag force and the local shear stress, the following equation may be derived as:

$$d\tau = \frac{1}{2}\rho C_D U^2(z) d\Phi(z), \tag{4}$$

where d is the differential operator, τ is the shear stress, $U(z)$ is the spatially-averaged velocity profile and $\Phi(z)$ is a nondimensional geometric factor that accounts for the different shapes of the obstacles e.g. the trees in Cionco's work. By using a Prandtl type closure and introducing a parameter B equal to $\frac{d\Phi}{dz}$, representing the effective aerodynamic surface of the trees per unit volume Cionco (1965), Eq. 4 can be re-written as:

$$\frac{d}{dz}\left(l_{\mathrm{vc}}(z)\frac{dU(z)}{dz}\right)^2 = \frac{1}{2}C_D B(z) U^2(z) \tag{5}$$

where $l_{\mathrm{vc}}(z)$ is the mixing length within the vegetative canopy and $U(z)$, the velocity profile within the canopy, is the velocity of air in the gaps between the trees. From the fluid dynamic point of view it can be shown by means of a control volume approach that there is a volume flux $A_{\mathrm{canopy}}V$ through the canopy, where A_{canopy} is the cross-sectional area. This defines the bulk velocity V. This is related to the spatially-averaged velocity bearing in mind that the measured velocity within the canopy is the velocity within the gaps between the obstacles at a particular cross-section. The measured velocity will have an average value of $U = \frac{A_{\mathrm{canopy}}V}{A_{\mathrm{canopy}} - A_{\mathrm{obstacles}}}$, where $A_{\mathrm{obstacles}}$ is the area occupied by the obstacles at a particular cross-section. Thus, U is proportional to the bulk velocity within the canopy.

Returning to Cionco's model derivation, if $l_{\mathrm{vc}}(z)$ and $B(z)$ are both constant with height and if we introduce σ_f to be

$$\sigma_f = \frac{C_D B}{2}, \tag{6}$$

Equation 5 can be re-written in the following way:

$$2U'(z)U''(z) = \frac{\sigma_f}{l_{\mathrm{vc}}^2}\left(U^2(z)\right) \tag{7}$$

where U' and U'' indicate the first and the second derivatives of U respectively. The integration of Eq. 7 leads to an exponential solution within the canopy that is

$$U(z) = U_H \exp\left(a\left(\frac{z}{H} - 1\right)\right), \tag{8}$$

where H is the vegetative canopy mean height and $U_H = U(H)$. The term a is the attenuation parameter introduced by Cionco and is defined in terms of the variables already introduced as:

$$a = \sqrt[3]{\frac{\sigma_f}{2l_{\mathrm{vc}}^2}}H. \tag{9}$$

The parameter a defined by Eq. 9 is not a universal constant, but is however limited in range and varies with the height of the roughness elements within the canopy, increasing with density and tree flexibility. For example, in Cionco (1972), this parameter is calculated for different types of vegetative canopy and falls in the range 2–3. The widely used dispersion model SCIPUFF (Sykes et al. 2004) assumes a default value for a equal to 2. Whether this is appropriate for urban canopies is however unclear. The determination of each type of canopy is based on the size of the planar surface of the leaves, height of the trees and canopy density. Equation 8 has been extensively validated against vegetative field data Cionco (1972) and proved to predict well the spatially-averaged wind profiles within vegetative canopies. Therefore the question we raise here is whether we can use Cionco's model for the prediction of the flow within the urban canopy. Given the simplicity of the model formulation it is straightforward to reinterpret the meaning of $B(z)$ and $\Phi(z)$ to describe the morphometry of the buildings.

2.2 Macdonald's Model

In Macdonald's work, buildings are treated as two-dimensional cylinders or cubes with sectional drag coefficient $C_D(z)$. It is assumed that at each horizontal cross-section there is a balance between the drag force due to the buildings and the local shear stress expressing the momentum loss. With the notation already introduced previously, Eq. 4 becomes:

$$d\tau = \frac{1}{2}\rho C_D(z)U^2\frac{dA_f}{A_T} \tag{10}$$

where $d\tau$ is the change in shear stress and dA_f is the portion of frontal area of the buildings between levels z and $z + dz$, and $C_D(z)$ is the drag coefficient at height z. A_T is the lot area per building or total ground surface area divided by the number of buildings. For buildings with a uniform cross-section we may write:

$$dA_f = \frac{dz}{H}A_f \tag{11}$$

where A_f is the total frontal area of the building. It is recognised that at the top of a homogeneous canopy, a shear layer forms. As a first approximation, we can make the assumption that the turbulent shear stresses within the urban canopy can be described by Prandtl's mixing length model. With this assumption, the momentum transport in the canopy, Eq. 10, can again be written as an ordinary differential equation:

$$\frac{d}{dz}\left[\left(l_c(z)\frac{dU}{dz}\right)^2\right] = \sigma_f(z)U^2(z) \tag{12}$$

with

$$\sigma_f = \frac{C_D(z)\lambda_f}{2H} \tag{13}$$

where we have now replaced $\frac{A_f}{A_T}$ with the parameter λ_f known as the building frontal area packing density (Grimmond and Oke 1999); $l_c(z)$ is the mixing length within the urban canopy. If $l_c(z)$ and $C_D(z)$ are taken as constant then Eq. 12 admits the same exponential solution, (Eq. 8), previously proposed by Cionco (1965). Equation 12 and its solutions (see Eq. 8), as already seen, depend on the attenuation coefficient a given formally by Eq. 9 but with the new σ_f given by Eq. 13. From his experiments Macdonald (2000) found an empirical relation for a, that is:

$$a = 9.6\,\lambda_f. \tag{14}$$

The above set of equations forms the basis of Macdonald's model and allows the prediction of wind profiles over arrays of three-dimensional surface buildings (cubes).

Figure 1 shows various profiles of the spatially-averaged wind velocity $U(z)$ within the urban canopy for different values of the parameter a. The curves in Fig. 1 refer to cubical buildings and show that as a increases (and therefore as λ_f increases), the drag force is larger, resulting in smaller velocity values close to the ground. By looking at Fig. 1 we observe that

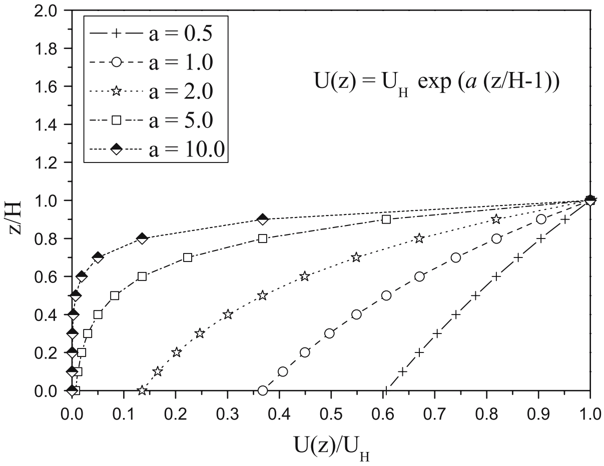

Fig. 1 Typical velocity profiles within the urban canopy as predicted by the exponential solution for different values of the attenuation coefficient *a*

the curves do not satisfy the no-slip condition at the ground level, as they must, because it has not been imposed as a boundary condition. In reality there will be a relatively thin (around $0.1H$ or $0.2H$) boundary layer near the ground to satisfy the no-slip boundary conditions. To pragmatically account for this effect we might assume a linear decrease in U below $0.1H$ or $0.2H$. Further CFD results would enable specific quantitative evaluation of this effect. From the figure we observe that for $a > 5$ there is a very small flow in the lower canopy so that the velocity values at ground level are almost zero. Macdonald's model has been validated against measurements of spatially-averaged velocity profiles within and over regular obstacle arrays with different packing densities. The comparison showed that the exponential solution fits data very well for relatively small area packing densities (Grimmond and Oke 1999). For large packing density (i.e. $\lambda_f > 0.30$), corresponding to situations close to the skimming flow regime (Oke 1978), the exponential solution is less satisfactory. However, we should keep in mind that these observations are made for flow over arrays of regular geometries and that different behaviour might occur in real cases. As mentioned earlier skimming flow is unlikely to occur in real cities because of their significant height variability.

3 Model for Real Urban Canopies

In Macdonald's work, spatially-averaged wind profiles in idealised urban-type building configurations can be obtained from the modification of Cionco's model for a vegetative canopy. Macdonald's extension is effective in predicting spatially-averaged mean profiles within cubical obstacle arrays. However, there are still a few difficulties that need to be addressed for a full application of the model to real urban areas. All Cionco types of models admit an exponential curve as a solution within the urban canopy. This derives from having adopted $U(H) = U_H$ as a boundary condition. The variable U_H is a poor choice of boundary condition as it is set in the shear layer characterised by large gradients of all measurable quantities. This is particularly true for real cities. Even though U_H is an important variable for setting the in-canyon layer velocity profile, its measurement close to the top of the buildings in real cities is problematic and typically affected by large errors. The estimation of the spatially-averaged velocity is difficult to be made meaningfully in the shear layer region of real cities as it is strictly dependent on the building spatial distribution and morphometry of the neighbourhood area considered. Furthermore, if the definition of U_H is well defined for arrays of cubes it is not for real cities. In this case we would need a dense distribution of meteorological stations within the city that would allow us to estimate the spatially-averaged velocity at the mean building height and this is not commonly available. Arrays of cubes are too crude a simplification of a real urban canopy as the buildings in real cities have different shapes and heights. Building heights and building height variability affect the flow field. This cannot be neglected when modelling the flow over a real urban area. A useful way of accounting for the spatial variability of building height is through the analysis of detailed building morphological data (Müller 1999; Ratti et al. 2002). An image based analysis technique of digital elevation models (DEMs) can be used to obtain parameters that are fluid-dynamically relevant, provided that the neighbourhood area has been correctly identified (Ratti et al. 2006). In the context of the development of a model for real cities, the estimation of λ_f as a function of z can be used as a key parameter to quantify the vertical building height variability over the neighbourhood area. Other parameters could be employed to quantify the vertical height variability such as the standard deviation of the building height. The parameter λ_f and $\lambda_f(z)$ is directly linked to the porosity of the city and therefore to the dilution potential of a given site. In particular $\lambda_f(z)$ is a powerful parameter as it still retains information about the vertical distribution of the buildings without much loss of details while for instance the standard deviation of the building height does not have this "property". More explicitly $\lambda_f(z)$ goes to zero at $H_{\max}$ that is the height of the tallest building within the chosen neighbourhood area. This leads to an elegant approach avoiding the use of U_H as a boundary condition but still accounting for vertical height variability. An important point here is that the model then allows for a direct connection with the wind flow well above the urban canopy rather than forcing a match within the urban canopy. Upper level wind data are easily accessible from numerical weather prediction (NWP) models for use as a model input.

As in Macdonald, the spatially-averaged wind profile is obtained by solving an equation for the variation in shear stress due to the drag forces on building elements. It reads as:

$$A_T d\tau = \frac{1}{2}\rho C_D U^2(z)\, dA_f \tag{15}$$

where $dA_f = (\sum L_f)dz$ with $\sum L_f$ the overall width of the buildings in the region dz and in the direction perpendicular to the flow. That means that we apply Cionco's type of argument but just for a thin layer of height dz. Figure 2 shows these quantities. The other symbols have the usual meanings.

Equation 15 can be re-written using a mixing length closure model for the turbulent transport in the canopy as:

$$\frac{d}{dz}\left(l(z)\frac{dU}{dz}\right)^2 = \frac{1}{2}C_D U^2(z)\frac{\lambda_f(z)}{H} \tag{16}$$

where we have used $\lambda_f(z) = \frac{H\sum L_f}{A_T}$.

Equation 16 is an ordinary second-order, non-linear differential equation with non-constant coefficients. Its solution depends upon the mixing length and in particular upon its variation with height. Besides, and more importantly the solution of Eq. 16 depends upon the shape of the λ_f vertical profile, $\lambda_f(z)$. To solve the equation and complete the model, a choice about the mixing length should be made. Conversely, in our approach $\lambda_f(z)$ is directly derived from the analysis of DEMs. Even though the present model is essentially the same as those seen before, it has advantages described below:

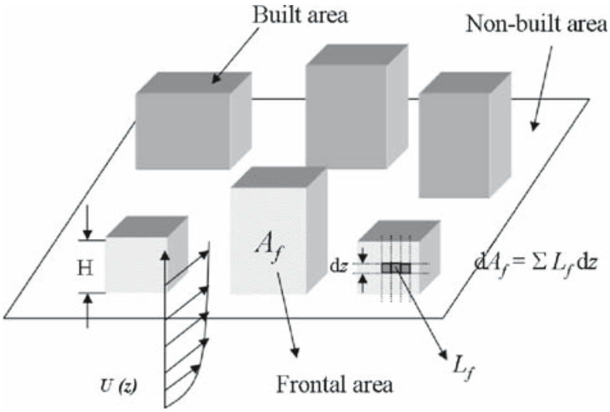

Fig. 2 Schematic of building distribution showing the meaning of the parameters used in our model. In particular dA_f is a portion of the building frontal area. This quantity is calculated for each building by estimating for each building the width of the building perpendicular to the wind direction in the region dz

– It uses as a boundary condition the wind velocity measured at the top of the domain, which can be chosen to be well above the shear layer region. In fact, we could use meteorological measurements in the inertial sublayer above the buildings or from mesoscale meteorological models or from numerical weather prediction (NWP) models. These may be less then perfect but they are becoming routinely combined with smaller scale models and used for prediction within urban areas. Thus, it avoids the use of the wind velocity values at the building height.
– It incorporates information on the spatially-averaged building height variability through the parameter $\lambda_f(z)$.

3.1 Discussion on the Mixing Length Closure Model

As mentioned earlier, solutions of Eq. 16 depend on the chosen mixing length. Different mixing length formulations are available, based on either wind-tunnel or water flume experiments over arrays of cubes Macdonald (2000). Even though data from full scale experiments are becoming increasingly available there are still no indications from the literature as to which mixing length should be used within and above the real urban canopy. However analysis and interpretation of available data from those experiments and theoretical works on flow over cube arrays can provide information on this point. Outside the canopy the mixing length is a parameter linked to the thickness and the vertical velocity gradients of the shear layer that is the region between the top of the canopy layer and the start of the inertial or logarithmic layer. That is because it depends on how effective the turbulent transport of momentum is down into the canopy. It can be shown that for dense obstacle arrays representing an urban area, if the obstacle density is increased, then the drag will increase thus resulting in a smaller l_c. In this case, the wake turbulence increases, increasing the turbulent dissipation rate and reducing the mixing length. This will lead to lower velocities higher up in the gaps between obstacles, and increased mean shear in the region over which longitudinal velocity varies. This stronger shear has some important effects (Hunt and Durbin 1999). Firstly, shear sheltering occurs, blocking the large boundary-layer eddies from intruding into the canopy (therefore the appropriate mixing length above the canopy becomes $l = \kappa(z - d)$, where κ is the von Karman's constant and d is the displacement height). If there are regions of channelled flow within the canopy, additional shear layers may also exist at the sides of obstacles, and these will also make a contribution to the turbulent kinetic energy levels. Due to these effects, the mixing length in dense arrays may be modelled as being constant with height within the canopy (Hunt and Durbin 1999). The mixing length should also be continuous at $z = H$. Therefore the simplest expression for the mixing length is as follows

$$l(z) = \kappa(z - d) \tag{17}$$

valid in the region $z > H$, and

$$l_c(z) = \kappa(H - d) \tag{18}$$

suitable when $z \leq H$.

A different formulation has been proposed by Macdonald (2000), whose arguments are summarised here. The mixing length within and above the urban canopy is composed of three parts: one suitable for the in-canopy layer, one for the logarithmic layer and one for the intermediate, matching region (shear layer). The starting point of his analysis is the definition of the total drag coefficient C_{DH} at the top of the obstacles. Therefore the balance between the drag forces and the shear stress can be written as:

$$\frac{1}{2}U_H^2 C_{\mathrm{DH}} \frac{A_f}{A_d} = K_t \left.\frac{\partial U}{\partial z}\right|_{z=H} = l_c^2 \left(\frac{a}{H} U_H\right)^2 \tag{19}$$

where a Prandtl type of approach has been used i.e.:

$$\tau(z) = K_t \frac{\partial U}{\partial z} = l_c^2 \left|\frac{\partial U}{\partial z}\right| \frac{\partial U}{\partial z} \tag{20}$$

and therefore:

$$a^2 = \frac{C_{\mathrm{DH}} \lambda_f H^2}{2l_c^2}. \tag{21}$$

Comparing this expression with that derived from the exponential solution $a^3 = \frac{H^3 \sigma_f}{2l_c^2}$, Macdonald derived the final relation, not depending on l_c, which links a to the morphological parameter λ_f:

$$a = \frac{H\sigma_f}{C_{\mathrm{DH}} \lambda_f}. \tag{22}$$

The relation above represents the link between the vegetative canopy and the urban one.

For a constant C_D, the mean drag coefficient can be expressed as

$$\overline{C_{\mathrm{DH}}} = \frac{C_D}{1 - e^{-2a}}. \tag{23}$$

From his experiments Macdonald (2000) proposed an average value for $C_{\mathrm{DH}} = 1.2$. It follows that the mixing length in the canopy l_c is derived starting from the expression for a:

$$a^3 = \frac{H^3 \sigma_f}{2l_c^2} = H^2 C_D(z) \frac{\lambda_f}{4l_c^2} \tag{24}$$

from which:

$$\frac{l_c}{H} = \sqrt{\frac{C_D(z)\lambda_f}{4a^3}}. \tag{25}$$

The resulting expression for l_c is:

$$\frac{l_c}{H} = \sqrt{\frac{\overline{C_{DH}}\lambda_f(1-e^{-2a})}{4a^3}}. \tag{26}$$

The above equation predicts that the mixing length within the canopy is z independent. In the logarithmic region the mixing length l is given by $l = \kappa(z-d)$ with d being the displacement height. While the value of l in the matching region depends on the choice of z_w (the height of the shear layer region), which is intermediate between H and the start of the logarithmic region, it is expected that z_w is larger for lower λ_f values to reflect the fact that the height of the shear layer is lower with the increase of the building packing density. Macdonald (2000) suggested an analytical method of obtaining the mixing length profile l_m in this region that is:

$$l_m(z) = l_c + \left(\frac{z-H}{z_w-H}\right)(\kappa\,(z_w-d)-l_c)\,. \tag{27}$$

With this expression the overall relation for l is given by Eqs. 18, 27 and 17.

Our model has only one adjustable constant, z_w and if necessary further refinement of the model could be developed through more sophisticated specification of the mixing length within the urban canopy.

4 Solution Method

Equation 16, which constitutes the main equation of our model, is solved numerically. To do this the equation is first re-written as a system of two first-order differential equations as:

$$U = u_1, \tag{28a}$$

$$\frac{dU}{dz} = u_2, \tag{28b}$$

and with this change of variables Eq. 16 reads:

$$\frac{du_2}{dz} = \frac{-l(z)\frac{dl}{dz}u_2^2 + \frac{1}{4}u_1^2\frac{\lambda_f(z)}{H}C_D}{l^2(z)u_2} \tag{29}$$

where all the terms have already been defined.

Equation 29 is then solved using the ODES (stiff ordinary differential equation) algorithms in Matlab© (MATLAB 1997) based on the Runge–Kutta numerical integration scheme (Jameson et al. 1981). The algorithms work in such a way that for a given integration interval $[z_a, z_b]$, an adaptive grid is internally created. This allows us to best calculate the solution in regions where there are changes of the signs of the function to be integrated and that of their derivatives. The algorithms require the initial conditions for both u_1 and u_2 are specified. In our case the integration interval was $[z = 0, z = bH]$ with $b > 2.5$. This choice of the parameter b enables us to calculate the solution in the logarithmic region as well as in the canopy layer. Initial conditions can be specified at the top of the computational domain at $z = bH$ corresponding to the unperturbed region of the flow, well above the canopy layer and where most wind velocity measurements are available or where they can at least be obtained. To summarise, the model requires as input the following quantities:

1. An initial estimate of the wind velocity at the top of the computational domain.

2. An initial estimate of the velocity gradient at the top of the domain. The solution is obtained iteratively but with a fixed velocity at a reference height within the inertial region. This will produce a u_* value that is consistent with the outer flow and with the drag force on the surface.
3. the frontal area packing density profile $\lambda_f(z)$;
4. the displacement height. This can be estimated from the morphological parameters λ_f and λ_p.
5. the mixing length $l(z)$.

These are possible input parameters for the model however other choices can also be selected as it will be discussed in the following sections. For example the velocity gradient at the top of the domain can be estimated from the logarithmic profile as $(u_*/\kappa)(z-d)^{-1}$, if data are available, but attention should be paid to the remaining input parameters to avoid over-prescribing the problem. The $\lambda_f(z)$ profile is read by the main Matlab© program in the form of a look-up table. The profile is then interpolated to obtain extra data at each of the point of the grid used to integrate the final solution.

In the next sections we analyse model results for arrays of cubes before applying the model to predict mean spatially-averaged velocity profiles over real urban areas.

5 Testing of Model with Data from Regular Arrays of Cubes

The experimental data used to verify our model are those by Macdonald et al. (2000) performed in a water flume. This study provides profiles of mean velocity, spatially-averaged mean velocity, turbulent intensities, and Reynolds stresses over regular cube arrays of different frontal area packing densities namely: $\lambda_f = 0.0625$, $\lambda_f = 0.16$ and $\lambda_f = 0.44$. These experiments were designed with the intention of studying different flow regimes in a sparse, intermediate and dense canopy that are representative, according to Oke's classification (Oke 1978), of isolated roughness, wake interference and skimming flow regimes, respectively. Given the nature of our model we are only interested in spatially-averaged mean velocity profiles. To run our model, as explained above, we need to specify the values of the wind velocity and its derivative at the top of the computational domain, which was for most cases equal to $8H$. This value can be much lower and it depends from where input data are derived. If input data are from a mesoscale meteorological model or NWP models $8H$ can be a reasonable choice otherwise if input data are from measurements it should be checked that they are taken within or just above the inertial layer, where the logarithmic velocity profile is expected. Model results are not sensitive to the choice of the top of the computational domain provided it is set above the region where the logarithmic region is expected. The model also requires that an adequate choice for the mixing length is made. To compare our results with measured data, we used the mixing length profile based on Eqs. 18, 27 and 17 as discussed in the previous section. This profile is composed of three parts each suitable for the in-canopy layer, the shear layer or matching layer and the logarithmic layer, respectively.

This particular mixing length formulation requires that a suitable choice of z_w is made. In our cases we used z_w equal to $2H$, $1.5H$ and $1.3H$ as suggested by Macdonald et al. (2000) from the analysis of different experiments over cube arrays in wind-tunnel. Other experiments by Cheng and Castro (2001), carried out in a wind-tunnel using cube arrays with $\lambda_f = 0.25$ in aligned and staggered arrangements, suggested that the vertical extent of the roughness sublayer for this case is around $1.8H$–$1.85H$. This is in agreement with Macdonald's work. The comparison between our model and the corresponding spatially-averaged

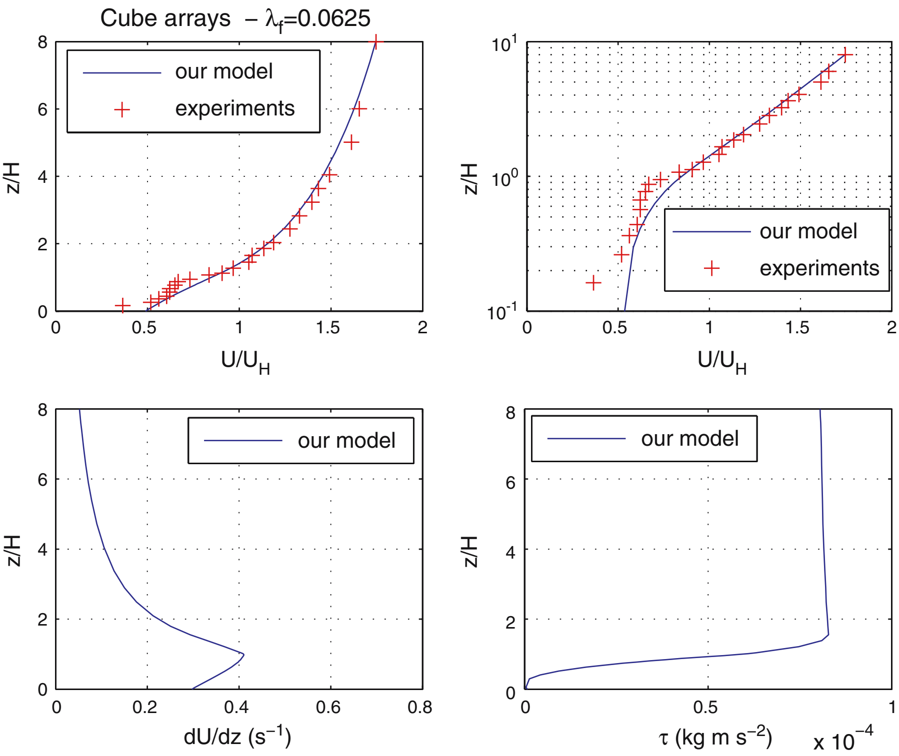

Fig. 3 $\lambda_f = 0.0625$—spatially-averaged wind profiles (upper plots); derivative of averaged wind velocity and shear stress profiles (lower plots)

measured wind profiles is shown in Figs. 3–5. Each figure shows spatially-averaged wind profiles (upper plots) presented both on a linear and a logarithmic scale and also the computed first derivative profiles and shear stress profile (lower plots) calculated as $\tau = \rho {l_c}^2 \frac{du}{dz} \mid \frac{du}{dz} \mid$. Looking firstly at the velocity plots we can see that the agreement between our model and measurements is generally good. The model shows a tendency to underestimate the velocity in the region above the building top and to overestimate it in the region below. Also we observe that the worst performance of our model is in the shear layer region. Looking at figures in more detail we can see that for the $\lambda_f = 0.0625$ case our model predictions conform very closely to those obtained from the experiment, except in the region near the building top and very close to the ground, where there is a tendency to overestimate the mean velocity by around 25%. The intermediate case $\lambda_f = 0.16$ shows that our model reproduces the experiment extremely well, showing a slight underestimation in the logarithmic region of less than 10%. At the larger packing canopy density of $\lambda_f = 0.44$ there is a slight overestimation which is more evident in a confined region close to the building top and in the in-canopy region where it is about 30%. The shear layer region is where the model performs the worst; this is to be expected given the the simple nature of the model. A better choice of the mixing length in this region could possibly provide better results. Hanna et al. (2002) attempted to simulate these cases using a large-eddy simulation (LES) technique, and found time-averaged velocity profiles that were only a fair approximation of the experimental results. Given the relative simplicity of the model, our results are in good agreement with experimental data.

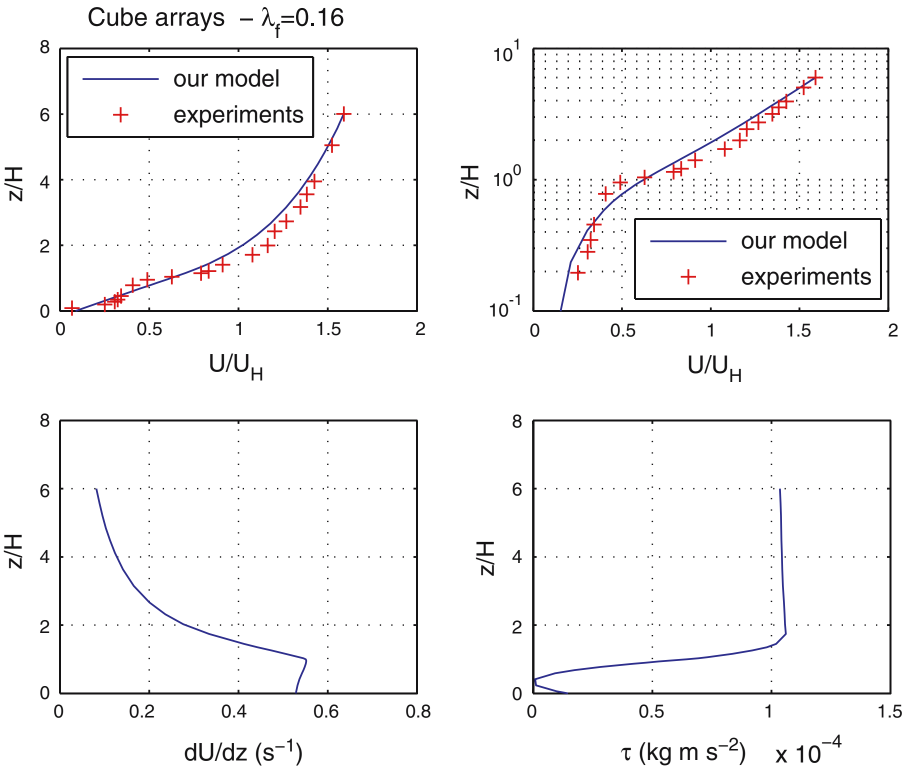

Fig. 4 $\lambda_f = 0.16$—spatially-averaged wind profiles (upper plots); derivative of averaged wind velocity and shear stress profiles (lower plots)

The quality of the agreement is no worse than that obtained by LES predictions in Hanna et al. (2002). The lower plots Figs. 3–5, which are shown for completeness, allow us to estimate the friction velocity from the computed shear stress profile (shown in the right-hand side of the plots). This is calculated from the velocity derivatives (shown at the left-hand side of the plots). To test our model further, we compared it against another set of data; that from the wind-tunnel experiments of Hall et al. (1998), Macdonald et al. (1998). We determined the ratio between the friction velocities and U_H i.e. the velocity at the building height from our model and compared this with the results from the experiment. This non-dimensional friction velocity is reported for three packing densities in Table 1. In these case, the agreement between our model results and quoted experimental data (estimated by using a log-law fit) is good. However, some doubts about these data exist (personal communication and the internal report, Macdonald (2000)). Different values of the friction velocities (around a factor of 1.7 smaller) were determined from the same experiments based on the direct measurements of the shear stress. The friction velocities estimated with this second method where thought to be of better accuracy, implying that our predictions are in fact larger, not smaller than the experimental results. A comparison using the water flume experiments from Macdonald et al. (2000) produced similar results. It should also be kept in mind that our model does not discriminate between staggered and non-staggered cube arrays as it is only the frontal area packing density (λ_f) that is the same for both staggered and non-staggered cube arrays. The general agreement between the model results and the laboratory experiments supports

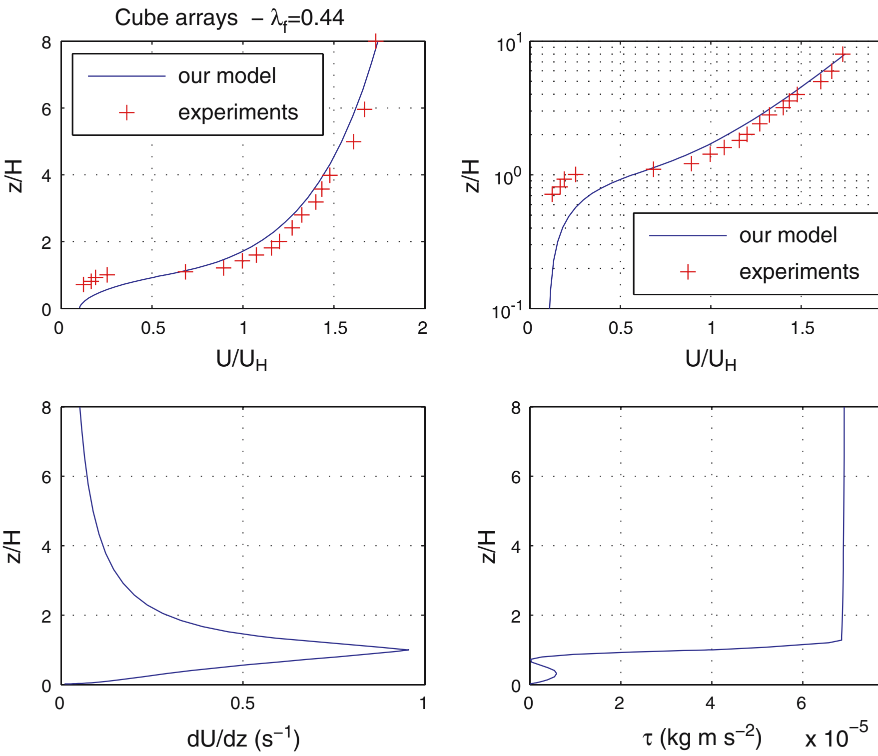

Fig. 5 $\lambda_f = 0.44$—spatially-averaged wind profiles (upper plots); derivative of averaged wind velocity and shear stress profiles (lower plots)

Table 1 Comparison between wind-tunnel measurements and our model predictions of scaled friction velocity values for different packing densities of arrays of cubes

	$\lambda_f = 0.11$	$\lambda_f = 0.20$	$\lambda_f = 0.33$
$\frac{u_*}{u_H}$ (from wind-tunnel experiments Hall et al. (1998)—Square)	0.20	0.23	0.26
$\frac{u_*}{u_H}$ (from wind-tunnel experiments Hall et al. (1998)—Staggered)	0.29	0.37	0.36
$\frac{u_*}{u_H}$ (from our model)	0.20	0.27	0.30

the view that the frontal area packing density λ_f is a fundamental parameter for modelling the flow over and through obstacle arrays. Furthermore, the mixing length approach seems to be sufficient to predict the flow in such configurations, though some refinement could be introduced to improve the friction velocity predictions.

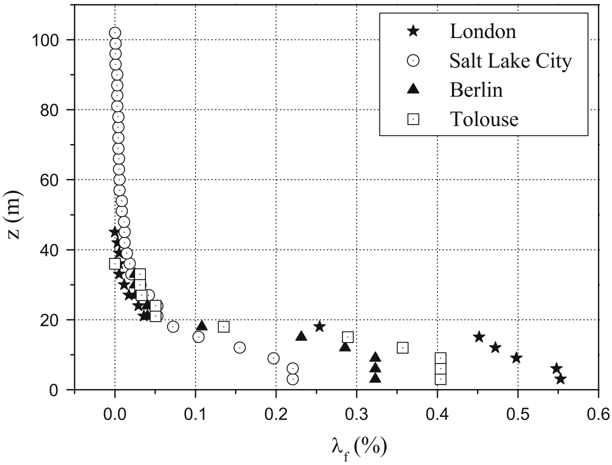

Fig. 6 Profile of $\lambda_f(z)$ for London, Toulouse, Berlin and Salt lake City calculated from DEM analysis

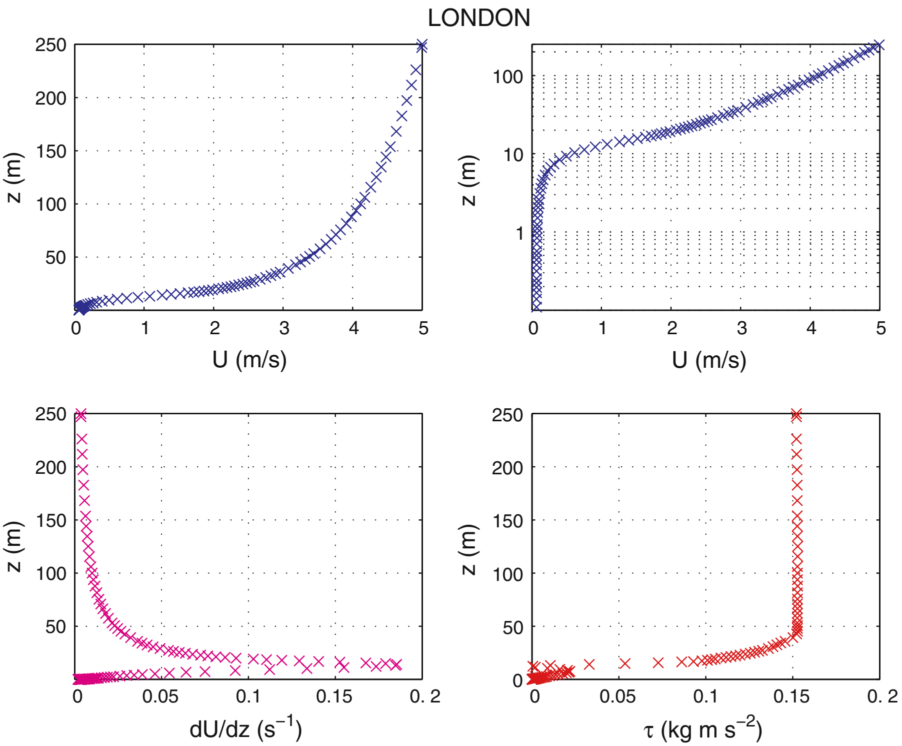

Fig. 7 Model results for London using λ_f variation with height. Mean velocity profiles (upper plots), mean velocity first derivative and shear stress profiles (lower plots)

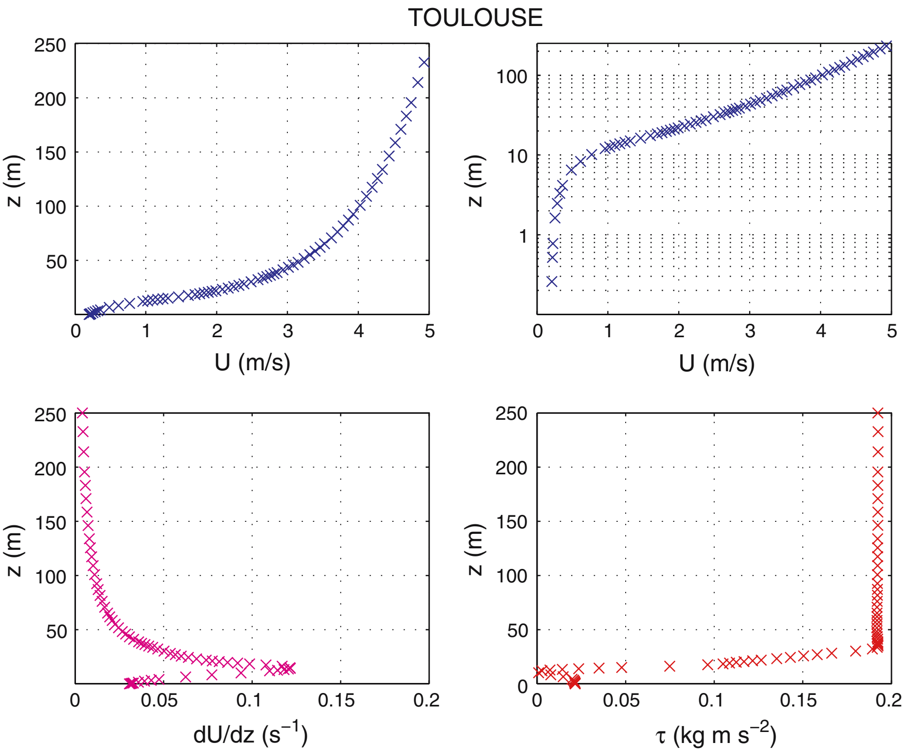

Fig. 8 Model results for Toulouse using λ_f variation with height. Mean velocity profiles (upper plots), mean velocity first derivative and shear stress profiles (lower plots)

6 Model Predictions for Portions of Full-size Cities

It is clear that the simple cube array geometries often used in experimental studies differ from real urban geometries in many ways. Building heights vary even on a neighbourhood scale where the requirement of spatial homogeneity is intrinsic to its definition. If we focus on morphological parameters, from the analysis of DEMs (Ratti et al. 2002), we see that cities typically have different λ_p and λ_f values. In city centres, buildings (at least in typical European cities and smaller North American cities) tend to be flatter, so for cube arrays λ_f is too large. Alternatively, in high-rise city centres where skyscrapers are common, the opposite is the case. The use of $\lambda_f(z)$ (the frontal area packing density vertical profile) calculated from DEMs, is capable of handling this difference.

Figure 6 shows $\lambda_f(z)$ curves for London, Toulouse, Berlin and Salt Lake City. Hence we can use our model to calculate spatially-averaged velocity profiles within and above real urban canopies.

Figs. 7–10 show the computed spatially-averaged wind profiles on both linear and logarithmic scales for each city, together with the wind velocity derivative and the computed shear stress. For all cases the same velocity value of $5\,\mathrm{m\,s^{-1}}$ is used at the top of the computational domain. To estimate the velocity gradient at the top of the computational domain we employed an iterative method. This is different from the case of cube arrays for which the friction velocity measurements were available. The model is repeatedly run with an initial guess of the velocity and its derivative at the bottom of the computational domain until it

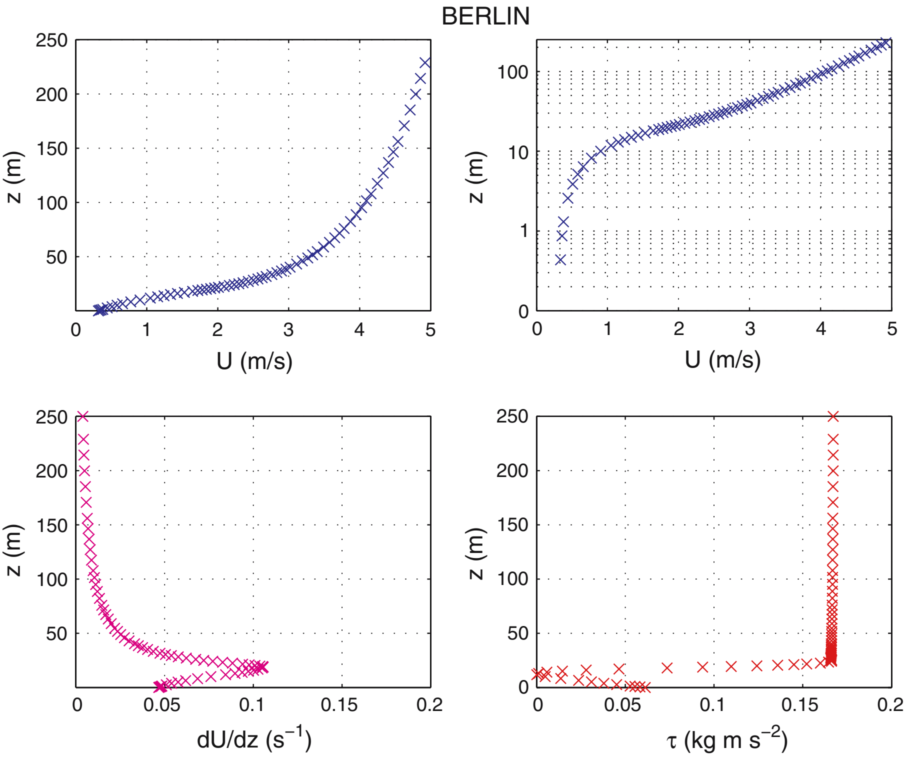

Fig. 9 Model results for Berlin using λ_f variation with height. Mean velocity profiles (upper plots), mean velocity first derivative and shear stress profiles (lower plots)

reaches the prescribed velocity value at the top of the domain. From Figs. 7–10 we can estimate the friction velocity u_* and the roughness length z_0 representative of neighbourhoods of London, Toulouse, Berlin and Salt lake City, respectively. Results are reported in Table 2 together with the displacement height d and the values of z_0 calculated for the four cities examined. The table shows that estimates of z_0 from our model are much larger than estimates of z_0 from DEMs using formulae derived from Macdonald et al. (1998). These formulae contain constant values for the morphometric parameters λ_p and λ_f. This is most probably one of the reasons for the low values of z_0 even though some vertical variability correction has been added. In estimating z_0 from the logarithmic part of the velocity profile predicted by our model we used the displacement height d estimated from DEMs. In doing so, we assumed that the d values were less affected by the spatial variability of the building height and therefore probably those values can be considered as a good estimate. The final conclusion that can be made from this comparison is that more suitable relations for the derivation of z_0 might be required if we would like to derive z_0 from DEMs directly. The other option is to use morphometric parameters and their variation with height from DEMs and to use our model to derive the roughness length at the neighbourhood scale.

7 Additional Thoughts

We have discussed an extension of Cionco's model for calculating spatially-averaged velocity profiles in real urban areas. The model is based on a balance equation between the obstacle

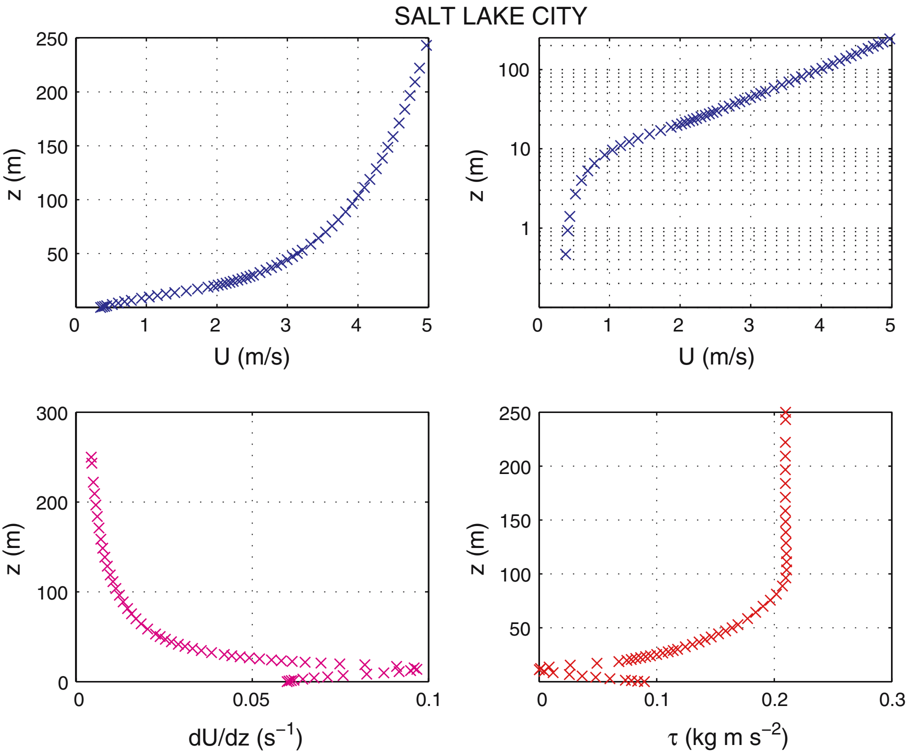

Fig. 10 Model results for Salt Lake City using λ_f variation with height. Mean velocity profiles (upper plots), mean velocity first derivative and shear stress profiles (lower plots)

Table 2 Morphometric parameters derived from DEMs for London, Toulouse, Berlin and Salt Lake City (SLC); roughness length values derived from both DEMs and our flow model and computed friction velocities

	H_{max} (m)	λ_p	d (m)	z_0 from DEMs (m)	z_0 from our model (m)	u_* from our model (m s^{-1})
London	40	0.55	11.9	0.30	0.92	0.36
Toulouse	32	0.40	10.9	0.92	1.6	0.40
Berlin	21	0.35	12.1	1.18	1.06	0.37
SLC	98	0.42	11.4	1.5	2.0	0.42

drag force and the local shear stress. The buildings within the canopy are represented as a canopy element drag, formulated in terms of the morphological parameters λ_f and λ_p. These parameters are obtained from the analysis of urban DEMs. The shear stress is parameterised using a mixing length approach. The model was validated against available experimental data over cube arrays. The use of $\lambda_f(z)$ removed some difficulties present in previous models especially in the boundary conditions. Also it is a useful way of accounting for building height variability. Our model results obtained by use of real frontal area densities taken from the analysis of DEMs show promise as a simple tool for predicting spatially-averaged velocity profiles in real urban areas at the neighbourhood scale. The nature of the model is such that it is suitable for inclusion into operational urban air quality models. In fact, if the morphometry of a city is known, the model only needs either a wind measurement (its derivative can be estimated by means of an iterative method) at a single height, or two wind measurements of which one could be in the logarithmic layer and the other within the urban canopy. For implementation into an operational air quality the model only needs a wind measurement at a single height or a wind prediction provided from a larger scale numerical model provided that they are in the logarithmic layer. The approach should be extendable to the provision of a single velocity anywhere.

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