Biotechnol Lett (2015) 37:2167–2171
DOI 10.1007/s10529-015-1899-9

ORIGINAL RESEARCH PAPER

Dimensionless equations to describe microalgal growth in a planar cultivation system

Clayton Jeffryes · Jian Li · Spiros N. Agathos

Received: 29 May 2015 / Accepted: 25 June 2015 / Published online: 2 July 2015

Abstract
Objectives To develop dimensionless equations to describe microalgal growth in planar photobioreactor or raceway pond systems that are generalized to all phototrophic species and reactor length scales.
Results Expressions for biomass growth and mean light intensity within a nutrient replete, well-mixed, phototrophic cell culture in a planar cultivation system were developed in terms of dimensionless variables for biomass, time and light intensity, plus two new dimensionless parameters. The first dimensionless parameter represents a species-specific physiological characteristic based on maximum growth rate and cell maintenance, while the second represents the light input. Optimal biomass productivities and photosynthetic conversion efficiencies are easily determined from the dimensionless expressions and system-specific performances can be easily determined by back substituting with the relevant cell culture and photobioreactor parameters.
Conclusion The dimensionless expressions are useful for understanding and determining the relevant bioprocess parameters in a generalized form applicable to all strains and reactor length scales.

Keywords Dimensionless equations · Irradiance · Microalgae · Modeling · Photobioreactor · Phototroph · Planar cultivation of algae · Raceway

Electronic supplementary material The online version of this article (doi:10.1007/s10529-015-1899-9) contains supplementary material, which is available to authorized users.

C. Jeffryes (✉) · J. Li · S. N. Agathos
Earth & Life Institute, Bioengineering Laboratory, Catholic University of Louvain, Place Croix du Sud 2, bte. L07.05.19, 1348 Louvain-La-Neuve, Belgium
e-mail: clayton.jeffryes@uclouvain.be

J. Li
e-mail: jli@supremebiotech.com

S. N. Agathos
e-mail: spiros.agathos@uclouvain.be

Present Address:
J. Li
Supreme Biotech, 1b Kotua Place, Stoke, PO Box 2450, Nelson 7011, New Zealand

Nomenclature

X	Cell culture biomass concentration (kg m^{-3})
t	Time (h)
μ_{max}	Maximum specific growth rate (h^{-1})
I_m	Mean photon flux within the cell culture (μmol photons m^{-2} s^{-1})
I_k	Light half saturation constant for cellular growth (μmol photons m^{-2} s^{-1})
M	Specific energetic loss rate (h^{-1})
I_o	Photon flux at the cell culture surface (μmol photons m^{-2} s^{-1})
K_c	Cell culture light attenuation constant (m^2 kg^{-1})

L	Light path length in cell culture, reactor length scale (m)
$X^* = Xk_cL$	Dimensionless biomass concentration
$t^* = t \cdot \mu_{max}$	Dimensionless time
$I^* = I_m/I_k$	Dimensionless light intensity
$\alpha_1 = M/\mu_{max}$	Dimensionless physiological parameter
$\alpha_2 = I_0/I_k$	Dimensionless light input

Introduction

Dimensionless equations can generalize problems in a way that reduces the number of variables and therefore the system complexity, allows the analysis of a system independent of the scale or units, facilitates scale-up, allows the determination of which groups of parameters are critical to the process and gives insight into parameters that can be treated approximately or neglected (Logan 2006). Dimensionless, generalized forms also simplify the solution using numerical methods and reduce the number of times a set of equations needs to be solved. Indeed, for these reasons the use of dimensionless equations is very common, even ubiquitous, in the study of problems in physics and engineering.

While algal bioprocesses have been in existence for decades, an explosion of interest and a nearly exponential increase in research within this field (Wijffels et al. 2013) has taken place since the publication of Chisti's review article on algal biodiesel (Chisti 2007). While many complex and eloquent studies on photobioreactor (PBR) modeling, which use sets of coupled, highly non-linear differential equations have been described (Acién Fernández 2013; Fernández et al. 2014), a dimensionless description of a PBR bioprocess has remained absent in the literature even though dimensionless expressions to describe microbial growth in fermenters have been established for decades (Dunn and Mor 1975) and the techniques to do so have become routine in bioprocess engineering textbooks (Blanch and Clark 1997). In this study, we draw inspiration from the dimensionless expressions of fermentation engineering to describe microalgal growth in commonly observed systems, such as light-limited, nutrient replete cultivations in a planar geometry (e.g. raceway ponds and flat panel PBRs). Specifically, this is achieved by replacing dimensionless substrate concentrations by dimensionless photon inputs and by modifying the growth expression to account for energetic losses intrinsic to the phototrophic metabolism. These expressions can be used for any algal cell culture and PBR length scale and facilitate the determination of biomass productivity, optimal biomass concentration and photosynthetic conversion efficiency (PCE) in terms of the dimensionless variables.

Methods

Development of dimensionless equations

When photons are the only growth-limiting substrate the change in biomass concentration with respect to time in a well-mixed, nutrient replete culture in the absence of photoinhibition can be described by (Li et al. 2015):

$$\frac{dX}{dt} = \left(\mu_{max}\frac{I_m}{I_k + I_m} - M\right)X \quad (1)$$

where X is the biomass concentration (kg m^{-3}), μ_{max} is the maximum growth rate in the absence of cellular losses (h^{-1}), M represents the energetic losses, such as due to photorespiration or cell maintenance (h^{-1}), and I_k and I_m represent the light half-saturation constant and the mean light intensity within the cell culture, both with units of μmol photons m^{-2} s^{-1}.

The value of M determines the mean light intensity at which cellular growth stops and its exclusion leads to erroneous results at low values of I_m. The mean light intensity within a planar geometry (e.g. a flat plate photobioreactor or a raceway pond) illuminated from one side can be estimated by (Acién Fernández 2013):

$$I_m = \frac{I_0}{k_cLX}[1 - \exp(-k_cLX)] \quad (2)$$

where I_o is the photon flux penetrating into the cell culture surface (μmol photons m^{-2} s^{-1}), K_c is the light attenuation constant of the cell culture between 400 and 700 nm of the given light source (m^2 kg^{-1}), which represents the portion of the light spectrum that can support photosynthesis, and L is the light path of the cell culture (m).

Because of the hyperbolic term in Eq. 1 and the exponential term in Eq. 2, non-dimensionalization by dimensional analysis (Rushton et al. 1950) was not

amenable to this system of equations. Therefore, a change of variables approach was used with the following substitutions: $X^* = Xk_cL$; $I^* = I_m/I_k$; $t^* = t \cdot \mu_{max}$, which yields:

$$\frac{dX^*}{dt^*} = \left(\frac{I^*}{1+I^*} - \alpha_1\right)X^* = \mu^* X^* \quad (3)$$

$$I^* = \frac{\alpha_2}{X^*}[1 - exp(-X^*)] \quad (4)$$

where α_1 is M/μ_{max}, a dimensionless metabolic descriptor of the algal species; α_2 is I_o/I_k, a dimensionless light input normalized to the species-specific half-saturation constant; and μ^* is the dimensionless growth rate, equivalent to the term within the parentheses in Eq. 3.

The change of variables is similar to that used for the non-dimensionalization of chemostats and fed-batch fermenters (Nelson and Sidhu 2005). The derivation of the substitution constants is found in Supporting Information. It is also interesting to note that if X^* is replaced by the first-order, rectilinear Thiele Modulus (Thiele 1939) then Eq. 4 becomes identical to the effectiveness factor in a diffusion–reaction system (Saltzman and Radomsky 1991).

Indeed, X^* can be thought of as the ability of the system to consume photons, μ^* as a type of temporal efficiency with respect to the available photons, I^* as the effectiveness of photon delivery within the volume of the cell culture and t^* a time scale normalized by the maximum growth rate of the species.

Results and discussion

The system performance for any cell culture can be quantified by examining μ^* as a function of the dimensionless light input, α_2 (Fig. 1). In this work, unless otherwise stated, α_1 has been fixed to 0.25, nearly identical to the value determined by Li et al. (2015) for an industrially grown strain of *Scenedesmus obliquus* and similar to the values determined for the marine diatom *Phaeodactylum tricornutum* (Liu et al. 2009) and the cyanobacterium *Nostoc flagelliforme* (Yu et al. 2009). The usefulness of the expression is further realized by calculation at different values of X^*, therefore describing the growth performance with respect to light input for any phototrophic species at any cell density and PBR light path.

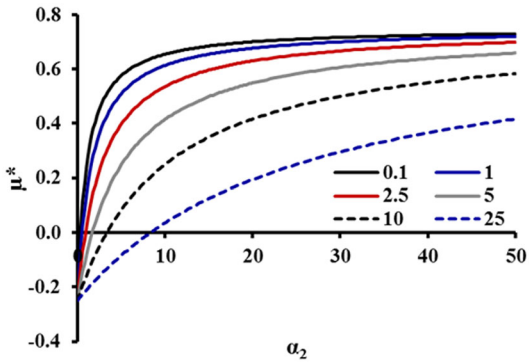

Fig. 1 The dimensionless growth parameter μ^* versus the dimensionless light input, α_2, plotted for different dimensionless biomass concentrations, X^*

The predicted X^* and μ^* with respect to t^* and as a function of the dimensionless light input (α_2) as well as the predicted and experimental data of Li et al. (2015) are presented in Fig. 2. The curves were generated by applying a classical Runge–Kutta method to Eqs. 3–4.

The predicted dimensionless biomass productivity, $\mu^* X^*$, as a function of X^* and α_2 is presented in Fig. 3. At a fixed growth rate the biomass productivity increases with X^*. However, with increasing X^* the growth rate, μ^*, will decrease (Eq. 3) because of a decrease in I^* (Eq. 4). There exists an X^* for every light input which optimizes biomass productivity. This value can be determined from the derivative of $\mu^* X^*$ with respect to X^* as follows:

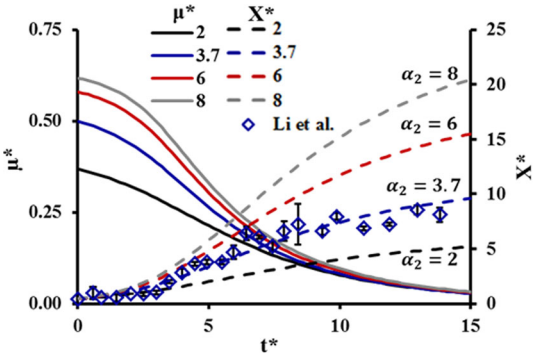

Fig. 2 Predicted X^* and μ^* with respect to t^* as a function of α_2. The *diamonds* are the cultivation data of Li et al. (2015) with $\alpha_2 = 3.7$, which represents an incident light intensity of 130 μmol photons m^{-2} s^{-1} to the cell culture at the inner reactor surface

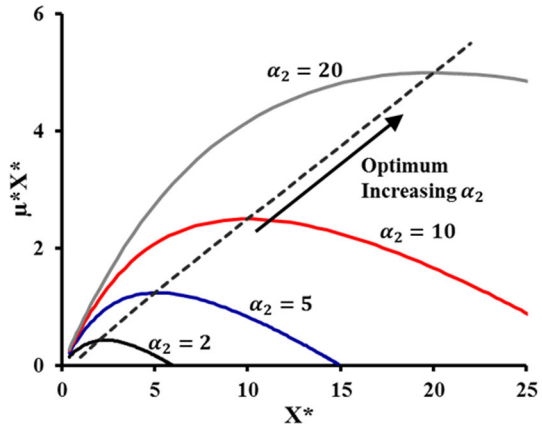

Fig. 3 Predicted dimensionless biomass productivity, $\mu^* X^*$, with respect to X^* and as a function of α_2. The *optimum* line represents the optimal dimensionless biomass productivity, $\mu^*_{opt} X^*_{opt}$, as a function of the optimal dimensionless biomass concentration, X^*_{opt}, with increasing α_2

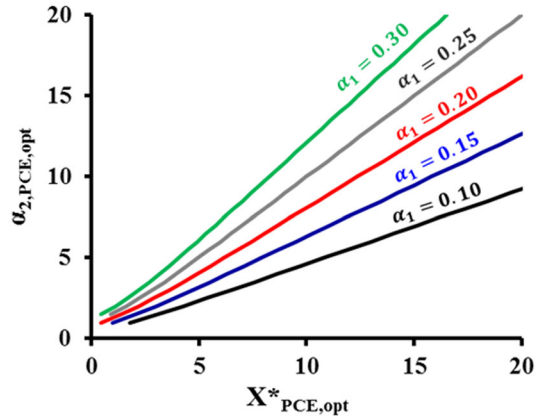

Fig. 4 Solution curves for $X^*_{PCE,opt}$ and $\alpha_{2,PCE,opt}$ which satisfy values of $I^*_{PCE,opt}$ as a function of α_1

$$\frac{d[\mu^*(I^*, X^*)X^*]}{dX^*} = \frac{d\left[\left(\frac{I^*(X^*)}{1+I^*(X^*)} - \alpha_1\right)X^*\right]}{dX^*} = 0 \qquad (5)$$

where μ^* is a function of I^* and X^* and I^* is itself a function of X^* (Eq. 4). When X^* is greater than four, the resulting implicit solution can be closely approximated by the surprisingly simple expression $X^*_{opt} = \alpha_2\left(\alpha_1^{-1/2} - 1\right)$, presented in Fig. 3 as the "optimum" line. This line depicts the maximum predicted biomass productivity, $\mu^*_{opt} X^*_{opt}$, with increasing illumination (α_2).

To recover the dimensioned growth expression and the predicted growth rate in a specific PBR system, the dimensionless variables are substituted into Eq. 3 as follows:

$$\frac{dX^*}{dt^*} = \frac{d(Xk_cL)}{d(t \cdot \mu_{max})} = \mu^*(Xk_cL) \rightarrow \frac{dX}{dt} = \mu^* \cdot \mu_{max} X = \mu_{predicted} X \qquad (6)$$

The light input that will give the optimal PCE can also be determined by finding the maximum growth rate, μ^*, per unit of light input, I^*, as follows:

$$\mu^* = \left(\frac{I^*}{I^*+1} - \alpha_1\right) \rightarrow \frac{d\left(\frac{\mu^*}{I^*}\right)}{dI^*} = \frac{d\left(\frac{1}{I^*+1} - \frac{\alpha_1}{I^*}\right)}{dI^*} = 0 \qquad (7)$$

$$I^*_{PCE,opt} = \sqrt{\gamma(\gamma-1)} - \gamma; \gamma = \frac{\alpha_1}{\alpha_1 - 1} \qquad (8)$$

An infinite number of combinations of $X^*_{PCE,opt}$ and $\alpha_{2,PCE,opt}$ can satisfy this value of $I^*_{PCE,opt}$ (Eq. 4), therefore one must be specified in terms of the other. Example curves of $\alpha_{2,PCE,opt}$ as a function of $X^*_{PCE,opt}$ at different values of the species-specific metabolic parameter α_1 are presented in Fig. 4.

Non-dimensional growth expressions for a well-mixed, nutrient replete, phototrophic culture in the absence of photoinhibition in a planar cultivation geometry were developed and the system performance is expressed in terms of only two dimensionless parameters: α_1, a factor related to the growth and maintenance; and α_2, a measure of the light input. Given the model assumptions, the dimensionless equations are valid for any phototrophic species and reactor path length. To solve for the growth characteristics of a specific system, a back substitution of cell culture and reactor parameters is easily achieved.

Acknowledgments This work was supported by the Belgian National Fund for Scientific Research (Fonds de la Recherche Scientifique—FNRS) project PHOTOFUNDS and by the Walloon Region, Belgium, project BEMA (Bio-Energy from Micro-Algae, contract no. 6161, TWEED Cluster).

Supporting information Dimensionless equations to describe microalgal growth in a planar cultivation system.

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