ORIGINAL RESEARCH



Consideration of second-order effects on plastic design of steel moment resisting frames

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Abstract

This work mainly aims to propose a new design procedure combining the benefits of the Performance-Based Plastic Design approach (PBPD) with a rigorous accounting of second-order effects. In fact, by exploiting the kinematic theorem of plastic collapse, second-order effects can be accounted for employing the concept of collapse mechanism equilibrium curve. The same tool constitutes the base of the Theory of Plastic Mechanism Control (TPMC) design approach. Besides, the paper reports a critical comparison between TPMC and PBPD, both having the scope to design structures exhibiting a collapse mechanism of global type. These two approaches are also compared with the refined PBPD where second-order effects are accounted for by the kinematic approach. Many steel moment resisting frames are designed according to PBPD, TPMC and refined PBPD and their performances have been compared on the bases of push-over analyses.

Keywords TPMC · PBPD · Global mechanism · MRFs · Plastic design

List of symbols

$\begin{array}{lll} h_{n_s} & \mbox{Top storey height} \\ h_1 & \mbox{First storey height} \\ h_i & \mbox{Storey height} \\ h_k & \mbox{kth storey height with respect to the foundation level} \\ B_2 & \mbox{Parameter to account for second-order effects (Eq. 14)} \\ C_e & \mbox{Spectral acceleration in the energy balance formulation} \\ d\theta & \mbox{Virtual rotation of column bases} \\ E & \mbox{Energy occurring in an elastic system} \\ E_e & \mbox{Earthquake input energy} \\ E_p & \mbox{Energy to be dissipated by hysteresis} \\ F_i & \mbox{Force acting at the ith storey} \\ F_k & \mbox{Seismic force al kth} \\ g & \mbox{Acceleration of gravity} \end{array}$	dv_k	Vertical virtual displacement occurring at kth storey
$ \begin{array}{lll} h_i & \mbox{First storey height} \\ h_i & \mbox{Storey height} \\ h_k & \mbox{kth storey height with respect to the foundation level} \\ B_2 & \mbox{Parameter to account for second-order effects (Eq. 14)} \\ C_e & \mbox{Spectral acceleration in the energy balance formulation} \\ d\theta & \mbox{Virtual rotation of column bases} \\ E & \mbox{Energy occurring in an elastic system} \\ E_e & \mbox{Earthquake input energy} \\ E_p & \mbox{Energy to be dissipated by hysteresis} \\ F_i & \mbox{Force acting at the ith storey} \\ F_k & \mbox{Seismic force al kth} \\ g & \mbox{Acceleration of gravity} \end{array} $	$h_{n_{n_{n_{n_{n_{n_{n_{n_{n_{n_{n_{n_{n_$	Top storey height
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E_p Energy to be dissipated by hysteresis F_i Force acting at the ith storey F_k Seismic force al kthgAcceleration of gravity	E_{e}	Earthquake input energy
F_i Force acting at the ith storey F_k Seismic force al kthgAcceleration of gravity	E_{n}	Energy to be dissipated by hysteresis
F_k Seismic force al kthgAcceleration of gravity	F_i^{r}	Force acting at the ith storey
g Acceleration of gravity	F_k	Seismic force al <i>k</i> th
	g	Acceleration of gravity

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l_i	Bay span
M	Building mass
M_{hik}	Plastic moment in the beam ends when plastic hinge has been developed
$M_{h in}$	Plastic moment of <i>j</i> th beam at the top storey
$M_{h ik}$	Plastic moment of <i>j</i> th beam at <i>k</i> th storey
$M_c(h)$	Moment in the column at the height h above the ground
M _{cil}	Plastic moment of <i>i</i> th column of the first storey reduced due to the simulta-
0.71	neous action of the axial force
M_{nhi}	Plastic moment of beams at the ith storey
M_{phr}	Required plastic moment of top storey beams
M_{pc}	Required plastic moment of columns at the first storey
$M_{\mu\nu}^{pc}(h)$	Final bending moment
n_{h}	Number of beams
n _o	Number of columns
n_	Number of storevs
P.	Gravity load on the column at <i>i</i> th floor level calculated using the seismic
- cg,i	load combination
<i>a</i> .,	Vertical load acting in the seismic load combination
$\mathbf{R}^{\mathbf{q}_{i,k}}$	Reduction factor
S	Spectral velocity of the expected seismic event
S S	Spectral velocity of the expected seismic event
T	Period of vibration
I V	Base shear
V	Seismic design shear at the ton storey
V_{n_s}	Storey shear at the ith storey
V _i V.	Seismic design shear at kth storey
V_k	Concentrated forces acting on columns at the kth storey coming from the
• lk	inner bays of the column
V	Storey shear at the top storey
W^{n}	Seismic weight
W	Virtual external work
W _e	Virtual external work
W	Total vertical load acting at k th storey
Λ / I	Maximum target drift
Δ / L	Rigid plastic analysis collapse mechanism multiplier
$\alpha^{(t)}$	First order collapse multiplier of undesired mechanisms
u_{i_m}	First order collapse multiplier of alabel mechanisms
α_0	First order contapse multiplier of global mechanism
α_0	First order contapse multiplier of norizontal seismic forces
α_{coef}	Coefficient provided by Eq. (7)
p_i	Shear proportioning factor
γ	Slope of the mechanism curve accounting for second-order effects
$\gamma^{(g)}_{(t)}$	Slope of the collapse mechanism equilibrium curve of global mechanism
$\gamma_{i_m}^{(c)}$	Slope of the collapse mechanism equilibrium curve of undesired
	mechanisms
γ_m	Modification factor based on Newmark and Hall studies (Newmark and
	Hall 1982)
δ_i	Step function
δ_k	Horizontal displacement occurring at the kth storey

δ_u	Ultimate design displacement
θ_p	Plastic rotation of dissipative members
λ	Distribution coefficient lower than 1
λ^*	Updating of the distribution coefficient λ
λ_i	Distribution coefficient
μ_s	Structural ductility factor which is equal to target drift divided by yield
MDE	uiiit Moment registing frames
MKFS	Moment resisting frames
PBPD	Performance based plastic design
TPMC	Theory of Plastic Mechanism Control
$\sum_{k=1}^{n_s} M_{b,i,k}$	Sum of beams plastic moments belonging to the left bay
$\sum_{k=1}^{n_s^{-1}} M_{b,i-1,k}$	Sum of beams plastic moments belonging to the right bay
$\overline{\sum}_{k=1}^{n_s} S_{i,k} \delta_k$	Axial forces at the kth storey in the <i>i</i> th column at the collapse state from
	the right bay
$\sum_{k=1}^{n_s} S_{i-1,k} \delta_k$	Axial forces at the <i>k</i> th storey in the <i>i</i> th column at the collapse state from $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 $
	the left bay

1 Introduction

The collapse mechanism of global type (Park 1986; Mazzolani and Piluso 1996; Bruneau et al. 2011) has been universally recognised as the best collapse typology because it assures the development of the most significant number of plastic hinges. In the case of moment resisting frames (MRFs), it is achieved when all the beam ends are involved in plastic range and the first storey column bases are yielded being all the column sections at the upper storeys in elastic range. In fact, "capacity design" principles, that constitute the stronghold of modern seismic design, require to design dissipative zones with internal actions coming from seismic combinations, while non-dissipative ones have to be proportioned on the bases of the maximum internal actions transmitted by dissipative zones in their fully yielded and strain-hardened state. Codes try to achieve the goal of a global mechanism by the applying approximated design rules based on capacity design principle. The rule devoted to the design of columns is the so-called beam-column hierarchy criterion, introduced also in Eurocode 8 (EN 1998-1 2004). Although the study on this topic began several decades ago, mostly devoted to concrete structures (Bertero and Popov 1977; Paulay 1977, 1979, 1980), design provisions given in Eurocode 8 (EN 1998-1 2004) as well as in other codes, are not able to promote the development of a global type mechanism (Piluso et al. 2015).

Many experimental and analytical studies have been conducted to investigate the seismic behaviour of MR-Frames for several decades and to develop new design procedures by which structures can behave in a predictable manner and to achieve a predictable target performance (Grigorian and Grigorian 2012, 2013; Grigorian and Kaveh 2013). This scope can be achieved by allowing the formation of a preselected yield mechanism, such as the global mechanism so that structures have an adequate strength and ductility during design level ground motions. Mazzolani and Piluso (1997) presented a rigorous design procedure based on the kinematic theorem of plastic collapse, whose goal was the development of a collapse mechanism characterized by the formation of plastic hinges at the beam ends only, leaving all the columns in elastic range. Starting from this work, the Theory of Plastic Mechanism Control (TPMC) (Piluso et al. 2015) has been rearranged in a closed form solution that makes it more suitable also for hand calculation, and that has outlined as a useful tool for the seismic design of structures. With reference to MR-Frames, this problem was faced also by Goel and Lee (2001). In this case the authors' design procedure was aimed to assure both a pre-selected yield mechanism and given target drift as required by Performance-Based Plastic Design (PBPD). In order to achieve this goal, the authors suggest to obtain the design base shear as a function of the target plastic drift. This goal is reached by means of the energy balance equation according to an approach proposed by Leelataviwat et al. (1999) with the introduction of an energy modification factor which is able to consider the relation between the ductility factor and the force reduction factor (Goel et al. 2010). In addition, this procedure adopts a different distribution of lateral design forces (Chao et al. 2007; Goel and Chao 2008; Liao and Goel 2012).

In order to show the practical application of the procedures, the seismic design of a building with perimeter MRFs is made. The considered seismic resistant frame has beams whose size is governed by the seismic load combination. Therefore, being known the seismic forces at each storey provided employing the energy approach, beam sections can be dimensioned by assuming a distribution of their size according to the storey shear distribution and by imposing the prevention of soft storey mechanism at first storey. The fulfilment of these two design conditions results in a system of two equations where the two unknowns of the problem are the plastic moment of beams at the top storey and the plastic moment of columns at first storey. This criterion for beam design, proposed in PBPD, is herein applied also for the preliminary design of beam sections of TPMC.

Concerning the design of columns at the upper storeys, the two design approaches are significantly different. In fact, within the framework of rigid-plastic theory, while TPMC is based on a kinematic approach, PBPD procedure is based on a static approach using freebody diagrams to derive the distribution of column bending moments along their height. Moreover, the influence of second-order effects is rigorously accounted for in TPMC using the concept of mechanism equilibrium curve joined with the application of the kinematic theorem of plastic collapse. Conversely, in the procedure suggested by Goel and Lee, second-order effects are considered by applying the moment amplification method to the columns bending moments which are derived employing a rigid-plastic analysis, rather than to moments calculated by elastic analyses as commonly made.

Given the above consideration, the study herein presented has the purpose to compare TPMC with the PBPD by highlighting strengths and weaknesses of both the methodologies and by investigating the influence of standard shapes causing an overstrength affecting the collapse mode of the structure. One other goal of this paper is to propose a design procedure combining the benefits of PBPD with a rigorous accounting of second-order effect. TPMC, PBPD and improved PBPD have also been applied to design several steel MRFs that have been furtherly investigated by push-over analyses. Also, serviceability requirements have been checked according to Eurocode 8 provisions (EN 1998-1 2004).

2 Performance-Based Plastic Design

In this paragraph, a summary of performance based plastic design approach is reported. It has been applied to several structural typologies (Goel and Chao 2008; Liao and Goel 2014; Chao and Goel 2008; Lee and Goel 2000) with the same goal that is the achievement of a collapse mechanism of global type. Performance based plastic design (PBPD) developed by Goel and Lee (2001) is to propose a seismic design procedure based on performance limit states using

target drift and yield mechanism for MRFs. The design procedure proposed is also aimed to provide the seismic design forces by means of balance equation (Lee and Goel 2000). Regarding a non-linear structural system, the earthquake input energy is evaluated as γ_m times the input energy occurring in an elastic system (Housner 1956):

$$E = \gamma_m \frac{1}{2} M S_v^2 \tag{1}$$

being M the building mass and S_v the spectral velocity of the expected seismic event.

The modification factor γ_m , based on Newmark and Hall (1982) studies, accounts for the expected ductility demand corresponding to the reduction factor R_{μ} and is given by:

$$\gamma = \frac{2\mu_s - 1}{R_{\mu}^2} \tag{2}$$

The amount of earthquake input energy which the structure can absorb in elastic range can be computed according to Akiyama (1985) as follows:

$$E_e = \frac{1}{2}MS_{v,y}^2 = \frac{1}{2}\frac{W}{g}\left(\frac{T}{2\pi}\frac{V}{W}g\right)^2 = \frac{WT^2}{8\pi^2 g}\left(\frac{V}{W}\right)^2 g^2$$
(3)

where $S_{v,y}$ is the spectral velocity value corresponding to yielding, V is the corresponding base shear, T is the period of vibration and W the seismic weight of the building.

The energy to be dissipated by hysteresis is therefore given by:

$$E_p = E - E_e = \frac{WT^2}{8\pi^2 g} \left[\gamma_m C_e^2 - \left(\frac{V}{W}\right)^2 g^2 \right]$$
(4)

where C_e is the spectral acceleration.

This energy is equal to the energy that the target yield mechanism can dissipate which, in turn, is equal to the external work of horizontal seismic forces:

$$E_p = \sum_{i=1}^{n} F_i h_i \theta_p = V \theta_p \sum_{i=1}^{n} \lambda_i h_i$$
(5)

being h_i the height of the *i*th floor from the foundation level and $F_i = \lambda_i V$ the corresponding seismic force depending on the distribution coefficient λ_i , that can be expressed in terms of weight of the structure at level *i* and the height of beam level *i* from the ground.

Employing the energy balance equation, i.e. by equating (4) and (5), the following relationship is obtained:

$$\frac{V}{W} = \frac{-\alpha_{coef} + \sqrt{\alpha_{coef}^2 + 4\gamma_m C_e^2}}{2} \tag{6}$$

where the coefficient α_{coef} is given by:

$$\alpha_{coef} = \left(\sum_{i=1}^{n} \lambda_i h_i\right) \frac{\theta_p 8\pi^2}{T^2 g}.$$
(7)

Starting from the knowledge of the seismic forces accomplished through the above energy balance approach, the design of the structural members, i.e. beam sections and column sections at each storeys addressed as follows. Beam sections are dimensioned by assuming a distribution of their size according to the storey shear distribution:

$$\beta_i = \left(\frac{V_i}{V_n}\right)^b \tag{8}$$

where V_i and V_n , respectively, are the static storey shears at level *i*th and the top level and the exponent value is a function of the period of the structure ($b = 0.5T^{-0.2}$).

Under this assumption and with reference to a one-bay MR-Frame, the work equation for the target collapse mechanism provides:

$$\theta_p \left(\sum_{i=1}^n 2\beta_i M_{pbr} + 2M_{pc} \right) = \theta_p \sum_{i=1}^n F_i h_i = \theta_p \sum_{i=1}^n \left(\beta_i - \beta_{i+1} \right) h_i F_n \tag{9}$$

where M_{pbr} is the reference plastic moment of beams (top storey), i.e. the plastic moment of top storey beam, and M_{pc} is the required plastic moment of columns at first storey. Moreover, by imposing the prevention of storey mechanism at the first storey, the following equation is obtained:

$$M_{pc} = \frac{1.1Vh_1}{4}$$
(10)

where V is the total base shear, h_1 is the height of the first storey, and factor 1.1 is the overstrength factor to account for possible overloading due to strain hardening.

Therefore, for a given base shear derived by means of Eq. (6), Eqs. (9) and (10) allow evaluating the required plastic moment of the first storey columns M_{pc} and the required plastic moment of top storey beam M_{pbr} . The plastic moment of beams at *i*th storey is determined as:

$$M_{pbi} = \beta_i M_{pbr} \tag{11}$$

Successively, the column sections are derived according to the bending moment computed through a static approach based on equilibrium equations written with reference to free body diagrams. After updating the lateral forces to account for overstrength of the beams after yielding, design moments of the column can be determined by treating the column as a cantilever and, starting from the top, the distribution of moments in columns is evaluated as follows:

$$M_{c}(h) = \sum_{i=1}^{n} \delta_{i} M_{pbi} - \sum_{i=1}^{n} \delta_{i} F_{i} (h_{i} - h)$$
(12)

where $M_c(h)$ is the moment in the column at the height *h* above the ground, δ_i is a step function defined as $\delta_i = 1$ if $h \le h_i$ and $\delta_i = 0$ if $h > h_i$ and F_{iu} is the force acting at the *i*th storey (Fig. 1).

Such bending moments are furtherly amplified according to the moment amplification method commonly applied to account for second-order effects in elastic analyses. For plastic design, according to Salmon and Johnson (1990), it is conservative to determine the final bending moment as follows:



Fig. 1 Free-body diagram of an external column

$$M_{\mu x}(h) = B_2 M_c(h) \tag{13}$$

where the factor B_2 is necessary to account for second-order effects in a simplified way and is calculated as:

$$B_2 = \frac{1}{1 - \frac{\sum_{i=k}^n P_{cg,i}}{\sum_{i=k}^n F_i} \left(\frac{\Delta}{L}\right)}$$
(14)

where $P_{cg,i}$ are the gravity load on the column at *i*th floor level calculated using the seismic load combination, F_i is the storey shear at the *i*th storey, and Δ/L is the maximum target drift (4%) for which the frame is designed. This procedure carried out for columns at each storey allows obtaining the final member size of each element of the frame.

3 Theory of Plastic Mechanism Control

The Theory of Plastic Mechanism Control (TPMC) works within the framework of limit analysis and pursues the aim to design structures with a global failure mode. This procedure originally proposed by Mazzolani and Piluso (1997) in the nineties has been recently updated (Piluso et al. 2015) leading to a closed-form solution allowing a more practical and easy procedure. It has been applied to design not only MRFs but also other steel (Mastrandrea et al. 2013; Longo et al. 2014a, b; Montuori et al. 2014, 2016, 2017; Longo et al. 2012; Piluso et al. 2019), reinforced concrete typologies (Montuori and Muscati 2017) and wooden resisting frames (Montuori and Sagarese 2018). TPMC exploits the kinematic theorem of plastic collapse by identifying three main typologies of collapse mechanism the structure can exhibit (Piluso et al. 2015; Krishnan and Muto 2012). They must be considered undesired, because not involving all the dissipative zones. However, in the elastic range structures can have high horizontal displacements, so that application of the kinematic theorem without second order effects is not able to assure a global collapse mechanism. In particular these effects are more significative for steel moment-resisting frames.

Therefore, the fundamental principle of TPMC is constituted by the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve. In particular, for each considered collapse mechanism, the equilibrium curve of the mechanism can be derived by equating the internal work to the external work, including in the external work also the second-order work due to vertical loads (Piluso et al. 2015). The collapse mechanism equilibrium curve is a straight line given by:

$$\alpha = \alpha_0 - \gamma \delta_u \tag{15}$$

where α_0 is the first order collapse multiplier of horizontal seismic forces, γ is the slope of the mechanism curve accounting for second-order effects; finally, δ_u is the ultimate design displacement which is selected to be compatible with the ductility capacity of the dissipative members. The theory states that the collapse mechanism equilibrium curve of the global mechanism has to be located below those corresponding to all the undesired mechanisms until a design displacement δ_u compatible with the available ductility of structural members, as depicted in Fig. 2:

$$\alpha_0^{(g)} - \gamma^{(g)}\delta_u \le \alpha_{i_m}^{(t)} - \gamma_{i_m}^{(t)}\delta_u \quad \text{for } i_m = 1, 2, 3, \dots, n_s; t = 1, 2, 3$$
(16)

In the case of beams parallel to the direction of the corrugation of the deck slab, the starting point of the procedure needs to design beam sections at last storey and column sections at first storey. In light of this, two conditions are needed: the first one is the prevention of the soft storey mechanism at first storey, while, the latter is the application of the principle of virtual work to the global mechanism (Piluso et al. 2015). Given the above, the column sections at each storey required to assure the development of a collapse mechanism of global type are provided by imposing the design statement. Finally, a technological



condition can be imposed by requiring that, starting from the top, the column sections cannot decrease along the building height.

4 Design approach based on free body diagrams and accounting for second-order effects

As pointed out in the previous sections, there are relevant differences between PBPD and TPMC. Within the framework of rigid-plastic theory, while TPMC is based on a kinematic approach, the Goel and Lee procedure is based on a kind of static approach using free-body diagrams to derive the distribution of column bending moments along their height. Moreover, the influence of second-order effects is rigorously accounted for in TPMC employing the concept of mechanism equilibrium curve and by the application of the kinematic theorem of plastic collapse. Conversely, in the procedure suggested by Goel and Lee (2001), second-order effects are considered by applying the moment amplification method to the columns bending moments which are derived through a rigid-plastic analysis, rather than to moments calculated through elastic analyses as commonly made.

Moment amplification method is a tool to consider second-order effects by increasing the design moment of columns by means of an amplifying coefficient that has a weak scientific basis. Besides, it tends to excessively amplify the stresses leading to structures able to achieve the design goal but not optimised concerning the ratio weight/performances. Therefore, in this paper, some modifications to the PBPD with the purpose of eliminating all the approximations existing in the procedure have been performed with particular reference to second order effects.

Without using the one-bay equivalent frame, the structure can be considered, in its maximum drift response state, as subjected to equivalent inertial forces, being achieved the global mechanism. This mechanism is based on the assumptions that all the plastic drift is mono-directional, and all the energy post-yielding is dissipated in plastic hinges only. Following the philosophy of strong column-weak beam, it is also assumed that the plastic hinges in a frame are limited only to beams and column bases sections at first storey, and the amount of plastic rotation θ_p over the height of the structure is uniform after yielding of the frame.

In case of global mechanism, the external work due to a virtual rotation $d\theta$ of columns plastic hinges is given by:

$$W_e = \alpha \sum_{k=1}^{n_s} F_k h_k d\theta + \frac{\delta_u}{h_{n_s}} \sum_{k=1}^{n_s} W_k h_k d\theta$$
(17)

where α is the multiplier of horizontal forces, F_k and h_k are, the seismic force al kth storey and the kth storey height with respect to the foundation level, respectively, h_{ns} is the value of h_k at the top storey, δ_u is the design top sway displacement and W_k is the total vertical load acting at kth storey.

In this case, the assumption of $\alpha = 1$ means that α assumes the unitary value in correspondence of the ultimate design displacement δ_u . The design of structure with seismic forces multiplied by that coefficient ensures the structure to withstands such forces and consequent stresses up to collapse conditions.

The first term of Eq. (17) is the external work due to horizontal forces, while the second term represents the second order work due to vertical loads. To compute the slope of the

mechanism equilibrium curve, it is needed to evaluate the second order work due to vertical loads. In Fig. 3, the horizontal displacement of the kth storey involved in the generic mechanism is given by $u_k = r_k \sin \theta$, where r_k is the distance of the kth storey from the base C and θ the rotation angle. The top sway displacement is equal to $\delta = h_n \sin \theta$.

The relationship between horizontal and vertical virtual displacements is given by $dv_k \tan \theta \approx du_k \sin \theta$. It is clear that, as the ratio dv_k/du_k is independent of the storey, vertical and horizontal virtual displacement vectors have the same shape. The virtual horizontal displacements are given by $du_k = r_k \cos\theta d\theta \approx r_k d\theta$, where r_k defines the shape of the virtual horizontal displacement vector, while the virtual vertical displacements are given by $dv_k = \frac{\delta}{h_k} r_k d\theta$ and, therefore, they have the same shape r_k of the horizontal ones (Mazzolani and Piluso 1996). It can be concluded that:

$$dv_k = \frac{\delta_u}{h_{ns}} h_k d\theta \tag{18}$$

(20)

where dv_k is the vertical virtual displacement of kth storey.

The internal work due to a virtual rotation $d\theta$ of plastic hinges of columns is:

$$W_{i} = \left(\sum_{i=1}^{n_{c}} M_{c,i1} + 2\sum_{k=1}^{n_{s}} \sum_{j=1}^{n_{b}} M_{b,jk}\right) d\theta$$
(19)

where $M_{c,i1}$ is the plastic moment of *i*th column of the first storey reduced due to the simultaneous action of the axial force, $M_{b,jk}$ is the plastic moment of *j*th beam at *k*th storey, n_c, n_s and n_b are the number of columns, storeys and beams, respectively.

In the same way, as Goel and Lee perform in PBPD, in the case of no significant vertical loads acting on the beams, it is desirable to impose a distribution of their flexural strength along the building height that is similar to the distribution of the design storey shears. This condition can be achieved employing a shear proportioning factor defined as follow:

 $\beta_k = \frac{V_k}{V_n}$



Fig. 3 Second order vertical displacements

where V_k is the seismic design shear at *k*th storey and V_{n_s} is the seismic design shear at the top storey. Therefore, thanks to this design strategy, the beam plastic moments of the other storeys are derived as:

$$M_{b,jk} = \beta_k M_{b,jn_a} \tag{21}$$

where $M_{b,jk}$ is the plastic moment of *j*th beam at *k*th storey, and M_{b,jn_s} is the plastic moment of *j*th beam at the top storey. Then, updating the term related to beams, the equation of the principle of virtual works can be written equating Eqs. (17) and (19):

$$\sum_{k=1}^{n_s} F_k h_k d\theta + \frac{\delta_u}{h_{ns}} \sum_{k=1}^{n_s} W_k h_k d\theta = \left(\sum_{i=1}^{n_c} M_{c,i1} + 2\sum_{k=1}^{n_s} \beta_k \sum_{j=1}^{n_b} M_{b,jn_s}\right) d\theta$$
(22)

where the only unknown terms are the sum of plastic moments of first storey column (i.e. $\sum_{i=1}^{n_c} M_{c.i1}$), and the sum of plastic moments of top storey beams (i.e. $\sum_{j=1}^{n_b} M_{b.jn_s}$). However, a second equation is needed, and it is the design conditions to be fulfilled

However, a second equation is needed, and it is the design conditions to be fulfilled to avoid type-3 mechanism at the first storey. According to the kinematic theorem of plastic collapse extended to the mechanism equilibrium curve concept, such design condition is the following:

$$\frac{\sum_{i=1}^{n_c} M_{c,i1} + 2\sum_{k=1}^{n_s} \beta_k \sum_{j=1}^{n_b} M_{b,jn_s}}{\sum_{k=1}^{n_s} F_k h_k} - \frac{\delta_u}{h_{n_s}} \frac{\sum_{k=1}^{n_s} W_k h_k}{\sum_{k=1}^{n_s} F_k h_k} \le \frac{2\sum_{i=1}^{n_c} M_{c,i1}}{h_1 \sum_{k=1}^{n_s} F_k} - \frac{\delta_u}{h_1} \frac{\sum_{k=1}^{n_s} W_k}{\sum_{k=1}^{n_s} F_k}$$
(23)

where h_1 is the first storey height.

The design conditions (Eq. 24) to be fulfilled to avoid all the partial collapse mechanisms require that the mechanism equilibrium curve of the global mechanism has to lay below those corresponding to all the undesired mechanisms until a displacement δ_u compatible with the available ductility of structural members.

This design condition can be expressed in the following compact form:

$$\alpha_0^{(g)} - \gamma^{(g)} \delta_u \le \alpha_1^{(3)} - \gamma^{(3)} \delta_u \tag{24}$$

where $\alpha_0^{(g)}$ is the kinematically admissible multiplier of horizontal forces corresponding to the global mechanism according to first order rigid-plastic analysis, $\gamma^{(g)}$ is the slope of the global mechanism equilibrium curve, $\alpha_1^{(3)}$ is the kinematically admissible multiplier of horizontal forces with reference to the mechanism of type-3 (soft-storey) at the first storey, and $\gamma^{(3)}$ is the slope of the type-3 mechanism equilibrium curve.

Equations (22) and (23) solved as an equation, constitute a system with two unknowns, $\sum_{i=1}^{n_c} M_{c,i1}$ and $\sum_{j=1}^{n_b} M_{b,jn_s}$. It has to be noted that, even if this procedure is similar to the one proposed by Goel et al., two important differences can be highlighted. First of all, in Eq. (22) second order effects are accounted for a given level of drift. Besides, second-order effects that occur when ultimate design displacement is reached are explicitly and rigorously evaluated by means of a kinematic approach without resorting to the amplification of moment method.

Being known the value of the sum of the plastic moments of top storey beams, it is possible to design beams of the other storey by the β_k factors. Furthermore, being known the values of the sum of the reduced plastic moments of first storey columns and the axial load acting at collapse state in the columns, it is possible to design the first storey columns.

With this background, in the spirit of PBPD columns are designed singularly through a static approach by which the bending moments along the column height are evaluated employing free-body diagrams.

At this point, a distribution coefficient λ lower than 1 has to be computed to provide the rate of seismic forces and vertical loads affecting the second order effects coming from the inner leaning part designed only for gravity loads. To achieve this purpose, we need to refer to each column in its maximum drift response state according to the plastic rotation of dissipative members, assumed in this study equal to 0.04 rad. In this condition, according to a global collapse mechanism, all the plastic hinges in the beam ends and first storey column bases are activated in yielding.

Assuming as positive the clockwise rotation, the equation of equilibrium to the rotation around the bottom of the single column is estimated as follows:

$$\lambda \sum_{k=1}^{n_s} F_k h_k + \lambda \sum_{k=1}^{n_s} V_{lk} \delta_k + \sum_{k=1}^{n_s} N_{ci,k} \delta_k + \sum_{k=1}^{n_s} S_{i-1,k} \delta_k + \sum_{k=1}^{n_s} S_{i,k} \delta_k$$

$$= \lambda \sum_{i=1}^{n_c} M_{c,i,1} + \sum_{k=1}^{n_s} M_{b,i-1,k} + \sum_{k=1}^{n_s} M_{b,i,k}$$
(25)

where $\lambda \sum_{k=1}^{n_s} F_k h_k$ is the sum of bending moments due to seismic forces from *k*th storey to the top storey, $\lambda \sum_{k=1}^{n_s} V_{lk} \delta_k$ is the sum of bending moments due to second-order effects coming from the inner bays of that column and δ_k is the horizontal displacement occurring at the *k*th storey. From *k*th storey to the top storey, $\sum_{k=1}^{n_s} N_{ci,k} \delta_k$ is the sum of bending moments due to secondary beams axial contributes according to the influence area of the *i*th column from *k*th storey to the top storey, $\lambda \sum_{i=1}^{n_c} M_{c,i,1}$ is the required moment rate at the *i*th column base, $\sum_{k=1}^{n_s} M_{b,i-1,k}$ and $\sum_{k=1}^{n_s} M_{b,i,k}$ are the sum of beams plastic moments to the right and to the left bay, respectively, from *k*th storey to the top store, $\sum_{k=1}^{n_s} S_{i-1,k} \delta_k$ and $\sum_{k=1}^{n_s} S_{i,k} \delta_k$ are the axial forces at the *k*th storey in the *i*th column at the collapse state. According to the global mechanism, axial forces in the columns at the collapse state have to be evaluated by considering both distributed loads acting on the beams and shears due to the development of plastic hinges at the beam ends. So that, the total load transmitted by the beams to the columns is expressed as:

$$S_{i,k} = \frac{q_{i,k}l_i}{2} \pm \frac{2M_{b,i,k}}{l_i}$$
(26)

where $q_{i,k}$ is the vertical load acting in the seismic load combination, $M_{b,i,k}$ is the plastic moment in the beam ends when plastic hinge has been developed, and l_i is the bay span. It is important to underline that these two contributions combination are influenced by seismic actions direction.

By solving Eq. (25) for the distribution coefficient λ , the following expression returns:

$$\lambda = \frac{\sum_{k=1}^{n_s} M_{b,i-1,k} + \sum_{k=1}^{n_s} M_{b,i,k} - \sum_{k=1}^{n_s} N_{ci,k} \delta_k - \sum_{k=1}^{n_s} S_{i-1,k} \delta_k - \sum_{k=1}^{n_s} S_{i,k} \delta_k}{\sum_{k=1}^{n_s} F_k h_k + \sum_{k=1}^{n_s} V_{lk} \delta_k - \sum_{i=1}^{n_c} M_{c,i,1}}$$
(27)

Thanks to the above distribution coefficient, the bending moment at the bottom of column by means of the equation of equilibrium to the rotation around the bottom point of the column, is evaluated as follows:

$$M_{c,i,1} = \sum_{m=1}^{n_s} M_{b,i-1,m} + \sum_{m=1}^{n_s} M_{b,i,m} - \frac{\delta_u}{h_{ns}} \sum_{m=1}^{n_s} S_{i-1,m} (h_m - h_{k-1}) + - \frac{\delta_u}{h_{ns}} \sum_{m=1}^{n_s} S_{i,m} (h_m - h_{k-1}) - \frac{\delta_u}{h_{ns}} \sum_{m=1}^{n_s} N_{ci,m} (h_m - h_{k-1}) + - \lambda \frac{\delta_u}{h_{ns}} \sum_{m=1}^{n_s} V_{l,m} (h_m - h_{k-1}) - \lambda \sum_{m=1}^{n_s} F_m (h_m - h_{k-1})$$
(28)

Set the value of the plastic moment at the bottom of first storey column $M_{c,i,1}^*$, the value of the distribution coefficient λ is updated as follows:

$$\lambda^{*} = \frac{M_{c,i,1}^{*} + \sum_{k=1}^{n_{s}} M_{b,i-1,k} + \sum_{k=1}^{n_{s}} M_{b,i,k}}{\sum_{k=1}^{n_{s}} F_{k} h_{k} + \sum_{k=1}^{n_{s}} V_{lk} \delta_{k}} - \frac{\sum_{k=1}^{n_{s}} N_{ci,k} \delta_{k} - \sum_{k=1}^{n_{s}} S_{i-1,k} \delta_{k} - \sum_{k=1}^{n_{s}} S_{i,k} \delta_{k}}{\sum_{k=1}^{n_{s}} F_{k} h_{k} + \sum_{k=1}^{n_{s}} V_{lk} \delta_{k}}$$
(29)

Definitely, this updated distribution coefficient allows to design the columns sections at each storey by means of the free-body diagrams, as depicted in Fig. 4, evaluating the bending moment at the top and bottom of the column on each storey by means of the following equations providing the top and bottom design moment of column at each storey:



Fig. 4 Free-body diagram of an exterior column

$$M_{c,i,k}^{top} = \sum_{m=k}^{n_s} M_{b,i-1,m} + \sum_{m=k}^{n_s} M_{b,i,m} - \frac{\delta_u}{h_{ns}} \sum_{m=k+1}^{n_s} S_{i-1,m} (h_m - h_k) + + \frac{\delta_u}{h_{ns}} \sum_{m=k+1}^{n_s} S_{i,m} (h_m - h_k) - \frac{\delta_u}{h_{ns}} \sum_{m=k+1}^{n_s} N_{ci,m} (h_m - h_k) + + \lambda^* \frac{\delta_u}{h_{ns}} \sum_{m=k+1}^{n_s} V_{l,m} (h_m - h_k) - \lambda^* \sum_{m=k+1}^{n_s} F_m (h_m - h_k)$$
(30)

$$M_{c,i,k}^{bottom} = \sum_{m=k}^{n_s} M_{b,i-1,m} + \sum_{m=k}^{n_s} M_{b,i,m} - \frac{\delta_u}{h_{ns}} \sum_{m=k}^{n_s} S_{i-1,m} (h_m - h_{k-1}) + + \frac{\delta_u}{h_{ns}} \sum_{m=k}^{n_s} S_{i,m} (h_m - h_{k-1}) - \frac{\delta_u}{h_{ns}} \sum_{m=k}^{n_s} N_{ci,m} (h_m - h_{k-1}) + + \lambda^* \frac{\delta_u}{h_{ns}} \sum_{m=k}^{n_s} V_{l,m} (h_m - h_{k-1}) - \lambda^* \sum_{m=k}^{n_s} F_m (h_m - h_{k-1})$$
(31)

The design moment is the maximum among those provided by Eqs. (30) and (31). Finally, column sections have to be selected from standard shapes.

Given the above, it is possible to make a summary of the TPMC and PBPD procedures starting from the same set of equation constituting the system to provide $\sum_{i=1}^{n_c} M_{c.i.1}$ and $\sum_{i=1}^{n_b} M_{b.ins}$. The flow chart of the procedures is reported in Fig. 5.

5 Case study

The study cases reported is referred to a building having a plan configuration reported in Fig. 6. The structural system is a perimeter seismic resistant system constituted by MR-Frames, while the inner bays are pinned, and they are designed to bear only vertical loads. The building constituting the study cases are constituted by a 4, 6 and 8 storeys and have been designed according to three methodologies: PBPD, TPMC and the approach herein proposed.

Being known the seismic forces at each storey, beam sections can be dimensioned by assuming a distribution of their size according to the storey shear distribution. In a purely ideal perspective, this strategy leads to contemporary involvement of dissipative zones in plastic range under the seismic design forces. Conversely, first storey column sections are obtained by imposing the prevention of storey mechanism at the first storey. About the design of columns at the upper storeys TPMC follows a kinematic approach while PBPD follows a static approach using free-body diagrams to derive the distribution of column bending moments along the structure height. Moreover, the influence of second-order effects is rigorously accounted for in TPMC by means of the concept of mechanism equilibrium curve. Conversely, PBPD uses the moment amplification method to account for



Fig. 5 Flowchart of TPMC and PBPD taking in account for second order effect rigorously

second-order effects to columns provided by means of a rigid-plastic analysis, rather than to moments derived using elastic analyses as usual.

The seismic response of the buildings herein analysed refers only to the transversal direction, i.e. where beams are parallel to the direction of the corrugation of the deck slab. The corresponding seismic resistant scheme for each case is depicted in Fig. 6, where there



Fig. 6 Plan and elevation configuration of the 6-storey building

Storey	$h_{k}(m)$	F_{C1} (kN)	F_{C2} (kN)	F _{lc} (kN)	Floor masses (kN/m ²)	Weight (kN)
1	4.50	30.60	61.20	550.80	4.80	1555.20
2	8.00	30.60	61.20	550.80	4.80	1555.20
3	11.50	30.60	61.20	550.80	4.80	1555.20
4	15.00	30.60	61.20	550.80	4.80	1555.20
5	18.50	30.60	61.20	550.80	4.80	1555.20
6	22.00	30.60	61.20	550.80	4.98	1613.52

 Table 1
 Heights, distributed loads, forces, floor masses and weight of 6-storey frame

is also the leaning column considered in structural model in order to consider the second order effects due to the gravity load acting in the internal part of the structure. The aim of the leaning column is to account the gravity loads acting in the leaning part of the structure. In fact, they cannot be negligible both in terms of second order effects than in terms of structural seismic masses.

The bay span is equal to 6.00 m; the interstorey height is equal to 4.50 m at the first storey and to 3.50 m at the upper storeys. The characteristic values of the vertical loads are equal to 4.50 and 2.00 kN/m² for permanent (G_k) and live (Q_k) actions, respectively. Consequently, with reference to the seismic load combination provided by Eurocode 8 (EN 1998-1 2004), the vertical loads acting on the floor are evaluated as follows:

$$G_k + \psi_2 Q_k = 5.10 \text{ kN/m}^2$$
 (32)

where ψ_2 , equal to 0.3 for residential buildings, is the coefficient for the quasi-permanent value of the variable actions. In addition, the floor weights and masses of 6-storey structure



Fig. 7 Sections of the PBPD designed structures



Fig. 8 Sections of the TPMC designed structures

are delivered in Table 1. According to Eurocode 8, the period of vibration to be used for preliminary design is given by:

$$T = CH^{3/4}$$
 (33)

where C = 0.085 for steel MR-Frames and H is the total height of the frame.

The steel grade adopted is S355 without using a partial safety factor, so the nominal yield stress is 355 MPa for beams and for columns. The beams of the MR-Frames have been checked in terms of serviceability requirements under vertical loads.

The approaches herein investigated show two different ways to evaluate the seismic design base shear, but in this study the lateral seismic forces used to design the structures has been calculated with reference to the design spectrum for soil class C of EC8 and by assuming a behaviour factor q equal to 6.5, a peak ground acceleration equal to 0.35 g and 2% damping. In addition, torsional effects have been accounted for by means of the δ parameter, evaluated according to EC8 provisions as $\delta = 1 + 0.6x/L_e$, where x is the considered frame distance



Fig. 9 Sections of the modified PBPD designed structures

from building plan center of gravity measured perpendicular to the direction of the seismic action considered, and L_e is the distance between the most external resisting elements measured perpendicular to the direction of the seismic action considered. The ultimate design displacement has been assumed as follows:

$$\delta_u = \theta_p h_{n_s} \tag{34}$$

where θ_p is the plastic rotation of dissipative members, assumed in this study equal to 0.04 rad, and h_{n_s} is the building height. $\theta_p = 0.04$ rad has also been assumed in the application of Eqs. (5) and (7). Finally, in Figs. 7, 8 and 9, the results of the application of the considered design procedures are reported.

6 Performance evaluation and comparisons

6.1 Weight of the structures

The first comparison regards the weight of the structures. In Figs. 10 and 11 the free body diagrams of moment for PBPD and modified PBPD for external and internal columns, respectively, are depicted with reference to the 4-storey structure. On the vertical axis, the height of the building is reported. It should be observed that the rigorous evaluation of the second order effects of the procedure herein proposed leads to structures lighter than those designed by considering second order effects in an approximated way, namely by means of the moment amplification method. Consequently, the comparison between structural weights between PBPD and modified PBPD, accounting for a force distribution in agreement with Eurocode 8 shows a glaring difference to the advantage of the methodology here proposed in all the analysed cases.

In addition, if the comparison is extended also to TPMC designed structures, it can be observed (Fig. 12) that, within the same distribution of seismic forces, structures designed according to TPMC are always lighter than those obtained by PBPD, while, those designed with the approach herein presented have almost the same weight of TPMC ones.



Fig. 10 Comparison between free body diagrams of external columns of the structures



Fig. 11 Comparison between free body diagrams of internal columns of the structures



Fig. 12 Comparison between free body diagrams of the internal column



Fig. 13 Push-over curves for the 4-storey frames

6.2 Push-over analyses

With reference to the seismic resistant system depicted in Fig. 6, push-over analyses have been carried out by means of SAP2000 computer program. The primary aim of these analyses is to confirm the development of the desired collapse mechanism typology (i.e. the global one). Regarding the structural modelling, all the members are modelled by beamcolumn elements whose mechanical nonlinearities have been concentrated at their ends by means of plastic hinge elements without rigid offset to account for the panel zone dimension. In particular, with reference to beams, plastic hinge properties are defined in pure bending (M3 hinges), while in case of columns plastic hinge properties are defined in order to account for the interaction between axial force and bending moment (P-M3 hinges). The constitutive law of such plastic hinge elements is provided by a rigid plastic moment-rotation curve without hardening and degradation. In addition, nominal properties of materials are considered. Push-over analyses have been led under displacement control taking in



Fig. 14 Push-over curves for the 6-storey frames



Fig. 15 Push-over curves for the 8-storey frames

account both geometrical and mechanical non-linearities. In addition, out-of-plane stability checks have been performed step by step in the non-linear analysis for all the analysed structures. The results given by the push-over analyses are represented in Figs. 13, 14 and 15, where both the push-over curves and the mechanism equilibrium curves corresponding



Fig. 16 Push-over hinge patter 4S—PBPD at δ_{ij} using standard shapes (**a**) and ideal hinges (**b**)



Fig. 17 Push-over hinge patter 4S—TPMC at δ_{μ} using standard shapes (a) and ideal hinges (b)

to the global mechanism are depicted even if only in the case of TPMC and modified PBPD. In addition, two straight lines representing the plastic rotation of the most involved hinge both in columns than in beams are reported. In this way, it is possible to evaluate if the ultimate plastic rotation $\theta_p = 0.04$ rad is achieved first or after the top sway displacement reach the design ultimate displacement δ_u .

From the results obtained by the analyses it can be seen that the softening branch of the push-over curve corresponding to the structure designed by means of the Theory of Plastic Mechanism Control and modified PBPD procedures is similar to the mechanism equilibrium curve given by second order rigid-plastic analysis, even if in the second case, the push-over curve points out the trend to a partial type collapse mechanism for displacement value higher than the ultimate design displacement. Regarding the push-over curves of structures designed by PBPD, it can be observed that they have a stiffness and strength higher than the TPMC designed structure. However, the structure designed by PBPD results oversized if compared to the TPMC one because the



Fig. 18 Push-over hinge patter 4S—REFINED PBPD at δ_u using standard shapes (a) and ideal hinges (b)



Fig. 19 Push-over hinge patter 6S—PBPD at δ_u using standard shapes (a) and ideal hinges (b)

maximum base shear achieved under static loading is about two time greater than the design base shear.

It has also been observed, starting from the push-over curves, that, sometimes, the most involved hinge reaches the ultimate target displacement (0.04 rad) for a value of the displacement lower than the ultimate one. It means that the ductility supply for the most involved member is achieved first of the complete development of the collapse mechanism.



Fig. 20 Push-over hinge patter 6S—TPMC δ_{μ} using standard shapes (a) and ideal hinges (b)



Fig. 21 Push-over hinge patter 6S—REFINED PBPD δ_u using standard shapes (a) and ideal hinges (b)

6.3 Influence of standard shapes

Standard shapes significantly affect the performance of structures designed under seismic design actions due to the over-resistance they give; this could undermine the judgment on the design methodologies. In this work, it has been considered to use "ideal" shapes, i.e. ideal plastic hinges, to capture the outcome of the design procedures in a "pure" way. These ideal shapes, in the same way as dog-bones elements, confer to elements a resistance



Fig. 22 Push-over hinge patter 8S—PBPD δ_u using standard shapes (a) and ideal hinges (b)



Fig. 23 Push-over hinge patter 8S—TPMC δ_u using standard shapes (a) and ideal hinges (b)



Fig. 24 Push-over hinge patter 8S—REFINED PBPD δ_{u} using standard shapes (a) and ideal hinges (b)

equal to the actual stress. This approach allows to evaluate the procedures outcome net of benefits belonging to the over-resistance intrinsically owned by the standard shapes.

Using a rigid-plastic analysis, the results of non-linear static analyses show that PBPD, even if refined to account for second order effects rigorously, is not accurate. In fact, the design target, i.e. a collapse mechanism of global type, is not achieved. It can also be observed the formation of dangerous plastic hinges at column ends that undermines the goodness of the yielding process and in worst cases leads to partial collapse mechanisms, as already check in previous study on the procedure (Dell'Aglio et al. 2017). Conversely, structures designed by TPMC and PBPD, both subjected to the Eurocode lateral force distribution, have push-over hinge patterns in perfect agreement with the global type mechanism. Regarding the push-over pattern of structures designed by means of Performance-Based Plastic Design, this can be justified by recalling that the stresses in the elements are very high through the moment amplification method, so it is virtually impossible for a partial mechanism to be activated. The push-over hinge patterns at the achievement of the ultimate design displacement δ_u in cases of the standard and the "ideal" shapes are reported in Figs. 16, 17, 18, 19, 20, 21, 22, 23 and 24.

The comparison between the performances of structures obtained by the application of ideal shapes confirms the Theory of Plastic Mechanism Control as a reliable and valid procedure leading to a collapse mechanism of global type. Conversely, such overstrength in Performance-Based Plastic Design procedure has a significant contribution to "correct" the collapse mechanism that, as pointed out in the figures referring to ideal standard shapes is not always of global type.

7 Conclusions

The work herein presented is devoted to compare different approaches for the design of MR-Frames. In particular, two design procedures are herein investigated: Performance-Based Plastic Design (PBPD) and the Theory of Plastic Mechanism Control (TPMC). Some criticisms have been highlighted being the two approaches different from several aspects. The most relevant one is the way to account for second order effects. In fact, PBPD uses the so-called moment amplification method at the end of the design procedure while TPMC accounts for the second order effects by means of the concept of collapse mechanism equilibrium curve since the starting point of the procedure. However, other methods to take in account the second order effects are proposed by codes, such as Eurocode 8, that introduces the so-called θ factor (Cassiano et al. 2016; Tenchini et al. 2014; Montuori et al. 2018; Isaincu et al. 2018). Unfortunately, this factor, usually leads to structures with oversized beam and column sections, therefore some modifications have been recently proposed (Vigh et al. 2016; Tartaglia et al. 2018).

In addition, in this study, PBPD has been refined to take in account second order effects in a rigorous way, i.e. by the kinematic theorem of plastic collapse. Furthermore, the effective rates of seismic stresses as well as second-order effects coming from the inner bays for each column, are evaluated in relation to their location in the frame (external or internal columns). This strategy allows to avoid unnecessary and over-budget over-dimensioning.

Aiming at the comparison among these design approaches, MRFs with 4, 6 and 8 storeys have been designed using the lateral force distribution as provided by Eurocode 8. Push-over analyses have been carried out to compare the investigated design procedures in terms of weight and seismic performances. The results show the achievement of the design goal by TPMC and PBPD, both in the case of standard shapes than in the case of the design value for the definition of plastic hinge properties. However, in case of PBPD, this result is attributable to an overestimation of the design moment in plastic hinges and over dimensioning of the frame elements. This affects also the structural weight and consequently the construction costs. Conversely, as already stated in a previous study (Dell'Aglio et al. 2017), free-body diagram static approach, though optimised to account for second order effects, is not a useful tool for designing frame columns because it is not able to promote the development of a collapse mechanism of global type. Furthermore, the results of non-linear static analyses show that PBPD, even if refined to account for second order effects are not accurate. In fact, the design target, i.e. a collapse mechanism of global type, is not achieved. It can also be observed the formation of dangerous plastic hinges at column ends that undermines the goodness of the yielding process and in worst cases leads to partial collapse mechanisms. In conclusion, TPMC remains the more useful tool for the design of steel MRFs collapsing with a global mechanism also for irregular (Montuori et al. 2017) and high-rise buildings. However, even if PBPD and TPMC have been severally investigated by Incremental Dynamic Analyses, further investigations on the outcomes of refined PBPD will be shown in forthcoming studies.

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