Bulletin of Earthquake Engineering (2019) 17:2627–2645
https://doi.org/10.1007/s10518-018-00552-6

ORIGINAL RESEARCH

A semi-analytical formulation for estimating the fundamental vibration frequency of historical masonry towers

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Received: 23 August 2018 / Accepted: 26 December 2018 / Published online: 3 January 2019

Abstract

A semi-analytical formulation is proposed for estimating the fundamental vibration frequency of historical masonry towers. In this method, in addition to general tower geometrical properties such as its effective height, cross-sectional width, and wall thickness, the influence of openings in perimeter walls including opening size and configuration are explicitly considered. For this purpose, a comprehensive numerical study is carried out using the ABAQUS software to evaluate the effects of various geometrical and material parameters on the fundamental frequency of towers. Then, a semi-analytical formulation is developed through statistical analysis of the generated database. To evaluate the accuracy of the proposed formulation, a number of towers, for which dynamic test results or numerical results are available, are selected and their fundamental frequencies are determined employing the proposed method. The comparison of the frequencies obtained using the semi-analytical formulation with those reported in the literature shows that the proposed formulation evaluates the fundamental frequency of masonry towers with an acceptable accuracy.

Keywords Fundamental frequency · Historical tower · Masonry · Semi-analytical formulation

1 Introduction

Masonry towers can be found in different parts of the world. Bell towers of churches and minarets of mosques are some examples of this type of construction and are of historical value. As a result, damage assessment of towers affected by environmental conditions through time, seismic performance assessment, and repair and retrofitting of historical

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towers have been subjects of much interest in recent years. On this basis, some researchers have used non-destructive techniques to monitor damage and crack patterns on the historical buildings and have presented retrofitting techniques to protect them against seismic loads (Baraccani et al. 2016; Foti et al. 2016; Carnimeo et al. 2014; Valente et al. 2017).

Discerning the dynamic properties of masonry towers provides a realistic assessment of the structural response under seismic loads. Consequently, to estimate these parameters, in situ static and dynamic tests accompanied by analytical modeling of the structure were employed. One of the principle dynamic properties of the towers is the fundamental frequency of vibration. A significant number of studies have been devoted to finding the fundamental frequency of existing towers using in situ experiments such as ambient vibration test (Gentile and Saisi 2007; Foti et al. 2012; Diaferio et al. 2014) to record the tower's response under environmental loads and Interferometric Radar (IR) method (Pieraccini et al. 2014) to remotely assess their dynamic characteristics. In the ambient vibration test, several points of the tower are selected to install accelerometer sensors to record the acceleration response of the tower under ambient effects. Then, using modal identification techniques such as the Frequency Domain Decomposition (FDD) technique or the Stochastic Subspace Identification (SSI) technique, the natural vibration frequencies of towers are determined (Cimellaro et al. 2011; Diaferio et al. 2011). In historical towers in which points of interest for installing the accelerometers are not accessible, the interferometric radar test can be utilized. In this method, certain locations along the height of the tower are selected to record their displacements under ambient excitation. By using recorded displacements, mode shapes and natural frequencies of the tower can be obtained. Also, some researchers have used both of these methods to find dynamic properties of structures and compared the results obtained from these two methods (Diaferio et al. 2015). Another experimental technique to determine dynamic properties of towers is forced vibration. In this method, a number of shakers are installed on the tower that generate vibration at controlled frequencies. A frequency sweep is then carried out using these shakers and the response is recorded through accelerometers to determine the resonance frequencies (Bartoli et al. 2013; Diaferio et al. 2018a, b).

Usually, an analytical study is necessary to support dynamic identification. To determine the dynamic properties of masonry towers and to assess the performance of these structures during earthquakes, a large number of analytical studies have been carried out using Finite Element software such as ABAQUS, Diana, and SAP2000 (Peña et al. 2010; Kouris and weber 2011; Clemente et al. 2015; Bartoli et al. 2016; Cakir et al. 2016; Valente and Milani 2016; Foti et al. 2015; Castellazzi et al. 2018). In these studies, towers are simulated with their actual geometry and appropriate material models. Free vibration analysis and time history analysis are carried out to assess the dynamic properties of the structures and to evaluate their response to ground shaking. Among these, some studies have focused on the effects of soil deformability on the dynamic response of towers by modeling soil layers in 3D finite element models using appropriate continuum elements or simplified equivalent spring elements (Camata et al. 2008; Ivorra et al. 2010; Casolo et al. 2017; de Silva et al. 2018).

Due to their complexities, high costs, and lack of accessibility, in situ dynamic tests are not always possible to perform. Therefore, some approximate methods and relations have been proposed in codes and in the literature to estimate the fundamental vibration frequency or vibration period of historical masonry towers. These methods and formulations have diverse accuracies and are usually developed for each region of the world according to its own specific construction practice. Three different approaches are usually used to develop these formulations. The most commonly used approach is to develop empirical equations. For the development of such relations, a collection of databases is compiled which includes dynamic characteristics obtained from modal identification tests and mechanical and geometrical properties of those constructions gathered from in situ surveys presented in the literature. Then, based on a statistical study of the available databases, empirical relations are proposed. The suggested equations in design codes for the estimation of fundamental frequency or vibration period of towers such as Italian Technical Code for Construction (NTC2008 2008) and Spanish Standard (NCSE-02 2002) are some examples of empirical formulations presented in Eqs. (1) and (2), respectively.

$$f_1 = \frac{1}{0.05H^{\frac{3}{4}}} \tag{1}$$

$$f_1 = \frac{\sqrt{L}}{0.06H\sqrt{\frac{H}{2L+H}}} \tag{2}$$

where f_1 denotes the fundamental natural frequency of vibration of the tower and H and L are the tower's height and cross-sectional width, respectively. Similar empirical equations have been proposed by other researchers for some regions of the world (Rainieri and Fabbrocino 2011; Kouris 2012; Shakya et al. 2014). In a recent study, Diaferio et al. (2018a, b) gathered information from 17 attached towers and 7 isolated towers. The authors proposed Eqs. (3a) and (3b) for estimation of the fundamental frequency of isolated towers and attached towers, respectively:

$$f_1 = 208.54L_{\min}^{0.55}H^{-1.73} \tag{3a}$$

$$f_1 = 14.61L_{\min}^{-0.254}H_{eff}^{-0.341}H^{-0.216} \tag{3b}$$

where L_{min} and H_{eff} are the minimum cross-sectional width and the effective height of the tower, respectively.

The second approach is a derivation of the analytical equations, based on theories of structural dynamics. For instance, the fundamental vibration frequency of a cantilever prismatic Euler beam can be determined exactly using Eq. (4):

$$f_1 = \frac{1.875^2}{2\pi.L^2}\sqrt{\frac{E.I}{\rho.A}} \tag{4}$$

where L is the length of the beam, E is the modulus of elasticity, I is moment of inertia of beam section, ρ is mass density, and A is the cross-sectional area of the beam. After modeling a masonry tower as a cantilever column with a fixed base, Eq. (4) can be used to estimate the vibration frequency of the structure.

Since many parameters such as openings in walls and the interaction of the tower with adjacent structures can affect the dynamic characteristics of the tower, the development of analytical formulations, which consider these influential parameters, is a complex issue. Consequently, a third approach has been employed in recent studies to enhance the accuracy of the results, whereby, analytical relations are modified using a number of coefficients according to available test data. For instance, Bartoli et al. (2017) gathered a large database from 11 towers located in San Gimignano (Siena, Italy) and 32 towers from the available literature. The database contains fundamental natural vibration frequency and material and geometrical properties of those towers. Accordingly, they improved Eq. (4) as follows:

$$f_1 = \frac{0.2 \times a}{H_{eff}^2} \times (1 - n) \times \sqrt{\frac{E}{\rho}} \tag{5}$$

where $n = s/a$; s is the thickness of the tower's wall, and a is the side length of tower section. Also, H_{eff} is the effective height of the tower (height of the tower regardless of the part interacting with adjacent structures). By applying this equation to the aforementioned database and comparing the global errors, a significant improvement in the results was observed compared to other empirical formulas (Bartoli et al. 2017).

Historical masonry towers are located in diverse regions of the world featuring varied architectural and material characteristics. Environmental conditions, such as moisture condition, may also affect the strength characteristics and behavior of towers (Maheri et al. 2011). However, for simplicity, some parameters, which may affect the fundamental vibration frequency of towers, are neglected in the proposed equations. This may be the reason for obtaining diverse results when using these equations for different towers around the world. As an important parameter, the presence of large openings in perimeter walls of towers can affect the global dynamic response of masonry towers. Openings in walls reduce the effective mass and effective cross-sectional stiffness of towers, both of which can directly affect their dynamic characteristics. These parameters are not explicitly considered in existing empirical equations. Neglecting the effects of an opening may be justified if the dimensions of the opening are small. However, with increasing opening size, its effects on the fundamental frequency of the towers may not be negligible.

With the aim of developing a more representative formulation, which considers some additional effective parameters on the vibration frequency of towers, a new semi-analytical methodology is proposed in this article to estimate the fundamental vibration frequency of historical masonry towers. For this purpose, a comprehensive parametric study is carried out on free vibration response of towers with different geometrical and material properties using the FE software, ABAQUS. Cross-sectional dimensions of the towers, their total height, opening size and configurations in perimeter walls, and modulus of elasticity of the masonry materials are the parameters considered as variables in the analyses. A formulation is then developed using a statistical analysis on the numerical results. Finally, the proposed methodology is employed to estimate the fundamental vibration frequency of a number of masonry towers from different regions which were identified using either in situ tests or analytical modeling. Moreover, the results are compared with some other equations available in the literature.

2 Numerical study

2.1 Verification of the finite element model

To verify the FE numerical models for free vibration analysis of the towers, Qutb Minar which is a historical minaret in India was selected. This tower has previously been examined using in situ tests and its seismic performance has been assessed in an analytical study by Peña et al. (2010). The Qutb Minar is one of the tallest historical monuments in the world. The height of the structure is about 70 m and it has been constructed using stone masonry to withstand the self-weight of the minaret. The tower has a square 18.6 m by 18.6 m stone foundation, 9.3 m thick. The circular cross section of the tower varies along its height and its diameter is 14.07 m at base and 3.13 m at top of the tower. A schematic view of the structure has been shown in Fig. 1. The perimeter walls of the tower have been constructed in three layers with different materials. The material properties of each layer, determined in an earlier investigation by Mendes (2006), have been listed in Table 1.

The 3D model of the minaret simulated in ABAQUS software has been shown in Fig. 2. The tower was simulated according to the geometrical and material properties reported and a free vibration analysis was carried out to determine the vibration mode shapes as well as their corresponding frequencies. In total, 35,320 linear brick (C3D8R) elements were used to discretize the model. C3D8R is an 8-node brick element which provides 3 translational degrees of freedom in each node. The foundation base of the minaret was considered as fully fixed. The analysis results, including the first ten vibration frequencies of the tower, have been listed in Table 2. The comparison of the numerical results with those obtained from in situ tests indicates that the adopted numerical model can properly simulate the dynamic behavior of the tower.

2.2 Parametric study

A comprehensive numerical study was conducted to evaluate the effects of geometrical and material properties, as well as openings in perimeter walls on the fundamental vibration

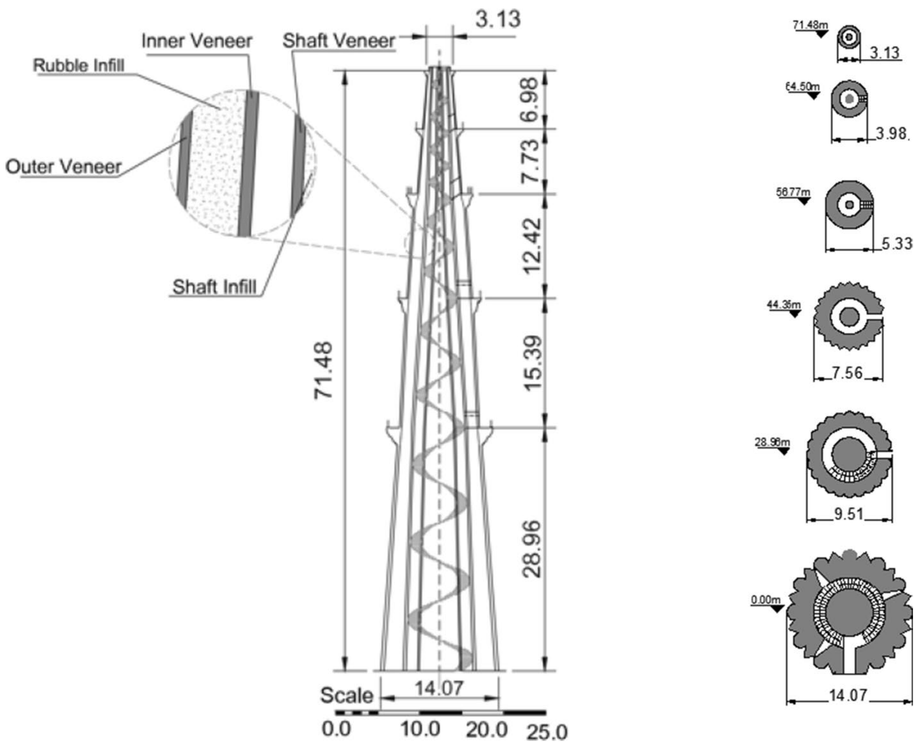


Fig. 1 A vertical section of Qutb Minar Tower (Pena et al. 2010)

Table 1 Material properties of Qutb Minar Tower employed in the finite element model (Mendes 2006)

	Density (kg/m^3)	Modulus of elasticity (GPa)	Poisson ratio
Shaft infill 1–3	1800	2.0	0.2
Shaft infill 4–5	1800	0.6	0.2
Shaft veneer	2600	5.21	0.2
Stairs	2000	3.69	0.2
Inner veneer layer 1–3	2600	5.21	0.2
Inner veneer layer 4–5	2300	2.5	0.2
Rubble infill layer 1–3	1800	2.0	0.2
Rubble infill layer 4–5	1800	0.6	0.2
Outer veneer layer 1–3	2300	2.5	0.2
Outer veneer layer 4–5	2600	3.0	0.2

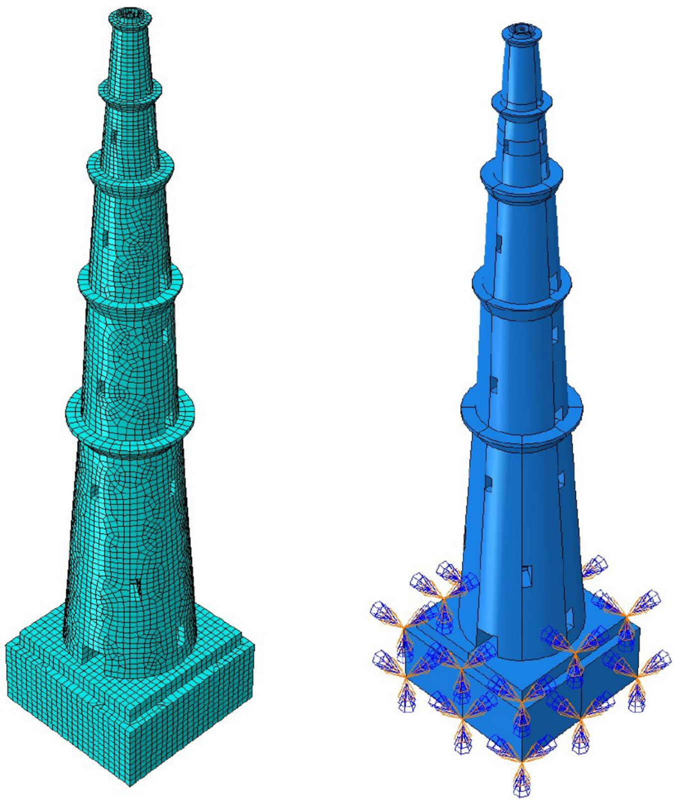


Fig. 2 3D FE model of Qutb Minar Tower

frequency of towers. The generated analysis results were also utilized for the development of a formulation for estimating the fundamental frequency of an arbitrary tower. A schematic view of the base tower with key geometrical properties has been illustrated in Fig. 3.

Table 2 The first ten vibration frequencies of the structure obtained from numerical study and in situ tests

Mode shape	Experimental (Hz)	Analytical model (Hz)
1	0.79	0.70
2	0.81	0.71
3	1.95	1.96
4	2.01	1.97
5	3.74	3.33
6	3.86	3.33
7	4.44	4.68
8	5.99	5.52
9	6.11	5.59
10	6.28	5.63

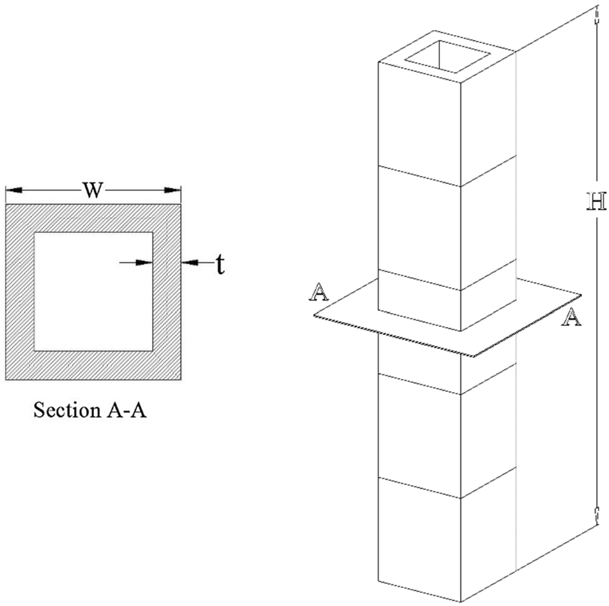


Fig. 3 A schematic view of the base tower in the numerical study

According to the significant number of experimental, analytical, and numerical studies on the dynamic response of masonry towers, it can be stated that the effective height, cross-sectional width, and thickness of perimeter walls are the major geometrical properties affecting the free vibration response of these structures. It is clear from the available database that in most cases the height of towers (*H*) ranges from 15 to 40 m. Available database and preliminary analysis results show that the effect of geometrical parameters on the fundamental frequency of towers can be considered by means of two geometrical ratios: (1) the ratio of effective height to cross-sectional width (*H/W*) (usually denoted as tower slenderness ratio) and (2) the ratio of cross-sectional width to thickness of the perimeter walls (*W/t*). On this basis, different values were considered for *H/W* and *W/t* ratios, ranging from 3 to 7.

In the case of openings in perimeter walls, opening size and location are the variables influencing the dynamic response of the tower. In this regard, the tower is divided into 5 equal sections in height and a pair of openings are inserted in each section, (see Fig. 4a). Variation in the size of the opening is defined by a parameter, termed: Opening Ratio (*OR*), as the ratio of the total opening area in each section to the External surface area of that section. The *OR* parameter ranges from 0.014 to 0.26 in this study (see Fig. 4b). To consider the effects of multiple openings in the height of towers, the openings were also inserted into the quintet parts simultaneously and the fundamental vibration frequency of the towers was determined.

Another parameter affecting the fundamental frequency of towers is the elastic modulus of masonry materials. A review of previous studies shows a wide range for this material property, ranging from about 500 MPa to 20 GPa. However, in most towers constructed with masonry materials, a range of 1.5 GPa to 6 GPa is reported. In this study, three values of elastic modulus are considered, including, 2 GPa, 3.5 GPa, and 5 GPa.

A summary of the parameters considered in the parametric study has been presented in Table 3.

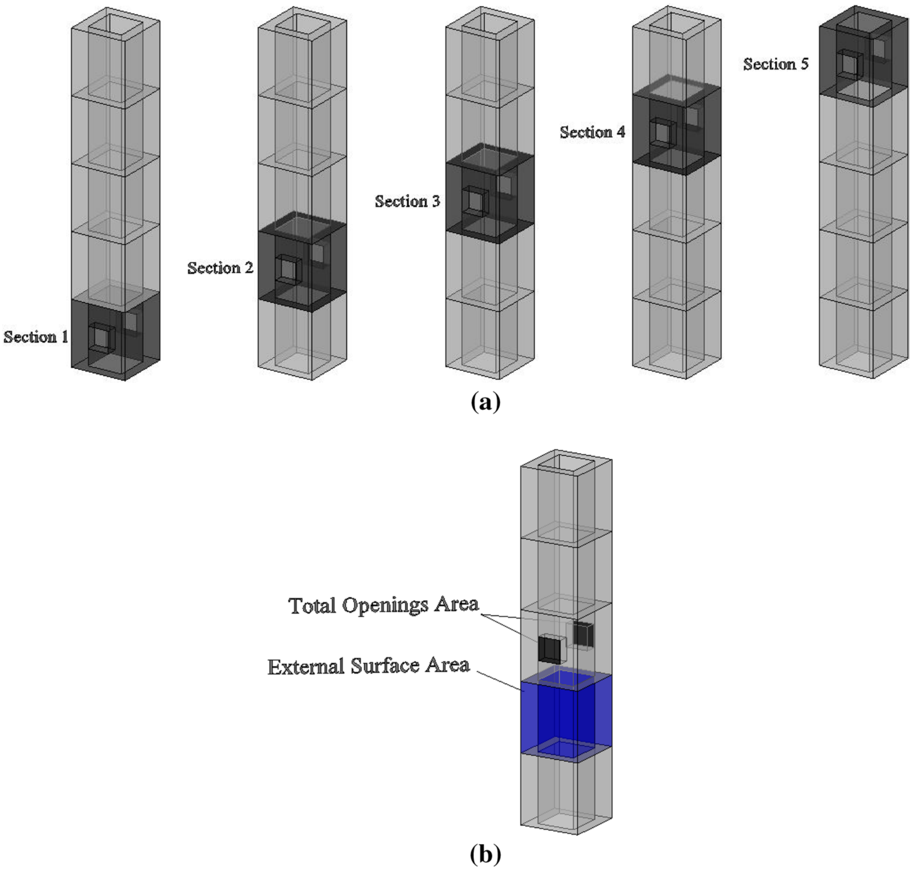


Fig. 4 **a** Opening location in the perimeter walls of the towers, **b** Definition of the Opening Ratio (*OR*) parameter

Table 3 Values of parameters considered in the parametric study

Parameter	Values
Height, *H* (m)	20, 25, 30, 40, 50
Cross sectional width, *W* (m)	2.86, 3.57, 4, 4.29, 5, 5.71, 6.0, 6.67, 7.14, 7.5, 8.33, 10, 12.5
Perimeter wall thickness, *t* (m)	0.57, 0.71, 0.8, 0.82, 0.86, 0.95, 1, 1.19, 1.33, 1.43, 1.5, 1.79, 2.08
Elastic modulus of masonry, *E* (MPa)	2000, 3500, 5000
Opening ratio, *OR*	0.014, 0.06, 0.09, 0.13, 0.17, 0.21, 0.26

For a better understanding of the impact of openings in perimeter walls on the fundamental vibration frequency of towers, a summary of the results of analyses performed on towers with two opening sizes (*OR* = 0.09 and *OR* = 0.21) has been presented in Figs. 5, 6, 7 and 8. The ranges of geometrical properties used for the FE models of towers with results reported in Figs. 5, 6, 7 and 8 are listed in Table 4. Furthermore, the elastic modulus of all of the towers is assumed to be 5 GPa. In Figs. 5 and 6, the ratio of vibration frequencies of the towers with openings in only one of the quintet sections (f) to the vibration frequency of a similar tower with solid walls (f_0), denoted as Frequency Ratio (f/f_0), has been plotted against the slenderness ratio (*H/L*) of the towers. The free vibration analysis results indicate that the existence of opening with *OR* of 0.09 in only one section of a tower may lead to an increase or decrease (depending on the effects on mass and stiffness) in the vibration frequency compared to an identical tower without an opening. This change in vibration frequency depends on the location of the opening and geometrical ratios (*W/t* and *H/W*). However, the change in vibration frequency due to the existence of opening with *OR* = 0.09 in only one section may reduce the frequency up to 15% or increase it up to around 5%. In case

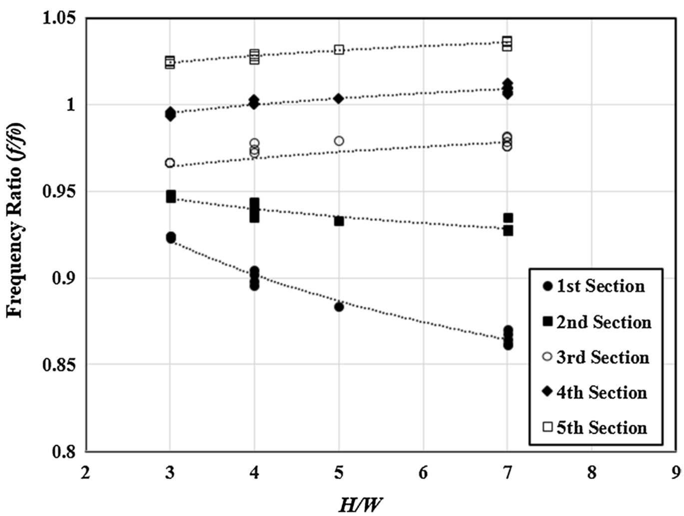


Fig. 5 Frequency ratio of towers versus Height/Width (*H/W*) ratio of towers with *OR* = 0.09 in only one section

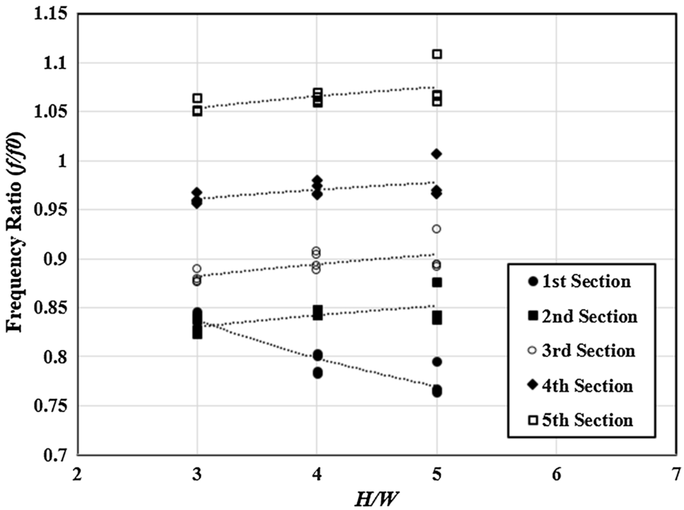


Fig. 6 Frequency ratio of towers versus Height/Width (*H/W*) ratio of towers with *OR* = 0.21 in only one section

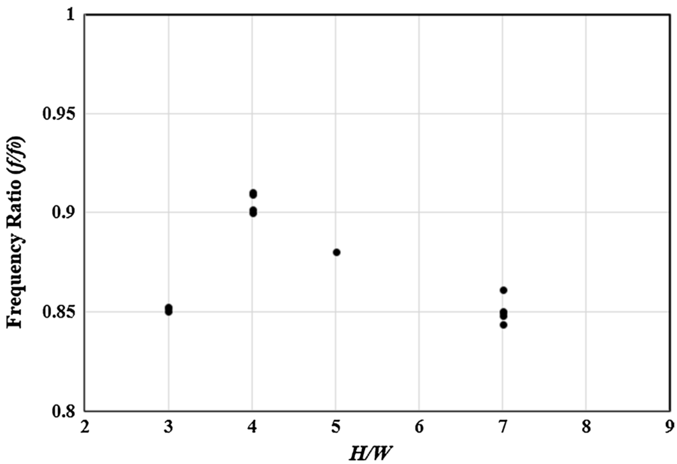


Fig. 7 Frequency ratio of towers versus Height/Width (*H/W*) ratio of towers with *OR* = 0.09 in all sections

of towers having openings with *OR* of 0.21 in one section, the vibration frequency may reduce by up to 25% or increase by up to 10%. These results confirm that the influence of openings on the vibration frequency of towers may be profound. It should be noted that position of openings in Sect. 4, has the least influence on the fundamental vibration frequency of the towers.

The results of another extreme condition, in which openings exist in all quintet sections in the height of the tower show that the decrease in vibration frequency due to openings in perimeter walls can be up to around 65% for towers with *OR* of 0.21 (see Figs. 7, 8).

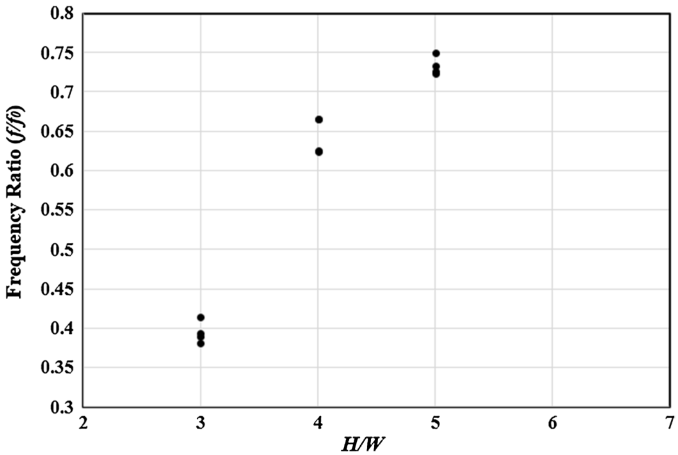


Fig. 8 Frequency ratio of towers versus Height/Width (*H/W*) ratio of towers with *OR* = 0.21 in all sections

Table 4 Geometrical properties of the towers, results of which are illustrated in Figs. 5, 6, 7 and 8

Height (m)	20, 25, 30, 40, 50
Cross-sectional width (m)	2.86, 3.57, 4, 4.29, 5, 5.71, 6.0, 6.67, 7.14, 7.5, 8.33, 10, 12.5
Perimeter wall thickness, *t* (m)	0.57, 0.71, 0.8, 0.82, 0.86, 0.95, 1, 1.19, 1.43, 1.5, 1.79, 2.08

3 The proposed methodology for the estimation of fundamental frequency

A database was compiled from the characteristics of numerical models including geometrical and mechanical features, as well as, the frequency of towers obtained from free vibration analysis of around 1000 towers as described in the previous section. A statistical nonlinear regression analysis was performed in SPSS software on the collected database to achieve an approximate formulation to determine the fundamental frequency of towers.

In statistical studies, there are two methods including linear regression analysis and nonlinear regression analysis which estimate the value of the dependent variable by changing the values of the independent parameters and show a relationship between these variables. The general form of this relationship has been shown in Eq. (6):

$$Y = h(x_1, x_2, \ldots, x_m; \gamma_1, \gamma_2, \ldots, \gamma_p) + e \tag{6}$$

where Y is the dependent variable, x_1, …, x_m are the predictors (independent variables), γ_1, …, γ_p are the parameters, and h is an appropriate function of the predictors and e is the error term. In a linear regression, it is necessary to have a relation between independent variables and the dependent variable in linear form. Contrary to a linear regression, in a nonlinear regression, various forms of equations can be used to find a nonlinear relationship between the dependent variable and a group of independent variables. Also, more than one parameter for every independent variable can be used.

The proposed formulation is based on the modification of Eq. 4, which gives the exact value of the fundamental frequency of a cantilever beam. In real cases, several openings may exist along the height of towers and the empirical correlation must be able to take into account their influence on the dynamic properties of towers. The general form of the proposed equation for estimating the fundamental vibration frequency is as follows:

$$f = \left(\beta_1 \times \alpha_1 + \beta_2 \times \alpha_2 + \beta_3 \times \alpha_3 + \beta_4 \times \alpha_4 + \beta_5 \times \alpha_5\right) \times f_0 \tag{7}$$

where β_i are the coefficients obtained from the nonlinear regression based on the database resulted from free vibration analysis of different towers with different opening configurations along the height (see Table 5), αi are the ratios of fundamental frequency of a tower with an opening in *i*th section to the fundamental frequency of the tower without any opening in the perimeter walls:

$$\alpha_i = f_i / f_0 \quad i = 1 \text{ to } 5 \tag{8}$$

where f_0 can be determined using Eq. 4, while f_i can be estimated by means of the following empirical equation derived using a nonlinear regression on the vibration frequencies of towers with openings of different sizes located in only one of the quintet sections along the height of the towers:

$$f_i = \left(1 + \left(\left(\lambda_1 \times OR_i^{0.1}\right) + \left(\lambda_2 \times i^{0.25}\right)\right) \times \left(\left(\lambda_3 \times \frac{H}{W}\right) + \left(\lambda_4 \times \frac{W}{t}\right)\right)\right) \times f_0 \tag{9}$$

where λ_j are the coefficients determined from the nonlinear regression and are tabulated in Table 4, OR_i is the opening ratio which is the ratio of the opening area in *i*th section to the perimeter area of the section, *i* is the section number which ranges from 1 to 5 and *H/W* and *W/t* are the geometrical ratios, defined before. The independent variables in Eq. (9) are selected according to the generated in the parametric analysis of Sect. 2.2.

The procedure of the proposed methodology in estimating the fundamental vibration frequency of a masonry tower has been explained by means of a flowchart, illustrated in Fig. 9.

Considering the insignificant effect of an opening on the fundamental frequency of tower, when the opening is positioned in the sections with minor influence (such as section four), or when it is small, to reduce computational cost, it can be omitted in the analysis.

4 Verification of the proposed method

To evaluate the accuracy of the proposed method for real masonry towers, 33 towers were selected (see Table 6) and their fundamental frequencies were determined by means of the proposed method and also by four other equations developed previously in the literature.

Table 5 Values of the coefficients of the empirical equations

Coefficient no.	1	2	3	4	5
β	0.412	1.058	1.895	−1.124	−1.237
λ	−0.353	0.334	−0.032	0.112	–

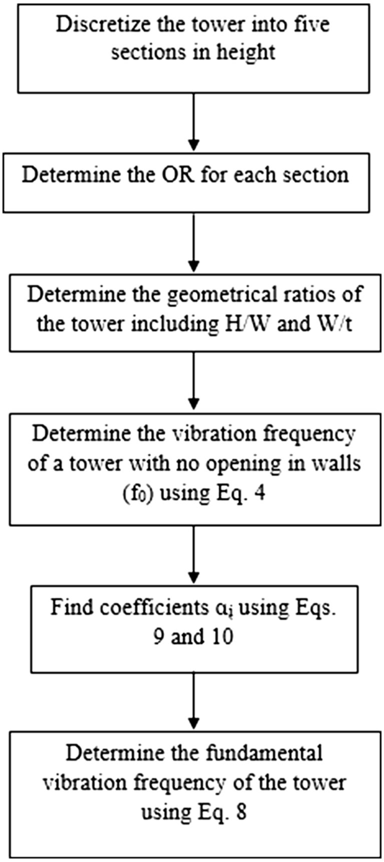


Fig. 9 Flowchart for determining the fundamental frequency of a historical masonry tower using the proposed method

From these, two equations were suggested by design codes of European countries with a significant number of historical towers (REF) and two equations were recently developed based on empirical and semi-analytical approaches by Diaferio et al. (2018a, b) and Bartoli et al. (2017), respectively. The selected towers had either been subjected to in situ dynamic tests or were analyzed through detailed finite element models, therefore, reasonably accurate results for their fundamental frequency of vibration were available. Since in real cases, the towers may be non-prismatic in height, the average geometrical properties of the cross-section is employed in formulation. The vibration frequencies of the selected towers obtained from the proposed semi-analytical relations were compared with the exact values, as well as, those obtained using the four other available empirical relations, in Table 7. The comparison of the vibration frequencies obtained from the proposed method with the actual frequencies (those obtained from experiments or numerical analysis) indicates that the formulation technique has a satisfactory accuracy for the estimation of the fundamental frequency of existing towers. To better compare the efficiency of the proposed formulation, two parameters including average error and the coefficient of determination, denoted as R^2, may be suitable. The average error can be determined as follows:

$$\bar{e} = \frac{\sum_{i=1}^{n} \frac{|(f_1)_i - (f_{actual})_i|}{(f_{actual})_i}}{n} \tag{10}$$

Table 6 Towers used for the verification of the proposed formulation

Tower/minaret no.	Tower/minaret	References	Cross section type
1	Ideal case study in northern Italy	Casolo et al. (2017)	Quadrilateral
2	Tower of Treves castle, Italy	Valente and Milani (2016)	Quadrilateral
3	Clock tower in Trecenta, Italy	Valente and Milani (2016)	Quadrilateral
4	Maistra tower of praetorian palace, Italy	Valente and Milani (2016)	Quadrilateral
5	Pighin tower, Italy	Valente and Milani (2016)	Quadrilateral
6	Clock tower in Lendinara, Italy	Valente and Milani (2016)	Quadrilateral
7	Bell tower of San Giacomo church, Italy	Valente and Milani (2016)	Quadrilateral
8	The bell tower of the Church of Santas Justa, Spain	Ivorra et al. (2010)	Quadrilateral
9	The bell tower of Announziata, Greece	Foti et al. (2015)	Quadrilateral
10	The bell tower of Roccaverano, Italy	Bonato et al. (2000)	Quadrilateral
11	The bell tower of “Nuestra Sra. De la Misericordia Church”, Spain	Ivorra and Pallarés (2006)	Quadrilateral
12	Civic tower of in L’Aquila, Italy	Cimellaro et al. (2011)	Quadrilateral
13	The “Matildea” Bell Tower, Italy	Milani et al. (2012)	Quadrilateral
14	Tower of the University of Coimbra, Portugal	Júlio et al. (2008)	Quadrilateral
15	The Collegiata of San Vittore bell tower, Italy	Gentile et al. (2009)	Quadrilateral
16	The civic tower of Soncino, Italy	Casciati and Al-Saleh (2010)	Quadrilateral
17	The torre Grossa masonry tower, Italy	Bartoli et al. (2013)	Quadrilateral
18	The bell tower of Cathedral of Monza, Italy	Gentile and Saisi (2007)	Quadrilateral
19	Mogadouro Clock Tower, Portugal	Ramos et al. (2010)	Quadrilateral
20	Aversa’s Dome bell tower, Italy	Abruzzese et al. (2008)	Quadrilateral
21	Capocci’s Tower, Italy	Abruzzese et al. (2008)	Quadrilateral
22	The bell tower in North East Italy	Casolo et al. (2013)	Quadrilateral
23	The bell tower in North East Italy	Casolo et al. (2013)	Quadrilateral
24	The bell tower in North East Italy	Casolo et al. (2013)	Quadrilateral
25	The bell tower in North East Italy	Casolo et al. (2013)	Quadrilateral
26	Ziar Minaret in Isfahan, Iran	Hejazi et al. (2015)	Circular
27	Ali Minaret in Isfahan, Iran	Hejazi et al. (2015)	Circular
28	Chehel-Dukhtaran Minaret in Isfahan, Iran	Hejazi et al. (2015)	Circular
29	Iskenderpasa Minaret in Turkey	Altunisik (2011)	Circular
30	Bell Tower on Corfu Island, Greece	Kouris (2012)	Quadrilateral
31	Historical Minaret in Turkey	Dogangun et al. (2008)	Circular
32	Historical Minaret in Turkey	Dogangun et al. (2008)	Circular
33	Historical Minaret in Turkey	Dogangun et al. (2008)	Circular

where $(f_1)_i$ is the estimated fundamental frequency corresponding to ith tower, $(f_{actual})_i$ is the actual fundamental frequency of the ith tower and n is the number of towers.

R^2 or “R squared” parameter indicates the accuracy of the formulation which estimates the value of the dependent variable based on the value of the independent variable(s). The closer R^2 to one, the greater the accuracy of the relationship. The values of average error and R^2 were

Table 7 Fundamental frequencies of the towers estimated using five different formulas

Tower/minaret no.	Actual (experimental/numerical) frequency	Proposed method	NTC2008 (2008) Equation (1)	NCSE-02 (2002) Equation (2)	Diaferio et al. (2018a, b) Equation (3)	Bartoli et al. (2017) Equation (5)
1	1.96	1.75	1.69	1.67	1.74	1.6
2	1.47	1.58	1.86	2.38	2.57	1.93
3	1.79	2.22	1.97	2.41	2.76	2.13
4	1.43	1.83	1.72	2.47	2.45	2.12
5	1.75	2.86	1.98	2.72	3.06	2.5
6	1.67	1.75	1.75	2.17	2.24	1.83
7	1.12	1.06	1.76	1.64	1.78	1.14
8	2.15	2.5	1.38	1.49	1.44	2.18
9	2.62	2.58	2.12	1.81	2.32	2.42
10	1.66	1.79	1.74	1.43	2.07	1.78
11	1.29	1.31	1.23	1.09	1.36	1.32
12	1.48	1.66	1.19	1.13	1.33	1.35
13	2.44	2.37	1.64	1.96	1.93	2.11
14	2.13	2.21	1.44	1.36	1.54	2.23
15	1.21	1.35	1.39	1.31	1.6	1.39
16	1.05	1.28	1.28	1.18	1.33	0.975
17	1.3	2.18	0.99	1.08	1.05	1.66
18	0.59	0.535	0.81	0.83	0.82	0.504
19	2.56	2.72	2.12	2.13	2.68	2.18
20	1.38	1.05	1.01	1.43	0.9	0.991
21	2	1.59	1.36	1.59	1.33	1.52
22	3.45	3.86	2.5	3.66	4.9	4.88
23	2.63	2.18	2.12	2.04	2.58	1.8
24	1.79	1.83	1.64	1.88	1.87	1.52
25	1.64	1.49	1.56	1.7	1.63	1.29
26	0.58	0.668	1.58	1.07	1.09	0.767
27	0.78	0.893	1.4	1.34	1.19	0.889
28	0.79	0.51	1.1	0.95	0.69	0.534
29	1.09	0.779	2.11	1.37	1.81	0.864
30	1.24	0.826	1.66	1.29	1.37	0.651
31	0.71	0.551	1.67	0.95	1.02	0.483
32	1.11	0.83	1.94	1.18	1.44	0.721
33	1.96	1.4	2.34	1.43	2.06	1.19

calculated for four mentioned equations as well as the proposed semi-analytical formulation using Eq. (11), see Table 8.

Table 8 The values of average error and R^2 corresponding to different equations

	Proposed method	NTC2008 (2008) Equation (1)	NSCE-02 (2002) Equation (2)	Diaferio et al. (2018a, b) Equation (3)	Bartoli et al. (2017) Equation (5)
R^2	0.79	0.33	0.49	0.55	0.71
Average error	18.1%	36.0%	27.8%	32.0%	21.5%

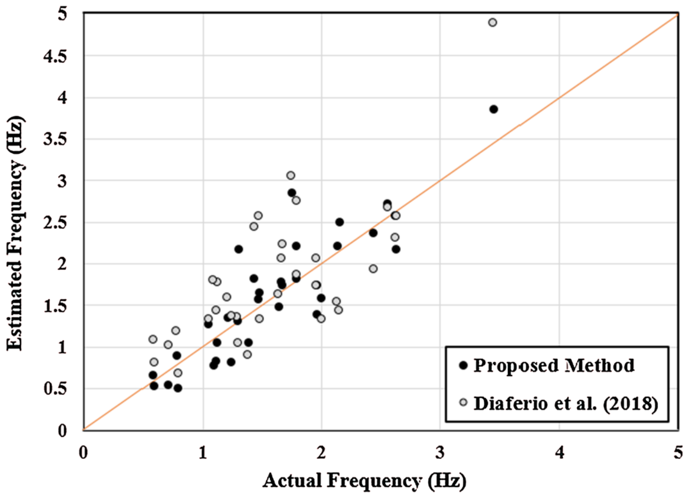
$$R^2 = \left(\frac{n(\sum_{i=1}^{n} (f_1)_i (f_{actual})_i) - \left(\sum_{i=1}^{n} (f_1)_i\right)\left(\sum_{i=1}^{n} (f_{actual})_i\right)}{\sqrt{\left[n\sum_{i=1}^{n} (f_1)_i^2 - \left(\sum_{i=1}^{n} (f_1)_i\right)^2\right] \times \left[n\sum_{i=1}^{n} (f_{actual})_i^2 - \left(\sum_{i=1}^{n} (f_{actual})_i\right)^2\right]}} \right)^2 \quad (11)$$

where $(f_1)_i$ is the estimated fundamental frequency corresponding to ith tower, $(f_{actual})_i$ is the actual fundamental frequency of the ith tower and n is the number of towers.

The calculated values for both of the parameters indicate that the accuracy of the simple equations suggested in Italian and Spanish codes is less than that of recently developed empirical and semi-analytical formulations and the proposed methodology in this article This is due to the fact that, the equations recommended by design codes, for simplicity, only consider the major geometrical properties of a tower such as its effective height and cross-sectional width. On the other hand, the formulation proposed by Bartoli et al. (2017), which considers additional influential parameters on dynamic characteristics of towers such as elastic modulus and density of the masonry, appears to be more accurate than the code equations and the empirical equation proposed by Diaferio et al. (2018a, b). However, in the semi-analytical method proposed in the present article, the values of the average error and R^2 (18.1% and 0.79, respectively) indicate that the proposed method produces more accurate and more coherent results compared to all the other four approximate formulations. This can be attributed to the fact that in the proposed method, more parameters affecting the fundamental frequency of a tower, including its height, cross-sectional width, perimeter walls thickness, openings in the walls and elastic modulus of masonry are taken into consideration. To further compare the efficiency of the proposed semi-analytical formulation and the equations developed by Batroli et al. (2017) and Diaferio et al. (2018a, b), the estimated frequencies of 33 towers by means of these three methods are plotted against the actual frequencies in Fig. 10.

5 Conclusions

A semi-analytical formulation was proposed using a generated database for estimating the fundamental vibration frequency of historical towers, as a key dynamic property of these structures. In deriving the semi-analytical formulation, the effects of symmetrical openings



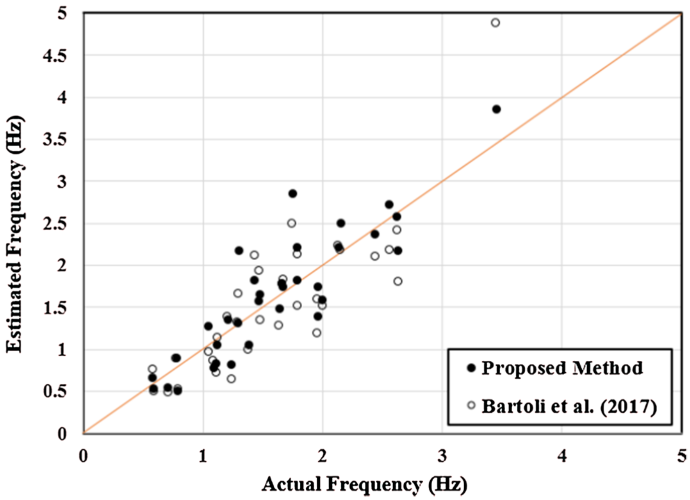


Fig. 10 Estimated Frequency versus Actual Frequency corresponding to the proposed semi-analytical method and equations of Diaferio et al. (2018a, b) and Bartoli et al. (2017)

in the perimeter wall of the tower, as well as, the geometrical and material properties of tower were considered. The extensive parametric analysis conducted showed that the presence of openings has a substantial influence on the fundamental frequency of a tower. This point was further highlighted when the results of the proposed semi-analytical formulation were compared with those from other available approximate formulations, which generally do not consider the effects of openings. The comparative results showed that the proposed semi-analytical formulation is able to predict the fundamental vibration frequency of a masonry tower more accurately than the simple code-recommended approaches and other approximate formulations available in the literature. It should be noted that since in the numerical study, it was assumed that the towers are prismatic with square cross-section and the openings are symmetrically positioned in the walls, it is expected that the proposed formulation lead to more accurate results for towers with these characteristics.

References

Abruzzese D, Ferraioli M, Miccoli L, Vari A (2008) Seismic improvement of masonry towers. In: Proceedings of 8th international seminar on structural masonry 2008, Istanbul, pp 395–403
Altunişik AC (2011) Dynamic response of masonry minarets strengthened with Fiber Reinforced Polymer (FRP) composites. Nat Hazards Earth Syst Sci 11(7):2011–2019
Baraccani S, Palermo M, Gasparini G, Silvestri S, Trombetti T, Azzara RM (2016) The static and dynamic monitoring of the Asinelli tower in Bologna, Italy. In: Proceedings of the 8th European workshop On structural health monitoring (EWSHM 2016), 5–8 July 2016, Spain, Bilbao
Bartoli G, Betti M, Giordano S (2013) In situ static and dynamic investigations on the "Torre Grossa" masonry tower. Eng Struct 52:718–733
Bartoli G, Betti M, Vignoli A (2016) A numerical study on seismic risk assessment of historic masonry towers: a case study in San Gimignano. Bull Earthq Eng 14(6):1475–1518
Bartoli G, Betti M, Marra AM, Monchetti S (2017) Semiempirical formulations for estimating the main frequency of slender masonry towers. J Perform Constr Facil 31(4):04017025
Bonato P, Ceravolo R, De Stefano A, Molinari F (2000) Cross-time frequency techniques for the identification of masonry buildings. Mech Syst Signal Process 14(1):91–109
Cakir F, Uckan E, Shen J, Seker S, Akbas B (2016) Seismic performance evaluation of slender masonry towers: a case study. Struct Des Tall Spec Build 25(4):193–212
Camata G, Cifelli L, Spacone E, Conte J, Torrese P (2008). Safety analysis of the bell tower of S. Maria Maggiore Cathedral in Guardiagrele (Italy). In: Proceedings of the 14th world conference on earthquake engineering, Beijing, pp 12–17
Carnimeo L, Foti D, Ivorra S (2014) On modeling an innovative monitoring network for protecting and managing cultural heritage from risk events. Key Eng Mater 628:243
Casciati S, Al-Saleh R (2010) Dynamic behavior of a masonry civic belfry under operational conditions. Acta Mech 215(1–4):211–224
Casolo S, Milani G, Uva G, Alessandri C (2013) Comparative seismic vulnerability analysis on ten masonry towers in the coastal Po Valley in Italy. Eng Struct 49:465–490
Casolo S, Diana V, Uva G (2017) Influence of soil deformability on the seismic response of a masonry tower. Bull Earthq Eng 15(5):1991–2014
Castellazzi G, D'Altri AM, de Miranda S, Chiozzi A, Tralli A (2018) Numerical insights on the seismic behavior of a non-isolated historical masonry tower. Bull Earthq Eng 16(2):933–961
Cimellaro GP, Piantà S, De Stefano A (2011) Output-only modal identification of ancient L'Aquila city hall and civic tower. J Struct Eng 138(4):481–491
Clemente P, Saitta F, Buffarini G, Platania L (2015) Stability and seismic analyses of leaning towers: the case of the minaret in Jam. Struct Des Tall Spec Build 24(1):40–58
de Silva F, Ceroni F, Sica S, Silvestri F (2018) Non-linear analysis of the Carmine bell tower under seismic actions accounting for soil–foundation–structure interaction. Bull Earthq Eng 16(7):2775–2808
Diaferio M, Foti D, Mongelli M, Giannoccaro NI, Andersen P (2011) Operational modal analysis of a historic tower in Bari. In: Chávez M (ed) Civil engineering topics, vol 4. Springer, New York, pp 335–342
Diaferio M, Foti D, Giannoccaro NI, Ivorra S (2014) Optimal model through identified frequencies of a masonry building structure with wooden floors. Int J Mech 8(1):282
Diaferio M, Foti D, Gentile C, Giannoccaro NI, Saisi A (2015) Dynamic testing of a historical slender building using accelerometers and radar. In: 6th International operational modal analysis conference (IOMAC 2015). Ediciones Univ Oviedo, Serv Publicaciones Univ Oviedo, Campus Humanidades Edificio Servicios, pp 1–10
Diaferio M, Foti D, Giannoccaro NI, Ivorra S (2018a) Measuring the modal parameters of a cultural heritage tower by using strong-motion signals. Acta IMEKO 7(3):86–94. https://doi.org/10.21014/acta_imeko.v7i3.601
Diaferio M, Foti D, Potenza F (2018b) Prediction of the fundamental frequencies and modal shapes of historic masonry towers by empirical equations based on experimental data. Eng Struct 156:433–442
Dogangun A, Acar R, Sezen H, Livaoglu R (2008) Investigation of dynamic response of masonry minaret structures. Bull Earthq Eng 6(3):505–517
Foti D (2014) Non-destructive techniques and monitoring for the evolutive damage detection of an ancient masonry structure. Key Eng Mater 628:168–177
Foti D, Ivorra S, Sabbà MF (2012) Dynamic investigation of an ancient masonry bell tower with operational modal analysis: a non-destructive experimental technique to obtain the dynamic characteristics of a structure. Open Constr Build Technol J 6:384–391

Foti D, Diaferio M, Giannoccaro N I, Ivorra S (2015) Structural identification and numerical models for slender historical structures. In: Asteris P, Plevris V (eds) Handbook of research on seismic assessment and rehabilitation of historic structures. Chapter 23. Engineering Science Reference, Hershey, pp 674–703. ISBN 13: 9781466682863, ISBN 10: 1466682868. https://doi.org/10.4018/978-1-4666-8286-3.ch023

Foti D, Diaferio M, Giannoccaro NI, Ivorra S (2016) Structural identification and numerical models for slender historical structures. In: Information Resources Management Association (ed) Civil and environmental engineering: concepts, methodologies, tools, and applications. IGI Global, Hershey, pp 196–222

Gentile C, Saisi A (2007) Ambient vibration testing of historic masonry towers for structural identification and damage assessment. Constr Build Mater 21(6):1311–1321

Gentile C, Saisi A, Gallino N (2009) Operational modal analysis and FE modelling of a masonry tower. In: 3rd International operational modal analysis conference, pp 499–506

Hejazi M, Moayedian SM, Daei M (2015) Structural analysis of Persian historical brick masonry minarets. J Perform Constr Facil 30(2):04015009

Ivorra S, Pallarés FJ (2006) Dynamic investigations on a masonry bell tower. Eng Struct 28(5):660–667

Ivorra S, Pallarés FJ, Adam JM, Tomás R (2010) An evaluation of the incidence of soil subsidence on the dynamic behaviour of a Gothic bell tower. Eng Struct 32(8):2318–2325

Júlio EN, da Silva Rebelo CA, Dias-da DA (2008) Structural assessment of the tower of the University of Coimbra by modal identification. Eng Struct 30(12):3468–3477

Kouris SS (2012) Applied earthquake engineering in the research of vulnerable masonry structures. J Civ Eng Sci 1(4):39–46

Kouris SS, Weber MK (2011) Numerical analysis of masonry bell-towers under dynamic loading. J Civ Eng Archit 5(8):715–722

Maheri MR, Motiollahi F, Najafgholipour MA (2011) The effects of pre and post construction moisture condition on the in-plane and out-of-plane strength of brick walls. Mater Struct 44(2):541–559

Mendes N (2006) Analise estrutural do minarete Qutub Minar. Universidade do Minho, Guimaraes **(In Portuguese)**

Milani G, Casolo S, Naliato A, Tralli A (2012) Seismic assessment of a medieval masonry tower in Northern Italy by limit, nonlinear static, and full dynamic analyses. Int J Archit Herit 6(5):489–524

Ministerio de Fomento (2002) Norma de Construcciòn Sismorresistente Parte General y Edificaciòn (Spanish Standard). NCSE 2002, Madrid (**in Spanish**)

Ministero delle Infrastrutture e dei Trasporti (2008) Nuove Norme Tecniche per le Costruzioni. NTC 2008, D.M. del Ministero delle Infrastrutture e dei Trasporti del 14/01/2008. G.U. n. 29 del 04.02.2008, S.O. n. 30, Roma (**in Italian**)

Peña F, Lourenço PB, Mendes N, Oliveira DV (2010) Numerical models for the seismic assessment of an old masonry tower. Eng Struct 32(5):1466–1478

Pieraccini M, Dei D, Betti M, Bartoli G, Tucci G, Guardini N (2014) Dynamic identification of historic masonry towers through an expeditious and no-contact approach: application to the "Torre del Mangia" in Siena (Italy). J Cult Herit 15(3):275–282

Rainieri C, Fabbrocino G (2011) Predictive correlations for the estimation of the elastic period of masonry towers. In: Proceedings of the 4th international conference on experimental vibration analysis for civil engineering structures—EVACES 2011

Ramos LF, Marques L, Lourenço PB, De Roeck G, Campos-Costa A, Roque J (2010) Monitoring historical masonry structures with operational modal analysis: two case studies. Mech Syst Signal Process 24(5):1291–1305

Shakya M, Varum H, Vicente R, Costa A (2014) Predictive formulation for the estimation of the fundamental frequency of slender masonry structures. In: Second European conference on earthquake engineering and seismology, Istanbul AUG, pp 25–29

Valente M, Milani G (2016) Seismic assessment of historical masonry towers by means of simplified approaches and standard FEM. Constr Build Mater 108:74–104

Valente M, Barbieri G, Biolzi L (2017) Damage assessment of three medieval churches after the 2012 Emilia earthquake. Bull Earthq Eng 15(7):2939–2980