ORIGINAL RESEARCH PAPER



### Seismic safety evaluation of reinforced concrete masonry infilled frames using macro modelling approach

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Received: 25 March 2016/Accepted: 5 March 2017/Published online: 17 March 2017 © Springer Science+Business Media Dordrecht 2017

**Abstract** Many reinforced concrete buildings have been built with masonry infill walls for architectural needs without considering their mechanical contribution. However, ignoring the structural influence of infills may lead to significant inaccuracies in the prediction of the actual seismic capabilities of the structure. Aiming at providing numerical tools suitable for engineering practice, simplified methodologies for predicting the nonlinear seismic behaviour of infilled frame structures (IFS) have been proposed, mostly considering the contribution of the infill as an equivalent diagonal strut element. In this paper, an alternative plane macro-element approach for the seismic assessment of IFS is proposed, validated and applied to a benchmark prototype building. The model validation is focused on recent experimental and numerical results that investigate the influence of non-structural infills, also in the presence of different openings layouts. As a benchmark investigation, a multi-storey plane frame prototype, for which the results of pseudo-dynamic tests are available, is investigated and compared to the results obtained by using a commonly adopted single-strut model. The merits and drawbacks of the considered numerical approaches are highlighted.

Keywords Infilled frame structures (IFS)  $\cdot$  Masonry infilled reinforced concrete frame (MIRC)  $\cdot$  Macro-element  $\cdot$  Discrete element approach  $\cdot$  Seismic vulnerability  $\cdot$  DIANA  $\cdot$  3DMacro

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#### 1 Introduction

Infilled frame structures (IFS) represent a significant percentage of the existing and new buildings in the south European and Mediterranean areas. According to the geographical location and age of the construction, two main typologies of buildings can be identified: buildings designed for vertical loads only and buildings designed according to a seismic code. In both cases, a large number of the buildings have been built with masonry infill walls for non-structural reasons, their role being not only to divide space but also to provide thermal and acoustic insulation and protection against weather and fire. In these cases, the structural contribution of masonry infill panels is generally neglected in structural analysis, leading to a significant inaccuracy in the prediction of the lateral stiffness, strength and ductility of the structure. As highlighted by many authors (Mehrabi et al. 1996; Buonopane and White 1999; Asteris 2003; Kunnath et al. 1990; Negro and Colombo 1997; Kakaletsis and Karayannis 2008; Dolsek and Fajfar 2001), ignoring the role of frame-infill panel interaction is not always safe, resulting in a possible change of the seismic demand and a substantial alteration of the actual structural scheme to be considered. The presence of infill walls can significantly modify the stiffness resistance and ductility spatial distribution along the structure. Furthermore, strong irregularity in the plan location or distribution of the openings in the infills may cause unconsidered torsional effects during seismic events. However, the highly nonlinear masonry infill response and the ever-changing contact conditions along the frame-infill interfaces make the simulation of the nonlinear behaviour of infilled frame buildings a challenging computational problem. A rigorous simulation of the complex nonlinear behaviour of infilled frames requires the use of computationally expensive nonlinear finite element models (Mehrabi and Shing 1997; Madan et al. 1997; Ghosh and Amde 2002; Harpal et al. 1998; Asteris 2008; Stavridis and Shing 2010), capable of reproducing the degrading behaviour of the masonry and the complex interaction between the frame and infill through e.g. accurate nonlinear interface elements (D'Ayala et al. 2009; Macorini and Izzuddin 2011). However, detailed approaches require computational resources that are hardly feasible for large buildings and are difficult to apply in real cases.

Aiming to provide numerical tools suitable for engineering practice, many authors developed simplified methodologies for predicting the nonlinear seismic behaviour of IFS based on a macro-model strategy, in which the infills are modelled according to equivalent simplified schemes that are capable of accounting for their influence on the structural response (Ellul and D'Ayala 2012; Asteris et al. 2015; Rodrigues et al. 2010).

The most commonly used practical approach is the so-called 'diagonal strut model'. According to this approach, the infilled masonry is represented by a diagonal strut under compression. Since the first proposals (Polyakov 1960; Holmes 1984), many alternatives have been proposed for the evaluation of the equivalent strut width, considering also the presence of openings (Liauw and Kwan 1984). More recently, with the aim of obtaining a better description of the effect of the infill in the surrounding frame, some authors proposed models wherein the infills are replaced by a set of equivalent struts (Thiruvengadam 1985; El-Dakhakhni et al. 2003). A review on existing numerical modelling strategies for infilled frame structures can be found in Asteris et al. (2011), where the advantages and disadvantages of single- and multi-strut models are pointed out.

An alternative innovative approach for the simulation of the seismic behaviour of IFS, suitable for research and engineering practice applications, has recently been proposed and validated numerically and experimentally by the authors (Caliò and Pantò 2014; Marques and

Lourenço 2014). In this approach, the infills are modelled by 2D geometrically consistent equivalent mechanical macro-models capable of simulating the in-plane nonlinear response of unreinforced masonry buildings (Caliò et al. 2012), while the reinforced concrete frames are modelled by concentrated-plasticity beam–columns. The adopted term 'geometrically consistent' is used to emphasize that the geometry of the macro-element is coherent with the actual two-dimensional geometry of the infill. Also, in the presence of openings the geometrical consistence is assured through a mesh of macro-elements. This novel approach has been applied to mixed reinforced concrete masonry structures (Marques and Lourenço 2011) using the software 3DMacro (Macro 2015) in which the macro-model has been implemented.

In this paper, the proposed macro-element approach is experimentally validated by using a recently published experimental campaign on several infilled frame structures (Pereira 2013) and a numerical comparison in the presence of different opening layouts (Akhoundi et al. 2015), by adopting the software DIANA (CEST). As a further contribution, this geometrically consistent approach is compared with the well-known single-strut model. The latter computational strategy suffers from an inevitable geometric inconsistency that, as highlighted in the paper, is a source of several drawbacks. Finally, the nonlinear behaviour of a multi-storey prototype infilled frame has been investigated under pushover analysis. The prototype has been built at real scale and subjected to pseudo dynamic tests at the ELSA laboratory in ISPRA (Carvalho and Coelho 2001), where some numerical simulations have already been performed according to the single-strut model strategy (Dolšek and Fajfar 2008). The prototype has been designed to be representative of a typical residential construction designed for vertical load only, and is hence characterized by non-ductile reinforcement details.

The purpose of this paper is manifold and can be summarized in the following goals:

- To validate an innovative approach, recently proposed in the literature (Caliò and Pantò 2014), through a comparison with recent experimental results that consider the presence of openings in masonry infills;
- To provide a critical appraisal of two different numerical approaches, suitable for engineering practice, through a comparison with experimental results performed on a multi-storey prototype, which has been tested experimentally and numerically, being representative of a wide class of infilled frame structures not designed to resist earthquake actions;
- To carry out a further investigation on the role of structural and non-structural infills by investigating the linear and nonlinear behaviour of the prototype buildings and performing a seismic assessment evaluation consistent with the Eurocode provisions.

The results here reported highlight that the standard European procedure for the assessment of reinforced concrete structures could be significantly influenced by the presence of non-structural infills. The explicit modelling of the infills by the innovative macro-model approach appears to capture a more realistic response, which is of interest mainly for complex geometries and in the presence of openings.

#### 2 The proposed macro-element approach

The proposed model simulates the infill frame by using a hybrid approach in which the skeleton frame is modelled according to a concentrated plasticity beam-column element, while the masonry infill is simulated by means of a plane discrete element (Caliò and Pantò

2014; Caliò et al. 2012). This element can be represented through a simple mechanical scheme, which is an articulated quadrilateral with rigid edges connected by four hinges and two diagonal nonlinear springs. Each side of the quadrilateral can interact with other elements or supports by means of a zero thickness interface constituted by a discrete distribution of nonlinear springs. Namely, each interface is discretised by n nonlinear orthogonal springs, perpendicular to the panel side plus a longitudinal spring, parallel to the panel edge. In spite of its great simplicity, such a basic mechanical scheme is able to simulate the main in-plane failures of a portion of masonry wall subjected to in-plane horizontal and vertical loads (Caliò et al. 2012). It is worth noticing that each macro-element inherits the plane geometrical properties of the corresponding modelled masonry portion. As a consequence, contrary to the simplified models based on the equivalent strut element approach, there is no need to define an effective dimension of the equivalent element (see Fig. 1).

The surrounding frame interacts with the masonry infill by means of the nonlinear-links distribution along the macro-element interfaces. In order to evaluate the nonlinear behaviour of the frame element, it has been assumed that plastic hinges can occur along the beam span in several cross-sections, uniformly spread along the length of the frame. In particular, the cross-sections that can potentially exhibit a plastic behaviour are those in which the frame element is joined to the masonry infill through the orthogonal links or to other elements. This latter assumption provides a reliable modelling of the frame elements since it allows the simulation of the formation of plastic hinges in different positions along the beam length, consistently with the adopted level of infill discretization. The inelastic behaviour of the element, concentrated in the rigid-plastic hinges, is governed by the interaction surfaces of the reinforced concrete cross-sections. In the application reported in the following, the interaction surfaces have been evaluated according to the theory of plasticity following the approach reported in Caliò and Pantò (2014).

The effectiveness of the simulation of the nonlinear behaviour relies on a suitable choice of the mechanical parameters of the model, inferred by an equivalence between the masonry medium and a reference continuous model characterised by simple but



**Fig. 1** Modelling of infilled frame with and without openings according to different mesh discretization: **a** the geometrical layout; **b** basic mesh; **c** more refined mesh resolution

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reliable constitutive laws as reported in Caliò and Pantò (2014). This equivalence is based on a straightforward fibre calibration procedure, and is based only on the main mechanical parameters that characterise the masonry according to an orthotropic homogeneous medium (Caliò et al. 2012).

In the considered approach, the macro-element inherits the geometry of the masonry portion that is modelled. This aspect constitutes a great advantage that is not common to all the simplified approaches based on a macro-element discretization. Furthermore, the consistent geometry of the element makes it possible to implement models with an irregular distribution of the openings and to implement models characterised by different levels of discretization associated with the mesh resolution and with the fine-tuning of nonlinear links in the interfaces. In the presence of openings, the minimum mesh discretization that is compatible with the opening geometry (Fig. 1b) is suitable to simulate the complete non-linear response of the structure. However, a more refined mesh could be considered, as proposed in the following analyses, in order to improve the detail of the masonry–frame interaction through a richer kinematics. More details on the influence of the mesh size discretization on the infilled frame response can be found in Caliò and Pantò (2014), where a sensitivity analysis of the discretization parameter of the model is reported.

#### **3** The considered strut model

In the numerical simulations, the macro-model described in Sect. 2 has been compared with a strut model approach, which is used and recommended in engineering practice due to its low computational cost and ease of use. Some formulations suggest the use of the strut model also in the presence of openings by proposing a reduced stiffness and strength. Figure 2 shows an example of a single bay infill frame with a central opening; in the plane, the macro-element model discretization of the window opening is described by considering an appropriate mesh of the macro-element that is consistent with the actual geometry, while in the strut model the influence due to the presence of the opening is considered simply by considering a different calibration of the diagonal struts.

In the application reported in the following, for the calibration of the equivalent strut model a simplified bilinear curve, which does not take into account the explicit dependency on the axial load of the flexural and shear behaviour of the panels, has been considered. The equivalent stiffness has been evaluated according to Eq. (1) as proposed in Farid (1996), while the strength has been assigned by considering Eq. (2), following the approach proposed by Žarnić and Gostič (1997).



Fig. 2 Infill frame with opening (a); macro model (b); strut model scheme (c)

$$K = \frac{G_w L_w t_w}{H_w} \tag{1}$$

$$F_{\max} = 0.818 \frac{L_w t_w f_t}{1.925 L_w / H_w} \left( 1 + \sqrt{(1.925 L_w / H_w)^2 + 1} \right)$$
(2)

In Eqs. (1) and (2),  $L_w$ ,  $H_w$ ,  $t_w$ , are the length, height and thickness of the panel,  $G_w$  is the shear modulus and  $f_t$  is the tensile strength obtained through a diagonal test.

The ultimate drift has been fixed equal to 0.3%, while the presence of the opening is considered by a reduction factor ( $\lambda_0$ ), as proposed in Dawe and Seah (1988), that is applied to the stiffness and strength and given by

$$\lambda_0 = 1 - \frac{1.5L_0}{L_w} \tag{3}$$

 $L_0$  being the width of the opening.

Among the different approaches already proposed in the literature, the strut model here considered has been chosen because it is based on a very simple formulation, independent from the mechanical characteristics of the frames that are widely applied in practical engineering applications. Furthermore, this approach is the same as the one that has been used in the numerical simulation already performed for the benchmark (Dolsek and Fajfar 2005) here considered.

#### 4 Validation of the proposed approach

The adopted macro-element approach has already received some experimental and numerical validation (Caliò and Pantò 2014; Marques and Lourenço 2014). However, the capability of the model to provide a suitable prediction also in the presence of non-structural infills, characterised by quasi-brittle masonry with different openings distributions, has not yet been compared with experimental results. With this aim, a recent experimental campaign performed on several infilled frame prototypes (Pereira 2013) is considered. Furthermore, detailed numerical simulations in the presence of regular and irregular opening distributions (Akhoundi et al. 2015) are taken into account.

In the first sub-section, experimental tests on an infilled frame without openings but considering the further contribution of the rendering is investigated, while the influence of a regular and irregular opening distribution is explored in the Sect. 4.2.

### 4.1 Experimental validation of a full infilled frame with and without rendering

The considered prototypes have been constructed to be representative of typical nonstructural infilled reinforced concrete structures built in the South European and Mediterranean areas. Two groups of prototypes are considered, with and without rendering, for which experimental and numerical results are available in the literature (Pereira 2013; Akhoundi et al. 2015). The experimental campaign has been carried at the University of Minho and performed by Pereira (2013). Three single-bay and one-storey reinforced concrete infill frames were tested under in-plane lateral load. The infill panels were constituted by a single leaf of brick masonry with a high percentage of horizontal holes and mortar joints. Here, the two experimental tests relative to an unreinforced infill with (Wall Ref-02) and without (Wall Ref-01) rendering are considered.

The tests were carried out considering cyclic static loads, by means of a horizontal actuator placed at the top of the frame. The layout of the geometrical characteristics of the infill frame and the geometrical reinforcing bars are summarised in Fig. 3.

The damage scenarios observed during the experimental tests are sketched in Fig. 4. For both specimens no significant damage was observed in the surrounding frame. It can be observed that, in the absence of rendering, the cracks are mainly concentrated along the mortar joints, while, in the presence of rendering, the crack distribution in the plaster surface follows a different path.

In the numerical simulation, reported in the following, the stress/strain relationship for concrete in compression has been assumed to be of parabolic type up to the strain  $\varepsilon_{co}$ , and of rectangular type with peak compressive stress ( $f_c$ ) up to the ultimate strain  $\varepsilon_{cu}$ . The initial modulus of normal deformation is  $E_c$ , while the tensile strength is  $f_{ct}$ .

The stress/strain relationship for steel has been taken as elastic-perfectly plastic with normal deformation modulus ( $E_s$ ), yield stress ( $f_y$ ) and ultimate strain ( $\varepsilon_u$ ). The adopted mechanical parameters for the reinforced concrete and steel are summarised in Table 1, according to Pereira (2013).

The mechanical properties of the masonry infills are reported in Table 2. These have been obtained by considering the experimental tests reported in Pereira (2013) through an inverse curve fitting procedure. In particular, uniaxial compression tests have been



Fig. 3 Geometrical layout and reinforcement details from Pereira (2013) (dimensions in cm)



Fig. 4 Damage distribution in the specimens: a Wall Ref-01, b Wall Ref-02

<b>Table 1</b> Mechanical characteristics of concrete and steel										
Concrete		Steel								
$E_c$ (MPa)	$f_{ct}$ (MPa)	$f_c$ (MPa)	$\varepsilon_{co}$ (%)	$\varepsilon_{cu}$ (%)	$E_s$ (MPa)	$f_y$ (MPa)	$\varepsilon_u$ (%)			
31,500	2.35	31.5	0.2	0.35	210,000	500	10			

Table 2 Mechanical characteristics of masonry infill

Flexural behaviour					Shear behaviour				
E (MPa)	$\sigma_t$ (MPa)	$\sigma_c$ (MPa)	G <sub>t</sub> (N/mm)	G <sub>c</sub> (N/mm)	G (MPa)	$f_{\nu 0}$ (MPa)	$tan(\phi)$ (–)	G <sub>sh</sub> (N/mm)	
Wall Ref-	01								
1580	0.10	1.00	0.02	1.00	700	0.07	0.58	0.10	
Wall Ref-02									
3600	0.25	1.26	0.03	1.00	1550	0.27	0.35	0.50	



Fig. 5 Constitutive law for the flexural (a) and diagonal shear (b) behaviour

performed on masonry prisms to estimate the initial elastic modulus and the ultimate compression strength of the masonry. The tensile properties, associated with the bed mortar joints, have been obtained considering the results of the out-of-plane bending tests, while triplet tests have been used to estimate the cohesion, friction coefficient and fracture energy associated with the sliding and diagonal shear mechanisms.

With the aim of considering a constitutive law that is as simple as possible, an elastoplastic behaviour with tensile strength ( $\sigma_i$ ) and compressive strength ( $\sigma_c$ ), modulus of normal deformation (E), with limited ductility, has been considered for the orthogonal links of the interfaces, while a Mohr-Coulomb criterion has been assumed for the diagonal nonlinear links, in which (G) is the shear deformation modulus  $(f_{v0})$  the shear strength without axial load and  $(\phi)$  the internal shear angle. The ductility capacities, for flexural and diagonal shear, are ruled by a fracture energy criterion in which  $G_t$ ,  $G_c$  and  $G_{sh}$  represent respectively the fracture energy in tension, compression and diagonal shear. The corresponding adopted uniaxial envelope curves are plotted in Fig. 5.

The sliding along the mortar joints has been modelled by considering a rigid-plastic Coulomb friction behaviour with cohesion c = 0.4 MPa and tangent of friction angle  $tan(\phi) = 0.7$ . The adopted discretization is relative to a regular mesh characterised by an



Fig. 6 Numerical and experimental curves: a Wall Ref-01; b Wall Ref-02



**Fig. 7** Damage in the infill at the collapse for the Wall Ref-01 (**a**) and Wall Ref-02 (**b**); bending moment in the frame for the Wall Ref-01 (**c**) and Wall Ref-02 (**d**)

element size  $0.50 \times 0.50$  m and a distance between the orthogonal springs of the interfaces of 5 cm. The same geometrical description is considered for the two prototypes. In the Wall-Ref-02 the contribution of the rendering has been taken into account by attributing different homogenised mechanical properties to the infills (Table 2).

Figure 6 reports the comparison of the capacity curves of the walls in terms of the total lateral force versus the top displacement. Figure 7a, c show the damage scenarios, while Fig. 7b, d report the bending moment distribution in the frame corresponding to the last step of the analysis. The pushover curves are in good agreement with the experimental and numerical results reported in Akhoundi et al. (2015). The latter have been obtained by using a nonlinear FEM model implemented in DIANA (CEST), in which the masonry infill and its surrounding frame are represented by four-noded plane-stress elements. The non-linear behaviour of the infills has been modelled by means of a Total Strain Crack Model based on the fixed stress–strain law concept available in DIANA. After the peak value of



Fig. 8 Lateral forces in the columns and infills: a Wall Ref-01; b Wall Ref-02

the lateral resistance, the discrete model shows an abrupt loss of resistance due to the brittle shear behaviour assumed in the numerical analyses. This behaviour corresponds to the rupture of the infills, as shown in Fig. 8.

The macro-element representation of damage (Fig. 7a) highlights that the collapse mechanisms, at the macro-scale, are associated with a composite shear failure mechanism due to a combination of tensile cracking and shear-diagonal mechanisms. By comparing the results with and without the rendering contribution (Fig. 8a, b), it can be observed that the presence of rendering increases the maximum resistance of the infilled frame to about double. In both cases, the shear contribution of the interacting frame is influenced by the interaction with the infilled panel. Since the presence of the infill modifies the behaviour of the surrounding frame, generally anticipating its entrance in the nonlinear range due to the interaction, after the rupture of the infill the resistance of the structure does not necessarily follow the bare frame skeleton curve, as observed in Fig. 8.

#### 4.2 Numerical validation of windowed infilled frame

The presence of openings in the infills can strongly modify their contribution in terms of resistance and ductility. In this section, a numerical validation of the proposed model is performed by considering numerical results recently obtained by using nonlinear FEM

analyses according to an anisotropic continuum model (Akhoundi et al. 2015). The numerical investigation here considered is not supported by any experimental comparison. However, this further comparison of finite element versus discrete element models is considered with the aim of showing the ability of the discrete model to simulate the response of infill frames in the presence of an irregular disposition of openings. The results are relative to the Wall Ref-01 considered in the previous sub-paragraph, in the presence of different opening layouts. The mechanical characterization of the continuous FEM, as described in Akhoundi et al. (2015), is based on the same experimental campaign reported in Pereira (2013), already used to calibrate the discrete model, as described in the previous sub-section.

The first investigation, reported in Fig. 9, analyses the response of the structure due to the presence of central window openings for different sizes. Each geometrical layout is identified by the ratio  $A_O/A_m$ , where  $A_O$  is the area of the opening and  $A_m$  the area of the masonry, as reported in Table 3. The dimensions of the infill are  $3.50 \times 1.70$  m. Also, the behaviour of a bare frame, as the limit case, has been considered.

Figure 9 reports a comparison in terms of base shear versus top horizontal displacement. Again, a good agreement is observed between the macro-model and the continuum model. The collapse mechanisms obtained by the macro-model approach are reported in Fig. 10.

Although the compared numerical approaches are quite different, a satisfactory agreement in terms of limit values and ductility is observed. The FEM approach provides a more regular (smooth) nonlinear response, while the pushover curves of the discrete model are characterised by several discontinuities due to rupture along the interfaces between the macro-elements. The differences between the two approaches are always less than 15%. It can be observed that for case C, characterised by the larger central opening, the presence of



Fig. 9 Capacity curves of the opening models

	Model	b' (cm)	h' (cm)	A <sub>o</sub> /A <sub>m</sub> (%)
h'	A	1575	765	20
	B	2100	1020	36
	C	2450	1190	49

#### Table 3 Geometrical parameter model



Fig. 10 Collapse mechanisms of the windowed models

the infill produces a limited increase of resistance, when compared to the bare frame, showing that, in this case, the influence of the infill can be neglected in the structural analysis. A significant difference in terms of initial stiffness can be observed in Fig. 9 for the bare frame case. This is due to the different approaches used for the nonlinear frame in the two models. 3DMacro considers a lumped plasticity frame model with an initial reduced stiffness of the section, while the DIANA model is based on a continuous two-dimensional inelastic model.

The next comparisons are relative to the irregular disposition of openings in Wall Ref-01. The considered layouts, reported in Fig. 11, are intended to represent typical distributions of buildings in the European Mediterranean area. These geometries have also been investigated by means of non-linear finite element analysis, as reported in Akhoundi et al. (2015). Figure 12 shows a comparison in terms of capacity curves between the proposed macro-element strategy and the nonlinear FEM simulations. A good agreement can be found in terms of limit resistance and available ductility for all the investigated models. The damage scenarios, reported in Fig. 13, show the presence of damage in the frame and in the infills, while rocking mechanisms can be observed in the piers that separate adjacent openings.



Fig. 11 Geometrical scheme of irregular windowed infilled models (dimensions in cm)



Fig. 12 Macro-model with irregular openings: capacity curves

## 5 Structural assessment of infilled frame buildings designed for vertical loads

In this section, the proposed macro-modelling approach is applied to investigate the nonlinear behaviour of a real scale prototype building subjected to pseudo dynamic tests in the ELSA laboratory (Carvalho and Coelho 2001). The frame prototype is a four-storey three-bay RC infilled frame, conceived to be representative of typical RC buildings designed without seismic provisions and built from the 1960s to the 1980s in Southern Europe and in the Mediterranean area. The three-bay geometrical layout of the prototype is sketched in Fig. 14, where the three considered configurations (without infills, with full infills, and with windowed infills) are reported. The permanent loads applied at each level are representative of reinforced concrete slabs 4.00 m wide and 0.15 m thick. The corresponding permanent loads have been set as equal to 36.4 kN/m at the first three levels and 32.0 kN/m at the fourth level. Different nonlinear numerical models of the prototypes



Fig. 13 Macro-model with irregular openings: damage distribution



Fig. 14 Geometry of the investigated structural schemes

are analysed and compared according to the values of mechanical parameters reported in Tables 4, 5.

The stress/strain relationship for concrete in compression is assumed to be of parabolic type, up to the peak stress  $f_c$  and strain  $\varepsilon_{c0}$ , and subsequently of rectangular type, up to the ultimate strain  $\varepsilon_{cu}$ ; the stress/strain relationship for concrete in tension is assumed linear until  $f_{ct}$ . The stress/strain relationship for steel has been taken to be elastic-perfectly plastic with yield stress  $f_y$ , yield strain  $\varepsilon_y$ , and ultimate strain  $\varepsilon_u$ . The mechanical properties of the masonry infills, composed of ceramic hollow blocks completed with 1.5 cm of render for each side, were estimated by compression tests on infill masonry walls (Pinto et al. 2001; Varum 2003). An elasto-plastic behaviour, with limited stress in tension  $f_t$  and in compression  $f_{ct}$  and limited ductility governed by a fracture energy in tension and compression ( $G_t$ ,  $G_c$ ) has been considered for the orthogonal springs of the interfaces ruling the axial/

Concrete					Steel		
$E_c$ (MPa)	$f_{ct}$ (MPa)	$f_c$ (MPa)	$\varepsilon_{c0}$ (%)	$\varepsilon_{cu}$ (%)	E (MPa)	$f_y$ (MPa)	$\varepsilon_u$ (%)
22,200	1.60	1.60	0.2	0.35	204,000	34.4	3.0

Table 4 Mechanical characteristics of concrete and steel

Table 5 Mechanical characteristics of masonry infill

Flexural				Diagonal shear				Sliding shear	
E <sub>s</sub> (MPa)	$\sigma_t$ (MPa)	$\sigma_c$ (MPa)	G <sub>t</sub> (N/mm)	G (MPa)	<i>f</i> <sub>ν0</sub> (MPa)	$tan(\phi)$ (–)	G <sub>sh</sub> (N/mm)	c (MPa)	μ (-)
2900	0.59	1.33	0.02	1171	0.38	0.4	0.5	0.5	0.7

flexural response, while an elasto-plastic behaviour with elastic shear modulus (G) and the Coulomb criterion has been assumed for the diagonal and sliding nonlinear springs, governing shear-diagonal strength by  $(f_{v0}, \tan(\phi))$  and shear-sliding mechanisms by  $(c, \mu)$ . A limited ductility is considered for the diagonal shear mechanism governed by a fixed fracture energy  $(G_{sh})$ , while an unlimited ductility is considered for the sliding mechanism.

With the aim of investigating both the linear and the nonlinear behaviour of the prototypes and to compare their respective responses, firstly the modal properties of the reference linear systems have been evaluated. Then, nonlinear static analyses, associated with the fundamental mode force distribution, have been performed. The nonlinear analyses have been performed in two phases: firstly all the vertical loads, including those of the infills' self-weight, have been applied to the bare frame structure; in the second step the horizontal loads have been applied to the entire structure: frame plus infills.

The bare frame and the full infill frame layouts (Fig. 14) have been subjected to experimental tests (Carvalho and Coelho 2001) while the presence of the opening has been numerically investigated by Dolsek and Fajfar (2005) by adopting an equivalent strut model. The analyses reported in the following have been obtained by using the software 3DMacro (Macro 2015), in which both the plane macro-element and the strut model have been implemented for comparison.

The first numerical investigation is focused on the dynamic properties of the linear elastic reference systems. The first three fundamental periods of the considered structural models are reported in Table 6. The high stiffening effect related to the contribution of infills with and without openings is evident.

The fundamental period (reported as I) ranges from 0.79 s for the bar frame to about 0.19 s in the full infill frame, with an intermediate value of about 0.24 s for the windowed infill model. The corresponding vibration modes of the considered structural models are reported in Fig. 15, which shows how the presence of the non-structural infills strongly modifies the modes. A satisfactory agreement is observed between the strut model and the plane macro-model in terms of vibration frequencies. However, important differences can be observed in terms of vibration modes, particularly at the higher frequencies, due to the different modelling of the frame-infill interaction in the strut model and in the plane macro-element.

The nonlinear behaviour under seismic loads has been investigated by performing nonlinear static analyses associated with horizontal distributions of loads consistent with the first mode of vibration. Aiming at providing an experimental comparison of the adopted

Vibration periods	Ι	II	III
Bare frame	0.789	0.260	0.152
Windowed infill			
Strut model	0.248	0.081	0.050
2D Macro model	0.231	0.079	0.049
Full infill			
Strut model	0.193	0.064	0.039
2D Macro model	0.183	0.062	0.048

 Table 6
 First three vibration periods of the considered structural models (in seconds)



Fig. 15 First three vibration modes of the considered structural models











Third mode



Fig. 16 Pushover curves: comparison of numerical and experimental results

numerical models, the pushover curves, relative to the full infilled frame, have been contrasted with the envelope curve obtained by the pseudo-dynamic experimental tests (Carvalho and Coelho 2001). Figure 16 reports a comparison in terms of base shear versus lateral drift at the first level. The strut and the plane macro-element provide comparable results in terms of initial stiffness. However, the plane macro-element provides better results in the post-peak behaviour.

A comparison between the different investigated models is reported in Fig. 17, where it can be observed how the presence of the infills produces a significant increment of stiffness and resistance that, as expected, is more pronounced for the full infilled frame. Both the strut and the plane macro-element provide comparable results in terms of initial stiffness. Again, some differences are found in the values of the limit loads as well as in the trend of the post-peak behaviour.

Confirming the experimental results (Carvalho and Coelho 2001), the numerical investigations show a post-peak behaviour characterised by a sharp strength degradation due to a sequence of yield and rupture of several infills. The residual ultimate ductile behaviour is governed by the frame contribution, the infill reactions for values of drift higher than 0.4% being negligible. In terms of numerical simulations, the post-peak phase is better described by the plane macro-model due to its capability to simulate the progressive degradation of the infills as well as the complex interaction at the interfaces between the infills and the surrounding frames.



Fig. 17 Pushover curves. a Full infill frame; b windowed infill frame

In Fig. 18 the damage scenarios for the bare frame model and the infill models in the presence of openings at the ultimate load are reported. It can be observed how the simulation based on the plane-macro-model predicts a partial collapse mechanism with a damage distribution mainly concentrated at the first level, while the model based on the equivalent diagonal strut shows a collapse mechanism in which the damage is distributed at all levels. A largely different distribution of damage can be observed in the bare frame, in which the plastic hinges are distributed in all levels, with the exception of the top one.

A key aspect in the numerical simulation of the infill frame structures is associated with the modelling of the complex interaction between the infills and the surrounding frames. This interaction is dominated by the low ductile behaviour of the masonry infills and handles the nonlinear bending response of both the infill and the frame elements. In the single-strut model, the interaction along the beam length is ignored since the struts are diagonally connected to the opposite nodes of the corresponding frames. Figure 19 shows the distributions of the bending moments for the windowed infill frame, obtained by means of the 2D macro model and the strut model.

Considerable differences can be found in the numerical predictions obtained by the two considered approaches, due to the difference in the modelling of the interaction between the infill and the surrounding frame. The strut model suffers from the abovementioned geometric inconsistency.



Fig. 18 Collapse mechanisms of the **a** bare frame model, **b** strut model with windowed infills, **c** plane macro-model with windowed infills



Fig. 19 Bending moment distribution at the collapse of the windowed infill frame; a strut model; b macro model

# 6 Assessment of the investigated building models according to Eurocode 8 (EC8)

Seismic code prescriptions for existing buildings guide engineers towards procedures that are able to provide a reliable structural assessment of the investigated building structure. An unreliable model of the structure, although consistent with the code prescriptions, could provide unrealistic or misleading results. This is particularly true for infilled frame structures, for which the choice of the numerical approach to be adopted can lead to substantially different results. With the aim of performing an investigation on the role of the numerical model in the seismic assessment of a low ductile reinforced concrete frame with non-structural infills, the pushover curves of the investigated prototypes are assumed as representative of the seismic assessment of structures in line with the EC8 prescriptions and in particular the N2 method (Fajfar and Gaspersic 1996). Since, in many cases, the non-structural infills are ignored in engineering practice, also the case of a bare frame is examined.

In accordance with the EC8 classification for existing buildings (EN 1998-3 2005a), three limit states have to be considered: DL (damage limitation), SD (significant damage), NC (near collapse). These limit states are associated respectively with the first yielding bending moment, with the 75% of the plastic rotation, and with the ultimate rotations in the concrete members, these latter evaluated by considering the expression reported in EC8-part III (Appendix A) (EN 1998-3 2005b). Table 7 reports the corresponding capacity displacements of the different implemented models for each limit state and the values of global ductility, identified by the ratio of displacements in the NC and DL states ( $\mu$ ).

By comparing the model in terms of global ductility, it can be observed how the presence of non-structural infills provides a ductility reduction, which is greater in the full frame model. The results obtained by the strut models are characterised by a slightly higher ductility than those associated with the plane macro-element model.

Figure 20 shows the absolute drift distributions of the considered models, relative to the different limit states. As stated, the results obtained by the strut and macro-element models are not in perfect agreement, which is due to the different collapse mechanism predicted by the two models. The N2 method implemented in Eurocode 8 (EC8) (EN 1998-3 2005b) has been extended to infilled frames by Dolsek and Fajfar (2005). The N2 method combines pushover analysis of a multiple-degrees-of-freedom (MDOF) model with the response spectrum analysis of an equivalent single-degree-of-freedom (SDOF) model. The base shear-top displacement relationship obtained by pushover analysis has to be idealized, usually by a bilinear (elasto-plastic) idealization. As suggested by Dolsek and Fajfar, in

	DL	SD	NC	μ
Bare frame	30.7	98.1	125.0	4.1
Full frame				
Strut model	7.8	19.3	22.6	2.9
Plane macro model	9.3	21.3	24.4	2.6
Windowed frame				
Strut model	7.4	21.7	25.1	3.4
Plane macro model	8.7	21.3	26.7	3.1

 Table 7 Top displacement capacity at the limit states (mm)



Fig. 20 Drift at the limit states: a bare frame; b opening frame; c full frame



Fig. 21 Definition of the envelope nonlinear behaviour of the single degree of freedom for the investigated models (Dolšek and Fajfar 2004)

order to apply the N2 method to infilled RC frames, two modifications need to be made to the basic (simple) version, as illustrated next.

Firstly, the pushover curve has to be idealized as a multi-linear force-displacement relation rather than simply elasto-plastic. Dolsek and Fajfar (2005) suggested the use of a typical idealized force-displacement envelope corresponding to an infilled RC frame as shown in Fig. 21. It can be divided into four parts. The first, equivalent elastic part represents both the initial elastic behaviour and the behaviour after cracking has occurred in both the frame and the infill. The second part, between points  $P_1$  and  $P_2$ , represents yielding. This part is typically short, due to the low ductility of infilled frames. In the third part, the strength degradation of the infill governs the structural response until point  $P_3$  is reached, where the infill contribution approaches zero. After this point, only the frame resists the horizontal loads.

Secondly, inelastic spectra have to be determined by using specific reduction factors (i.e. the R- $\mu$ -T relation) that are appropriate for infilled frames, e.g. those proposed by Dolšek and Fajfar (2004). The structural parameters determining the reduction factor, which are employed in addition to the parameters used in a usual, e.g. elasto-plastic, system (i.e. the initial period and global ductility) are the ductility at the beginning of strength degradation  $\mu_s = D_u/D_y$ , and the reduction of strength after the failure of the infills  $r_u = F_r/F_y$  (Fig. 21). The reduction factor also depends on the corner periods of the elastic demand spectrum (T<sub>B</sub>, T<sub>C</sub> and T<sub>D</sub> according to EC8).

	<i>m</i> (kNs <sup>2</sup> /mm)	Γ(-)	T (s)	$S_{\mathrm{ay}}\left(\mathrm{g}\right)$	$S_{\mathrm{ar}}\left(\mathrm{g}\right)$	$S_{\rm dy}~({\rm mm})$	$S_{\rm dr}~({\rm mm})$	μ <sub>s</sub> (–)
Bare frame	0.164	1.28	0.77	0.137	_	25.6	_	_
Full frame								
Strut model	0.179	1.25	0.22	0.430	0.096	10.5	31.2	2.18
Plane macro model	0.175	1.32	0.23	0.410	0.197	7.0	17.7	1.34
Windowed frame								
Strut model	0.176	1.28	0.19	0.448	0.295	7.3	22.8	2.89
Plane macro model	0.162	1.27	0.18	0.562	0.279	6.5	13.8	1.50

Table 8 Main parameters of the equivalent SDOF systems

The main parameters of the equivalent degrees of freedoms, plotted in terms of acceleration and displacement format (A–D), are reported in Table 8, namely: the equivalent mass *m*, the modal participation factor  $\Gamma$ , the equivalent period *T*, the spectral values of pseudo-acceleration  $S_a$  and displacement  $S_d$  associated with the elastic limit,  $S_{ay}$  and  $S_{dy}$ , and with the ultimate limit,  $S_{ar}$  and  $S_{dr}$ .

Knowing the ductility demand at the limit state  $(\mu_{SL})$  and the period of the system (T), the reduction factor  $R(\mu_{SL},T)$ , defined as the ratio between the elastic spectral acceleration demand and the maximum spectral acceleration of nonlinear system  $(S_{ay})$ , is computed by using the relations proposed in Dolšek and Fajfar (2004) for the infill model and the equal displacement rule ( $R = \mu$ ) for the bare frame. The PGA admissible for each limit state  $(a_{g,SL})$  of the system is computed by equating the acceleration of the system  $S_A(a_g,T)$  and the inelastic spectra acceleration  $S_A(a_g,T,R)$ , both associated with the fundamental period of the system. Figure 22 presents the equivalent multi-linear SDOF of the considered models superimposed on the inelastic spectra for each limit state, for the full infilled frame and the windowed frame. The results of the bare frame are also reported for comparison. Table 9 summarises the results for all the models and each limit state.

The results of seismic assessments of the considered models, expressed as PGA, are also expressed as a bar graph in Fig. 23. The results show different predictions for each numerical strategy. For the full frame prototype, the plane macro-element approach provides higher results for all the limit states with more pronounced differences when compared to the strut model. This particular behaviour cannot be generalised since the reduction of resistance of the bare frame is balanced by its greater ductility and in the strut model the different interaction mechanism between the frame and the infills produces major differences in the model predictions. The differences are lower for the windowed infilled frame, in which the presence of openings reduces the contribution of the non-structural infills.

This comparison, relative to a simple plane structure, draws attention to the need to provide further comparisons and validations of the numerical strategies currently adopted in engineering practice for the assessment of reinforced concrete frames with masonry infills.

#### 7 Summary and conclusions

Infilled frame structures represent a significant percentage of the existing and new buildings in the South European and Mediterranean areas. A large number of these buildings have been built with masonry infill walls for architectural needs, while neglecting their



Fig. 22 Admissible PGA assessment for the full infill frame (a) and for the open infill frame (b)

contribution in the structural design. However, as highlighted by many authors, ignoring the role of frame-infill panel interaction is not always safe, resulting in a possible change of the seismic demand and in a substantial alteration of the actual structural scheme to be considered. On the other hand, the highly nonlinear masonry infill response and the everchanging contact conditions along the frame–infill interfaces make the simulation of the nonlinear behaviour of infilled frame buildings a challenging computational problem.

In this paper, an innovative plane macro-element approach is proposed and experimentally and numerically validated by using a recently published experimental campaign on several infilled frame structures and numerical investigations in the presence of different opening layouts using the nonlinear finite element software DIANA. This geometrically consistent approach has also been compared with the well-known and widely used single-strut model, which suffers from an inevitable geometric inconsistency that is a source of several drawbacks, highlighted in the paper. Important differences in terms of vibration modes and collapse mechanisms between the strut and the plane macro-element models have been found. These are due to the different capabilities of the two models of considering the infill frame interaction along the beam lengths, which is completely ignored by the single-strut model.

	Limit state	D <sub>SL</sub> (mm)	$\mu_{SL} \ (-)$	R (-)	$S_{ae}(T)(g)$	a <sub>g,SL</sub> (g)
Bare frame	NC	97.7	3.82	3.82	0.502	0.401
	SD	76.6	2.99	2.99	0.412	0.315
	DL	1.4	1.00	1.00	0.137	0.105
Full frame						
Strut model	NC	17.7	2.41	1.56	0.699	0.280
	SD	15.1	2.06	1.42	0.636	0.254
	DL	6.1	1.00	1.00	0.448	0.179
Plane macro model	NC	19.1	2.95	2.20	1.24	0.495
	SD	16.7	2.57	1.92	1.09	0.436
	DL	7.3	1.12	1.12	0.66	0.263
Windowed frame						
Strut model	NC	17.9	1.71	1.27	0.545	0.218
	SD	15.8	1.51	1.20	0.506	0.202
	DL	9.3	1.00	1.00	0.430	0.172
Plane macro model	NC	20.2	2.69	1.98	0.80	0.322
	SD	16.4	2.19	1.65	0.68	0.270
	DL	6.6	1.00	1.00	0.41	0.164

Table 9 Admissible PGA assessment



Fig. 23 Limit peak ground acceleration assessment: a full infill frame; b windowed infill frame

The results here reported highlight that the standard European procedure for the assessment of reinforced concrete structures could be significantly influenced by the presence of non-structural infills. The explicit modelling of the infills by the innovative macro-model approach appears to capture a more realistic response, which is of interest mainly for complex geometries and in the presence of openings. The results highlight that the standard European procedure for the assessment of reinforced concrete structures, in which the influence of the infill masonry walls is neglected, could significantly influence the results, producing either unsafe or conservative results, compared with an explicit modelling of the non-structural elements. The better performance of the plane macro-element can be justified by its geometrical consistency together with its capability to simulate the highly nonlinear interaction between the masonry infill and the surrounding

beams and columns through nonlinear interfaces rather than elements that share forces by means of two diagonally opposite nodes.

The strut model appears to be a crude approximation of reality, mainly in the presence of the openings. Furthermore, the safety assessments performed by using the strut model, at least for the cases investigated, seem to be more conservative than the proposed plane macro-model approach.

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