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Seismic response of multi-frame bridges

Masoud Mehr¹ · Arash E. Zaghi²

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Abstract Long cast-in-place concrete bridges are often constructed in multiple frames separated by in-span hinges. The multi-frame system offers lower construction and maintenance costs, fewer adverse effects due to creep, post-tensioning, and thermal deformations as a few of its advantages. However, the seismic response of multi-frame bridges has been uncertain owing to the complexities of their discrete system. This study intends to improve the understanding of the seismic response of multi-frame bridge systems and evaluate the applicability of current design assumptions. Responses of multiframe bridges and comparable single-frame bridges of the same length are compared. Seismic demands on multi-frame bridge columns, abutments, and in-span hinges were investigated through high-fidelity analytical simulations. Approximately 3400 nonlinear time history analyses of prototype bridges with realistic designs were performed using the OpenSees platform. Analysis of variance was implemented along with a factorial design to study the effect of several independent factors, including the number of frames, substructure system, unequal column heights, soil type, ground motion intensity, and capacityto-demand ratio. It was observed for elastic dynamic analysis that a 90 % modal mass participation ratio is not adequate to accurately estimate dynamic responses. Seismic demands on columns in multi-frame bridges are typically smaller than those in comparable single-frame bridges. The multi-frame system is seismically more robust than the singleframe system, specifically for bridges spanning non-uniform valleys that include unequal column heights. To prevent longitudinal unseating at in-span hinges, it is critical to

Masoud Mehr m.mehrraoufi@gmail.com; mehrraoufim@pbworld.com Arash E. Zaghi

zaghi@engr.uconn.edu

¹ Division of Structures, WSP | Parsons Brinckerhoff, Glastonbury, CT, USA

This research was completed while Masoud Mehr was a Research Assistant at the University of Connecticut.

² Department of Civil and Environmental Engineering, University of Connecticut, 261 Glenbrook Rd, Unit 3037, Storrs, CT 06269, USA

consider the interaction of transverse and longitudinal responses. The seismic damage to abutment backwalls and backfills in multi-frame bridges is expected to be extensive owing to small expansion joints.

Keywords Multi-frame bridges · Cast-in-place concrete bridges · In-span hinge · Nonlinear seismic response · Equal displacement analysis

1 Introduction

A single-frame bridge, shown in Fig. 1a, is an integrated column-superstructure system offering construction and maintenance cost advantages by eliminating the bearings at bents. Single-frame bridges with multiple bents provide structural redundancy for seismic design. This system is used extensively in the State of California and in other states with high seismicity. However, in long, single-frame, concrete box-girder bridges, creep and thermal deformation may impose large demands on columns, bearings, and expansion joints. Therefore, long concrete bridges are often constructed in multiple frames separated by in-span hinges in their superstructure (Fig. 1b). Multi-frame construction facilitates post-tensioning of the superstructure and reduces the adverse effects of creep deformation. In-span hinges allow the superstructure to experience longitudinal expansion and contraction without inducing large forces in supporting columns (Hube and Mosalam 2008). The transverse integrity of adjacent frames in a multi-frame bridge system is maintained by shear key elements placed within the in-span hinges.

There is interest in using multi-frame systems for long concrete bridges because of their simple construction and cost advantages. However, the seismic response of multi-frame bridges is more complex than that of single-framed bridges and is not well understood. Unseating at in-span hinges caused several bridge collapses during the 1971 San Fernando earthquake (Elnashai and Di Sarno 2008). This led to the application of longitudinal cable restrainers at in-span hinges. The failure of some of these restrainers in the 1989 Loma Prieta and the 1994 Northridge earthquakes encouraged studies on the longitudinal seismic response of multi-frame bridges. The goal of these studies was to determine the minimum gap size or seat width required to avoid significant pounding and unseating caused by the out-of-phase longitudinal movement of adjacent frames (Fenves and Ellery 1998; Hao and Chouw 2008; Singh 1994; DesRoches and Muthukumar 2004; Shrestha et al. 2013). Some other studies focused on the design of hinge restrainers as a retrofit measure for bridges with inadequate seat width (DesRoches and Fenves 1998, 2001; Trochalakis et al. 1997; Tegos and Markogiannaki 2014). Jeon et al. (2015) studied the seismic fragility of long



Fig. 1 Schematics of a a single-frame bridge, and b a multi-frame bridge

multi-frame bridges subjected to multi-support excitations. The spatial variation of ground motions may increase the relative displacement of adjacent frames, increasing the risk of unseating. Abbasi et al. (2015) demonstrated that, in this system, non-uniform column heights increase seismic vulnerability. Through a series of elastic response history analyses, Singh (1994) found that the longitudinal gap size between adjacent frames affects the transverse response because of the coupling of longitudinal and transverse responses.

There is still a lack of understanding of the seismic response of multi-frame bridges, especially in the transverse direction. The transverse seismic response of multi-frame bridges could be more complicated than its longitudinal equivalent due to the coupling of transverse displacement and rotation about the vertical axis of individual frames (Priestly et al. 1996). Relative displacements at in-span hinges caused by the rotation of frames about the vertical axis increase the longitudinal gap opening and hence the likelihood of unseating. The aforementioned studies did not consider this relationship between transverse frame response and hinge unseating.

This paper attempts to address differences between seismic responses of multi-frame and comparable single-frame systems through comprehensive nonlinear time history (NTH) analyses of two sets bridge models. Each set contains fifty-two prototype bridges designed in accordance with Caltrans seismic design criteria, SDC v1.7 (Caltrans 2013). The large number of prototype bridges allows for the study of the effects of different geometric and design parameters. To compare the responses, each multi-frame prototype has a comparable single-frame prototype. A suite of 33 ground motions with different soil classes and magnitudes is used to accommodate the variability of ground motions. The scope of this study was limited to the investigation of seismic responses under horizontal components of earthquake ground motion. It should be noted that combination of horizontal and vertical components of ground motions may cause possible detrimental effects. The main contribution of this paper is comparison of the seismic response of multi-frame and comparable single-frame bridges through comprehensive NTH analyses of these systems. It is expected that this contribution is theoretically and practically relevant because there is still a lack of understanding of the seismic response of multi-frame bridges, especially in the transverse direction, and the results obtained inform bridge designers regarding design and performance differences between these two systems.

Approximately 3400 NTH analyses were performed using the OpenSees seismic simulation platform (McKenna and Fenves 2014). The longitudinal and transverse responses of the multi-frame prototypes were compared with their comparable single-frame counterparts using established statistical analysis methods. Within this global comparison, several other seismic response and design characteristics were studied. The applicability of the equal displacement method (Caltrans 2013) for seismic design of multi-frame bridges was investigated. The drift and displacement ductility demands on columns, relative displacement demand on in-span hinges, displacement demands on abutment backwalls, and deformation demands on abutment shear keys were evaluated. These are important response parameters in the seismic vulnerability study of bridges.

The analysis of variance (ANOVA) was used to compare the aforementioned response parameters of the two bridge systems. ANOVA is a robust method for testing whether there is a statistically meaningful difference between data obtained from two different groups. This method has yet to be advanced in civil engineering. Except in certain areas, such as environmental engineering (Lye 2002), the application of this robust method has not been fully realized. In addition, a factor analysis (Montgomery et al. 2010) was conducted to describe variability among seismic response parameters. This statistical analysis method is beneficial where a large number of factors affect responses. The six factors studied herein are: (1) the number of frames, (2) the substructure system, (3) non-uniform column heights, (4) soil type, (5) ground motion intensity, and (6) column capacity-to-demand ratio.

The findings of this research are expected to expand the understanding of the longitudinal and transverse seismic responses of multi-frame bridges. The analyses show that the multi-frame system is more seismically robust than its single-frame counterpart if seat width in in-span hinges and gap size in abutments are adequately large.

2 Analysis methodology

Padgett and DesRoches (2007) found that, in general, the seismic response of bridges is more sensitive to geometric and ground motion variability than to other modeling parameters. To this end, the authors paid careful attention to the design of bridge models and the selection of ground motions.

2.1 Geometry of prototype bridges

A suite of 52 multi-frame bridge models was deliberately defined to represent a variety of realistic bridge geometries. This included straight bridge modes with two, three, four, and five frames. A span length of 200 ft (60.9 m) was selected as a practical length for posttensioned multi-frame bridges. Post-tensioned box-girder multi-frame bridges are designed with comparable frame lengths. In this study, the length of frames was assumed to be 600–760 ft (182.9–231.6 m), following practice. The position of the in-span hinges was assumed to be at one-fifth of the span after consultation with bridge engineers at the California Department of Transportation (Caltrans). These locations are near the points of contraflexure under dead loads. Twenty-six realistic ground profiles, referred to here as 'valley shaped,' were included to study the effect of unequal column height. Figure 2 shows the valley shapes assumed for each set of bridges with the same number of frames. The shallow, intermediate, and deep valleys corresponded to column clear heights of 20, 30, and 40 ft (6.0, 9.1, and 12.2 m), respectively. One prototype in each set had nonuniform column height within the frame to account for eccentricity between the centers of stiffness and mass. For example, in the sixth valley shape (V6) of the four-frame prototype set, the column heights from left to right were 30, 30, 40, and 20 ft (9.1, 9.1, 12.2, and 6.0 m) (Fig. 2).

Two superstructure widths of 40 and 64 ft (12.2 and 19.5 m), corresponding to threeand five-lane bridges, respectively, were selected (Fig. 3a, b). Single-column extended pile-shaft bents (Fig. 3a) and pinned-base two-column bents (Fig. 3b) were used for the 40-ft- and 64-ft-wide superstructure, respectively. The bents included an integral cap-beam that is 2 ft (0.61 m) wider than the column diameter. This cap-beam extended along the width of the soffit. A seat-type abutment was assumed, including a backwall and two exterior shear keys sized to fail under a design-level earthquake (Caltrans 2013). Figure 3 shows the dimensions of the superstructures. The thickness of the deck slab, webs, and soffit were determined according to MTD-10-20 (Caltrans 2008). The size of the longitudinal gaps at the in-span hinges and at the abutments was 2.0 in. (5.08 cm) according to AASHTO provisions (AASHTO 2012). The transverse gap on the sides of the in-span concrete shear key block was 1 in. (2.54 cm). The design of the prototype bridges is discussed later.



Fig. 2 Prototype sets showing geometries and valley shapes for \mathbf{a} two-frame, \mathbf{b} three-frame, \mathbf{c} four-frame, and \mathbf{d} five-frame prototypes



Fig. 3 a A single-column extended pile-shaft, and b a two-column bent with pinned-base

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2.2 Analytical modeling of prototypes

Three-dimensional inelastic spine models as shown in Fig. 4 were created using the OpenSees v2.4.4 platform (McKenna and Fenves 2014). The superstructures, diaphragms, and cap-beams were expected to remain elastic, and thus were modeled using "elasticBeamColumn" elements with appropriate flexural and torsional stiffness values recommended by Caltrans (2013). Detailed multi-component models, shown in Fig. 5, were developed for the abutments and in-span hinges. Their assemblies were composed of elements representing diaphragms, bearings, shear keys, gaps, a backwall, and soil backfill.

All components of the in-span hinges and abutments, except diaphragms, were modeled using an assembly of "zeroLength" elements. Impact on the shear keys and backwalls and longitudinal pounding were simulated through use of the uniaxial "ImpactMaterial" developed based on the Hertz model (Muthukumar and DesRoches 2006). Steel-reinforced elastomer bearings were modeled using the uniaxial "Elastic" material in the horizontal directions. In the vertical direction, the uniaxial elastic no-tension material "ENT" was used to allow for uplift. The transverse and vertical stiffnesses of bearings were determined according to Section 14.6.5.3 of the Caltrans Bridge Design Specifications (Caltrans 2008). The transverse capacity of the in-span shear keys was assumed to be equal to the transverse capacity of the bent closest to the hinge. Caltrans commonly used this limit for the lateral capacity of in-span shear keys to prevent the transfer of large transverse forces between adjacent frames. The transverse stiffness of the in-span shear keys was found to be 3200 kip/in. (5600 kN/cm) for a 15×15 in. (38×38 cm) concrete block (Megalley et al. 2002). The sensitivity of displacement responses to the shear key stiffness value was investigated by changing it in one of the three-frame prototypes. The effect of increasing or decreasing the value of the stiffness by 50 % was found to be <3 % of total displacements. To model in-span shear keys, a uniaxial "Steel02" material was assigned to the shear direction of the zero-length element shown in Fig. 5. The gap on the sides of the shear key was modeled using two zero-length elements of uniaxial "ImpactMaterial" in series with it.

The two exterior shear keys in the abutment assembly were assumed to be elasto-plastic elements (Bozorgzadeh et al. 2007). One shear key assembly was modeled on each side of the end diaphragm (Fig. 5). They were modeled using uniaxial "Steel02" material in series with "ImpactMaterial." The capacity of the abutment shear keys was assumed to be equal to the dead load reaction at the abutment (Caltrans 2013). The soil backfill in the abutment assembly was modeled using the uniaxial "HyperbolicGapMaterial." The initial stiffness



Fig. 4 The schematic of the spine model developed for multi-frame bridges



Fig. 5 Abutment and in-span hinge modeling scheme

and ultimate passive force were calculated according to Sec. 7.8.1 of SDC 1.7 (Caltrans 2013).

The column modeling scheme is shown in Fig. 6. The values of torsional and in-plane bending stiffness of the cap-beams in the two-column bents were modified to account for integrity of the cap-beam and superstructure by factors of 100 and 5, respectively (Aviram et al. 2008). The single-column bent was modeled as a fixed-base cantilever with an equivalent depth of fixity according to AASHTO 10.7.3.13.4 (AASHTO 2012). The columns of the two-column bent were modeled with a pinned base. The columns of single-and two-column bents were modeled using single nonlinear "forceBeamColumn" elements with seven and six "Mid-Distance" integration points, respectively. This formulation minimizes errors (Scott 2011), allows for the definition of a plastic hinge within the element, and eliminates the need for discretizing the column element. This feature is indispensable for modeling the extended pile-shaft (i.e., the single column) as it develops a plastic hinge in between the ground level and the point of fixity (Fig. 6). The equilibrium of the passive soil pressure on the column and the lateral load at the top was used to



Fig. 6 Column modeling scheme in single- and two-column bents

calculate the location of the maximum moment. The plastic hinge length was determined according to SDC 1.7.

An inelastic fiber section with aggregated torsional stiffness was assigned to the column element. To control the position of the plastic hinge in the single-column bents, a fiber section with elastic material for steel and elastic compression only for concrete was assigned to the first integration point to prevent yielding at the point of fixity. The unconfined concrete strength f_c was assumed to be 5.0 ksi (34.5 MPa). The Mander model (Mander et al. 1988) was implemented for confined concrete properties using the uniaxial "Concrete04" material model. This material model captures tensile cracking of crosssections and reduction of flexural stiffnesses. Longitudinal reinforcing bars were modeled using uniaxial "ReinforcingSteel" material with a 60 ksi (413.7 MPa) yield strength. The behavior of an individual column element was verified using shake table test data (Zaghi and Saiidi 2010a; Zaghi and Saiidi 2010b; Zaghi et al. 2012). Because OpenSees 2.4.4 does not allow for aggregation of both shear and torsional stiffnesses, effects of shear deformations of the columns were not included. Including shear deformations was found to have negligible impact on the response of the prototype column owing to its length-to-diameter ratio, i.e., the shear-span ratio of approximately 5. Shear-spans of columns of the studied bridges, except in a few cases, were between 3.75 and 8.2. Shear deformations become significant for columns with shear span-ratios <3 (Vosooghi and Saiidi 2013; Lee and Elnashai 2001). However, because of the significance of torsional modes of vibration in multi-frame bridge systems, the torsional stiffnesses were necessary to be included in the analytical models. The masses were assigned to all nodes for all six degrees-of-freedom. The mass of the superstructure was 12,500 (182 kN/m) and 19,500 lb/ft (285 kN/m) for 40- (12.2-m-) and 64-ft-wide (19.5-m-wide) superstructures, respectively. Live load is not included in the seismic analysis. The "Pdelta" coordinate transformation object was used to account for P-Delta effects. A 5 % Rayleigh damping (Chopra 2001) was associated with the periods of 0.5 and 2.0 s to ensure similar damping values over a wide range of periods of vibration. Special caution was taken that the proportional damping formulation did not dampen higher modes of vibration unrealistically (Charney 2008; Della Corte et al. 2013). The modeling method was verified using the results of an analogous model developed in SAP2000 ver. 15.1 (CSI 2011).

2.3 Selection of ground motions

A diverse collection of eleven acceleration response spectra (ARS), generated following Appendix B of SDC 1.7, was used for the design of prototype models. To ensure that the findings of this study are applicable to different hazard levels and site specifications, as defined by Caltrans, the set of ARSs included three hazards levels as shown in Table 1. The spectrum set is shown in Fig. 7a.

Hogond lovel	Magnitude (M _w)	PGA (g)	ARS label ^a
Hazaru level			
Moderate	6.25-6.75	0.2	B11, C11, D11, and E11
Large	7.00-7.75	0.4	B22, C22, D22, and E22
Severe	7.75-8.25	0.6	B33, C33, and D33

Table 1 Selected seismic hazard levels

^a The first letter in the ARS label denotes the soil type followed by numbers indicating magnitude and PGA levels, respectively



Fig. 7 a ARSs used for designing bridge models and b the acceleration spectrum for one of the matched ground motions

The NGA ground motion database of the Pacific Earthquake Engineering Research Center (PEER 2011) was used to select ground motions for NTH analyses. To obtain realistic bridge responses, the ground motions need to be compatible with the design demands obtained from ARSs. To that end, eleven sets of three biaxial ground motions with soil types, magnitudes, and intensity levels comparable to those of the corresponding ARSs were selected. Each set included one or two pulse-like motion(s), except that $M_w = 6.25-6.27$, to account for the near-field effect such as high-velocity pulses that may cause permanent deformations. The PEER-NGA search engine was used to find three ground motions with acceleration spectra similar to each target ARS between periods of 0.1 and 2.0 s. Subsequently, SeismoMatch spectral matching software (Seismosoft 2011) was used to generate a library of thirty-three spectrum-matched biaxial acceleration histories. This software utilizes a wavelet-based algorithm to match acceleration histories to a target acceleration spectrum. Where a set of spectrally matched ground motions are used for NTH analyses of a structure, a lower degree of dispersion in responses can be expected. In addition, the mean of response values obtained from a small set of motions are comparable to the corresponding values obtained when a large set of motions is used (NIST 2011). To maintain the non-stationary characteristics of the ground motions, the number of iterations used for spectrum matching was limited to a maximum of five iterations. For the same reason, the maximum spectrum misfit tolerance between periods of 0.1 and 2.0 s was set to 30 %. Figure 7b shows an example of spectrum-matched ground motion. Spectrum matching was limited to periods <2.0 s to preserve the pulse characteristics of motions as shown in Fig. 7b (NIST 2011). Details of ground motion properties were presented by Mehrraoufi (2015). The horizontal components of the ground motions were randomly applied in the longitudinal and transverse directions. To design bridges that are located near fault sites, Caltrans amplifies design spectra by a factor of 1.2 (Caltrans 2013). In this study, the same bridge model was used for analyses under the three motions of each set; motions with pulse-like characteristics were reduced using a factor of 1/1.2 to remain consistent. For design of ordinary bridges, SDC 1.7 (Caltrans 2013) does not require consideration of the combined effects of vertical and horizontal components of ground motions. Thus, the vertical components of ground motions were not included in the analysis. The vertical component, however, may affect the response of bridges in near-field sites owing to variation of axial forces in columns. Displacement responses may increase or decrease (Kim et al. 2011) and shear demand-to-capacity ratio may be affected by large variation of axial forces in concrete columns (Di Sarno et al. 2011). One should be mindful of this limitation of this study.

2.4 Seismic design of bridge models

The prototype bridge models were designed using the equal displacement method in accordance with Caltrans SDC v1.7 (Caltrans 2013). This method assumes that the maximum displacement of an inelastic structure is approximately equal to that of an equivalent elastic structure (Fig. 8). This assumption is valid for medium- to long-period structures with minimal strength and stiffness degradation and insignificant P-delta effects (NCHRP 2013). Elastic displacements can be estimated using the equivalent static analysis (ESA) method if dynamic analysis does not give further insight into structural behavior. Elastic dynamic analysis (EDA) is used for bridges that did not satisfy the ESA requirement (Caltrans 2013).

Because of the large number of models, a systematic and automated methodology for designing the models was indispensable. To this end, a Tcl script (ActiveState 2013) was developed and integrated into the OpenSees analysis script. The design displacement demands were found using EDA. The configuration and properties of the elastic models used in EDA were similar to those of the inelastic models with a few exceptions: (1) the columns were modeled using "elasticBeamColumn" elements, as opposed to the "forceBeamColumn" elements used for inelastic models (OpenSees Wiki 2014); (2) column members were divided into four segments (Caltrans 2013); and (3) all the transverse and longitudinal gaps were closed. The effective flexural stiffness of a cracked section was assigned to the elastic column member.

The following assumptions were made in the design of prototype bridges. (1) Column diameters were the same in each bridge. Initial diameters were determined based on an axial load ratio of $0.1 f_c$. The range of variability of shear-span ratios of columns, defined as ratios of distances between point of contraflexure and fixed support to column diameter, were approximately 4.6–8.2 and 3.5–6.3 for the single-column and two-column bents, respectively. (2) The column reinforcement ratio was kept the same within the frames. Column reinforcement ratios could be different in frames with different column heights. In accordance with SDC 1.7, columns were designed by controlling the following criteria. (1) The local displacement ductility capacity of any member was <3.0. The displacement ductility capacity is the ratio of ultimate displacement capacity to yield displacement. Ultimate and yield displacement of columns were found through inelastic analysis of





stand-alone bents. (2) It was considered that global displacement capacity of the system should be larger than the displacement demand. The global displacement capacity was found through inelastic static analysis of the bridge. (3) It was considered that the design displacement demand for any column should be less than the target values of 4 and 5 for single- and multiple-column bents, respectively. (4) Minimum plastic moment of a column had to be greater than 10 % of the axial gravitational load multiplied by the height of the column. (5) The second-order moment due to P-Delta had to be <20 % of a column's plastic moment.

In both multi- and single-frame systems, the 96-in.-diameter (2.44-m-diameter) columns were found to be adequate for the majority of single-column bents. A small number of columns in multi-frame bridges with irregular valley shapes required 102-in.-diameter (2.59-m-diameter) columns for severe hazard levels. The diameter of the columns in the two-column bents was 86 in. (2.18 m) in both multi-frame and single-frame bridges. In a few cases, the longitudinal reinforcement ratio was slightly larger in the multi-frame system than that of single-frame bridges. The longitudinal reinforcement ratio ranged between 0.01 and 0.025. The transverse reinforcement volumetric ratio was 0.016 on average. In general, the total volume of concrete and the total weight of reinforcing steel in the columns of multi-frame bridges were similar to those of the single-frame bridges.

2.5 Response history analysis

A step-by-step direct integration method was used for the NTH analyses. Newmark's time integration method with $\gamma = 0.5$ and $\beta = 0.25$ was implemented. In the case of a convergence problem, the central difference, Hilber–Hughes–Taylor, and TRBDF2 methods were tried automatically. The maximum time step was set as 0.005 s and, in the case of convergence problems, the time step was reduced by a factor of 0.5. This was done until convergence was achieved, while other integration methods were tried instantaneously. Once a stable converged step was achieved, the integration method and the time step were automatically reset to their original settings. Data recorded from the NTH analyses include: (1) base accelerations and column base reactions in the transverse, longitudinal, and vertical directions, (2) longitudinal and transverse displacements at the tops of the columns, (3) transverse accelerations at the tops of the columns, (4) longitudinal and transverse displacements at the diaphragm nodes, (5) in-span hinge transverse accelerations, (6) inspan shear key forces and in-span shear key deformations, (7) transverse and longitudinal impact forces, and (8) longitudinal and transverse abutment forces and deformations. All the calculations were made by an in-house MATLAB (Mathworks 2012) algorithm.

3 Data analysis method

3.1 Overview of ANOVA

A large dataset was generated from approximately 3400 NTH analyses. To understand the effects of geometric and design parameters, the application of systematic statistical analysis methods was necessary. Because of inherently large standard deviation values of response measures that were obtained from the time history analyses, direct comparison of mean values of responses may yield misleading observations. Therefore, a statistical analysis methodology was utilized to investigate if the differences between the means of different variables are meaningful. Analysis of variance (ANOVA) was used to compare

the seismic behavior of single- and multi-frame bridge systems. In its simplest form, ANOVA is used to understand whether there is a statistically meaningful difference between the mean values of the responses of two systems (Montgomery 2001). For example, a hypothesis could state that the maximum column drifts in multi-frame bridges are different from those of single-frame bridges for a given soil type. In the ANOVA analysis, the ratio of variance of data among groups to the variance within groups is calculated. The variance is the square of the standard deviation. This ratio is used to calculate the probability of expected response values, such as maximum drift values. This probability is called the *p* value. The *p* value can be interpreted as the probability that the variation of the response values of the bridges due to different variables, occurs by chance. A small probability value means that there is indeed evidence that the variation of the expected response values. Traditionally, the threshold for the *p* value is 0.05, which corresponds to a confidence level of 95 %.

In statistical modelling, there are dependent and independent variables. The models investigate how the former depend on the latter. The dependent variables represent the output or outcome whose variation is being studied while the independent variables represent inputs or causes, i.e., potential reasons for variation. Variables may be continuous or categorical. If a variable can take on two particular real values such that it can also take on all real values between them, then it is a continuous variable. A categorical variable is a variable that can take on one of a limited, and usually fixed, number of possible values, thus assigning each individual to a particular group or "category" (Yates et al. 2003). MATLAB (Mathworks 2012) was used to run six-way ANOVA to understand the effect of the bridge system, i.e., multi-frame and single-frame bridges, on several seismic response parameters with respect to the independent variables. In addition, the dual interaction of these variables with the bridge system type was assessed. Interaction means that a specific independent variable has different effects on different groups. For example, a hypothesis could be "soil type has a similar effect on column drift response in multi-frame and singleframe bridges," meaning there is a 95 % probability that there is no interaction of soil type with the bridge system. If the p value is >0.05, then it can be concluded that the hypothesis is not true. In other words, the effect of soil type on column drift responses in single-frame bridges is different from its effect on column drift responses in multi-frame bridges.

3.2 Definition of independent variables

The six independent variables defined to compare the seismic responses of single- and multi-frame bridge systems are listed in Table 2. Variables 1, 2, and 4 are inherently categorical with two defined subgroups while Variables 5, and 6, i.e., ground motion intensity and capacity-to-demand ratio (C/D), are continuous variables. To address this difference, they are converted to categorical variables by defining two subgroups based on their medians. The variable C/D is included to account for the strength of a structure with respect to design demands. This variable is quantified as the average of the ratios of lateral capacities, C, to lateral force demands, D, for all the columns in the bridge. If this value is equal to 1.0, then the bridge has an approximately balanced design. The third variable (valley shape) is qualitative. Qualitative data is a categorical measurement expressed not in terms of numbers, but rather by means of a natural language description. The coefficient of variation (CV) of the values of stiffness of columns in the bridge is used to quantify the valley shape with a two-subgroup categorical variable. CV is equal to the standard deviation of the stiffness of all columns in the bridge divided by the ratio of standard deviation

	Variable name	Variable level
1	Bridge length	Short (1400, 2000 ft) (427, 610 m)
		Long (2600, 3200 ft) (792, 975 m)
2	Substructure type	Single-column bent
		Two-column bent
3	Valley shape	Uniform (CV < 0.35)
		Non-uniform (CV > 0.35)
4	Soil type	Stiff (Type B, C)
		Soft (Type D, E)
5	Ground motion intensity	Low (PGA < 0.36)
		High (PGA > 0.36)
6	Capacity-demand ratio (C/D)	Low $(C/D < 1.6)$
		High $(C/D > 1.6)$

Table 2 Independent variables used to compare seismic responses of multi-frame and single-frame bridges

to the mean of the stiffness of all columns in the bridge. CV is a measure of the irregularity of stiffness values in the columns in the bridge. A smaller CV denotes a uniform column stiffness and vice versa. If the height and size of all the columns in a frame or bridge are the same, the value of CV is equal to zero.

3.3 Basic assumptions of ANOVA

The basic requirements of the factorial ANOVA are normality, homoscedasticity, and no multicollinearity of data (Montgomery 2001). The histograms of response values were checked for normality. Homoscedasticity requires that error variances of responses associated with independent variables be equal. It was verified that in most cases standard deviations of responses were comparable. Finally, no multicollinearity requires that responses be mutually independent from each other, e.g., no repeated measurements, and that the independent variables be independent from responses. Multicollinearity occurs when independent variables are inter-correlated and not independent. The variables and responses used in this study were independent in nature.

4 Results and discussion

The following sections present the major observation made in this study. It is realized that many of the conclusions are well-understood concepts in seismic analysis and design of structures. However, this study was intended to investigate whether the same seismic analysis and design concepts hold for heterogeneous multi-frame bridge systems.

4.1 Fundamental period

The fundamental period of a bridge identifies its dynamic characteristics and affects the seismic design demands obtained from a design spectrum. In this study, modal analysis was performed to determine the fundamental periods of the prototype bridges. Figure 9 displays the mean value of the fundamental periods of both systems with respect to the



Fig. 9 ANOVA results for the fundamental period of prototype bridges

independent variables. In this figure, the diamond markers show that the median values and error bars represent the standard deviations. The p-values, indicating the statistical significance of the difference between the periods of multi- and single-frame bridges for individual variables, are listed in the first row. p values showing the statistical significance of the interaction between the independent variables and the bridge system are listed in the second row at the bottom. The p values that are <0.05 are shown in boldface to emphasize the statistical significance of the differences or interactions. The statistically meaningful observations are discussed herein. It should be noted that a p value slightly <0.05 indicates that there is a statistical difference between two groups of data; however, the difference may be negligible from an engineering point of view.

The multi-frame bridges have consistently longer periods compared to the single-frame ones. The average of the differences of the periods of these two bridge systems is 0.2 s (12 % of the average fundamental period). This is a result of the additional flexural stiffness of the continuous superstructure in single-frame bridges. This difference is larger in bridges with two-column bents because the longitudinal period of some of the frames in multi-frame systems may be notably larger than other frames. Valley shape was found to have considerable interaction with the bridge system. Again, this is due to vibration of the softest standalone frame in the longitudinal direction. Another important observation is that bridges with lower capacity-to-demand ratios have longer fundamental period than those with higher capacity-to-demand ratios. Similar results have been found in numerical studies for RC buildings (Elnashai and Di Sarno 2008). This is valid for both multi-frame and single-frame systems, and could be related to the equal displacement concept used in seismic design discussed in the following section. Given the *p* values for interaction, the length of bridges, soil type, intensity of motion, and capacity-to-demand ratio do not affect the difference between the periods of single- and multi-frame bridges.

4.2 Applicability of the equal displacement method

Current Caltrans (Caltrans 2013) and AASHTO seismic design criteria (AASHTO 2011) utilize the equal displacement method to determine displacement demands at the system level.

A multi-frame bridge is a discrete system. This study investigated whether the equal displacement method holds for this system by comparing the displacement responses obtained from EDA and NTH analysis methods. For EDA, modal response analysis was performed using the acceleration spectra of the thirty-three ground motions. The correlation between the maximum transverse displacements of all columns from NTH analysis and those determined by the equal displacement method is presented in Fig. 10. Both the single- and multi-frame systems show very similar trends. For low-intensity ground motions, scatter in the data is smaller. The equal displacement method tends to overestimate displacements. For ground motions of high intensity, the data are more scattered but the average response is satisfactory in both multi- and single-frame bridges.

To quantify these results, the ratio of transverse displacement from the EDA method to that of NTH analysis method with respect to the six independent variables is shown in Fig. 11. A ratio of close to 1.0 with a small standard deviation implies the reliability of the EDA. On average, the EDA method slightly overestimates displacements in multi-frame bridges and slightly underestimates displacements for single-frame bridges. In general, the EDA method is more reliable for bridges with single-column bents than for those with twocolumn bents. This may be related to the longer fundamental period of bridges with singlecolumn bents. This method underestimates displacements of two-column-bent bridges, especially in single-frame systems. Ground motion intensity has a significant effect on the reliability of the method. For low-intensity motions, multi-frame systems show a 13 % over-prediction in displacement. For high-intensity motions, both systems show an underestimation of approximately 10 %. Figure 10 confirms this observation. Figure 11 shows that the EDA method is more reliable where the capacity-to-demand ratio is smaller. In the previous section, it was discussed that bridges with smaller capacity-to-demand ratio have longer fundamental periods. These findings are consistent with the assumption that the concept of equal displacement in seismic design works better for medium- to longperiod structures (NCHRP 2013).

4.3 Required cumulative mass partition ratio for EDA

In the EDA method, spectral responses from multiple modes are combined to estimate the dynamic response of a structure. The minimum number of modes required to achieve an accurate result depends on the number of degrees of freedom and the complexity of a structure. The value of the cumulative mass participation ratio is used as a measure to



Fig. 10 Comparison of transverse displacements from EDA and NTH analysis



Fig. 11 ANOVA results for the applicability evaluation of the equal displacement method

determine the number of modes that should be considered. Currently, seismic design codes require a sufficient number of modes to achieve a minimum mass participation ratio of 0.9 in each direction (Caltrans 2013; AASHTO 2011). Recent research suggests that responses of dynamic modes with periods as small as 0.05 s should be included (NIST 2012). In this study, the minimum mass participation ratio needed to obtain sufficiently accurate transverse displacements was investigated for both bridge systems. A sufficiently accurate displacement is defined as 90 % of the displacement corresponding to a mass participation ratio of 97 %.

The results presented in Fig. 12 show that multi-frame bridges require approximately 2 % larger cumulative mass participation ratio compared to single-frame bridges. For longer bridges, bridges with two-column bents, and bridges with non-uniform valley shapes, a larger number of modes should be considered owing to their irregular responses. The minimum cumulative mass ratio for low-intensity motion and hard soil type is larger



Fig. 12 ANOVA results for the minimum required cumulative mass participation ratio for estimation of transverse displacements

because of the larger energy content at higher frequencies. On average, the minimum required cumulative mass participation ratio is approximately 90 %; however, the large standard deviation denotes the possible inaccuracy of this value. It was observed that higher modes with small mass participation (even <5 %) may contribute to the overall response of multi-frame bridges. This is due to the vibration of individual frames, especially in long bridges with multiple frames. The design implication of this observation is that a considerably larger number of modes should be included for seismic analyses of multi-frame bridges.

4.4 Column drift and displacement ductility demands

The drift and displacement ductility of columns are widely used as measures of damage in bridge columns. Drift is defined as the ratio of the lateral displacement of the top of a column to the column's height. Displacement ductility is the ratio of lateral displacement to the idealized yield displacement. In general, drift is not a conclusive measure of damage because of its dependence on the column length.

The maxima of "longitudinal" drift and displacement ductility demands are presented in Figs. 13 and 14, respectively. The effects of the six independent variables are similar for both response parameters. The multi-frame systems show consistently smaller demand values with smaller dispersion compared to the single-frame systems. In multi-frame bridges, frames are separated using in-span hinges. The probability of synchronized longitudinal vibration of these frames is small. Thus, a single-frame bridge with larger end gap size at its abutments may experience larger longitudinal drifts. The longitudinal demands placed on two-column bents are approximately twice those for single-column bents. This is because the latter bend with double curvature while the pinned-base columns of the former behave as cantilevers. Non-uniform valley shape has a pronounced effect on the difference of the drifts between the two bridge systems.

The maxima of "transverse" drift and displacement ductility demands are presented in Figs. 15 and 16, respectively. As opposed to the longitudinal response, for some of the independent variables, the variation in transverse drift does not follow that of transverse ductility demand. The mean drift demands in single- and multi-frame bridges are



Fig. 13 ANOVA results for column drift demand in the longitudinal direction



Fig. 14 ANOVA results for column displacement ductility demand in the longitudinal direction



Fig. 15 ANOVA results for column drift demands in the transverse direction

approximately the same for the first four independent variables. However, this is not true for displacement ductility demands.

The transverse displacement ductility demands in multi-frame bridges are approximately the same as in single-frame bridges. The ductility demands in two-column bents are approximately twice those in single-column bents. The effects of other independent variables appear trivial.

4.5 Demands of abutment backwalls and shear keys

Damage to abutment backwalls due to longitudinal pounding of the superstructure is concurrent with permanent deformation of the abutment backfill. This damage may interrupt bridge service due to the failure of expansion joints and upward deformation of the approach slab. The maximum longitudinal displacement demands on abutment backwalls are presented in Fig. 17. The displacement is measured at the centerline of the



Fig. 16 ANOVA results for column displacement ductility demands in the transverse direction



Fig. 17 ANOVA results for displacement demands on abutment backwalls

bridge. The backwall damage in multi-frame bridges is significantly larger than in singleframe bridges due to a smaller expansion gap size. In the multi-frame prototypes, the gap size is 2.0 in. (5.08 cm) in accordance with AASHTO Sec. 3.12. This size was calculated based on the length of one frame, which was approximately the same in all the bridges. For single-frame bridges, the gap size varies over 3.5–7.5 in. (8.9–19 cm) depending on the length of the bridge. In both multi- and single-frame systems, shorter bridges suffer more damage to abutment backwalls owing to smaller gap sizes. The bent type and non-uniform column height did not show considerable effects. Soil type, however, had a significant effect; soft soils caused twice as much damage to abutment backwalls as stiff soils. Bridges with higher capacity-to-demand ratios have more reserve capacity in their columns; thus, the demands on their abutments are lower. To reduce damage in abutment backwalls, the authors recommend implementing a larger gap size require special expansion joint details, which are usually more expensive. Special expansion joints also increase maintenance costs. The exterior shear key elements in the abutments restrain the superstructure in the transverse direction. They are typically designed to yield or fail at a specific load in large earthquakes in order to control the damage to abutment foundations and wing walls (Fig. 5). On the other hand, excessive displacement may impair the serviceability of a bridge after an earthquake unless the expansion joint can accommodate large transverse movements and permanent transverse displacements are negligible. The maximum transverse deformation of the abutment shear keys is shown in Fig. 18. The deformation demands on shear keys in multi-frame systems are comparable to those of single-frame ones. Except for bent type, other independent variables did not contribute to differences in shear key demands in single- and multi-frame systems. Two-column-bent systems experience approximately 70 % higher demands as the in-plane stiffness of superstructures in two-column bent bridges is larger than that of single-column ones. In addition, the mass of the abutment diaphragm is much larger in two-column bent bridges. In this study, abutment shear keys in two-column bent bridges have 60 % larger capacity, corresponding to their larger dead load reaction.

4.6 Interaction of longitudinal and transverse responses at in-span hinges

As explained in the introduction, unseating due to large relative longitudinal displacements at in-span hinges has been a major source of seismic vulnerability in multi-frame bridges. The combination of large rotations about the vertical axis and longitudinal displacement may increase the displacement demand on bearings.

A schematic of the displacement at an in-span hinge is shown in Fig. 19. The gap opening at the centerline of the bridge, g_{avg} , is associated with longitudinal response while the maximum gap opening at the edge of the superstructure, g_{max} , is a result of the interaction between the longitudinal and transverse responses. Figure 20 shows the distribution of g_{max} for all single-column and two-column bent prototypes under low- and high-intensity ground motion. This distribution is close to lognormal with mean values of 4.8 and 5.28 in. (12.2 and 13.4 cm) for single-column and two-column bent prototypes, respectively. The ANOVA results for g_{max} with respect to all other variables are presented in Fig. 21. Because this response is particular to multi-frame bridges, two new groups



Fig. 18 ANOVA results for deformation demand on abutment shear keys



Fig. 20 Distribution of the maximum gap opening, gmax, in in-span hinges in multi-frame bridges

based on substructure systems, i.e., single-column and two-column bents, are defined. The maximum gap opening for in-span hinges in two-column bents is consistently larger than that of single-column systems. This is because of larger longitudinal displacement response (Fig. 13) and wider superstructure. All variables had a considerable effect on this response.

To study the importance of biaxial analysis of multi-frame bridges, as opposed to uniaxial analysis, the interaction between the transverse response and the maximum gap opening was studied through the ratio of g_{max} to g_{avg} as presented in Fig. 22. On average, the maximum gap opening at in-span hinges is 8 % larger than those calculated at the centerline, but may be larger by as much as 60 %. Bridge length has a greater effect on this ratio than the other factors. In short bridges, transverse displacement causes larger relative rotation in the superstructure. This rotation magnifies the gap opening. Non-uniform valley shapes have smaller effects than uniform valley shapes on the interaction between transverse response and maximum gap opening. Given that relative rotation of the frame is a result of the synchronized rotation of adjacent frames, different rotational stiffnesses of adjacent frames results in smaller relative rotations and, consequently, smaller corner gap openings.

The relative transverse displacement at in-span hinges was studied as the final performance parameter in this research. This response is due to the yielding of the ductile in-span shear key discussed in the modeling section. The expected relative transverse displacement



Fig. 21 ANOVA results of the maximum gap opening at in-span hinges



Fig. 22 ANOVA results of the interaction of transverse and longitudinal responses in in-span hinges

found was 1.4 in. (3.5 cm). The initial transverse gap was 1.0 in. (2.54 cm); thus, on average, the pure deformation of the in-span shear key was approximately 0.4 in. (1.0 cm). This deformation was negligible in comparison with the transverse displacement of the superstructure (Fig. 10); thus, it would not impair post-earthquake performance of a bridge. It should be noted that this observation is only valid for in-span shear keys that are designed for a force demand equal to the capacity of the adjacent bent and that are not brittle. The seismic force demands on elastic in-span shear keys are presented by the authors extensively in Mehrraoufi (2015).

5 Conclusions

Seismic response parameters of a large set of multi-frame bridges were compared with their comparable single-frame counterparts. The bridge prototypes were designed according Caltrans's seismic design criteria (SDC). High-fidelity NTH analyses performed using a suite of thirty-three ground motions. The response parameters of multi- and single-frame bridges were compared using ANOVA. The effects of the following independent factors on the responses of each system as well as the difference of responses between the two systems were investigated, including: (1) the number of frames, (2) substructure system, (3) valley shape, (4) soil type, (5) intensity of motion, and (6) design capacity-to-demand ratio. The following summarizes the observations and conclusions drawn from this research:

- Design according to Caltrans SDC or AASHTO seismic provisions yields almost the same column diameter and reinforcement for single-frame and multi-frame bridges.
- The concept of equal displacement is applicable to multi-frame systems. It tends to provide robust results for flexible systems.
- For EDA, the minimum required cumulative modal mass participation is slightly larger for multi-frame bridges than for single-frame bridges. For design purposes, this study suggests that dynamic modes with frequencies <15 Hz are included in analysis and modes with modal masses >5 % are accounted for.
- In bridges with non-uniform column heights, the longitudinal ductility demands on columns are smaller in multi-frame bridges compared to those in single-frame bridges. On the other hand, in the transverse direction, ductility demands on columns of multi-frame and single-frame systems are comparable, even for non-uniform valley shapes, i.e., uneven ground profiles. Thus, this study suggests that multi-frame systems are better options for design of a bridge where use of different column heights cannot be avoided.
- To reduce column displacement demands in multi-frame bridges with non-uniform valley shapes, it is recommended to position in-span hinges such that columns within each frame have equal stiffness.
- The ductility demand on two-column, pinned-base bents is approximately twice that on single-columns bents. Thus, the seismic detailing for the plastic hinge region of pinned-base columns must be performed more diligently.
- Displacement demands on abutment backwalls in multi-frame bridges are significantly larger than in single-frame bridges owing to smaller longitudinal gap sizes in multi-frame bridges. Gap sizes designed in accordance with AASHTO provisions for service and temperature loads are not adequate for accommodating seismic displacement demands in the abutments of multi-frame bridges. It is recommended that, in multi-frame systems, the gap sizes be increased by a factor of 2.0 to maintain the same level of damage to abutment backwalls as single-frame systems. The designers, however, should be aware of increased costs of expansion joint elements for larger gap sizes.
- Two-column bent bridges experience larger demands on abutment shear keys compared to single-column bent bridges. The approach of current codes in the United States (e.g., SDC 1.7) for design of abutment shear keys may lead to large transverse displacements at abutments of bridges with two-column bents. Therefore, it is suggested that a larger gap size is used for design of bridge with multiple-column bents where superstructures have large in plane stiffnesses.

- Relative longitudinal displacements of adjacent frames at the in-span hinges, and correspondingly, longitudinal gap openings are approximately 75 % larger in pinned-base, two-column bent bridges with columns because of smaller longitudinal stiffnesses. Therefore, the risk of unseating for multi-frame bridges with pinned-base columns is larger than for those with extended pile-shafts.
- The interaction of the transverse and longitudinal responses increases the gap opening in in-span hinges by an average of 8 %. However, the increase can be as high as 60 % in certain cases. Further studies are needed to quantify the effects of the vertical component of ground motion on in-span hinge openings.

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