

# Role of inclination in the seismic vulnerability of bell towers: FE models and simplified approaches

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**Abstract** The role played by inclination in both the load carrying capacity and seismic assessment of masonry bell towers is investigated through the results obtained from different analyses on three case studies. The Italian Code for the built heritage is not explicit about the influence of inclination on the seismic assessment of towers, leaving to practitioners the task to properly consider it, potentially leading to both an overestimation of the capacity and an underestimation of the horizontal actions. The case studies investigated in this paper are three leaning masonry bell towers, all exhibiting a quite meaningful inclination and all located in Emilia-Romagna region (Italy), recently (2012) stricken by a moderate/high intensity seismic sequence. This study compares the procedure provided by the Italian Code with the finite element (FE) results obtained through non-linear static analyses and proposes a modification of the Italian code simplified mechanical model (SMM), which explicitly accounts for the actual inclination of the towers within a cantilever beam approach. The FE results show that inclination may considerably reduce the load carrying capacity, increasing the seismic vulnerability of the structures. The SMM approach proposed properly takes into account the role played by inclination, always providing results on the safe side with respect to FEM.

**Keywords** Bell tower · Masonry · Inclination · Pushover analysis · Non-linear shell element · Simplified mechanical model (SMM)

## 1 Introduction

Masonry towers are quite widespread in Italy and represent an important portion of the built Heritage that must be preserved, especially in high-seismicity regions. Very often, such masonry towers exhibit unique peculiar morphologic and typological characteristics, which might affect their different structural behaviors under horizontal loads.

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Traditionally masonry towers were conceived to withstand vertical loads only, but in recent years national and international standards have imposed the evaluation of their structural performance under horizontal loads simulating earthquake excitations and they encourage the use of sophisticated non-linear methods of analysis.

A main issue in the seismic behavior of masonry towers is the influence of the axial stresses induced by gravity loads, whose values are often of the same order of magnitude as the ultimate compression of the masonry material. Considering that historical masonry is typically characterized by a complex geometry, irregularities and a high degree of inhomogeneity, stress concentrations can occur, thus promoting local collapses. Hence, the structural failure can be driven even by a moderate increase in the stress level, which can occur during seismic events or in the presence of long term loads.

At present, a number of studies are available in the technical literature dealing with the numerical/experimental analysis of masonry towers. A wide tradition exists in Italy and in Europe in general, with a variety of analyses ranging from the utilization of non-linear finite element (FE) codes (Abruzzese et al. 2009; Carpinteri et al. 2006; Riva et al. 1998; Bernardeschi et al. 2004; Pena et al. 2010; Milani et al. 2012; Camata et al. 2008; Valente and Milani 2016a, b; Bayraktar et al. 2010; Pintucchi and Zani 2014; Bartoli et al. 2016) to the implementation of very specific fiber-element models for the 3D non-linear dynamic simulation of slender towers (Casolo 1998, 2001) or 2D rigid elements (Casolo et al. 2013), passing through combined eigenvalues and experimental identification studies (Ivorra and Pallares 2006; Russo et al. 2010), to 2D limit analyses performed by means of a no-tension material approach (as recommended by Heyman 1995), experimental and in situ tests (Anzani et al. 2010; Binda et al. 2005; Zanotti Fragonara et al. 2016), repairing and rehabilitation proposals (Modena et al. 2002).

To perform a full literature review of all the contributions regarding the experimental-numerical analysis of masonry towers is almost impossible and outside the aims of the present paper. However, in a partial review of the literature, it should be mentioned that the great part of the studies carried out does not put emphasis on the role played by inclination on the reduction of the load carrying capacity of towers under horizontal loads. In addition, the simplified seismic assessment required by DPCM (2011) does not suggest dedicated approaches in the evaluation of the load carrying capacity of tower transversal sections, whereas the role of inclination is only implicitly taken into account in the geometric discretization. This is a key issue, because it could potentially lead to an underestimation of the seismic vulnerability, since inclination is intuitively responsible for collapse at low levels of horizontal loads.

Heyman (1992) was probably the first to analytically tackle the issue of inclination on towers, assuming masonry as a material unable to withstand tensile stresses. The approach did not account for seismic actions and one of its main limitations was represented by the hypothesis of either rectangular full or thin-walled sections and the absence of any irregularity along the height, which is typically not realistic for practical cases. However, such simplifications allowed deriving a quite simple differential equation describing the crack curve delimiting the failure mechanism and providing very useful hints to have an insight into the limit inclination angle associated with the collapse of the structure for gravity loads only.

The procedure is here generalized through a commercial spreadsheet, taking into account the presence of irregularities of the cross section and the application of horizontal loads simulating the seismic action.

In particular, the flexural load bearing capacity of the transversal section is evaluated by means of a simplified equilibrium approach, where masonry is assumed unable to

withstand tensile stresses and the compression zone is simulated with a stress block, in agreement with the procedure provided by the Italian code DPCM (2011) in the absence of inclination.

Three cases of technical interest, relying on inclined bell towers located in a region recently stricken by a long seismic sequence (Emilia Romagna, north-east of Italy, May–June 2012 seismic sequence), are investigated in detail in order to have an insight into their seismic vulnerability.

An important issue for towers and tall structures in general is certainly the role played by the vertical component of the seismic action. Intuitively indeed, the effect of a vertical ground motion could be rather important, especially in the presence of bedrock foundation.

Another key issue is the soil–structure interaction in both the static and the dynamic range. In standard design, the soil type affects the definition of the response spectrum, whereas in non-linear dynamic analyses a meaningful portion of soil, modelled as an elastic continuum, could be introduced, basically with the aim of providing a reasonable condition of constraint for the foundation as well in order to add a minimum participant mass representing the soil below the foundation. Large scale sensitivity analyses should be carried out on each case study to have a better insight into the structural response when increasing portions of interacting soil are considered. In this study, attention is focused exclusively on the behavior of the structure, whereas the effects of soil–structure interaction are left out to considerably reduce the conceptual complexity of the engineering problem.

Casolo (1998) was probably the first to show that for slender towers the influence of the vertical component of the accelerogram tends to be negligible. Unfortunately, the extension of the results to leaning towers is not straightforward and, at least in-principle, the loss of verticality could be responsible for an amplification of the effects even in other cases.

To have an insight, the aforementioned towers are modeled into FEs and their seismic behavior is investigated by means of non-linear dynamic analyses, applying only a horizontal accelerogram or combined horizontal/vertical accelerograms. Spectrum compatible accelerograms are utilized for the simulations.

Negligible differences are found between the two cases (presence or absence of the vertical component), meaning that classic static approaches (e.g. pushover or limit analysis) are effective for towers with reasonable inclination angles and small slenderness.

In this paper the role played by inclination in the load carrying capacity and seismic assessment of masonry bell towers is investigated, proposing in particular three newly conceived simplified mechanical models (called hereafter SMM-1, SMM-2 and SMM-3), which are essentially an evolution of Heyman's approach (1992). In the so-called simplified mechanical model (SMM) proposed by the Italian code on the built heritage (DPCM 2011), a tower is seen to behave as a cantilever beam and subdivided into several rigid elements. In correspondence with the centroid of each rigid element, a static horizontal force simulating the seismic action is applied, typically with a triangular distribution intuitively derived from the first mode deformed shape of the cantilever. The seismic vulnerability is finally evaluated comparing the resisting and the acting bending moments in correspondence with each interface between adjoining elements. The resisting bending moment is typically estimated with simplified formulas, where masonry is considered unable to withstand tensile stresses. It is worth noting that in DPCM (2011) the possible inclination of the tower is not mentioned. In the present SMM-1, only the horizontal component of the seismic action (treated as a static force) is considered; in SMM-2 both the horizontal and vertical components of the seismic action are applied, whereas in SMM-3, as a consequence of the reasonable hypothesis of infinite axial stiffness of the tower, the

seismic action is applied perpendicularly to the tower axis. Such models appear to have a certain interest for common design and they are benchmarked on the three already discussed cases studies. For validation purposes, several analyses are carried out by using the newly proposed SMMs, Italian Code DPCM (2011) standard cantilever beam approach and plate/shell 3D FE pushover analyses, where masonry is modeled with a damage plasticity material exhibiting very low tensile strength.

The results of the FE analyses show that inclination may considerably reduce the load carrying capacity of a tower. The SMMs approaches properly account for the role played by inclination and always provide results on the safe side when compared to FEM.

## 2 Safety assessment with simplified approaches

The present section briefly reviews the most diffused approaches to provide a safety assessment of existing masonry towers in the presence of horizontal loads, focusing on all the methods that can be used in common design (e.g. modal analysis and simplified mechanical models). The following sections present the results obtained through pushover analyses, where masonry is treated as a softening damaging material and full 3D plate and shell FE discretizations are adopted. In addition, the results obtained through sophisticated non-linear dynamic analyses are also discussed in order to investigate the role played by the vertical component of the accelerogram in the seismic safety assessment of inclined towers. Here, some modifications implemented on existing simplified mechanical models to properly account for the real inclination of the towers are proposed and put at the disposal of practitioners in common design.

### 2.1 Modal response

Response spectrum analyses are admitted by the Italian Code NTC 2008 (2008) for existing buildings, but cannot provide specific insight on the actual behavior of masonry towers, not only because the masonry material is considered elastic, but especially because in such types of structures it is difficult to identify macro-elements where a safety assessment conducted using code of practice formulas may be applied.

On the contrary, the evaluation of the fundamental period/frequency of a tower is useful in safety assessments performed using both simplified mechanical models (based on equilibrium of a no tension material model) and pushover analyses. In the majority of the cases, there is no need to estimate natural frequencies by means of complex 3D FE discretizations of the structure. As a matter of fact, considering a cantilever elastic beam approximation (Zalka 2002), the fundamental transversal vibration period (Zalka 2002; Timoshenko 1938) is well approximated in both the inclined and vertical cases as follows:

$$T_1 = 2\pi \frac{H^2}{\alpha_1} \sqrt{\frac{\rho \cdot A}{E \cdot I}} \quad (1)$$

where  $E$  is the Young modulus,  $I$  the transversal inertia of the cantilever,  $H$  the length,  $A$  the cross section area,  $\rho$  the average density and  $\alpha_1 = 3.5156$  a numerical coefficient associated with the first vibration mode.

Obviously, irregularities, such as presence of internal stairs and vaults or the high perforation of the bell cell, cannot be accounted for in Eq. (1), but the estimated

fundamental period is almost always quite accurate, with deviations not exceeding 5% from those evaluated with full 3D FE discretizations, see for instance Milani et al. (2006a, b).

### 2.2 Simplified mechanical models (SMM) according to Italian Guidelines on the built heritage

In a safety assessment performed with the so called simplified mechanical model (SMM), in agreement with the Italian Guidelines for the built Heritage DPCM (2011), the tower is subdivided by means of several cross sections into rigid blocks subjected to both vertical gravity and equivalent static horizontal loads, see Fig. 1a.

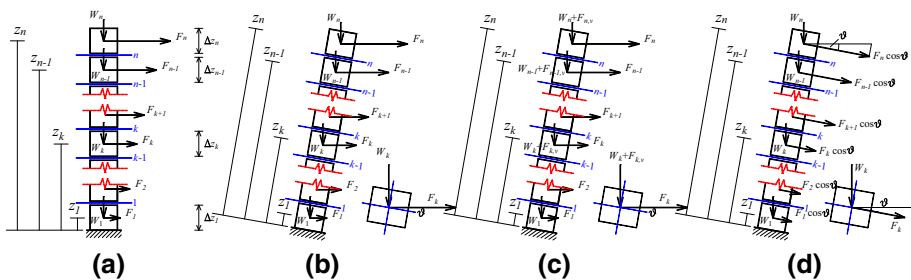
Cross sections between adjoining blocks are assumed as unable to withstand tensile stresses and the resisting bending moment is compared with the acting one. Limitations of the procedure are mainly due to the fact that failure occurs exclusively for bending actions, and in the majority of the cases strength deficiencies are found near the base sections.

The seismic base shear force  $F_{h,b}$ , and by generalization the horizontal force  $F_k$  acting on the  $k$ th rigid element, shall be determined using the following expression:

$$F_{h,b} = 0.85 \cdot \frac{S_e(T_1)}{q} \cdot \frac{W}{g} \tag{2}$$

where  $S_e(T_1)$  is the spectral acceleration of the elastic response spectrum at the fundamental vibration period of the structure in the direction under consideration,  $W$  the total weight of the tower,  $g$  the gravity acceleration and  $q$  the behavior factor that can be assumed equal to 3.6 for regular structures or 2.8 for irregular towers characterized by sudden changes in stiffness along the height or in the presence of adjacent structures in contact.

It is worth noting that in  $S_e(T_1)$  it is implicitly accounted for the soil type and the soil–structure interaction. In particular, the response spectrum is always amplified by a factor  $S = S_S S_T$ , where  $S_S$  is a coefficient accounting for the soil type and  $S_T$  is a factor considering the topography. For a soft soil (“D” type according to the Italian code)  $S_S$  depends on  $T_c^*$  [the reader is referred to Italian code NTC 2008 (2008) for further details] and it is equal to 1.8 for the geographic coordinates here considered (Ferrara municipality). Therefore, the response spectrum is implicitly amplified in order to properly consider the soft soil conditions under consideration. Such an amplification has obviously effect both in the safety assessments performed through both simplified mechanical models and pushover



**Fig. 1** Simplified mechanical model discretization into rigid elements. **a** Vertical case SMM, **b** inclined case with horizontal seismic load SMM-1, **c** inclined case with horizontal and vertical seismic loads SMM-2, **d** inclined case with seismic load perpendicular to the cross section SMM-3

analyses (within N2 method). In non-linear dynamic analyses, a portion of the soil under the foundation should be modeled, as done for instance in Casolo et al. (2013), in order to represent its actual characteristics more realistically. This matter is however outside the scopes of the present approach that focuses exclusively on simplified methods to adopt in the case of leaning towers.

Finally, it should be mentioned that, in the case of soft soils, at least in principle, the fundamental period  $T_1$  of a cantilever beam reasonably increases. Such a change of the period would have the consequence to decrease acting horizontal loads, but this is normally disregarded for the sake of safety.

### 2.3 SMM-1 with distribution of horizontal loads $F_{h,b}$

When the horizontal component of the seismic action is applied to a structure as a static force considering the seismic vertical component negligible, it is reasonable to assume, through the application of Newton's law, that the structure is subjected to a distribution of horizontal forces proportional to the mass, eventually with a shape that depends also on the first vibration mode. When such hypothesis is done, Fig. 1b, in the case of inclined towers  $F_{h,b}$  can be decomposed into shear and normal components, taking the tower axis as reference. In addition, it can be argued if a vertical component should be added, but the matter of the application of the seismic vertical component turns out to remain still controversial.

When dealing with the non-linear dynamic behavior of tall structures, it is well known that the effect of a vertical ground motion can be intuitively rather important, especially in the presence of bedrock foundation.

Casolo (1998) has shown that, by means of few DOFs cantilever beam models, the influence of the vertical accelerogram tends to be negligible for low slenderness towers. No information is however available in the case of towers with inclination. At least in-principle, indeed, the non-verticality could be responsible for an amplification of the effects induced by vertical accelerations, especially for inclination near the limit value for collapses under self-weight.

Another complication is the choice of a vertical component of the accelerogram to couple with the horizontal one, when spectrum-compatible signals are adopted, because of the difficulties of correlating the signals artificially generated.

In the case of vertical towers, the resultant horizontal action on the  $k$ th rigid element is distributed along the height proportionally to the mass and the position ( $z_k$ ) of the element  $k$  centroid, and can be evaluated by means of the following expression:

$$F_k = \frac{W_k \cdot z_k}{\sum_{r=1}^n W_r \cdot z_r} \cdot F_{h,b} \quad (3)$$

where  $F_k$  is the horizontal seismic action on the  $k$ th element,  $z_k$  is the distance of the mass center of the  $k$ th element from the base and  $W_k$  is the weight of the  $k$ th element.

Internal actions (shear and bending moment) in correspondence with a section  $j$  can be easily derived by equilibrium from (3), as follows:

$$V_j = \frac{\sum_{k=j+1}^n W_k \cdot z_k}{\sum_{k=1}^n W_k \cdot z_k} \cdot F_{h,b} \quad (4)$$

$$M_j = \sum_{k=j+1}^n F_k \cdot \left( z_k - z_j - \frac{\Delta z_j}{2} \right) = \sum_{k=j+1}^n \frac{W_k \cdot z_k}{\sum_{r=1}^n W_r \cdot z_r} \cdot F_{h,b} \cdot \left( z_k - z_j - \frac{\Delta z_j}{2} \right) \tag{5}$$

where  $\Delta z_j$  is the height of the  $j$ th block and  $z_j$  is the distance of the  $j$ th block centroid from the base.

In the case of non-zero inclination of the tower, assuming reasonably that the structure has high stiffness in the axial direction, the motion of the tower will be orthogonal to the direction of the axis, therefore the horizontal static seismic load can be decomposed into a tangential and normal component, see Fig. 1b, as follows:

$$\begin{aligned} F_{k,par} &= \cos \theta \frac{W_k \cdot z_k}{\sum_{r=1}^n W_r \cdot z_r} \cdot F_{h,b} \\ F_{k,per} &= \sin \theta \frac{W_k \cdot z_k}{\sum_{r=1}^n W_r \cdot z_r} \cdot F_{h,b} \end{aligned} \tag{6}$$

From equilibrium equations, internal actions are:

$$\left\{ \begin{aligned} N_{j,i=\theta} &= \cos \theta \sum_{k=j+1}^n W_k - \sin \theta \sum_{k=j+1}^n F_k = \cos \theta \cdot N_{j,i=0} - \sin \theta \cdot V_{j,i=0} \\ V_{j,i=\theta} &= \cos \theta \sum_{k=j+1}^n F_k + \sin \theta \sum_{k=j+1}^n W_k = \cos \theta \cdot V_{j,i=0} + \sin \theta \cdot N_{j,i=0} \\ M_{j,i=\theta} &= \cos \theta \sum_{k=j+1}^n F_k \cdot \left( z_k - z_j - \frac{\Delta z_j}{2} \right) + \sin \theta \sum_{k=j+1}^n W_k \cdot \left( z_k - z_j - \frac{\Delta z_j}{2} \right) \end{aligned} \right. \tag{7}$$

where  $N_{j,i=\theta}$ ,  $V_{j,i=\theta}$  and  $M_{j,i=\theta}$  are respectively the normal action, shear force and bending moment on the  $j$ th section of the inclined tower ( $i = \theta$ ). Rearranging Eq. (7) by means of (4) and (5) (not-inclined case), the following expressions for shear and bending moments at a generic section  $j$  are obtained:

$$N_{j,i=\theta} = \cos \theta \cdot \sum_{k=j+1}^n W_k - \sin \theta \frac{\sum_{k=j+1}^n W_k \cdot z_k}{\sum_{k=1}^n W_k \cdot z_k} \cdot F_{h,b} \tag{8}$$

$$V_{j,i=\theta} = \sin \theta \cdot \sum_{k=1}^n W_k + \cos \theta \frac{\sum_{k=j+1}^n W_k \cdot z_k}{\sum_{k=1}^n W_k \cdot z_k} \cdot F_{h,b} \tag{9}$$

$$\begin{aligned} M_{j,i=\theta} &= \sin \theta \cdot \sum_{k=j+1}^n W_k \cdot \left( z_k - z_j - \frac{\Delta z_j}{2} \right) + \cos \theta \sum_{k=j+1}^n \frac{W_k \cdot z_k}{\sum_{r=1}^n W_r \cdot z_r} \cdot \left( z_k - z_j - \frac{\Delta z_j}{2} \right) \\ &\quad \cdot F_{h,b} \end{aligned} \tag{10}$$

In order to have a quantitative insight into the increase of internal actions in the case of inclination, the following amplification factor  $\alpha$  for bending moments, defined as the ratio between the values of the inclined and vertical cases, is introduced:

$$\alpha = \frac{M_{j,i=\theta}}{M_{j,i=0}} = \frac{\sin \theta \cdot \sum_{k=1}^n W_k \cdot z_k + \cos \theta \sum_{k=1}^n \frac{W_k z_k^2}{\sum_{r=1}^n W_r \cdot z_r} \cdot 0.85 \cdot \frac{S_e(T_1)}{q} \cdot \frac{W}{g}}{\sum_{k=1}^n \frac{W_k \cdot z_k^2}{\sum_{r=1}^n W_r \cdot z_r} \cdot 0.85 \cdot \frac{S_e(T_1)}{q} \cdot \frac{W}{g}} \tag{11}$$

$$\alpha = \sin \theta \frac{qg}{0.85 S_e(T_1) W \sum_{k=1}^n W_k \cdot z_k^2} (\sum_{k=1}^n W_k \cdot z_k)^2 + \cos \theta$$

$$\alpha = \sin \theta \frac{qg}{0.85 S_e(T_1)} C_1 + \cos \theta$$

It is interesting to notice that, in the case of regular towers with masses regularly distributed along the height,  $\alpha$  is independent from their actual characteristics, indeed it turns out that:

$$C_1 = \frac{(\sum_{k=1}^n W_k \cdot z_k)^2}{W \sum_{k=1}^n W_k \cdot z_k^2} = \frac{1}{n} \frac{W^2 (\sum_{k=1}^n z_k)^2}{W^2 \sum_{k=1}^n z_k^2} = \frac{3}{4} \tag{12}$$

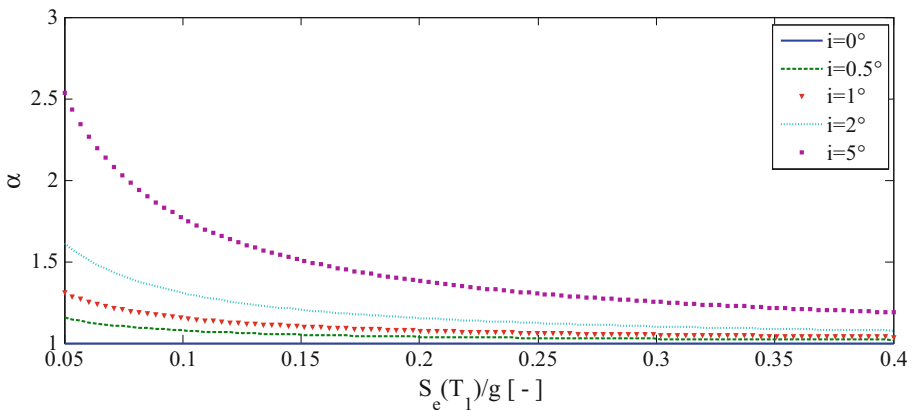
Figure 2 plots the amplification parameter  $\alpha$  as a function of  $S_e(T_1)$ , at different inclination angles, with typical values of acceleration ranging between 0.05 and 0.4 g, for  $q = 1$ .

As can be noted, for low values of  $S_e(T_1)$ , which are typical for slender masonry towers, the amplification is rather high, meaning that inclination may play a crucial role in a seismic vulnerability increase.

Following the same procedure, it is also interesting to evaluate how the axial load decreases with inclination. Let us indicate with factor  $\beta$  the ratio between the values of normal action at the base in the inclined and vertical cases:

$$\beta = \frac{N_{j,i=\theta}}{N_{j,i=0}} = \frac{\cos \theta \cdot \sum_{k=1}^n W_k - \sin \theta \frac{\sum_{k=1}^n W_k \cdot z_k}{\sum_{k=1}^n W_k} \cdot 0.85 \cdot \frac{S_e(T_1)}{q} \cdot \frac{W}{g}}{\sum_{k=1}^n W_k} \tag{13}$$

$$\beta = \cos \theta - \sin \theta \frac{0.85 S_e(T_1)}{qg}$$



**Fig. 2** Variation of the moment amplification factor  $\alpha$  with inclination angle and spectral acceleration  $S_e(T_1)$



As can be noted, the difference on axial load between vertical and inclined cases is almost always negligible and higher than 5% only for large inclination angles and spectral accelerations (Fig. 3).

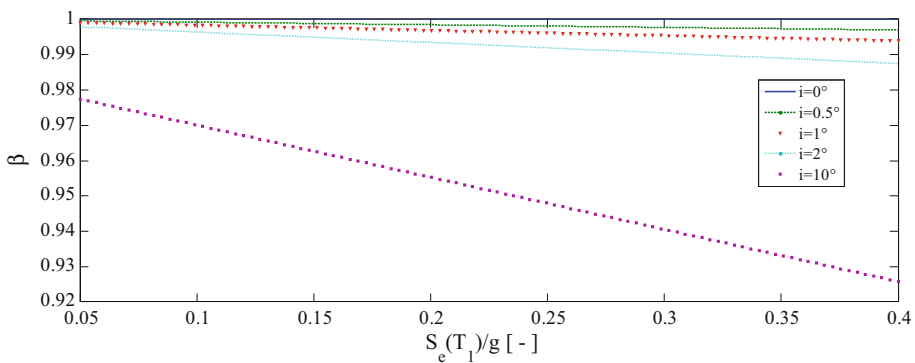
### 2.4 SMM-2 with horizontal ( $F_{h,b}$ ) and vertical ( $F_{v,b}$ ) distributions of loads

In some specific cases, Italian Code NTC 2008 (2008) suggests considering the vertical component of the seismic action through the vertical response spectrum  $S_v(T_v)$ , assuming a behavior factor  $q_v$  equal to 1.5. However, the applicability into SMM methods, which are essentially based on lower bound limit analysis concepts (equilibrium and admissibility) is not explicitly required by the Italian Guidelines for the built heritage (DPCM 2011), both for tall and inclined towers. However, it can make sense to consider them in a simplified mechanical modelling (hereafter called SMM-2 for the sake of clarity) in case of inclined towers, because a seismic vertical component increases the acting bending moments.

It is straightforward to assume that the vertical component is similar to the horizontal one, considering always that Italian code requirements do not mention the need of introducing such forces. Referring to NTC 2008 (2008), the vertical component of the seismic action should be taken into account for (1) long span structures, (2) pre-stressed elements, (3) cantilever beams longer than 4 m, (4) structures with constraints subjected to considerable horizontal actions, (5) beams supporting columns and (6) buildings with suspended floors. No mention is given to tall or inclined structures. In addition, different load combinations, typically within modal response spectrum analysis, should be considered, including the so called “100-30” combination rule ( $E = E_x + 0.3 \cdot E_z$ ), which is the only one analyzed hereafter for the sake of simplicity. The corresponding formula considered for the vertical action is the following:

$$F_{v,b} = \psi \cdot \lambda \cdot \frac{S_v(T_v)}{q_v} \cdot \frac{W}{g} \tag{14}$$

where  $S_v(T_v)$  is the spectral acceleration of the vertical response spectrum at the fundamental axial vibration period of the tower  $T_v$ ,  $q_v$  is the vertical behavior factor equal to 1.5,  $\lambda$  is the correction factor for vertical forces considered equal to 1.0 and  $\psi$  is the combination factor equal to 0.3.



**Fig. 3** Variation of axial force at the base ( $\beta$  parameter) with inclination angle and spectral acceleration  $S_e(T_1)$

Similarly to the transversal case,  $T_v$  is evaluated by means of the simplified formula  $T_v = 4H\sqrt{\frac{\mu}{gE}}$ , where  $E$  is the Young modulus,  $H$  the height,  $\mu$  the average specific weight and  $g$  the gravity acceleration.

In case of vertical component, as indicated in Fig. 1c, internal actions are re-written as follows:

$$N_{j,i=\theta} = \cos \theta \cdot \sum_{k=j+1}^n W_k - \sin \theta \frac{\sum_{k=j+1}^n W_k \cdot z_k}{\sum_{k=1}^n W_k \cdot z_k} \cdot F_{h,b} + \cos \theta \frac{\sum_{k=j+1}^n W_k \cdot z_k}{\sum_{k=1}^n W_k \cdot z_k} \cdot F_{v,b} \quad (15)$$

$$V_{j,i=\theta} = \sin \theta \cdot \sum_{k=1}^n W_k + \cos \theta \frac{\sum_{k=j+1}^n W_k \cdot z_k}{\sum_{k=1}^n W_k \cdot z_k} \cdot F_{h,b} + \sin \theta \frac{\sum_{k=j+1}^n W_k \cdot z_k}{\sum_{k=1}^n W_k \cdot z_k} \cdot F_{v,b} \quad (16)$$

$$M_{j,i=\theta} = \sin \theta \cdot \sum_{k=j+1}^n W_k \cdot \left( z_k - z_j - \frac{\Delta z_j}{2} \right) + \cos \theta \sum_{k=j+1}^n \frac{W_k \cdot z_k}{\sum_{r=1}^n W_r \cdot z_r} \cdot \left( z_k - z_j - \frac{\Delta z_j}{2} \right) \cdot F_{h,b} \\ + \sin \theta \sum_{k=j+1}^n \frac{W_k \cdot z_k}{\sum_{r=1}^n W_r \cdot z_r} \cdot \left( z_k - z_j - \frac{\Delta z_j}{2} \right) \cdot F_{v,b} \quad (17)$$

It is worth noting that in practice the bending moment due to the vertical component modifies the overall acting bending moment within a small percentage (1–5%), because  $S_v(T_v)$  is typically not so high and the lever arm of the forces is small, especially for small inclination angles.

### 2.5 SMM-3 with load distribution perpendicular to the cross section ( $F_{p,b}$ )

When the effect of the application—in a static way—of the seismic action is considered on a structure that reasonably can be assumed as infinitely rigid in the axial direction, it makes sense, at least from a practical standpoint, to apply a load distribution of seismic actions perpendicular to the cross section, Fig. 1d, instead of horizontally oriented as rigorously deduced through the Newton’s law.

When such hypothesis is done, Eqs. (8)–(10) modify as follows:

$$N_{j,i=\theta} = \cos \theta \cdot \sum_{k=j+1}^n W_k \quad (18)$$

$$V_{j,i=\theta} = \sin \theta \cdot \sum_{k=1}^n W_k + \frac{\sum_{k=j+1}^n W_k \cdot z_k}{\sum_{k=1}^n W_k \cdot z_k} \frac{F_{h,b}}{\cos \theta} \quad (19)$$

$$M_{j,i=\theta} = \sin \theta \cdot \sum_{k=j+1}^n W_k \cdot \left( z_k - z_j - \frac{\Delta z_j}{2} \right) + \sum_{k=j+1}^n \frac{W_k \cdot z_k}{\sum_{r=1}^n W_r \cdot z_r} \cdot \left( z_k - z_j - \frac{\Delta z_j}{2} \right) \frac{F_{h,b}}{\cos \theta} \quad (20)$$

Consequently the factor  $\alpha$ , the ratio between the bending moment values of the inclined and vertical cases, takes the following expression:

$$\alpha = \sin \theta \frac{qg}{0.85S_e(T_1)} C_1 + \frac{1}{\cos \theta} \tag{21}$$

Figure 4 plots the amplification parameter  $\alpha$  as a function of  $S_e(T_1)$  evaluated with model SMM-1 (continuous lines) and model SMM-3 (dashed lines) for two medium/high inclination angles, namely  $5^\circ$  and  $10^\circ$ . As can be seen, the difference is negligible, since curves are almost superimposable. Furthermore, the gap obviously tends to vanish decreasing the inclination (for null inclination the methods formally coincide). As a consequence, it can be affirmed that for practical purposes it is possible to use equally SMM-1 or SMM-3, without committing errors of engineering relevance.

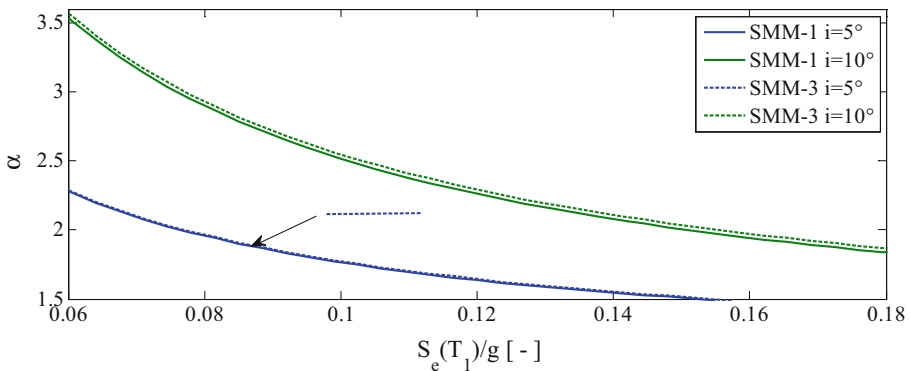
### 2.6 Evaluation of resisting bending moments of cross sections

The Italian Guidelines DPCM (2011) provide the following simplified formula for the resisting bending moment of the  $i$ th section:

$$M_{u,j} = \left( A_j \cdot \frac{\sigma_{0j}}{2} \right) \left( b_j - \frac{A_j \cdot \sigma_{0j}}{0.85 \cdot f_d \cdot a_j} \right) \tag{22}$$

where  $A_j$  is the cross area of the tower on the  $j$ th section,  $a_j$  and  $b_j$  are, respectively, the outer width and length of the section (excluding possible openings),  $f_d$  is the ultimate strength of the material in compression and  $\sigma_{0j} = \sum_{k=j+1}^n W_k/A_j$  is the average normal stress derived from the self-weight. It is notable that the bending moment capacity does not depend on any factor related to inclination, and this appears reasonable because of the negligible variation of the axial load with inclination angle, see Fig. 3. It can be noted also that no shear capacity check is required by DPCM (2011), where it is assumed that the collapse always occurs for the formation of flexural hinges, as the structures themselves are generally more similar to slender cantilevers.

It is interesting to notice from Eq. (22) that SMM method accounts for windows (and irregularities in general) in a simplified manner, modifying the resisting bending moment. However, SMM failure is always due to the formation of flexural hinges, in most of the cases occurring at the base of the structures. For the case studies analyzed in the present paper, FE pushover models turn out to fail in a similar way (shear-bending failure with



**Fig. 4** Variation of the moment amplification factor  $\alpha$  evaluated with models SMM-1 and SMM-3 in case of medium–high inclination angles

plasticization near the base), because non-linear static analyses are typically unable to activate higher modes leading to collapse of the weak upper part (e.g. bell cell), or with a failure mechanism formed by vertical shear cracks.

Whilst it has been shown in many previous papers presenting the results of non-linear dynamic analyses, (e.g. see Pena et al. 2010; Milani et al. 2012; Valente and Milani 2016a, b; Casolo et al. 2013), that local mechanisms seem to be most probable in the collapse of towers under seismic excitation, the application of static horizontal loads is almost always characterized by such a type of failure modes.

In the present modified simplified mechanical model (MSMM), the cross section is schematized as hollow rectangular section as in Fig. 5, where the vertical pre-compression is  $N_j$ .

At failure, it is assumed that the ultimate compression zone obeys a stress-block approximation, where the position of the neutral axis is deduced from the equilibrium of vertical actions. The ultimate bending moment is evaluated by integration, once known the position of the neutral axis:

$$M_{u,j}^{inc} = N_w \cdot e_w + N_f \cdot e_f \tag{23}$$

where  $N_w$  and  $N_f$  represent the ultimate forces respectively on the web and the flange, and  $e_w$ ,  $e_f$  are the eccentricities of internal stresses  $N_w$  and  $N_f$ , respectively.

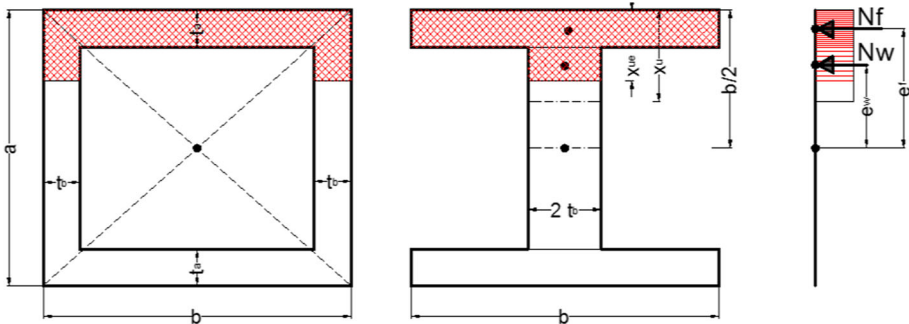
If the compressed tow is constituted by one flange and part of the web, the position of the neutral axis  $x_{ue}$  is determined by axial equilibrium as follows:

$$N_j = W_j \cos \theta = f_d [a_j t_a + 2(x_{ue} - t_a) t_b] \tag{24}$$

$$x_{ue} = \frac{W_j \cos \theta - f_d (a_j t_a - 2t_a t_b)}{2f_d t_b}$$

It is worth noting that in the previous equation it has been made the simplifying hypothesis that the axial load is given exclusively by the axial component of the weight (a condition that holds rigorously only for SMM-3), to directly compare the formula by Italian Guidelines and present simplified approach. In the computations reported in the sequel, the ultimate bending capacity is however estimated with the real axial load acting in the different cases.

Eccentricities  $e_w$  and  $e_f$  are:



**Fig. 5** Limit state and ultimate resisting capacity of a rectangular hollow section: geometrical properties, equivalent section and equilibrium of the section

$$\begin{aligned}
 e_w &= \frac{b_j}{2} - t_a - \frac{x_{ue} - t_a}{2} \\
 e_f &= \frac{b_j}{2} - \frac{t_a}{2}
 \end{aligned}
 \tag{25}$$

If the compressed tow is constituted by a portion of one flange, then the position of the neutral axis  $x_{ue}$  is evaluated as follows:

$$\begin{aligned}
 N_j &= W_j \cos \theta = x_{ue} f_d a_j \\
 x_{ue} &= \frac{W_j \cos \theta}{f_d a_j}
 \end{aligned}
 \tag{26}$$

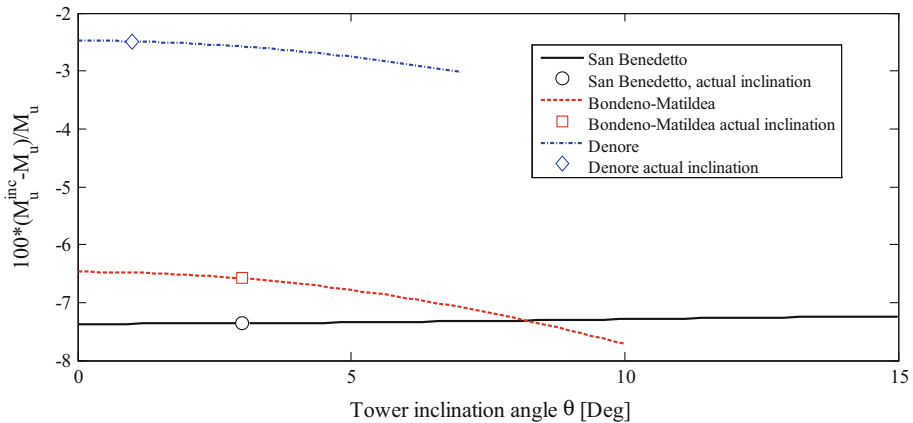
Eccentricity  $e_f$  is:

$$e_f = \frac{b_j}{2} - \frac{x_{ue}}{2}
 \tag{27}$$

Previous formulas are different from Eq. (22) even for vertical towers, whereas their variability as a function of the structure inclination is rather moderate, since the only difference is linked to the variation of the vertical pre-compression  $N_j$ . Figure 6 compares the ultimate moments (percentage relative deviation of the ultimate bending moment obtained through the formula here proposed and code of practice approach) for the three towers under study.

### 3 Pushover FE limit analyses

To have a deep insight into the real structural behavior of the towers, the structures are modeled by means of the computer code SAP2000 (CSI 2013), which provides a wide gallery of material models and elements. Plate and shell FEs are used to discretize the structures, a choice which appears a reasonable compromise between numerical efficiency (when compared to full 3D approaches) and reliability of the geometric approximation. By

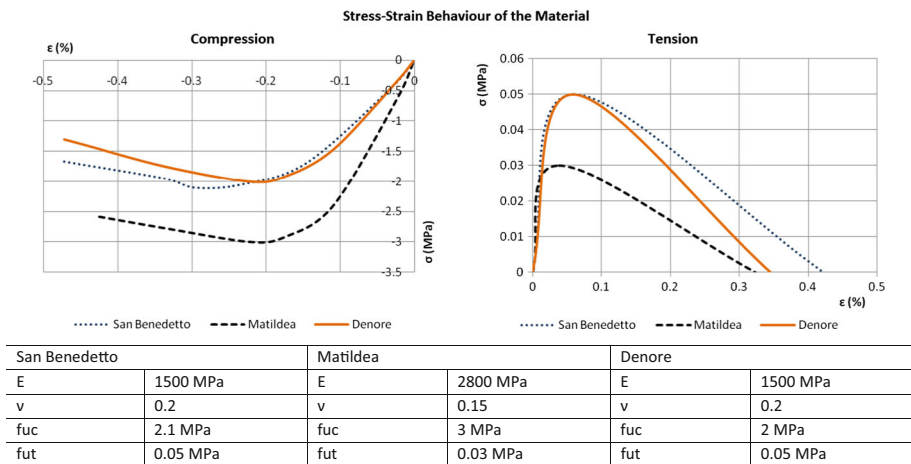


**Fig. 6** Percentage relative deviation between the ultimate bending moment obtained by the Italian Code ( $M_u$ ) and that proposed in the present paper ( $M_u^{inc}$ ) for different inclinations of the towers under study

means of a plate and shell discretization, indeed, it is possible to suitably model openings, presence of vaults and domes, diaphragms and internal stairs. The material model assumed for masonry in FE analyses is isotropic and obeys a smeared crack concrete softening damage-plasticity model, ruled by a mono-axial stress–strain behavior depicted in Fig. 7. It is worth mentioning that such relationships are assumed either in agreement with Italian Code (NTC 2008) specifics in the case of limited knowledge of the structure (LC1 knowledge level), as in the case of San Benedetto and Denore towers, or in agreement with comprehensive experimental/numerical campaigns carried out in previous studies (Milani et al. 2012), as in the case of Matildea Tower. In particular, according to Italian Code (NTC 2008), Chapter 8, and subsequent Explicative Notes (2009), masonry elastic moduli have been taken in agreement with Table C8A.2.1 of the Explicative Notes (2009) assuming a masonry typology constituted by clay bricks (approximate dimensions  $210 \times 52 \times 100 \text{ mm}^3$ ) with poor mechanical properties of the joint and quite regular courses. In the case of Matildea Tower, the parameters used in Milani et al. (2012) have been adopted for comparison purposes. It is worth noting that, with the lowest knowledge level LC (confidence factor  $FC = 1.35$ ), Italian code requires to select, in Table C8A.2.1, the lower bound values for strength and the average values between lower and upper bound for elastic moduli.

It should be also noted that masonry is a material that exhibits distinct directional properties due to the mortar joints, acting as planes of weakness. Depending on the level of accuracy and simplicity desired, it is possible to use the following modeling strategies:

1. Micro-modeling. Units and mortar in the joints are represented by continuum elements, whereas the unit mortar interface is represented by discontinuous elements. To limit the computational effort, in simplified micro-modeling, units are expanded and modeled by continuum elements, whereas the behavior of the mortar joints and unit-mortar interface is lumped in discontinuous elements (Lourenço et al. 1997). For the problem at hand, micro-modeling is inapplicable, due to the need of limiting the degrees of freedom in non-linear analyses.



**Fig. 7** Mono-axial stress–strain behavior and mechanical properties of the material used for the FE models of the towers

2. Homogenization (Luciano and Sacco 1998; Milani et al. 2006a, b). It replaces the complex geometry of the basic cell using, at a structural level, a fictitious homogeneous orthotropic material, with mechanical properties deduced from a suitable boundary value problem solved on a suitable unit cell, which generates by repetition the entire structure. For the problem at hand, where multi-leaf and multi-head walls are present, the identification of a unit cell is however questionable and the utilization of orthotropic models would complicate the structural analysis without a sufficiently based hypothesis on the actual texture. Furthermore, it is expected that orthotropy is a crucial issue for masonry walls loaded within their plane or façade eminently loaded out-of-plane. In this case, the behavior is more similar to a squat cantilever beam. It is therefore preferable to use isotropic materials, also in agreement with existing literature.
3. Macro-modeling (Lourenço et al. 1997; Milani and Valente 2015a, b). Units, mortar and unit-mortar interface are directly smeared into a continuum (either isotropic or orthotropic), with mechanical properties deduced at the micro-scale by means of experimental data available. This latter approach is probably the most suitable in this case.

The structural analysis conducted by means of the computer code SAP2000 (CSI 2013) is performed assuming a smeared crack total strain material model for masonry, which allows for an investigation of the non-linear behavior of the bell tower. Although the model is specifically suited for a fragile isotropic material (as is the case of concrete), its basic constitutive laws can be adapted for reproducing masonry properties in the inelastic range. The “concrete model” basic characteristics, indeed, well reproduce the uniaxial masonry behavior near collapse, as for instance:

1. Tensile failure due to cracking and consequent softening branch;
2. Compression crushing failure (with compressive strength higher than tensile strength, as is the case of masonry);
3. Strain softening during compression crushing until an ultimate strain value, at which the material totally fails, is reached.

The biaxial behavior of the material is uncoupled, meaning that cracks propagate perpendicularly to the direction of the principal positive stress. When a crack opens, the crack direction remains fixed.

Since typical brickwork anisotropy at failure is not reproducible with the model at hand, average values between horizontal and vertical strengths are adopted with reference to some specific data available in technical literature. The assumption of an isotropic model is justified by the following considerations:

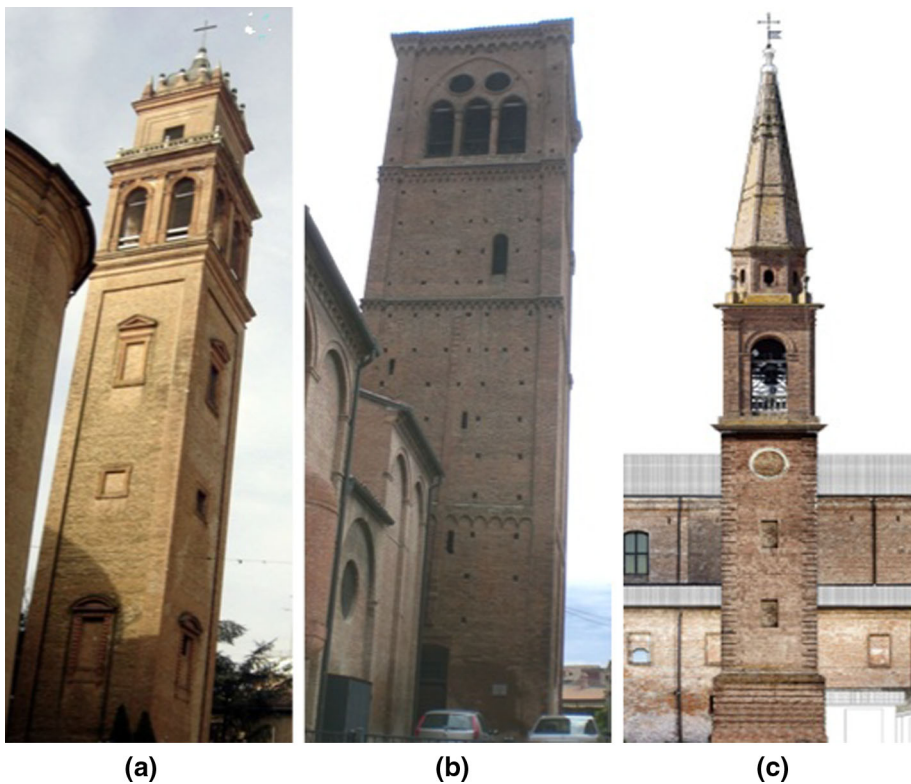
1. 2D elements are used. A full characterization of the compressive-tensile behavior of masonry along the geometrical  $x$ - $y$ - $z$  axes is not available and would require costly experimental campaigns.
2. The texture of the walls is a multi-leaf one and does not maintain constant all over the tower. Therefore, different macroscopically equivalent materials should be used in several different zones of the tower.
3. Little differences between the present model and sophisticated softening orthotropic models are expected in this case. Indeed, the non-linear behavior of the tower is strongly influenced by masonry self-weight and differences in the tensile resistance do not significantly affect the global behavior of the structure under increasing static loads, as it will be pointed out later on in the paper.

## 4 Geometrical description

### 4.1 San Benedetto Tower (inclined bell tower of Ferrara)

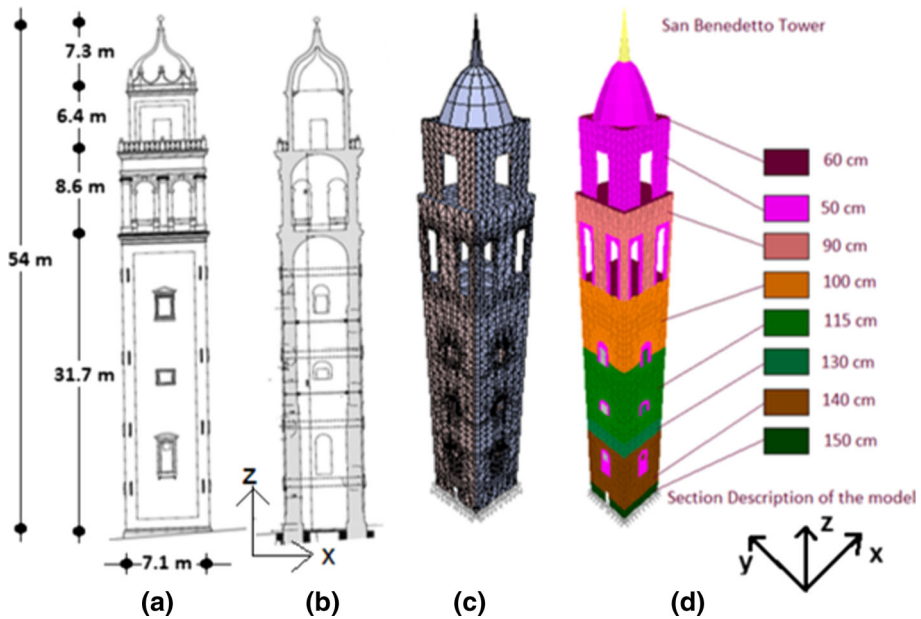
The bell tower of San Benedetto Church (Figs. 8a, 9) is located in Ferrara and presents a particularly high inclination. The construction of the church started in 1496 and the works concluded in 1553, but the bell tower of the church was completed only in 1621, due to the development of foundation settlements at a very early stage of the construction process. Now the tower has a notable inclination in one of the geometrical direction ( $0.5^\circ$  North–South and  $3.07^\circ$  East–West plan). The horizontal displacement of the centroid of the top section of the structure (52.45 m from the base) is equal to 49 cm along the Y direction (N–S) and almost 282 cm along the X direction (E–W). Recently equilibrium conditions have been assessed carefully in static conditions to have an insight into both the stability of the structure and the residual capacity against possible, albeit small, seismic events.

The tower has a rectangular cross section, quite regular from the base to the top, exception made for the irregularity represented by the bell cell. The thickness of the walls varies from 150 to 30 cm. North–South façades show three openings located at different heights, as illustrated in Fig. 8. Such openings were also present in the east–west façades, but are now closed and filled with masonry. From 32 to 40 m along the height, there is a



**Fig. 8** The case studies; **a** San Benedetto Tower in Ferrara, **b** Matildea Tower in Bondeno, **c** San Giovanni Battista Tower in Denore





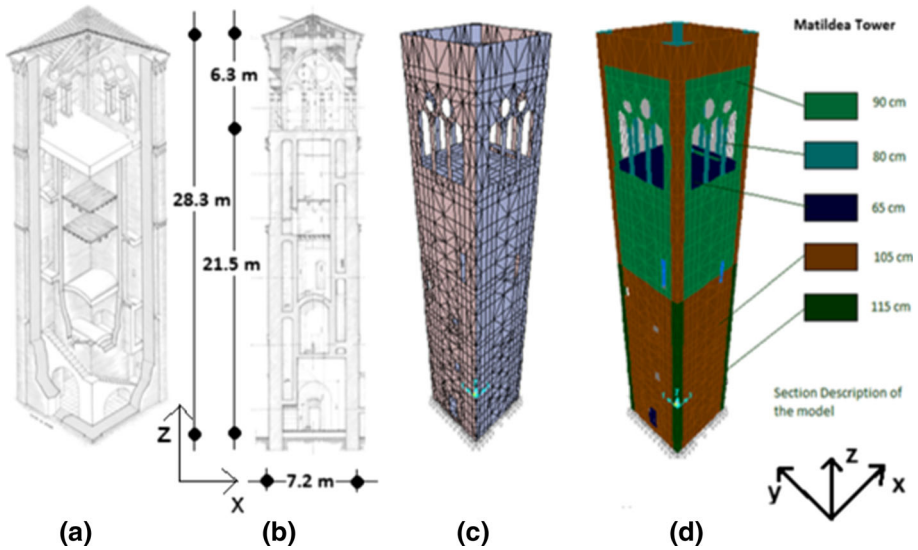
**Fig. 9** San Benedetto Tower: geometry and FE model (6546 elements). **a** Front view, **b** section view, **c** mesh of the FE model, **d** section properties view of the FE model

wide bell cell with large arched openings, surmounted (from 40 to 46 m) by a smaller cell having small openings. Two barrel vaults carry the floors of the first and second cell. A Venetian style small dome closes the second bell cell. The structure is modeled into FEs by means of 6546 shell elements. All the aforementioned details and irregularities are taken into account in the discretization.

#### 4.2 Matildea Tower (Bondeno Tower)

The Matildea Tower (Figs. 8b, 10) rises in Bondeno, a small town located near Ferrara and strategically important in the Medieval Age. A moderate inclination on the vertical is present, due to foundation differential settlements, namely  $1.6^\circ$  West–East and  $3^\circ$  North–South. The inclination results in an out-of-center displacement of the top section centroid of approximately 160 and 85 cm along the X and Y directions, respectively. It is expected that this inclination slightly reduces the resistance of the tower for horizontal loads. The height is approximately equal to 30 m and, considering a side of around 7.2 m at the base, the slenderness ratio is equal to 4. The tower may be considered as an isolated construction, with very little connection at the base with the contiguous church, in agreement with the medieval building practice in Italy, where the tradition of isolating the church survived until the Renaissance age. Nowadays the connection is carried out by a small modern corridor built with thick masonry.

The internal structure is typical of this region and may be found in several other towers around that region (e.g. San Mercuriale in Forlì, San Marco in Venice), and relies on two coaxial vertical box structures. Stairs are built between the internal and external walls by means of small masonry barrel vaults, conceived with both a sustaining and ceiling role.



**Fig. 10** Matilde Tower: geometry and FE model (5754 elements). **a** 3D cut view, **b** section view **c** mesh of the FE model, **d** section properties view of the FE model

The internal space is subdivided along the height into four superimposed floors, which can be accessed by means of the stair system. Two floors (the first and the second) are constituted by masonry barrel vaults, whereas the remaining (the third and fourth) consist of two light timber structures. The internal floors probably had the original role to host the guard-house. The construction of the tower started around the twelfth century and was concluded in the early of the fourteenth century. The original function of the tower was twofold: defense and sighting. Then, two additional stories were added to reach the height of 30 m and the structure was converted into a bell tower.

The first and second levels (up to a height of around 14 m) date back to the twelfth century, when the structure was used as a sighting tower. The base is approximately square, with angular reinforcing pillars. In correspondence with the second floor, the tower was built in multi-leaf with smaller bricks and a different technology, suggesting that a raising work was carried out during these years. The thickness of the walls diminishes from 1.5 to 0.9 m.

The bell cell was built later and architectonically draws features of the last gothic period. A large gothic arch is visible on all the sides. Under the arch, a mullioned window with three lights is present, supported by slender masonry pillars with cubic-shape capitals. Small circular holes between the arch and the window are also present, in order to both lighten the structure and allow for a better acoustic diffusion of the bell sound. The last level, indeed, was conceived and added with the aim of converting the structure, immediately near the new church.

A 12 m high spire was initially planned on the top but never built, as demonstrated by the presence of three small superimposed arches on the internal corners at the top of the tower, immediately under the roof. Instead, a very light wooden roof was used to cover the bell cell.

From a structural point of view, the tower is conceived as two coaxial cantilever beams, with approximately square section and interconnected one to each other by the stairs,

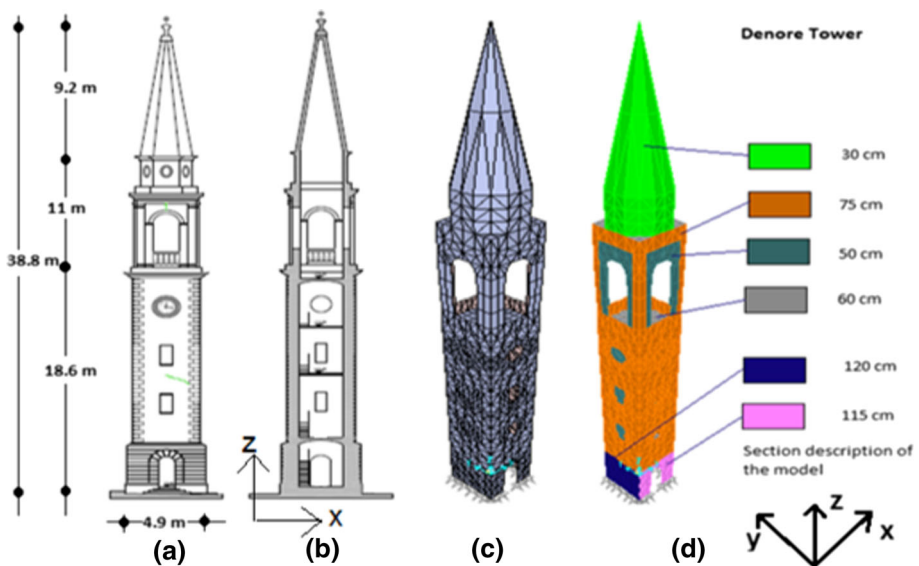
masonry vaults and timber vaults. The internal walls are supported by squat circular arches at the base, a feature that considerably increases the seismic vulnerability of the structure. The barrel vaults are approximately 20 cm thick, whereas the external walls are multi-leaf walls built with small “Bolognese” clay bricks of approximate dimensions  $28 \times 5 \times 10$  (length  $\times$  height  $\times$  thickness). Timber floors and masonry barrel vaults, arranged orthogonally one to another, guarantee a 3D behavior of the structure under possible horizontal loads. In the present study, the tower is discretized with 5754 shell elements of different thickness.

### 4.3 San Giovanni Battista Tower (Denore Tower)

Denore is a small town near Ferrara. The country has ancient origins and is mentioned in the Ferraresi statutes of 1287, but the parish church dates back to 1314. It was subsequently rebuilt in 1617 and dedicated to St. John the Baptist.

The bell tower (Figs. 8c, 11) is separated from the church and the construction does not interact with neighboring buildings. The total height of the tower is 38.8 m, including the octagonal conic dome. It is a structure of secondary importance from both an architectural and historical point of view, therefore little information can be collected on it. This notwithstanding, the tower was analyzed in the present study because it exhibits geometric features (such as slenderness and resisting transversal shear area) different from the previous examples. The East–West plan of the structure shows a slight inclination of  $1^\circ$ , which is progressively increasing as a consequence of differential settlements still acting.

The geometry of this tower is much simpler and less irregular than the previous towers. It can be reasonably schematized with a tubular square cross section with wall thickness equal to 75 cm. Two noticeable irregularities are present near the base (the portion up to 4 m), where perimeter walls have a thickness equal to 115–120 cm, and the bell cell. The



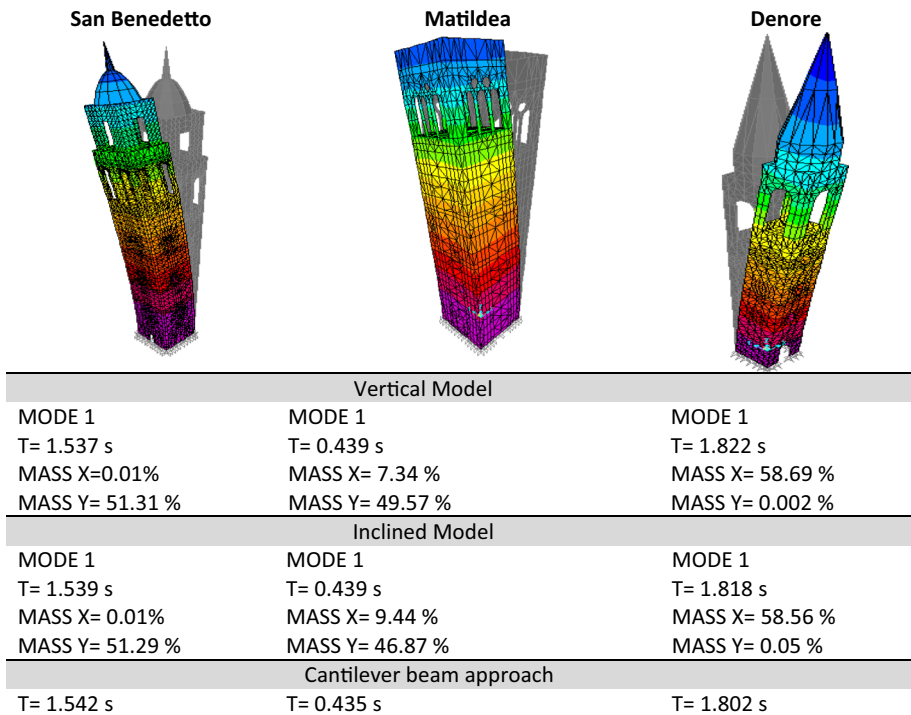
**Fig. 11** Denore Tower: geometry and FE model (4283 elements). **a** Front view, **b** section view, **c** mesh of the FE model, **d** section properties view of the FE model

structure ends with an unusual octagonal conic dome construction. Within the FE pushover approach used as validation, the tower is discretized by means of 4283 shell elements.

### 5 Modal analysis results and non-linear dynamic analyses

A preliminary modal analysis carried out on a refined and accurate discretization of the three towers under study shows that the fundamental period of the structures with and without inclination is the same, see Fig. 12, and in agreement with the simplified formulas previously provided when a cantilever beam approach is used.

In order to have an insight into the role played by the vertical component of the seismic action in the stability of leaning towers, non-linear dynamic analyses are performed on the same FE models shown in Fig. 12. It is worth mentioning that the adoption of such a sophisticated approach is necessary to quantitatively evaluate how the vertical accelerogram may influence the behavior of inclined slender masonry structures. As already pointed out, indeed, Italian code does not explicitly recommend accounting for the vertical component of the seismic action in static analyses like SMM and pushover. When dealing with non-linear dynamic analyses, the same smeared crack isotropic softening model used for pushover analyses is adopted, being also possible to model damage in the unloading phase. Spectrum-compatible accelerograms generated by means of the SIMQKE-II (Vanmarcke et al. 1999) software are utilized. The reference response spectra are those for



**Fig. 12** Deformed shapes of the first vibration mode with indication of corresponding period and excited masses

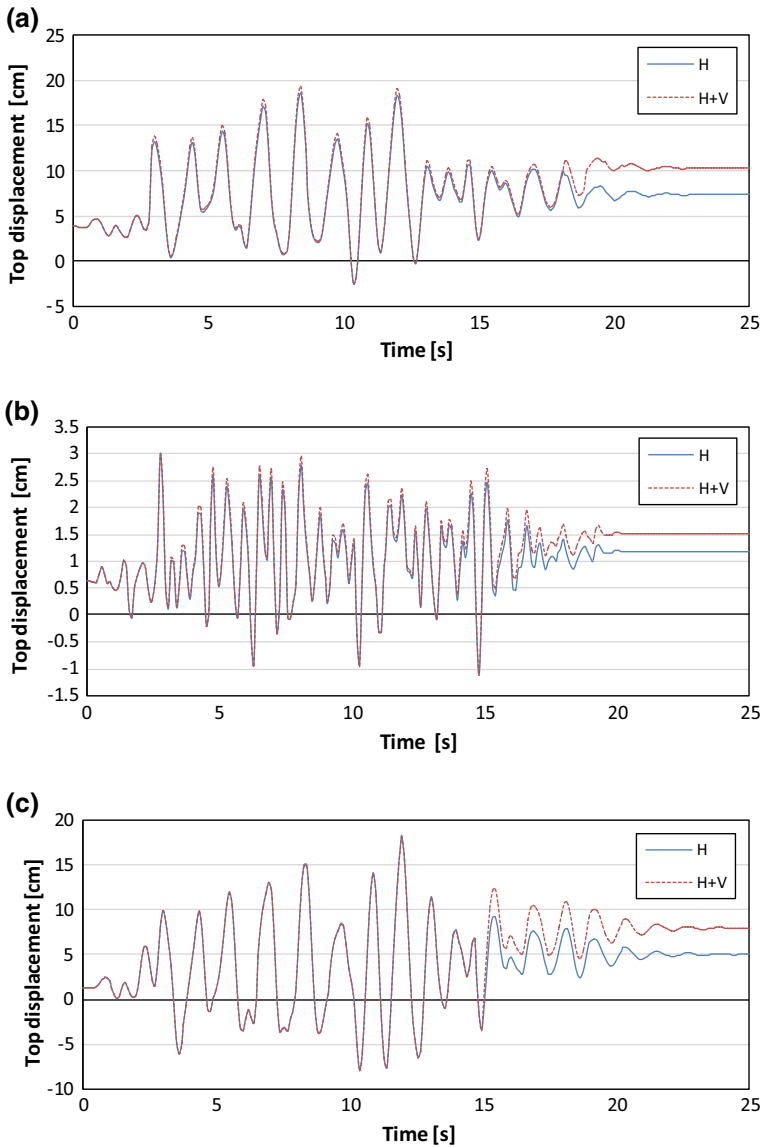
the Ultimate Limit State of Life Safety (SLV) in the Ferrara municipality [return period of the seismic action  $T_R = 475$  years,  $a_g = 0.137$  g,  $F_0 = 2.594$ ,  $T_c^* = 0.273$  s, as suggested by the Italian code NTC 2008 (2008)]. A type D soil and no topographic amplification effects were assumed. From a practical point of view, non-linear dynamic analyses have been performed as follows: in a first step, gravity loads were slowly applied to the structure; in a second phase, artificial accelerograms were applied at the base of the structure along with pre-existing gravity loads. As a rule, when there are some irregularities either in the geometry or in the mesh discretization, towers may exhibit non-zero horizontal displacements even under vertical loads. Intuitively, for high towers small irregularities at the base may result in perceivable displacements at the top. Usually, such displacements are negligible, but they can be sometimes appreciated.

Horizontal displacement time-history diagrams obtained at the top edge of the towers are depicted in Fig. 13 for the three case studies investigated in this paper, with and without the vertical component of the accelerogram. As can be noted, there is a perceivable residual displacement at the end of the simulations, meaning that the structures underwent an inelastic behavior without collapsing (even if displacements of 10–20 cm can reasonably suggest that a failure mechanism is active). It is also interesting to point out that the different behavior between the two cases (presence or absence of the vertical component of the accelerogram) is negligible from an engineering standpoint, meaning that the effect of the seismic vertical component does not seem to be paramount, even for inclined towers. Much more research is obviously needed on such a topic to draw general conclusions, which is, however, outside the scopes of the present paper.

## 6 Simplified mechanical model (SMM) results

The approach proposed by the Italian Code and the simplified mechanical models (SMM-1, SMM-2, SMM-3) conceived in the case of inclination are here utilized on the three towers analyzed as benchmark in order to evaluate, within a cantilever beam approach, the acting and resisting moments along the height of the structures, as well as the acting shear. Figures 14, 15 and 16 show the results for the three towers under study. In each figure, two sub-figures are reported: sub-figure a refers to bending moment, sub-figure b to shear force. In sub-figure a, the three continuous lines refer to the acting bending moment (1) in the case of no inclination [Italian code formula, Eq. (22)] and (2), (3) in the case of inclination. For curves (2) and (3), models SMM-1 and SMM-2 are respectively adopted, i.e. Equations (8)–(10) and (15)–(17) are utilized. It is worth noting that results provided by SMM-3 are left out, providing SMM-1 and SMM-3 almost superimposable moment distributions. The other curves (dashed lines) are for resisting moment capacities: both formulas provided by the Italian Code, see Eq. (22), and by the proposed SMMs [Eq. (23)] are used. Analogously, in sub-figure b, the curves refer to the acting shear in the case of no inclination and in the case of inclined model. The other curves are for shear capacities: in the absence of Italian Code indications, the shear capacities are evaluated using FE pushover analysis after bi-linearization of the capacity curve (Explicative Notes 2009; Fajfar 2000; Chopra and Goel 1999).

The results obtained for San Benedetto Tower are depicted in Fig. 14. As can be observed, the load carrying capacity seems to fulfill the requirements of the Italian Code, even when inclination is taken into consideration. On the other hand, the acting shear



**Fig. 13** Horizontal displacement time-history diagram (top edge node) from non-linear dynamic analyses in the presence (*dashed line*) and the absence (*continuous line*) of the vertical component of the accelerogram. **a** San Benedetto Tower, **b** Matildea Tower, **c** Denore Tower

forces are higher than the shear capacity associated with the collapse of the tower; once again it has to be pointed out that no limitation for shear is provided by the Italian Code.

The same conclusions can be drawn for Matildea Tower and the corresponding results are depicted in Fig. 15. It should be mentioned that in this case the reduction of the resisting bending moment at the base, visible in the figure, is mainly due to an inaccuracy in the proposed formula, related to the presence of perforations at the base of the structure;

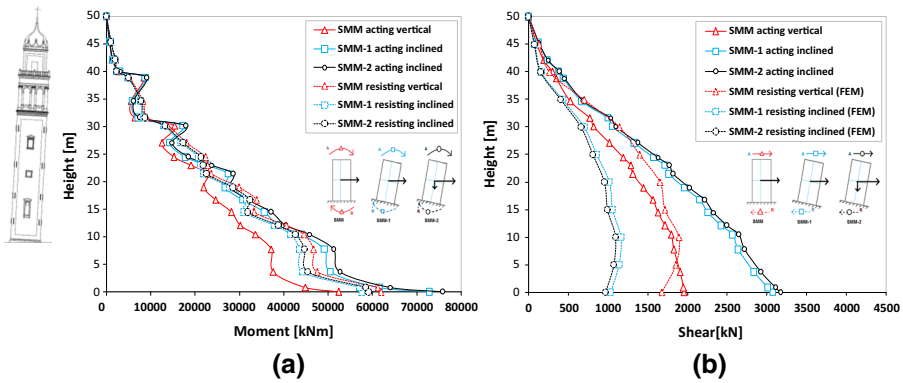


Fig. 14 SMM results for San Benedetto Tower. a Bending moment, b shear

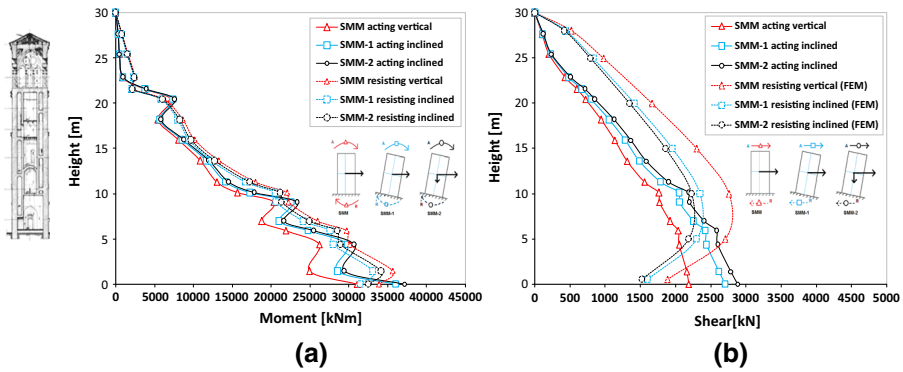


Fig. 15 SMM results for Matildea Tower. a Bending moment, b shear

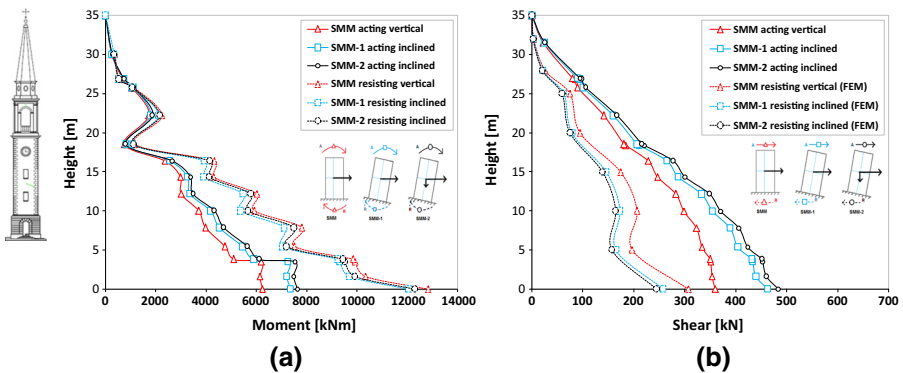


Fig. 16 SMM results for Denore Tower. a Bending moment, b shear

such a situation does not fit very well with the hypothesis of rectangular section. The same observation can be made for the bell cell of Denore Tower. In this latter case, see Fig. 16, it seems that the bending capacity is larger than the acting moment (without inclination) but, on the other side, the acting shears are higher than the resisting ones at the base.

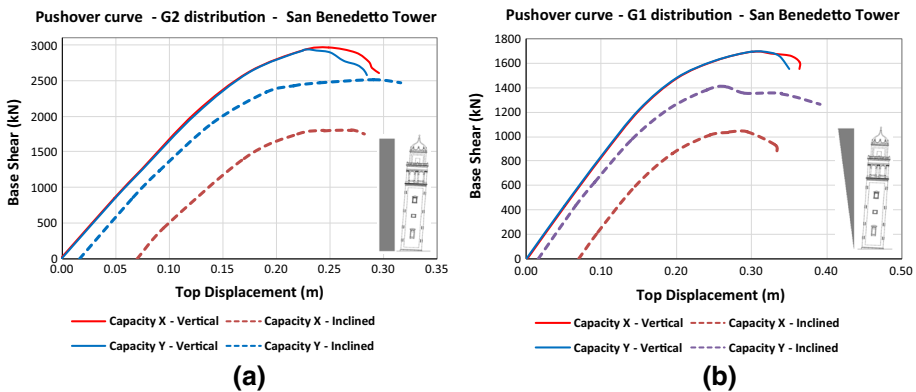
Another remark can be made about the amplification of the acting moments in the case of inclination, which clearly shows an increase of the vulnerability for leaning towers. As indicated, it is nothing else but the amplification factor  $\alpha$  shown in Fig. 2.

### 7 FE pushover analysis results

FE pushover sensitivity analyses are conducted varying the inclination of the three towers, comparing the results with those provided by the simplified models. The results obtained (pushover curves) without and with inclination are depicted respectively in Fig. 17 for San Benedetto Tower, Fig. 18 for Matildea Tower and Fig. 19 for Denore Tower. Subfigures (a) refer to G1 distributions of horizontal loads (i.e. triangular load pattern) and subfigures (b) refer to G2 distributions (i.e. uniform load pattern), in agreement with the provisions of the Italian Code. As expected, G1 distribution presents less conservative results and should be considered as reference.

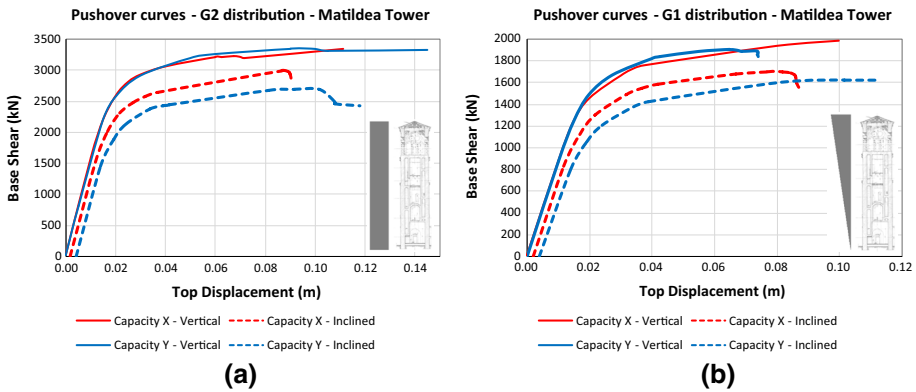
Table 1 summarizes the strength degradation obtained through FE pushover analyses and SMM-1 for the three case studies with the real inclination, when compared with the results of the same models without inclination. As can be noted, SMM strength degradation is systematically higher than that provided by FEs; this outcome proves that SMM method provides more conservative results than FE analyses, assumed as reference. Full sensitive analyses are repeated for different values of inclinations and the ratios between lateral capacities without and with inclinations are synoptically depicted in Fig. 20. The results are obtained re-running FEM simulations (red curves) for different values of inclination angles, applying the present simplified model (blue line) and an evolution of the Heyman’s approach on leaning towers.

The procedure is a limit analysis kinematic approach applied on a portion of the tower overturning around point P, as shown in Fig. 21. The hypotheses at the base of the model are that masonry is unable to withstand tensile stresses, the cross section is thin-walled, no thickness variations occur along the height and masses are homogeneously distributed, exception made for the possible presence of a mass concentrated at the top, representing bells. It is assumed that the first crack occurs in Sect. 1-1 located at  $H_1$  and that the crack pattern is linear (in Heyman 1992 it is shown that the crack pattern is non-linear, but

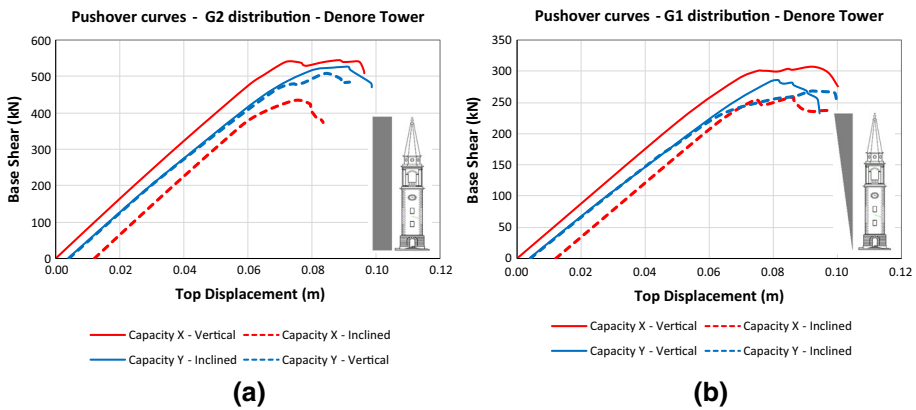


**Fig. 17** FE pushover curves for San Benedetto Tower; **a** G2 distribution, **b** G1 distribution; inclination parallel to the X direction is 3.08°; inclination parallel to the y direction is 0.5°





**Fig. 18** FE pushover curves for Matildea Tower; **a** G2 distribution, **b** G1 distribution; inclination parallel to the X direction is 3.06°; inclination parallel to the y direction is 1.6°



**Fig. 19** FE Pushover curves for Denore Tower; **a** G2 distribution, **b** G1 distribution; inclination parallel to the X direction is 1°; inclination parallel to the Y direction is zero

linearity approximation fits it reasonably well, especially for thin walled sections) and described by the angle  $\beta$ . The multiplier  $\lambda$  is thus numerically estimated solving the following non-linear equations system with the three unknowns  $H_1$ ,  $\beta$ ,  $\lambda$ :

$$\frac{(W_b + W_1)(\lambda \sin \theta - \cos \theta)}{A} + \frac{(W_b + \frac{W_1}{2})(\lambda \cos \theta + \sin \theta)}{I} \frac{a}{2} = 0$$

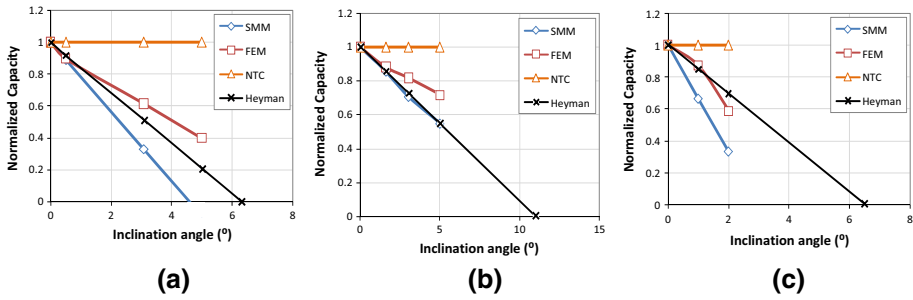
$$\tan \beta - \frac{a}{H - H_1} = 0$$

$$\lambda W_2^b \cos \theta \frac{H - H_1}{2} + (\lambda \cos \theta + \sin \theta) \left[ \frac{2}{3} W_2^a (H - H_1) + W_1 \left( H - \frac{H_1}{2} \right) + W_b H \right] - (\cos \theta - \lambda \sin \theta) \left[ W_2^a \frac{a}{3} + (W_1 + W_b) \frac{a}{2} \right] = 0 \tag{28}$$

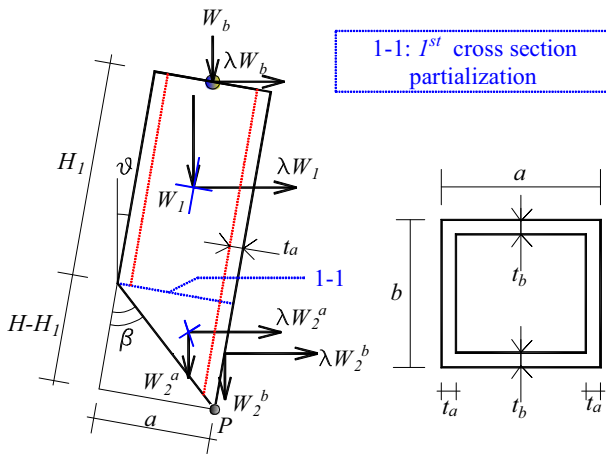
where  $A = ab - (a - 2t_a)(b - 2t_b)$ ,  $I = \frac{a^3 b}{12} - \frac{(a - 2t_a)^3 (b - 2t_b)}{12}$ ,  $W_1 = \gamma A H_1$ ,  $W_2^a = \gamma t_b a (H - H_1)$ ,  $W_2^b = \gamma t_a b (H - H_1)$  and  $\gamma$  is the specific weight of masonry.

**Table 1** Strength degradation (percentage) for the three cases under study: results from pushover analyses and the SMMs (for Denore tower only the results in the X direction are reported because inclination along the Y direction is zero)

Tower	San Benedetto		Matildea		Denore	
	x	y	x	y	x	y
Direction						
Angle	3.08°	0.5°	1.6°	3.06°		
Type	G1 G2	G1 G2	G1 G2	G1 G2	G1 G2	G1 G2
Strength degradation (%)	39.8	39.2	66.5	14.6	14.3	11.7
				12.6	15.8	18.6
				26.9	13.9	18.5
				28.5		



**Fig. 20** Comparison of the results obtained with Italian Code, FEM, Heyman and SMM methods for the three cases under study and different inclination angles: **a** San Benedetto Tower, **b** Matildea Tower, **c** Denore Tower

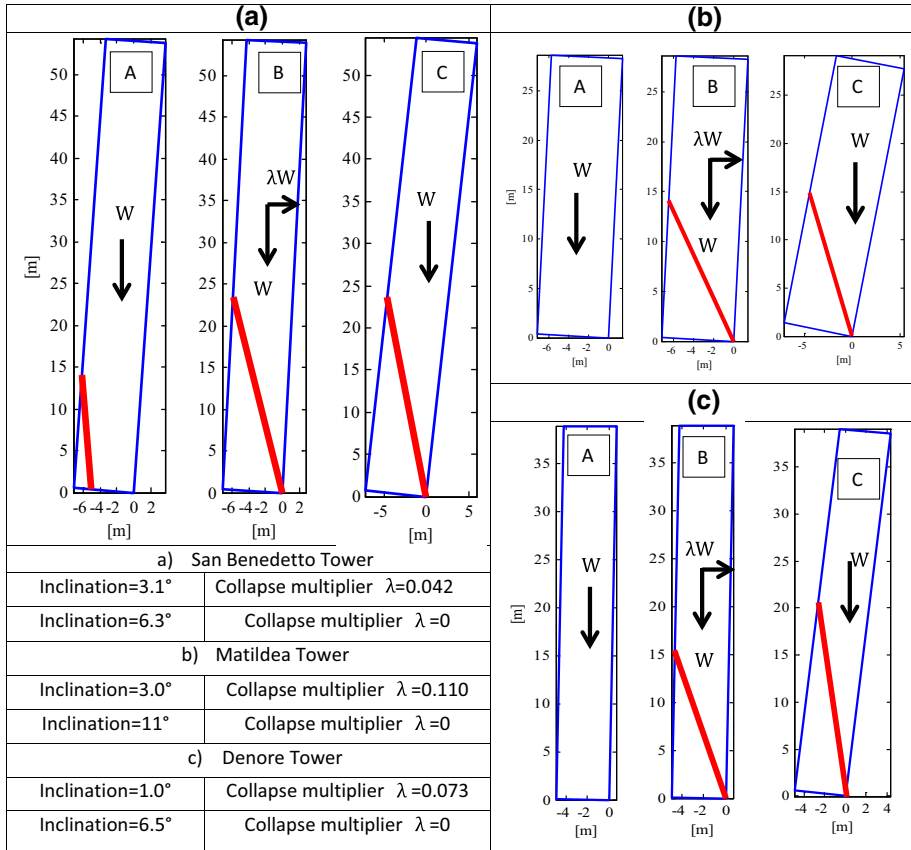


**Fig. 21** Modification of the Heyman's approach for the evaluation of the horizontal collapse multiplier  $\lambda$

Crack surfaces and ultimate horizontal loads can therefore be found for the three towers investigated in the paper from Eq. (24), disregarding geometric irregularities (presence of openings, bell cells, thickness variations etc.) and the limited compressive strength. Results obtained solving Eq. (24) are summarized in Fig. 22. Each tower is analyzed (A) without horizontal loads and the real inclination, (B) with horizontal loads up to collapse and the real inclination and (C) increasing the inclination up to a value of the collapse multiplier equal to zero. The thick red line in Fig. 22 represents the crack pattern.

Case (A) is useful to evaluate if there are pre-existing cracks due to vertical loads and lack of verticality. It is found that only San Benedetto tower presents geometric features to develop cracks (spreading up to 15 m). Case (B) provides the load carrying capacity against horizontal actions in the real situation, whereas case (C) predicts the limit inclination angle, i.e. the intersection of the black curves in Fig. 20 with the horizontal axis.

An estimation of such a limit inclination angle can be found directly in Heyman (1992), where it is suggested to use the following approximate formula for thin-walled sections and  $W_b = 0$ :



**Fig. 22** Crack surface (*thick red line*) found in the case of no-tension material model and thin walled section. **a** real inclination with vertical loads only, **b** real inclination with horizontal collapse loads, **c** limit inclination with vertical loads only. **a** San Benedetto Tower, **b** Matildea Tower, **c** Denore Tower

$$\tan \theta = 0.8418 \frac{a}{H} \tag{29}$$

From a detailed analysis of Fig. 20, it can be observed that the results provided by SMM appear roughly linear (as indeed expected for relative small inclination angles), but not with the same slope for all the structures. It is straightforward to believe that each structure exhibits its own degradation rate, based on geometric properties and load characteristics, like slenderness, ratio of the vertical actions, irregularities, etc. Such a feature cannot be easily predicted before a comprehensive analysis of the actual characteristics of the cross sections. Italian Code formula is obviously insensitive to inclination.

Compared to Heyman results, it seems that the proposed SMM method is more conservative for slender structures (San Benedetto and Denore cases). It could be possible that the inversion of the results observed for Matildea Tower (SMM also superimposable with Heyman’s approach) is due to the particularly complex geometry of the structure, which is essentially constituted by two coaxial thin walled structures interacting with internal vaults and stairs.

What should be mentioned at the end of the study is that inclination should be considered with particular care when assessing the load bearing capacity of leaning towers. The SMM formula proposed in this study is almost always conservative (especially when compared with FEM, assumed as reference), meaning that it could be profitably used for practical purposes.

## 8 Conclusions

The seismic vulnerability of masonry towers is usually evaluated by means of a cantilever beam schematization without the utilization of FE models. The role played by inclination is however largely under-investigated in the structural safety assessment against horizontal loads. The seismic performance of towers can be drastically undermined by inclination. This paper has investigated in detail the role of inclination on three existing masonry bell towers located in the same region and recently stricken by a moderate/high intensity seismic sequence. The analyses carried out in this study comprise: (1) the evaluation of resisting and acting moments and shears according to the Italian code; (2) FE pushover simulations on full 3D numerical models; (3) seismic safety assessment performed with a newly developed simplified mechanical model (SMM). The analyses are repeated hypothetically changing the inclination of the towers. From the results of the numerical simulations, the following key issues emerge:

- The modal analysis shows that inclination does not significantly affect the structural response, meaning that the distribution of horizontal loads is quite the same and that intuitively the main role played by inclination is on the horizontal projection of vertical actions.
- The simplified mechanical model (SMM) shows a degradation of the resisting capacity, roughly proportional to inclination (at least for small angles), slenderness and the inverse of ultimate acceleration capacity.
- The FE pushover analysis, which is considered as the reference solution for the present investigation, shows that the SMM seems more accurate than the standard approach proposed by the Italian Code.
- In all the case studies the proposed SMM formulas provide conservative results when compared with FE models and actual norms, meaning that they could be considered by any practitioners interested in a proper structural safety assessment of inclined towers subjected to horizontal loads.

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