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ORIGINAL RESEARCH PAPER

Shear strength degradation due to flexural ductility demand in circular RC columns

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Abstract An analytical model was developed to estimate the shear-strength degradation and the residual capacity of circular reinforced concrete (RC) columns subjected to seismic action. The proposed model is an upgrade of a previously proposed model for axial force N, bending moment M and shear force V (N–M–V) interaction domain evaluation for rectangular and circular cross-section RC elements subjected to static loading. The model was extended to the case of circular cross-sections subjected to seismic actions with limitation of the range of variability of the deviation angle between the directions of the stress fields and the crack inclinations, as a function of the amplitude of the flexural ductility demand. Numerical evaluation of resistance domains for circular RC columns having the current structural configuration, like bridge piers, highlights the increment in the risk level induced by shear strength degradation due to flexural ductility demand.

Keywords RC circular cross-section · N–M–V domains · Plastic approach · Shear strength degradation

1 Introduction

In literature, several models have been proposed (Ang et al. 1989; Schwartz 2002; Turmo et al. 2009; Rossi and Recupero 2013) to evaluate the shear strength of reinforced concrete

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(RC) circular columns, but they do not take into account the interaction effect among different internal actions (Bairan Garcia and Mari Bernat 2006). At the same time several research works have focused on the performance of RC elements, subjected to simultaneous actions of axial force N, bending moment M and shear force V. But most of the models have been proposed for static actions, and usually they do not take into account the effect of strength degradation due to flexural ductility demand and cyclic actions such as those produced in the critical region of the columns by earthquakes.

Modern approaches to structural analysis in seismic areas aim to evaluate the system's capacity related to large inelastic displacement, i.e. corresponding to a large ductility demand. Unfortunately, the classical formulations for assessment of shear strength are independent of the deformation undergone, leading to overestimation of shear capacity when the ductility demand is large.

Moreover, when the structural response is governed by tensile strength of concrete, the uncertainty of concrete cracking and the great sensitivity of the parameters involved in the constitutive behavior of material make very complex the formulation of a reliable assessment of the whole response. Therefore, the most frequent approaches suggested in the literature for the evaluation of the shear strength in RC members are not always developed on the basis of valid physical-mechanical models.

These issues have given rise to a large debate on the assessment of the response, in the case of a static action, while only a few models are available for shear strength prediction under the effect of seismic actions, when the demand of shear strength is often localized in sections that undergoes large flexural inelastic deformations also.

Several studies (Martin-Pérez and Pantazopoulou 1998, 2001) have suggested solving these drawbacks on the basis of smeared cracking non-linear models, such as the Modified Compression Field Theory (MCFT) (Vecchio and Collins 1986). Originally, the smeared cracking approach was developed for the analysis of elements subjected to a static load. However, despite their success in modeling several structural type behaviors (Colajanni et al. 2008a; Bertagnoli et al. 2011; Spinella et al. 2012) they do not appear suitable to handily provide relationships required for designers. Therefore, design codes prefer to adopt relationships based on simplified mechanic models, or formulations obtained through numerical regression of experimental results. A similar approach is followed in Colajanni et al. (2008b), Spinella et al. (2010), Colajanni et al. (2012), Cucchiara et al. (2012), Colajanni et al. (2014), and Spinella (2013).

Nevertheless, most of the models suggested by codes evaluate the shear strength neglecting the interaction among the internal forces; though in the literature numerous studies and researches on the behavior of RC elements under N–M–V forces are reported.

Recently, a general approach was employed (Recupero et al. 2003, 2005) to formulate a model able to provide generalized strength domains of RC structural elements by taking into account the internal force interaction effects (N–M–V). The model provided results that were successfully compared against laboratory tests, proving the accuracy of the whole procedure. However, it refers to static action only, and it is not able to consider the effect of strength degradation caused by large flexural deformation and cyclic action, like those due to seismic actions.

In this paper, it is observed that the models for static action derived by using the stress fields approach render possible an ample variation of the angle θ of inclination of the concrete stress field, which is, in general, different from the inclination θ_I of the first cracking surface. When large deformation and cyclic actions of wide intensity occur, the progressive roughness reduction limits the range of variation of θ, preventing the development of directions of yielding lines with a slope different from the first cracking one, θ_I.

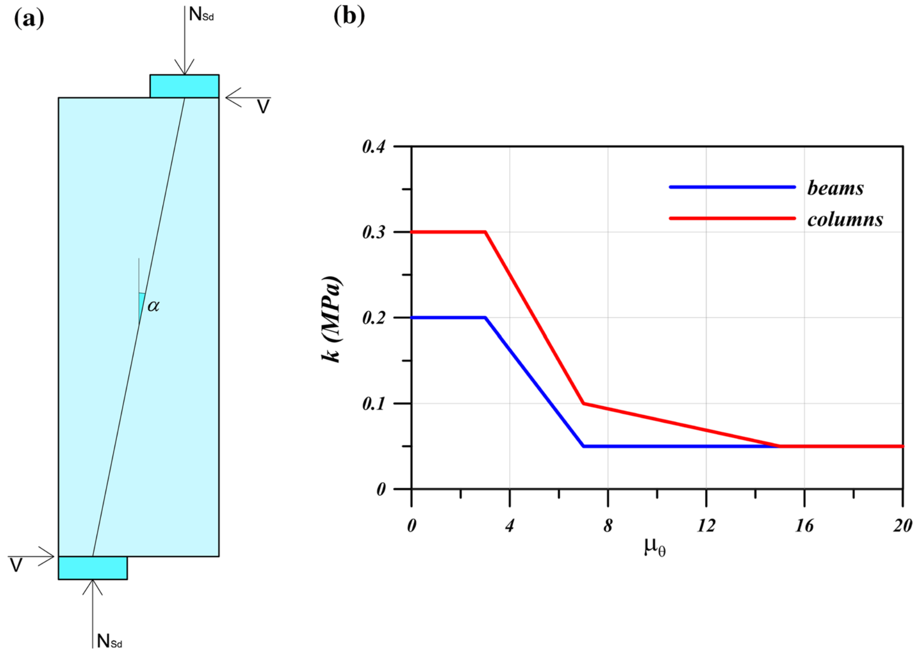

Fig. 1 **a** Contribution of axial force to column shear strength; **b** strength degradation coefficient

In this context a new proposal is formulated that allows the evaluation of N–M–V interaction domains for an assigned ductility by limiting the range of the deviation angle between the inclinations of the yield θ and the crack line θ_I.

2 Model of RC member shear strength under cyclical actions

Many shear strength code equations for RC members are known to be conservative and to give large scatter when used in predicting test results. Except for a new formulation in the Japanese Code (Watanabe and Ichinose 1991; Architectural Institute of Japan 1994), they do not consider the dependence of shear strength on flexural ductility.

In 1996, Priestley and Benzoni (1996) developed a model to take into account the reduction in shear strength due to the ductility demand, which provides close agreement with tests on simple RC members. In this model the shear strength of a member is obtained as the sum of three different contributions due to transverse reinforcement, compressed concrete, and axial load. Thus, in a circular cross-section the shear strength V_{Rd} can be evaluated as follows:

$$V_{Rd} \leq \frac{\pi}{2} \cdot \frac{A_{sw}}{s_w} \cdot (D - x) \cdot \frac{f_{yk}}{\gamma_s} \cdot ctg\vartheta_I + k(\mu) \cdot 0.8 \left(\frac{\pi D^2}{4}\right) \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} + N_{sd} \cdot \tan\alpha \quad (1)$$

where A_{sw} and s_w are the circular steel stirrup cross-section area and spacing, respectively; f_{yk} and f_{ck} are the characteristic strength of the steel and compressed concrete; γ_c and γ_s the safety coefficients for concrete and steel, respectively; $k(\mu)$ the strength degradation coefficient; D the diameter of the circular cross-section; x the neutral axis depth; finally, the contribution of the axial force to the column shear strength and the meaning of the term $N_{sd}\, tan\, \alpha$ are shown in Fig. 1a. By contrast, for beams and columns Fig. 1b shows the strength degradation coefficient $k(\mu)$ curves versus curvature ductility $\mu_\theta = \phi_u/\phi_y$ the latter being the ratio between the current curvature ϕ and the yielding curvature ϕ_y. The coefficient $k(\mu)$ governs the strength degradation due to the required flexural ductility.

Unfortunately, analogous proposals have not yet been formulated for extending the Bach et al.'s (1978) stress field model to the presence of cyclic seismic forces. Thus, the interaction resistance domains (N–M–V) proposed in (Recupero et al. 2003, 2005), which were formulated on the basis of the Bach et al. model, are not able to reproduce the strength degradation effects due to the bending ductility demand in the plastic hinge zones.

3 Stress field model and strength domains under static actions

When RC elements are simultaneously loaded by N, M and V forces, the stress distribution in the cross-section is complex; thus an analytical model able to predict the stress distribution cannot easily be derived. Nevertheless, the strength of a structural element can be evaluated according to Eurocode 2 (1992) under the following simplifying assumptions:

- longitudinal and transverse steel reinforcements are only subjected to axial forces; their action is expressed by smeared stress fields, assumed to be uniform;
- concrete in the external portion of the cross-section is only subjected to compressive stress fields, once again assumed to be uniform;
- the concrete stress field of the web central portion forms a θ angle in the longitudinal direction (yield surface), which may differ from the slope θ_I due to the aggregate interlock action transmitted along the shear fractures and dowel action;
- the failure mode of the structural member is due to concrete crushing, or to reinforcement yielding, or to both simultaneously.
- direct contributions of strength due to dowel action, the aggregate interlock action and the concrete tension resistance of the teeth are neglected, since they are included in the contribution due to concrete stress field of the web central portion by a suitable deviation of the θ angle from the initial cracking line orientation θ_I;

By these assumptions, the analytical model for rectangular, I, T, and circular cross-sections is a generalized truss model, and all internal forces (compressed chord, tension string, strut, and tie) are replaced by uniform stress fields.

The adopted general procedure for strength domain evaluation consists of dividing the cross-section into layers of depth y_i not identified a priori, and subjected to uniform normal (σ) and shear (τ) stress distributions to ensure the equilibrium with the internal actions N, M and V. More precisely, in our research the cross-section was divided into three concrete layers having areas S_{c1}, S_{c2} and S_{c3}, and the uniformly spaced longitudinal rebars were modeled as continuous elements divided into three different steel layers with areas S_{s1}, S_{s2} and S_{s3} (Fig. 2c).

With reference to the concrete element with a circular cross-section, obtained by two cuts, one with a plane orthogonal to the beam axis at the abscissa z, and another one parallel to the web concrete stress field at the abscissa $z + \Delta z$ (Fig. 2a), the following equilibrium equation in the y direction can be written:

$$V^* - q \cdot z = V_{sd} = 2\frac{A_{sw}}{s}\ ctg\theta \cdot R_c\ \sigma_{sw} \cdot \int_{\delta_1}^{\delta_2} \cos^2\alpha \cdot d\alpha$$
$$= \frac{A_{sw}}{s}\ ctg\theta \cdot R_c\ \sigma_{sw} \cdot (\cos\delta_2 \sin\delta_2 + \delta_2 - \cos\delta_1 \sin\delta_1 - \delta_1) \tag{2}$$

where $V*$ is the shear external action at the abscissa z; q is the distributed vertical load; σ_{sw} is the normal stress of the circular steel stirrup, R_c is the radius of the concrete core; and δ_1 and δ_2 the angles between the horizontal diameter of the circular cross-section and the chords that identify the layers S_{c1} and S_{c2}, respectively (Fig. 2c). Next, a new column segment is considered, obtained by cutting the element with two section planes with slope $\theta = 90°$ to the beam axis at the abscissae z and $z + \Delta z$ (Fig. 2b); thus the new equilibrium equation of the column segment in the y direction reads:

$$V^* - q \cdot \Delta z = V_{sd} = \sigma_{cw}\ S_{c3}\ \cos\theta\ \sin\theta \tag{3}$$

where σ_{cw} is the normal stress in the S_{c3} concrete layer. Furthermore, since N_{sd}, V_{sd} and M_{sd} are the axial, shear and bending moment internal design forces at the cross-section at the abscissa $z + \Delta z$, the expressions of the internal forces in the tension chord and in the compression chord are the following:

$$\sigma_{s1} \cdot \int_{S_{s1}} dS_s + \sigma_{s2} \cdot \int_{S_{s2}} dS_s + \sigma_{s3} \cdot \int_{S_{s3}} dS_s + \sigma_{c1} \int_{S_{c1}} dS_c + \sigma_{c2} \int_{S_{c2}} dS_c$$
$$= F_1 + F_2 + F_3 + C_1 + C_2 = N_{sd} + V_{sd} \cdot ctg\theta \tag{4a}$$

$$\sigma_{s1} \int_{S_{s1}} y_s dS_s + \sigma_{s2} \int_{S_{s2}} y_s dS_s + \sigma_{s3} \int_{S_{s3}} y_s dS_s + \sigma_{c1} \int_{S_{c1}} y_c dS_c + \sigma_{c2} \int_{S_{c2}} y_c dS_c$$
$$= M_{sd} + V_{sd} ctg\theta \left(\int_{S_{c3}} y_c dS_c \Big/ \int_{S_{c3}} dS_c \right) \tag{4b}$$

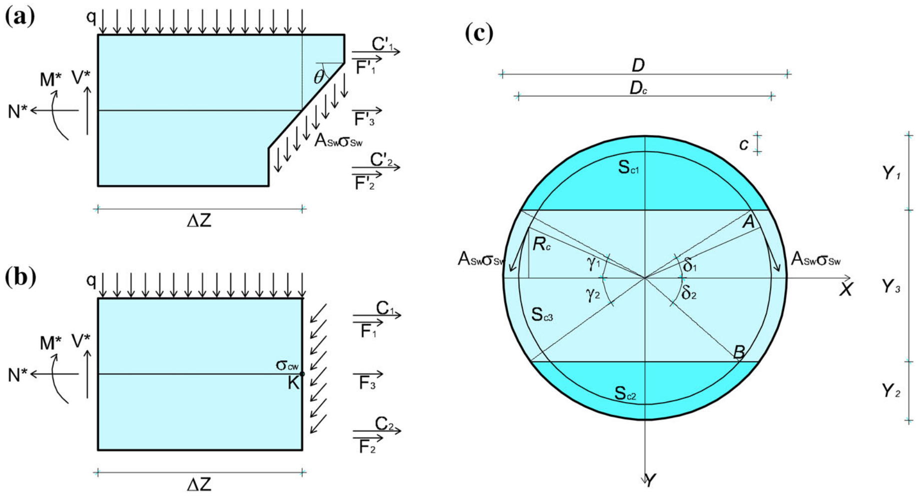

Fig. 2 Circular shaped cross-section

where σ_{ci} and σ_{si} are the normal stress of the concrete and steel related to the ith layer, respectively; C_i and F_i the resultant forces in the concrete and the steel related to the ith layer, respectively; and y_c, y_s are the lever arms (algebraic values) calculated starting from the central axis of the circular cross-section.

Using the static theorem of the plasticity theory, also known as the "*lower bound solution*", the maximum values of the external forces, i.e. the strength interaction domain, can be obtained by means of Eqs. (2), (3), (4a) and (4b), once the terms involved in these equations satisfy both the following geometrical condition and mechanical inequalities for the concrete and steel stress fields:

$$y_1 + y_2 + y_3 = D \tag{5a}$$

$$-f_{cd1} \leq \sigma_{ci} \leq 0 \qquad i = 1, 2 \tag{5b}$$

$$-f_{cd2} \leq \sigma_{cw} \leq 0 \tag{5c}$$

$$-f_{yd} \leq \sigma_{sj} \leq f_{yd} \quad j = 1, 2, 3 \tag{5d}$$

in which f_{yd} is the yield strength of the reinforcing steel, and f_{cd1} and f_{cd2} are the cylindrical strength of the concrete in the uniaxial and biaxial stress states.

In the proposed model the following assumptions are adopted:

- the slope of the yield surface θ and of the first crack surface θ_I are not equal;
- the angular deviation between the two inclinations $\Delta\theta = \theta_I - \theta$ is dependent on mechanical compatibility considerations, i.e. the angular deviation is linked to ability of the roughness of the surface and dowel action to deviate the direction of the stress transfer through the crack from that of the initial cracking line θ_I, but its variation range is limited by the indications in CEB-FIP (1993) based on the results of experimental tests under static load conditions.

The value of θ_I corresponds to the inclination of first cracking under the service loads. In detail, for a structural element with pure bending (beam), the slope θ_I is around 45°, while for columns it depends on the ratio between axial and shear forces under service loads.

In the case of static loads, the inclination angle of the yield surface is evaluated using the plastic approach, independently of the value of θ_I, as the angle in the range $0.4 \leq ctg\,\theta \leq 2.5$, $(22° \leq \theta \leq 68°)$ [as suggested by Eurocode 2, part. 1. (1992)] that allows the maximum shear strength.

A handy model was implemented to evaluate the strength domains N–M–V. Once the element's geometrical properties and longitudinal and transversal reinforcement area A_{sw} and A_{sl} are set, the N–M–V interaction domain can be determined by assigning a value of the axial force N, linking the value of the shear and bending moments by an assigned value of the shear span M/V and finding the maximum admissible values of the external forces through an optimization procedure based on static and geometrical equalities (2), (3), (4a), (4b) and (5a) and static inequalities (5b)–(5d). The whole strength domain can be evaluated by variation, in the above procedure, in the axial force and shear span length.

Useful references for details are reported in the literature (Recupero et al. 2003, 2005).

4 Stress field model and strength domains in presence of flexural ductility demand

A new procedure to evaluate strength domains under seismic loads is presented in this section. It predicts the variation in the angular deviation $\Delta\theta$ as a function of both geometrical and

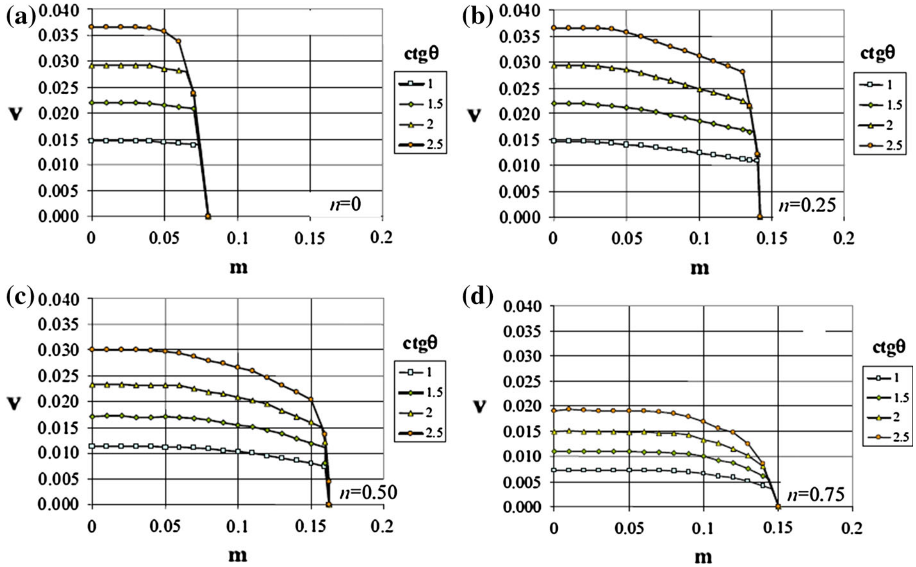

Fig. 3 Strength domains with angular deviation $\Delta\theta$ and $[cot\,\theta]_{max}$ assigned

mechanical characteristics and of the amount of flexural ductility demand (μ) undergone by the structural member under seismic action.

The proposed formulation aims to extend Priestley's suggestions to models based on inclined stress fields. Priestley's model and its extensions require that the strength contribution provided by the tensile strength of the concrete to the truss model is tuned as a function of the flexural ductility demand; its extension to models based on the diagonal stress fields is apparently unfeasible.

In the stress field model, the difference of the slope of the yield surface in comparison to that of the cracking surface is partly generated by the effects of aggregate interlock, which avoid slips along cracks, and are a function of the roughness of the crack sides in contact.

When the maximum deformations and/or the accumulated damage due to small amplitude of cyclic actions increase, the roughness of the sliding surfaces is reduced. Thus, the range of the deviation angle $\Delta\theta$ is limited. The proposed model assumes a limit value of the angle $\Delta\theta$ that should depends on a measure of the damage generated by the combined effects of amplitude of maximum flexural ductility demand and cumulated effect of cyclic action, i.e. on a damage index that should include both the two aforementioned contributes. As an example, the Park and Ang index (Park and Ang 1985) appears to be a suitable damage index for governing the limitation of the deviation angle $\Delta\theta$. However here, due to the lack of adequate amount of experimental data for investigating the effect of cyclic action, the limit value of the deviation angle $\Delta\theta$ is linked in a simpler way to the maximum value of the flexural ductility demand, i.e. the more the ductility demand increases, the more $\Delta\theta$ is reduced.

Aiming at stressing how such an assumption modifies the strength domains of RC members, the effects of the progressive reductions of the deviation angle $\Delta\theta$ on N–M–V domains are shown for circular cross-sections with the following geometrical and mechanical characteristics: $f_{sd}/f_{cd} = 320/20 = 16$, $\rho_l = A_{sl}/A_c = 0.009$, $A_{sw}/s_w = 0.26\,\text{mm}^2/\text{mm}$). The strength domains are shown in Fig. 3 for four limit values: $ctg\,\theta = 2.5(\theta \approx 22°)$, $ctg\,\theta = 2(\theta \approx 26°)$, $ctg\,\theta = 1.5(\theta \approx 34°)$ and $ctg\,\theta = 1(\theta = 45°)$ and for four normalized axial force values $n = N_{sd}/(f_{cd1}A_c) = 0, 0.25, 0.50$ and 0.75. The limit of the deviation angle $\Delta\theta$ becomes the corresponding limit of the inclination angle θ of the stress fields. Figure 3 shows that the progressive reduction in the yield surface inclination (angle θ) causes a major reduction in the maximum shear strength; by contrast, it does not have any influence on the ultimate bending moment.

In order to characterize the relation between angular deviation $\Delta\theta$ and flexural ductility demand on the basis of the indications provided by Priestley and Benzoni (1996), it is observed that the limit of the yield surface inclination influences the horizontal line of the strength domain corresponding to small values of the bending moment, for which the failure of the structural element is reached by attainment of shear strength:

$$\begin{aligned}[V_{sd}]_{\max} &= [ctg\theta]_{\max} \frac{A_{sw}}{s_w} R_c\ f_{yd}\ 2 \int_{\delta_1}^{\delta_2} \cos^2\delta\ \ d\delta \\ &= [ctg\theta]_{\max} \frac{A_{sw}}{s_w} R_c\ f_{yd}\ (\cos\delta \sin\delta + \delta)\big|_{\delta_1}^{\delta_2} \end{aligned} \tag{6}$$

When squat element are considered, besides the beam effect, represented by Eq. (6), the arch effect supplies supplementary capacity to carry shear force. If the additional contribution of the arch effect is estimated as done in Priestley and Benzoni (1996) as ($N_{sd}\ tan\ \alpha$), by comparison of Eq. (1) and Eq. (6) the following relation holds:

$$\begin{aligned} &[ctg\theta]_{\max} \frac{A_{sw}}{s_w} \left(\frac{D-2c}{2}\right) f_{yd}\ (\cos\delta \sin\delta + \delta)\big|_{\delta_1}^{\delta_2} + N_{sd} \tan\alpha \\ &= \frac{\pi}{2} \frac{A_{sw}}{s_w} (D-x) f_{yd} ctg\theta_I + k_p(\mu)\ 0.8 \left(\frac{\pi D^2}{4}\right) \frac{\sqrt{f_{ck}}}{\gamma_c} \tan\alpha + N_{sd} \tan\alpha \end{aligned} \tag{7}$$

The degradation coefficient of shear strength provided by the concrete $k = k_p(\mu)$ (Fig. 1b) is obtained using Priestley and Benzoni's model (1996), assuming $\theta_I = 30°(ctg\,\theta_I = 1.732)$ for the column and $\theta_I = 45°(ctg\,\theta_I = 1)$. Thus, by Eq. (7) the minimum slope of the yield line (i.e. the maximum value of $ctg\,\theta$) can be derived:

$$[ctg\theta]_{\max} = \frac{\left(\frac{\pi}{2}\frac{(D-x)}{R_c} ctg\theta_I + k_p(\mu)\cdot\left(\frac{s_w R_c}{A_{sw}}\frac{f_{cd2}}{f_{yd}}\right) 0.8\pi \left(\frac{R}{R_c}\right)^2 \frac{\sqrt{f_{ck}}}{f_{cd2}\gamma_c}\right)}{(\cos\delta \sin\delta + \delta)\big|_{\delta_1}^{\delta_2}} \tag{8}$$

where f_{cd1} and f_{cd2} are suggested in CEB-FIP (1993) and x, δ_1 and δ_2 must be evaluated according to the axial force level.

5 Model corroboration and numerical analysis

In order to corroborate the proposed model, experimental results reported in Ang et al. (1989) are reproduced by the proposed model. In Table 1 the geometrical and mechanical characteristics of the considered specimen are reported, and the following symbols are introduced: c = cover; L = column length; f'_c = concrete compressive strength; #, ϕ_l and f_{yl} number, diameter, and steel yielding stress of reinforcing longitudinal bars, respectively; s_w, ϕ_w and f_{yw} spacing, diameter and yielding stress of hoop reinforcement, respectively; and N_s = axial load. In Fig. 4 the theoretical predictions of the shear strength provided by the proposed model, when $[ctg\,\theta]_{max}$ is evaluated by Eq. (8) are presented. Only specimens

Table 1 Geometrical and mechanical characteristics of the specimens

Spec.	D (mm)	c (mm)	L (mm)	f'_c (MPa)	*#Long. Bars*	ϕ_l (mm)	f_{yl} (MPa)	s_w (mm)	ϕ_w (mm)	f_{yw} (MPa)	N_s (kN)
No 4	400	20	800	30.6	20	16	436.0	165	10	316	0
No. 6	400	18	600	30.1	20	16	436.0	60	6	328	0
No. 7	400	18	800	29.5	20	16	448.0	80	6	372	0
No. 16	400	18	800	33.4	20	16	436.0	60	6	326	420
No. 18	400	18	600	35.0	20	16	436.0	60	6	326	440
No. 19	400	18	600	34.4	20	16	436.0	80	6	326	432
No. 20	400	18	700	36.7	20	16	482.0	80	6	326	807
No. 21	400	18	800	33.2	20	16	436.0	80	6	326	0
No. 22	400	20	800	30.9	20	16	436.0	220	10	310	0

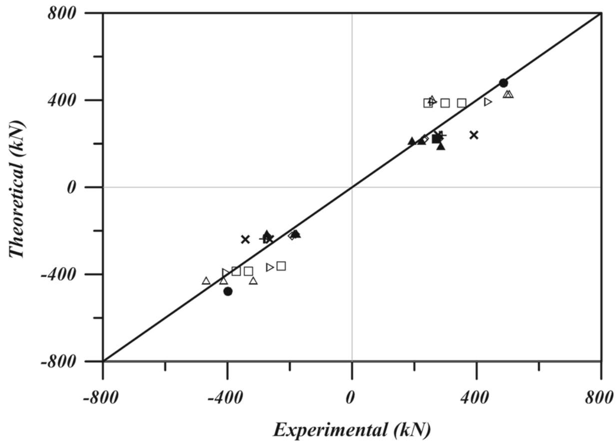

Fig. 4 Theoretical to experimental shear strength ratio for specimens tested in Ang et al. (1989) falling in shear

where the failure was due to combined action of shear and flexure were considered. Moreover, since the model is not able to take into account the effect of repeated load cycles, only results corresponding to the first attainment of a given ductility level are considered for positive or negative displacement, i.e. data corresponding to displacement smaller than those obtained in previous cycles are neglected. The mean value of the ratio of theoretical and experimental strength is equal to 1.03, and the coefficient of variation (COV) is 0.248. These values prove that the model is efficient, but an improvement is required for increasing its effectiveness and to make it able to reproduce the effect of cyclic load.

Moreover, some numerical analyses were carried out, in order to show how different geometrical and mechanical parameters and ductility demands can reduce the range of variability of the inclination of the stress fields of compressed concrete. In Fig. 5a, b, the curves of $[ctg\,\theta]_{\max}$ derived by Eq. (8) [providing the maximum value of the shear strength given by the Priestley and Benzoni's model (1996)] versus the flexural ductility demand (μ) are reported for three different values of the specific axial force $n = 0$, 0.25 and 0.50, for two values of the mechanical ratio of the stirrups $\omega_w = A_{sw} f_{yd}/(s_w R_c f_{cd2})$ equal to 0.05 and 0.10. It has to be stressed that, since in the Priestley and Benzoni's model (1996) the contribute provided by

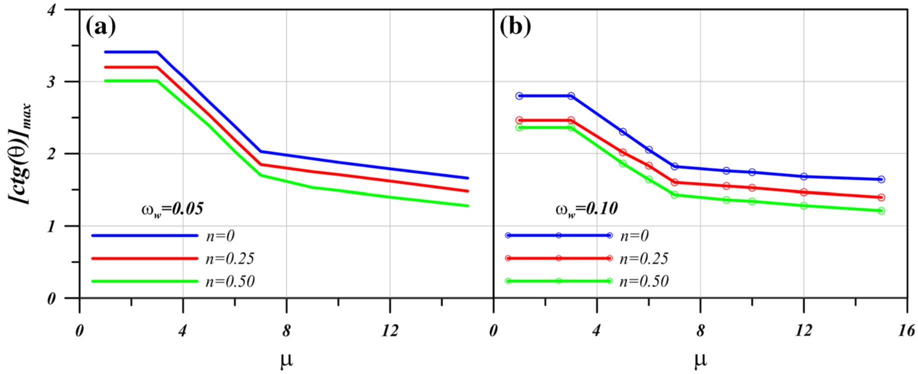

Fig. 5 $[cot\,\theta]_{max}$ versus flexural ductility demand (μ) for different value of axial force (n) and stirrups mechanical ratio (ω_w)

the concrete in not affected by the ductility demand value, whereas in the proposed model the ductility demand tune the whole shear strength (but the arch effect), the value of $[ctg\,\theta]_{max}$ depends on the mechanical ratio of the stirrups ω_w.

In the presence of low stirrups density in the proposed model the maximum values of shear are attained when the inclination of the stress fields is reduced in order to allow a larger number of stirrup legs to cross into the yield line, where the equilibrium is imposed.

No reduction of $[ctg\,\theta]_{max}$, i.e no increment of the minimum slope of the yield line, is obtained when flexural ductility demand is small ($\mu < 3$). For a small value of $\omega_w = 0.05$ (Fig. 5a) and $n = 0$ (beams) when the ductility demand increase in the range $3 < \mu < 7$ the value of $[ctg\,\theta]_{max}$ decrease in the range $3.41 > [ctg\,\theta]_{\max} > 2.03$, and the minimum inclination of the stress fields is included in the range $16.34° < \theta < 26.2°$. Therefore, the angular deviation in comparison to the conventional cracking angle adopted by Priestley ($\theta = 45°$) proves to be in the range $28.65° > \Delta\theta > 18.77°$. Larger values of the flexural ductility demand up to $\mu = 15$ produce a more gently reduction of $[ctg\,\theta]_{max}$ up to the value of 1.66.

Increasing the level of axial force ($n = 0.25$, columns) a wider reduction of $[ctg\,\theta]_{max}$ is found, with $[ctg\,\theta]_{max} = 3.2$ for flexural ductility demand $\mu < 3$; and a descending branch that in the range $3. < \mu < 7$ varies in the range $3.2 > [ctg\,\theta]_{max} > 1.85$ (i.e. $17.35° < \theta < 28.4°$, and angular deviation to the conventional cracking angle adopted by Priestley ($\theta_I = 30°$) in the range $12.65° > \Delta\theta > 1.6°$. However, it has to be stressed that a beneficial effect of the axial force on the shear strength is taken into account both in the values of the dimensions of the compressed chord and in the term of the arch effect ($N_{sd}\ tan\ \alpha$). The curve for $n = 0.50$ proves that the greater the axial force level, the greater the reduction of $[ctg\,\theta]_{max}$ that have to be assumed in order to reproduce the Priestley and Benzoni's results, whit values of $[ctg\,\theta]_{max} = 3$ for $\mu < 3$ and reduction up to $[ctg\,\theta]_{max} = 1.70$ and $[ctg\,\theta]_{max} = 1.28$ when μ increases up to 7 and 15 respectively.

In Fig. 5b the curve of the value of $[ctg\,\theta]_{max}$ versus the ductility demand derived by Eq. (8) for a larger value of the mechanical ratio of the stirrups $\omega_w = 0.10$ are reported. The curves show a trend similar than those in Fig. 5a. In order to compare how the ductility reduce the slope of the concrete field for the different values of mechanical ratio of the stirrups and axial force, in Fig. 6a the curves of $[ctg\,\theta]_{max}$, normalized with respect to the value for static load, versus the ductility demand are depicted. The curves show that the smaller the

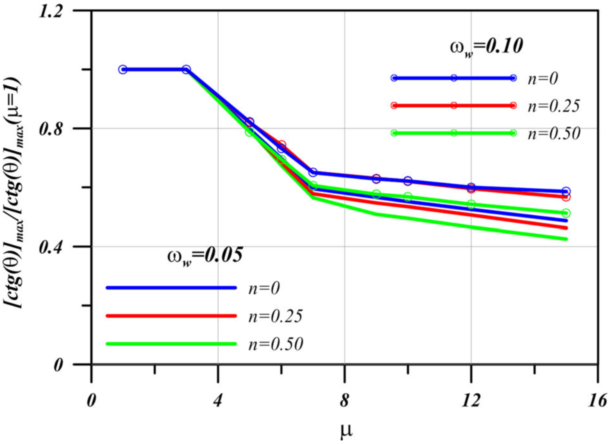

Fig. 6 Normalized curves of $[cot\,\theta]_{max}$ versus flexural ductility demand (μ) for different value of axial force (n) and stirrups mechanical ratio (ω_w)

mechanical ratio of the stirrups and the larger the axial force, the larger is the reduction of $[ctg\,\theta]_{max}$, i.e. the more the yield line resemble the initial cracking line.

6 Conclusions

In this paper, a theoretical model is proposed for assessment of the shear strength of RC elements with circular cross-sections under cyclic actions. The model is able to take into account the interaction between the internal forces, N_{sd} and M_{sd}, and the shear strength degradation due to external cyclic actions, such as those produced by earthquakes. The model is derived on the basis of the stress field approach, and is able to predict the reduction in shear strength related to concrete damage due to the attainment of large flexural curvature by linking the limitation of the angle of inclination of stress fields to the flexural ductility demand.

The predicted shear strength degradation is consistent with those predicted by Priestley and Benzoni's model, since the reduction of the angle of inclination of stress fields to the flexural ductility demand has been tuned according that model. However, in Priestley and Benzoni the shear strength reduction was obtained by a reduction in the concrete tensile strength as a function of the bending ductility demand, and the effects of interaction among internal forces were neglected.

Furthermore, the proposed method allows the drawing of makes it possible to evaluate N–M–V interaction strength domains for assigned values of flexural ductility demand that would have to be used in push-over analysis, instead of the well-known axial force-bending moment resistance domains.

Numerical analyses have shown that, the more the ductility demand increases, the more θ leads to first crack surface inclination θ_I, for any amount of stirrups. Moreover, for high strength compression concrete, the minimum value of the yield surface inclination becomes close to the limit value $\theta = 30°$. Thus, a rough approximation of the shear strength can be obtained by assuming such an inclination of the stress fields.

It has to be emphasized that, in order to take into account the effect of cyclic load, further investigation are needed in order to obtain a more effective calibration of the reduction of the inclination of the stress fields as a function of cumulative flexural ductility demand or damage indexes, tuned on results of experimental test rather than on existing models.

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