

Performance-based assessment of the Great Mosque of Algiers

M. Rossi · S. Cattari · S. Lagomarsino

Received: 30 May 2014 / Accepted: 16 September 2014 / Published online: 5 October 2014
© Springer Science+Business Media Dordrecht 2014

Abstract The seismic Performance-Based Assessment (PBA) of monumental buildings requires to consider safety and conservation objectives, including also the possible presence of artistic assets. In order to face these issues, the case study of the Great Mosque of Algiers is analysed in this paper: in fact, besides to be one of the remaining Almoravid architecture and the oldest mosque of the city, it is also characterized by the presence of a *mihrab*, a decorated arched niche that represents an interesting artistic asset to be included in the PBA. Within this context, particular attention has been paid to the choice of the most reliable modelling strategy for the application of the displacement approach in the PBA procedure, as a function of different possible seismic behaviours. In the case of Great Mosque both the current state of the building and a virtual strengthened condition are analysed. It is worth noting that, while in the current state the seismic behaviour of the asset is well described by a set of macroelements that may be analysed independently (through 2D models), in the second case the strengthening intervention leads to the adoption of a 3D global model (indeed, the roof bracing promotes a “box-type” behaviour). In the paper, the integrate use of three different modelling strategies of different complexity is discussed: the finite element model, the equivalent frame approach and the macro-block model. The results of nonlinear analyses performed (static and kinematic) constitute the main tool to interpret the seismic response of the asset, perform the PBA and address the choice on the rehabilitation decisions. These latter in this case are mainly affected by the occurrence of too high deformations in local portions of the building, including that in which the artistic asset is located.

Keywords Performance-based assessment · Masonry modelling · Nonlinear static analysis · Nonlinear kinematic analysis · Historical buildings

M. Rossi (✉) · S. Cattari · S. Lagomarsino
Department of Civil, Chemical and Environmental Engineering (DICCA),
University of Genoa, Via Montallegro 1, 16145 Genoa, Italy
e-mail: michela.rossi@unige.it

S. Cattari
e-mail: serena.cattari@unige.it

S. Lagomarsino
e-mail: sergio.lagomarsino@unige.it

1 Introduction

The Great Mosque of Algiers is one of the few remaining examples of Almoravid architecture and the oldest mosque of the city; its relevance is not only due to the historical and architectural value but also to the presence of an artistic asset, the *mihrâb*, which is a decorated arched niche. Considering the high seismic hazard of Algiers, the cultural value and the significant use of the building, its seismic Performance-Based Assessment (PBA) is a priority both for safety and conservation objectives.

The PBA of monumental masonry structures, in particular in case of churches, mosques and other special typologies, is a complex task, which requires the adoption of nonlinear analyses and detailed modelling techniques: in fact, on the one hand, some damages occur in such structures even for low seismic demand and, on the other one, they are complex and worth to be analysed in details.

The PBA is carried out according to a displacement-based approach by using overdamped spectra, performing nonlinear static and kinematic analyses and applying the procedure specifically outlined for monumental buildings in PERPETUATE project (Lagomarsino et al. 2010; Lagomarsino and Cattari 2014).

Within this context, particular attention is paid to the choice of the most reliable modelling strategy and the PBA procedure to be adopted, as a function of different possible seismic behaviours. In the case of Great Mosque both the current state of the building and a virtual strengthened condition (after the hypothesized intervention of roof bracing) are analysed. In fact, while in the current state the seismic behaviour of the asset is well described by a set of macroelements that may be analysed independently (through 2D models), in the second case the effect of the intervention requires the adoption of a 3D global model. In the paper, the use of three different modelling strategies, of different complexity, is discussed: the finite element model, the equivalent frame approach and the macro-block model (Calderini et al. 2010; Roca et al. 2010); particular attention has been paid to their integrate use and the coherent definition of different mechanical parameters which they are based on.

First of all, the PBA requirements for the Great Mosque are discussed and selected, in terms of performance levels to be checked (Sect. 2). Then, the most reliable modelling strategies to be adopted and the necessary mechanical parameters (Sect. 4), acquired during the as-built information phase (Sect. 3), are illustrated. Finally, the safety verification in the current state of the building is carried out (Sect. 5) and the effects on the seismic response of a possible strengthening intervention are described (Sect. 6).

2 Performance requirements for the Great Mosque

2.1 Classification

The first step for the PBA is a proper interpretation of the seismic behavior expected for the structure, which comes out from the as-built information: historical notes, observation of architectural and structural features, as well as data achieved from investigations (discussed in more detail in Sect. 3).

Figure 1 shows some external and internal views of the Great Mosque, while Fig. 2 a plan view and some sections.

The Great Mosque is characterized by a large hall partitioned by a set of orthogonal system of arcades, without any intermediate horizontal diaphragms, except the timber roof that is not so rigid to guarantee a “box behavior”. Thus, the independent response of the



Fig. 1 General external and internal view of the Great Mosque and particular on the timber roof

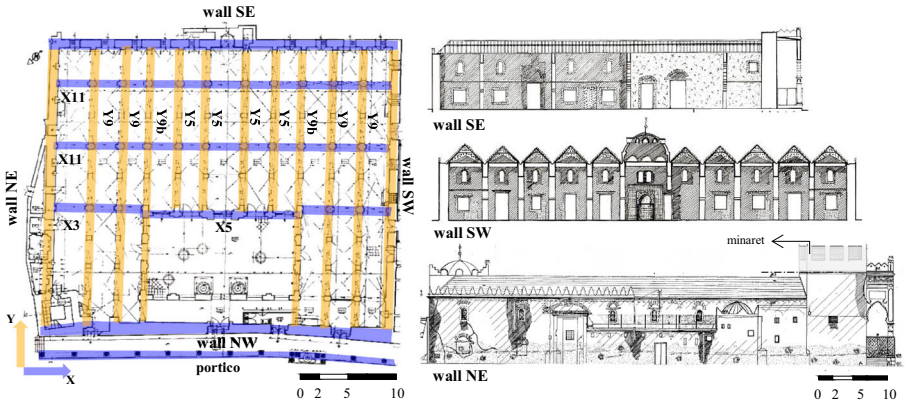


Fig. 2 Plan view (with the indication of macroelements that compose the building) and sections on the main façades of the Great Mosque

different macroelements that compose the building is considered. According to the two main orthogonal directions (X and Y as identified in Fig. 2), they are:

- the systems of arcades, classified by an alphanumeric code (ji) that identifies the seismic direction ($j = X, Y$) and the number of arches present ($i = 3, 5, 11, 9$). In case of the Y9 arcade type, a further boundary condition (named as “b”) has been considered to distinguish the behavior of the arcades that partially overlook the internal courtyard;
- the four external walls, oriented in North-East (NE), South-East (SE), North-West (NW) and South-West (SW) directions;
- the portico, forward the NW wall.

The monument is also characterized by the presence of a minaret, which however can be considered as an independent asset. Thus, it has not been included in the list of macroelements but it has been considered in the global model of the mosque (used for the seismic assessment in the examined strengthened state), in order to take into account the possible mutual interaction effects.

Thus, according to the classification proposed for architectural assets in Lagomarsino et al. (2011), which is based on the identification of the most relevant macroelements, the seismic behaviour and the prevailing damage type, the Great Mosque belongs to Class B—Assets analyzable by independent macroelements (sub-class B2-Mosques), which are subjected to

Table 1 Performance levels and reference target demand assumed for the PBA

PL (kn)	Use and human life		Building conservation	Artistic assets
	$2U$	$3U$	$3B$	$3A$
γ_n	1.2		1	0.8
\bar{T}_{kn} (years)	120	570	475	380
\bar{M}_{kn} (m/s ²)	1.96	3.84	3.55	3.22

PLs examined correspond to the following requirements (Lagomarsino and Cattari 2014): $2U$ —Immediate occupancy; $3U$ —Life safety; $3B$ —Significant but restorable damage; $3A$ —Severely damaged but still restorable

damage classes D (*arch structures loaded in their vertical plane*) and A (*in plane damage of vertical walls*).

2.2 Safety and conservation objectives

According to the procedure illustrated in Lagomarsino and Cattari (2014), PERPETUATE guidelines consider various performance levels identified by the alphanumeric code kn , where $k = 1, 4$ is the level and $n = U, B, A$ is related to three different categories of requirements: *use and human life* (U), *building conservation* (B) and *artistic assets conservation* (A). Table 1 shows those considered for the PBA of the Great Mosque, taking into account its relevant current public use ($2U$ and $3U$), its architectural value ($3B$) and the presence of the *mihhrâb* ($3A$). Moreover, also the $4B$ (*near collapse*) performance level has to be considered, being useful to assess the PBA according to the procedure adopted, as discussed more in detail in Sect. 5. Since only nonlinear static analyses have been performed, the seismic demand is defined in terms of elastic and overdamped spectra.

The intensity measure (IM) adopted for the seismic assessment is the peak ground acceleration (PGA). In Table 1 the return periods (\bar{T}_{kn}) and the corresponding values of PGA (\bar{M}_{kn}) assumed as target for each considered PL are shown. The values of \bar{M}_{kn} derive from the probabilistic seismic hazard analysis of the city of Algiers illustrated in Faouzi and Nasser (2013). Those of \bar{T}_{kn} have been defined by applying specific importance coefficients γ_n ($n = U, B, A$) to the basic values of the target return periods assumed for $k = 2, 3$ (equal to 100 and 475 years, respectively); these coefficients have been differentiated for each requirement, taking into account the relevance of each of them for the given asset ($\gamma_U = 1.2$, due to the possible crowding; $\gamma_B = 1$; $\gamma_A = 0.8$, since the artistic asset is not the original one).

3 As-built information

3.1 Geometry and constructive details

The Great Mosque of Algiers, also known as the El Jedid Mosque, is located on the Rue de la Marine in Algeria's capital city. It was built in 1097 under the direction of Sultan Ali ibn Yusuf (1106–1142) and it is the oldest mosque in Algiers, as well as one of the few remaining Almoravid architecture. According to an inscription on its base, in 1322 the Sultan of Tlemcen, Abu Tachfin, added the minaret at the north corner. At the beginning of the Colonial period, in 1840, a portico of columns and poly-lobed arches was added to the façade of the mosque, because of the new alignment of the street. Its architectural features and layout, with naves perpendicular to the *qibla* wall and its rectangular courtyard bordered

on both its narrower sides by a *riwaq* (gallery), became a model for Muslim architecture, particularly in al-Aqsa Maghreb mosque in Algeria.

The original *mihrâb*, an arched niche indicating the direction of Mecca, was destroyed during a conflict in 1682 and it has been replaced with a *mihrâb* decorated with two small spiral columns flanking it on both sides, which are characteristic of eighteenth-century in Algiers, as well as some stucco relieves, in the shape of lobed arches that underline its ogee arch.

The building is almost square in plan, measuring approximately 40 by 50 m (Fig. 2). The interior of the mosque is a series of hallways, passages and rooms, with the common theme of pillars and archways throughout the building, based on a 9 by 11 grid. Poly-lobed arches run parallel to the *mihrâb* and alternate with slightly broken horseshoe arches, running perpendicular to it. They rest on rectangular or cross-shaped pillars. The eleven naves that run perpendicular to the *qibla* are crowned by a double sloping wooden roof (Fig. 1). The prayer hall is aligned below the first five of the nine aisles that run parallel to the *qibla* wall.

The current state of the building, that has been recently restored, does not present any relevant crack pattern and structural damage; no information are available on the damage due to past earthquakes.

3.2 Mechanical properties of masonry

According to information directly acquired by the Ministry of Culture of Algiers, it is reasonable to assume the masonry type as similar to other buildings belonging to the same period. In particular, the masonry walls and arched structures are built-up with fired-clay bricks, whose dimensions are approximately: 10–12 cm wide, 22–30 cm long and 3.5–5 cm thick. The thickness of mortar joints is significant and similar to brick's height (almost 3 cm thick) and the quality is quite poor. It is a mortar made of clay, pozzolana, sand and lime in variable ratio. Since it was not possible to perform in-situ diagnostic tests on the Great Mosque, values of mechanical parameters necessary for the structural analysis (as discussed in Sect. 4) have been deduced from different tests performed on masonry (masonry wallets compressive test), mortar (three points bending and compressive test on mortar prisms) and joint (triplet test on brick-mortar assemblies) specimens built in laboratory (Fig. 3b/c). All these samples are considered representative of the specific masonry of the Great Mosque and, more in general, of many Algerian historical assets. In particular, they have been prepared for the mechanical characterization of a full scale masonry vault mock up (Fig. 3a), which reproduces the vaulted structure of the Algerian Mosque of Dey (located in the Citadel of the

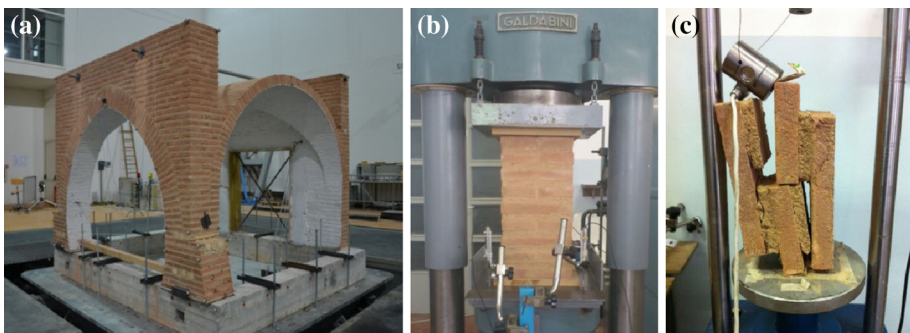


Fig. 3 a The vault masonry mock up tested on the shaking table at ENEA UTT-MAT QUAL Laboratory; b a masonry wallet for the compression test; c a masonry assembly for the triplet test

Casbah of Algiers), tested on the shaking table at UTTMAT-QUAL laboratories of ENEA Casaccia in Rome (Italy) during PERPETUATE project (De Canio et al. 2012).

4 Modelling

As introduced in Sect. 2.1, the Great Mosque belongs to Class B—*Assets analyzable by independent macroelements*, according to PERPETUATE architectural assets classification (Rossi et al. 2014; Lagomarsino et al. 2011). This means the seismic response of the building may be studied by analyzing independently each single macroelement that characterizes the structure (the arcades, the external walls and the portico). The timber roof, which is the only diaphragm present, has a negligible in-plane horizontal stiffness and is not able to effectively redistribute the seismic action between macroelements: thus, the setup of a 3D model is not necessary to examine the current state of the mosque.

The seismic assessment has been performed by considering the N_m capacity curves representative of each m -th macroelement ($m = 1, \dots, N_m$), obtained by adopting the most adequate modelling approach for each one, as listed in Sect. 2.1. Some macroelements have been studied by using different strategies in an integrate way, in order to validate and refine the definition of the capacity curve. In particular, according to the classification of models illustrated in Calderini et al. (2010), the approaches adopted may be summarized as follows:

- for the arcades macroelements: the Macro Block Model (MBM), aimed to perform a nonlinear kinematic analysis, and the Continuous Constitutive Law Model (CCLM), based on finite element modeling with nonlinear micromechanical constitutive laws, are used. The results of these two models have been combined in order to define the final capacity curve adopted for the PBA;
- for the external walls and the portico: a 2D Structural Element Model (SEM) is considered, which discretizes the wall by an equivalent frame of piers and spandrel elements and allows to perform a nonlinear static (pushover) analysis.

This different choice in the modeling strategy between the internal arcades and external walls is motivated by the fact that, while the latter are characterized by the in-plane seismic response of piers and spandrels, with failure modes ruled by the masonry material strength (either in shear or compression), the former are dominated by the response of arches and slender pillars, with loss of equilibrium that depends on the geometry.

The nonlinear kinematic analyses based on the MBM modelling strategy have been performed by using the MB-PERPETUATE software (Lagomarsino and Ottonelli 2012). According to the upper bound theorem of limit analysis, this approach is based on the *a priori* identification of the possible kinematisms, made by a proper set of rigid blocks, each one characterized by three degrees of freedom (two translations in X and Y coordinates and one rotation), and mutual constraints. While the preliminary identification of the plausible mechanisms is usually made on basis of damage observation on similar structures from post-earthquake survey, in the examined case it has been supported by the results from some nonlinear static analyses performed with the CCLM model. To this aim, also other modelling approaches could be adopted as that proposed in Milani et al. (2007) based on a simple micro-mechanical model for the homogenized FE limit analysis of masonry walls. Figure 4 illustrates the final mechanisms analysed for two recurrent arcades of the Great Mosque (X11 and Y5), where the different internal constraints (hinge, simple support, pendulum) have been defined on basis of results provided by the CCLM model.

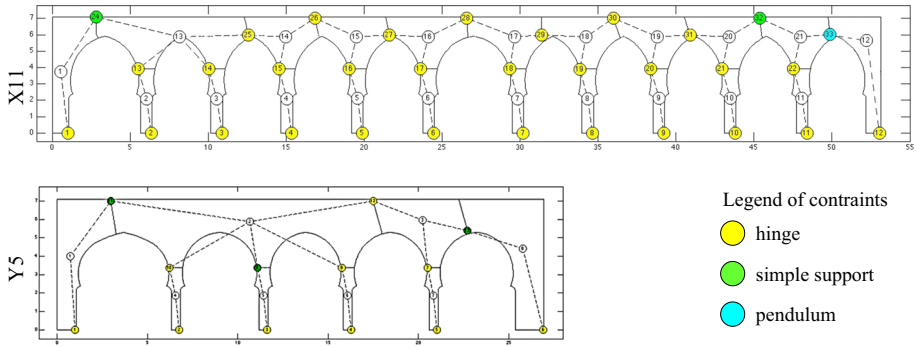


Fig. 4 MBM model: blocks geometry and constraints for X11 and Y5 arcade systems

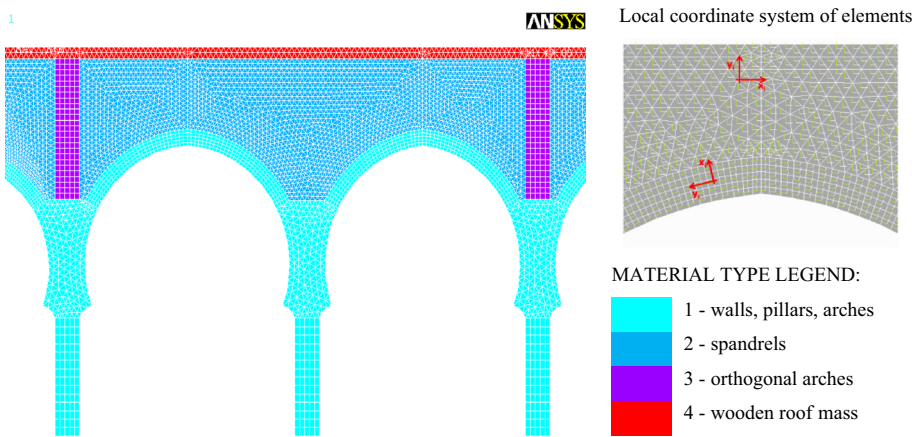


Fig. 5 CCLM model: mesh discretization and material map. On the right a zoom of the local coordinate systems of finite elements useful to define the orientation of bed joints in spandrels (modelled with the law by Calderini and Lagomarsino 2008) and arches (modelled with the law by Gambarotta and Lagomarsino 1997)

The analyses based on the CCLM approach have been performed by using ANSYS Finite Element software and by assuming the nonlinear constitutive laws for masonry proposed in Gambarotta and Lagomarsino (1997) and Calderini and Lagomarsino (2008). They are based on a multiscale approach by which the micromechanical behaviour is homogenized to an equivalent continuum in plane stress. The limited tensile and shear strength (with friction) of mortar joints and the crushing of masonry are considered. The orthotropic behaviour of masonry material is considered introducing the modelling of the bed and head mortar joints (without strength in the case of head joints). Moreover, a damage model allows to simulate the progressive loss of stiffness, up to a predefined strength value, and the following softening phase. The finite element modelling has used 2D plane stress elements (PLANE42 element type). The mesh details and the local coordinate system for elements are shown in Fig. 5; the model proposed in Calderini and Lagomarsino (2008) has been adopted only for spandrel portions where the correct simulation of a possible shear stair-stepped crack is relevant (namely material type No. 2 in Fig. 5).

The results achieved by analyses performed on such CCLM model have been very useful also to calibrate the initial branch of the capacity curves of examined macroelements.

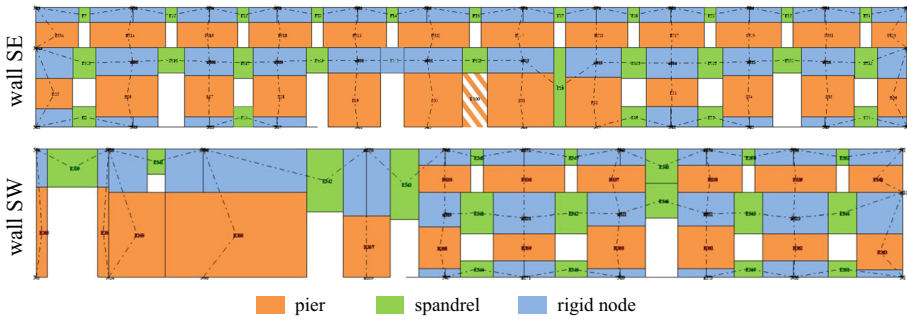


Fig. 6 SEM model: equivalent frame idealisation of SE and SW walls (the diagonal stripe pattern in the central pier no. E100 of wall SE indicates the *mihrâb* location)

Finally, the analyses based on the SEM approach have been performed by using the Tremuri software (Lagomarsino et al. 2012), originally developed at the University of Genoa (Lagomarsino et al. 2013) and further improved in PERPETUATE project by implementing new multi-linear constitutive laws for masonry panels (Cattari and Lagomarsino 2013, Fig. 8a). According to the SEM model, each wall is discretized by a set of masonry panels (piers and spandrels), in which the non linear response is concentrated, connected by rigid area (nodes). Figure 6 illustrates the equivalent frame mesh for SE and SW walls: indeed, due to the quite regular pattern of openings, the definition of elements geometry has a limited arbitrariness in these cases.

The PBA of the mosque in its current state showed (see Sect. 5) its partial inadequacy in fulfilling all the PLs; thus, the effect of a stiffening intervention of the roof has been analysed (see Sect. 6). It aims to promote the redistribution of actions among walls and achieve a global response of the building (“box-type” behaviour): consequently, the mosque behaviour passes from that described by Class B to that of Class A—*assets with a box behaviour* (Lagomarsino et al. 2011). In the latter case the PBA requires the adoption of a 3D global model, being the overall response well described by a single capacity curve. Among the different possible choices, the SEM approach is considered as particular effective due to its limited computational effort. The complete 3D model is obtained by assembling 2D walls, assuming the perfect connection between walls and condensing the degrees of freedom of two 2D incident nodes; the roof is modelled as a horizontal orthotropic membrane finite element with equivalent stiffness (Lagomarsino et al. 2013). Of course the reliability of such model in the examined case is strongly affected by its capability in capturing the actual response of the arcade system, for which the equivalent frame model represents a rough approximation. Thus, to guarantee an accurate evaluation of such system it is necessary to properly calibrate: (i) the geometry of the equivalent frame idealization; (ii) the mechanical parameters of masonry. To this aim, the results from the MBM and CCLM models have been properly exploited as described in more detail in Sect. 6 (Figs. 9, 10a). Figure 7 shows a general view of the 3D SEM model and clarifies the rules adopted to define the equivalent frame of the arcade system.

Moreover, as aforementioned, although the minaret has been considered an independent asset and was not analyzed as a macroelement of the Mosque, it has been included in the global 3D SEM model, in order to take into account the possible mutual interaction.

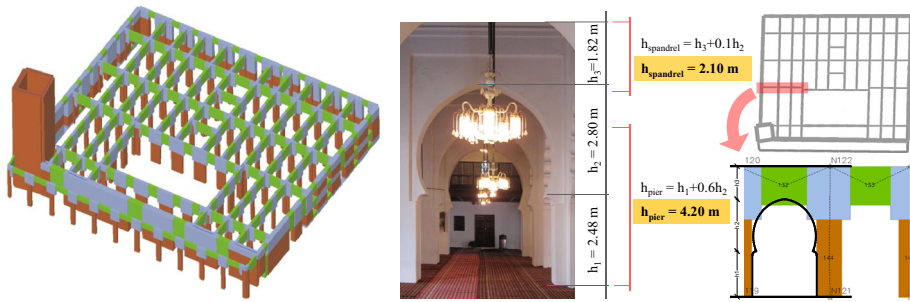


Fig. 7 3D SEM model of the mosque and rules adopted for the equivalent frame idealization of the arcade system

4.1 Mechanical parameters

Depending on the adopted modelling approaches, the mechanical parameters of masonry differ in number and also in the scale they aim to represent: single constituents, equivalent homogeneous masonry material or structural element (pier and spandrel). In the following, the three strategies considered are described with reference to the effort and details they require in the assignment of parameters.

In the case of the CCLM approach, the assumed constitutive laws (Gambarotta and Lagomarsino 1997; Calderini and Lagomarsino 2008) consider the nonlinear stress–strain relation in terms of mean stresses and mean strains in a reference cell of masonry. Thus, the mechanical parameters directly involved in the assessment are those aimed to describe the response of masonry at scale of both the single constituents (mortar joints and blocks) and the equivalent homogeneous material.

As summarized in the following tables, a proper set of parameters has to be defined in order to completely describe:

- the elastic phase (associated with a homogenized elastic continuum) through the Young (E) and Shear (G) Moduli and the Poisson ratio (ν) (see Table 2);
- the strength domains addressed to interpret the failure modes considered by the constitutive laws adopted (as illustrated in Sect. 4, see Table 3), which are: the masonry crushing (through the compressive strength of masonry f_M); the sliding of mortar joints and the opening/closing state of bed and head joints (through the friction coefficient μ , the cohesion c , the tensile strength f_{mt} and the interlocking ϕ due to the masonry pattern); the failure in tension of blocks (through the tensile strength f_{bt}).
- the inelastic strain contributions and the softening phase (see Table 4): through the inelastic compliance parameters of interfaces (c_{bm} , that represents the inelastic/elastic shear strain ratio at failure in mortar joints) and blocks (c_{bM} , inelastic/elastic normal strain ratio of masonry at failure in compression) and their corresponding softening coefficients (β_m and β_b).

They have been calibrated on basis of the experimental evidences from the tests mentioned in Sect. 3.2 and other ones available in literature for similar masonry (Anthoine et al. 1995; Page 1983; Binda et al. 1997).

In the case of the SEM approach, the masonry behaviour is directly described at the scale of structural elements, where the nonlinearity is concentrated. As known, since at panel scale the stress distribution is complex and not homogeneous, the most common strength

Table 2 Mechanical elastic parameters of materials

CCLM model					SEM model				
Material type	ρ (kg/m ³)	ν	E (MPa)	G (MPa)	Wall type	E (MPa)	G (MPa)	k_0	k_r
1	1,800	0.2	1,000	250	External walls	800 (600)	250 (200)	0.5 (0.5)	0.7 (0.7)
2	1,800	0.2	800	200	Internal arcades	800 (600)	250 (200)	0.66 (0.66)	0.7 (0.7)

CCLM: ρ = density; in case of material no. 3 and 4 (Fig. 5) an equivalent density equal to 10,000 and 5,800 kg/m³ have been assumed, respectively.

SEM: in brackets the values assumed in case of spandrels elements are indicated; in the case of internal walls, these values are adopted to simulate the arched spandrels (see Fig. 7)

Table 3 Mechanical parameters aimed to define the limit strength in CCLM and SEM models

CCLM model						SEM model				
Mortar/joints			Brick	Masonry		Piers/spandrels				
μ	f_{mt} (MPa)	c (MPa)	f_{bt} (MPa)	f_M (MPa)	ϕ	$\tilde{\mu}$	\tilde{c}	f_{bt} (MPa)	f_M (MPa)	f_{St} (MPa)
0.6	0.02	0.04	2	4	0.5	0.46	0.03	2	4	$f(\mu, \phi, c, f_{bt}, \sigma_0)$

m: mortar, M: masonry; b: brick

SEM: $\tilde{\mu} = \mu / (1 + \mu\phi) =$ equivalent friction coefficient; $\tilde{c} = c / (1 + \mu\phi) =$ equivalent cohesion coefficient

Table 4 Material parameters that describe the nonlinear phase in CCLM and SEM models

CCLM model				SEM model					
Mortar/joints		Brick	Masonry	Piers/spandrels (%)					
c_{bm}	β_m	β_b	c_{bM}		δ_{E3}	δ_{E4}	δ_{E5}	β_{E3}	β_{E4}
0.6	0.02	2	4	Flexural	0.6/0.3	1/0.8	1.5/2	-	15
				Shear	0.3/0.3	0.5/0.8	0.7/2	30	60

SEM: after collapse ($\delta > \delta_{E5}$) the element is supposed be able to support only the vertical load but no shear force

criteria proposed in literature and adopted in codes to interpret the failure modes that may occur (crushing, bed joint sliding, shear diagonal cracking) are based on the choice of a reference stress (either shear, normal or principal stress) and a reference point or section on which it is calculated (e.g. the end section or the central transversal cross section), as discussed in (Magenes and Calvi 1997; Calderini et al. 2009). These strength domains are based on parameters that in most of cases may be directly correlated to those defined at the material scale, as used in the CCLM model. In particular, in the examined case, for interpreting the shear response, the criterion proposed in Mann and Müller (1980) has been assumed as reference. It is a Coulomb-type criterion based on equivalent cohesion \tilde{c} and equivalent friction $\tilde{\mu}$ parameters, computed starting from those of mortar joints (μ, c) and including also the effect of masonry texture ϕ (see Table 3). In the case of flexural response, the criterion proposed in Eurocode 8 (CEN 2005) and Italian Building Code (NTC 2008)

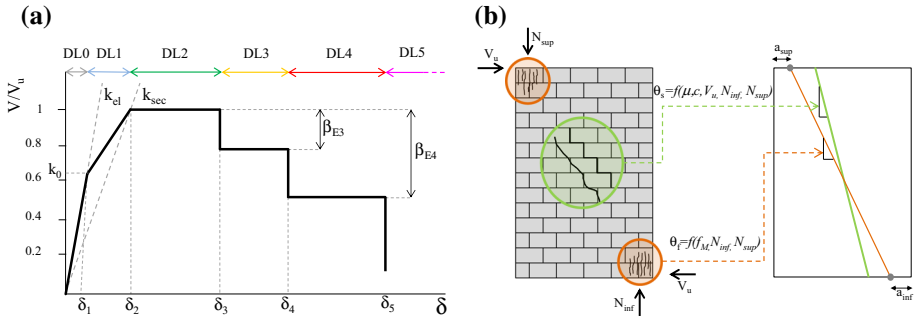


Fig. 8 **a** Multilinear constitutive laws adopted for masonry panels in the SEM model; **b** assumed strategy to take into account in the MBM modelling approach the limited strength of the material

has been assumed for piers, while in the case of spandrels the modified domain proposed in Calderini and Lagomarsino (2008) has been adopted. The first one (for piers) is calculated on the basis of the beam theory, neglecting the tensile strength of the material and assuming an appropriate normal stress distribution at the compressed toe; then the failure is associated with the attainment of the compressive strength of masonry normal to bed joints (f_M). The second one is based on an equivalent tensile strength of spandrels (f_{St}) that is due to the interlocking phenomena at the end sections with the adjacent masonry portions. This equivalent tensile strength is computed as a function of different failure modes that may involve either the block and the mortar joints; in particular, it depends on the tensile strength of blocks (f_{bt}), the local properties of mortar joints at the end sections of spandrel (μ, c, ϕ) and the compressive state on the masonry portions adjacent to spandrels (σ_0). The reliability of such assumption is testified by the evidence of some experimental campaigns and also recently discussed in Beyer and Mangalathu (2013).

Then, according to the multilinear constitutive laws adopted (Cattari and Lagomarsino 2013), the elastic phase is described according to the beam theory by defining the initial Young (E) and Shear (G) moduli; then the progressive degradation is computed in an approximate way by a secant stiffness (by assigning a proper ratio— k_r —between the initial k_{el} and secant k_{sec} stiffness at the point in which the maximum strength is reached, and a ratio— k_0 —between the shear at the end of the elastic phase and the shear strength, as illustrated in Fig. 8a).

Concerning the description of the nonlinear response, it is defined through subsequent strength decay (β_{Ei}) and drift limits (δ_{Ei}), which are associated to the achievement of reference damage levels ($i = 1, \dots, 5$). Such values are summarized in Table 4 and are different for the two considered failures modes (flexural or diagonal shear cracking) and the element type (pier or spandrel).

It is worth noting that, in the case of 3D SEM model representative of the mosque after strengthening, a proper tuning of parameters adopted to simulate the shear strength of arched spandrels (for the arcade system) has been done. In particular, according to the seismic response highlighted by the detailed CCLM analyses, the occurrence of the shear failure mode has been inhibited.

Finally, in the case of MBM approach, it is important to point out that, as known, it is based on the assumption of rigid blocks, that corresponds as stated by Heyman (1966) to: infinite compressive strength, no tensile strength and no sliding failure. As a consequence, the result of a limit analysis depends on geometry rather than material properties. Nevertheless,

the effect of the limited strength of the material may be taken into account in an approximate way by a proper placement of hinges. Usually, in order to include the effect of the limited masonry compressive strength, by assuming a stress block, a proper shifting of the hinges at the end sections is considered (as indicated in Fig. 8a through the a_{inf} and a_{sup} entities); it results a limitation in the slope θ_F of the compressed strut in the block. Analogously, in the case of shear failure, it is possible to limit the slope of the strut (θ_S), by assuming for example a Coulomb-type criterion like that proposed in Mann and Müller (1980) or other criteria aimed to interpret the diagonal shear cracking. Thus, at the scale of the single block, the maximum compatible slope θ may be computed as the minimum value between these latter ones, as follows (Fig. 8b):

$$\theta = \min(\theta_S, \theta_F) = \min \left[\arctg \left(\frac{2V_u}{N_{inf} + N_{sup}} \right), \arctg \left(\frac{N_{inf} + N_{sup}}{2Af_M} \right) \right] \quad (1)$$

where, N_{sup} and N_{inf} are the acting axial forces at two end sections of block; A is the transversal cross section of the block; V_u is the ultimate shear of the panel as obtained by the strength criterion adopted to interpret the shear failure.

It is evident from Tables 2, 3 and 4 that the numerical models depend on several parameters. Therefore, a sensitivity analysis would be useful to evaluate the role of uncertainties; the relevance of this topic for the as-built information and the improvement of the PBA reliability has been highlighted in Cattari et al. (2014). Despite this, the main attention of this paper is focused on the choice of the most reliable models and their integrate use, which does not require the sensitivity analysis.

5 Seismic assessment in the current state

The PBA of the Great Mosque in its current state requires the analysis of the seismic response of each m -th macroelement identified in Sect. 2 ($m = 1, \dots, N_m$); then, proper criteria are introduced to evaluate the maximum IM compatible with the fulfilment of the given PL (see Table 1) at scale of the whole asset ($IM_{kn,G}$), starting from the IM values computed at macroelement scale ($IM_{kn,m}$).

As regard the seismic response of the arcade system, Figs. 9 and 10 clarify the integrate use of CCLM and MBM models to evaluate the capacity curves of two of the most representative arcade types (Y5 and X11, as indicated in Fig. 2). In particular, as shown in Fig. 9, the damage pattern obtained through the CCLM model has been used to address the choice on the hinge position in the MBM model; the damage is expressed in terms of inelastic normal strain (opening) of mortar bed joints. Moreover, the results of nonlinear analyses with the CCLM model (Fig. 10b) have also supported the evaluation of the initial period (equal to 0.55 and 0.6 s in case of Y5 and X11 arcades, respectively) useful for the rising branch of capacity curve, that is joined with the descending branch evaluated through the MBM model, as depicted in Fig. 10a.

The position of PLs has been defined according to the criteria proposed in Lagomarsino (2014) and they are assumed to be coincident with the corresponding damage levels (DLk, $k = 1, 4$), in particular (Fig. 10a): performance level 2U corresponds to the intersection between the elastic branch and that from the incremental kinematic analysis; performance levels 3U and 3B (assumed to be coincident) correspond to a displacement capacity equal to $0.25d_0$, where d_0 is the displacement in which the spectral acceleration of the capacity curve is zero.

As regard the seismic response of perimetral masonry walls, modeled through the SEM approach, that of the SE wall is described in the following. It constitutes a significant example,

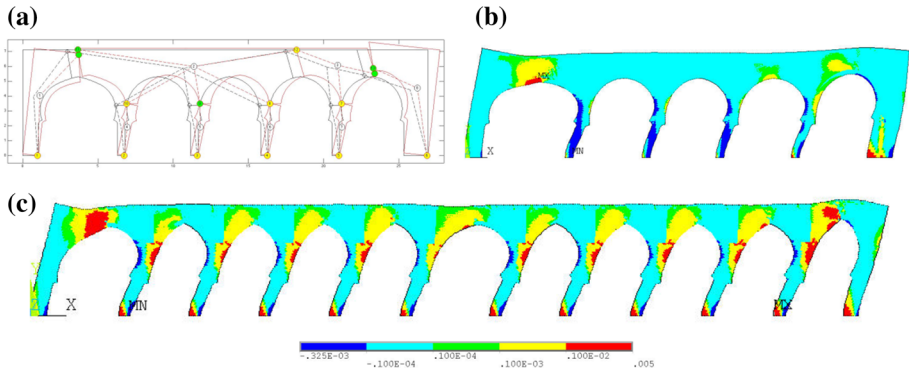


Fig. 9 **a** Kinematic analysed for the Y5 arcade trough the MBM model (hinge position—marked in yellow—and deformed shape; in green the simple support restraint is indicated); **b**, **c** inelastic stain opening of mortar bed joints obtained by means of the CCLM model for arcades Y5 and X11, respectively

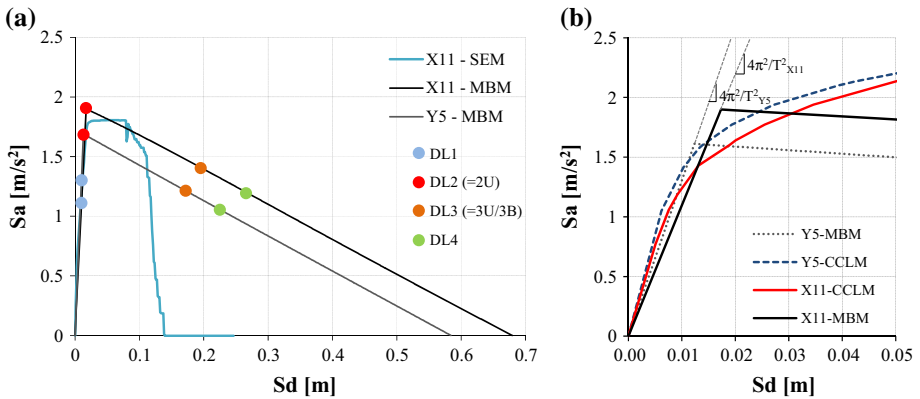


Fig. 10 **a** Capacity curves obtained by means of MBM model for arcade types X11 and Y5 and positioning of the PLs; **b** calibration of the initial branch of the capacity curve on basis of CCLM model results

as it is also characterized by the presence of the *mihrâb*, the most significant artistic asset in the Great Mosque. In particular, Fig. 11 illustrates the position of PLs on the pushover curve, which has been normalized to the maximum value of base shear of the wall. The definition of PLs related to the macroelement response follows the multiscale approach illustrated in Lagomarsino and Cattari (2014). In particular, the threshold displacement associated to each PL is defined as the minimum value resulting from the checks performed at these two different scales:

- the macroelement scale, where the attainment of PLs is monitored by the reaching of fixed rates of the base shear (in particular, the values equal to 1 and 0.6 have been assumed for the 2U and 3U/3B performance levels, respectively);
- the structural element scale, where the attainment of PLs is monitored by the achievement of the threshold equal to 3% of the cumulative rate of piers (Δ_{P,DL_i}) that reach a certain damage level DL_i ($i = 1 \dots, 5$, damage level monitored as scale of the structural element as indicated also in Fig. 8a). In particular, the damage levels 3 and 4 have been assumed as reference for 2U and 3U/3B performance levels: this means that a greater damage level is admitted at local level, for the definition at global scale.

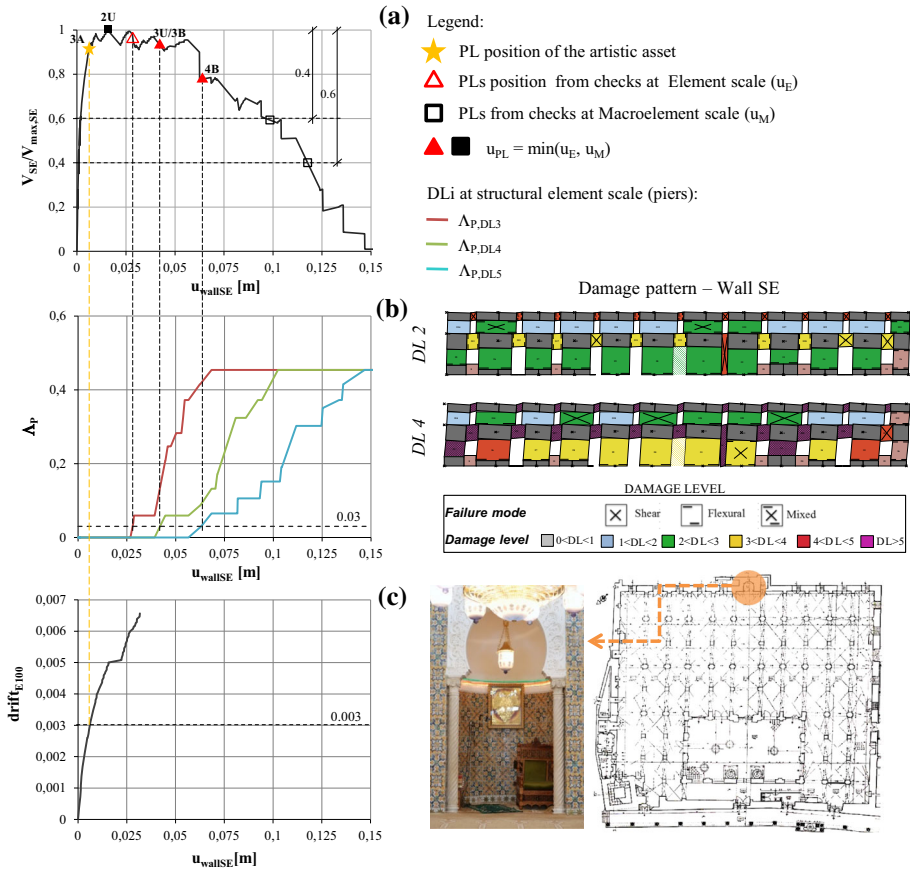


Fig. 11 Definition of the PLs on the pushover curve at Macroelement scale, including that of the artistic asset: **a** pushover curve of the wall SE and overview of the whole set of PLs position; **b** cumulative rate of damage reached in the pier elements; **c** drift curve of the pier with the artistic asset is connected to

As shown in Fig. 11, in the examined case, the check at macroelement scale prevails in case of $2U$, while that at structural element scale in case of $3U/3B$. In fact, this latter check aims to avoid the occurrence of too severe damage in local portions, which may not have a significant influence on the strength decay on the overall pushover curve.

Finally, the definition of the performance level of the artistic asset ($3A$) on the pushover curve of the whole macroelement is calculated starting from local checks on the structural element to which the *mihrâb* is connected. In fact, according to the classification proposed in Lagomarsino et al. (2011), the *mihrâb* belongs to Class Q—artifacts strictly connected to structural elements. Thus, as illustrated in Fig. 11, the attainment of $3A$ is monitored by the achievement of the drift threshold equal to 0.003 in the structural element no. E100 marked by the diagonal stripe pattern in Fig. 6; the drift is evaluated in terms of shear deformation, by considering the difference of horizontal displacements over the panel height and also taking into account the end rotations (Lagomarsino et al. 2013). Indeed, although the assumed value of the threshold is rather low for the structural response of the pier, it intends to consider that the *mihrâb* is composed by different types of vulnerable decorations, like as spiral columns,

Table 5 List of macroelements examined in two directions (X and Y) and corresponding $IM_{kn,m}$ values

Macroelement type	Dir. X					Dir. Y					
	No.	ρ_m	IM_{2U} (m/s ²)	$IM_{3U/3B}$ (m/s ²)	IM_{4B} (m/s ²)	No.	ρ_m	IM_{2U} (m/s ²)	$IM_{3U/3B}$ (m/s ²)	IM_{4B} (m/s ²)	
Arcade X11	2	0.125	1.23	7.57	10.08	Arcade Y9	4	0.083	1.26	7.8	11.1
Arcade X5	1	0.125	1	5.25	6.2	Arcade Y5	4	0.083	1.01	5.68	6.6
Arcade X3	2	0.125	1.05	5.3	6.33	Arcade Y9 _b	2	0.083	1.29	8.1	12
Wall SE	1	0.125	2.54	4.39	5.41	Wall NE	1	0.083	2.96	3.18	3.23
Wall NW	1	0.125	3.97	4.08	4.16	Wall SW	1	0.083	3.27	3.53	3.59
Portico	1	0.125	1.1	7.4	9.8						

stucco reliefs and mosaics: thus, the limit assumed aims to be comprehensive of the most punitive condition among them. The adopted value has to be intended conventional since very few experimental results are available on this issue, as recently discussed in [Calderini et al. \(2014\)](#).

Once the PLs have been fixed on the pushover curve of each macroelement, each $IM_{kn,m}$ value is computed according to the procedure illustrated in [Lagomarsino and Cattari \(2014\)](#), which uses the overdamped spectra. In particular, the conversion of the pushover curve (representative of the original MDOF) in the capacity curve (equivalent SDOF) is made through the participation coefficient (Γ) and the participation mass (m^*), according to the proposals originally illustrated in [Fajfar \(2000\)](#), in the case of nonlinear static analyses (SEM model), and in [Lagomarsino \(2014\)](#), in the case of nonlinear kinematic ones (MBM model). Table 5 summarizes the values obtained for each macroelement examined in two main directions (X and Y, as indicated in Fig. 2); for each type, the total number of macroelements that are present in the mosque, as well as the weight ρ_m ($\sum_1^{N_m} \rho_m = 1$), are reported. In the examined case an equal weight has been attributed to all macroelements, but more in general this parameter represents the size, role and relevance assumed by each macroelement with respect to the specific PL examined (e.g. for the safety of people or the protection of the whole asset).

Finally, the results have been combined in order to pass from the PBA of the single macroelement to that of the Great Mosque as a whole ($IM_{kn,G}$). According to the criteria illustrated in [Lagomarsino and Cattari \(2014\)](#), two approaches may be followed: the simplest one (approach a) is to assume $IM_{kn,G}$ as the minimum value from the values obtained for the whole set of macroelements ($IM_{kn,m}$, $m = 1, \dots, N_m$); the second one (approach b), that is consistent also with the multiscale approach proposed for buildings with a box behavior, is based on the computation and use of a fragility curve representative of the seismic performance of the whole asset. In particular, this latter is defined as:

$$P_{kn}(IM) = P(D \geq DL_k | IM) = \sum_{m=1}^{N_m} \rho_m H(IM - IM_{kn,m}) \tag{2}$$

where D represent the damage state and H is the Heaviside function (0 if $IM < IM_{PL,m}$; 1 otherwise). Then, the value of $IM_{kn,G}$ is obtained as the minimum of the following two conditions: (i) the lower value of IM for which the fragility curve has $P_{kn}(IM) = 0.5$; (ii) the value of IM for which the fragility curve of the performance level $(k + 1)n$ is greater than 0. The application of condition (ii) requires the definition also of the performance level 4B; it has been defined in case of macroelements analyzed by the MBM model as the displacement

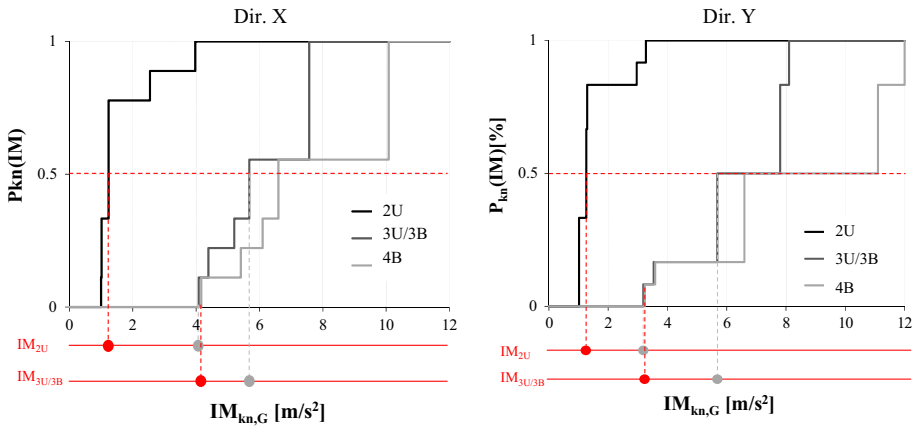


Fig. 12 Fragility curve representative of the seismic behavior of the whole asset in X and Y directions and computation of $IM_{kn,G}$ according to approach b

Table 6 Values of IM_{kn} and $T_{R,kn}$ corresponding to the different considered PLs and the corresponding safety factor in terms of return period

	Dir. X				Dir. Y		
	3A	2U	3U	3B	2U	3U	3B
IM_{kn} —approach a (m/s^2)	1.83	1	4.08	4.08	1.01	3.18	3.18
IM_{kn} —approach b (m/s^2)	1.83	1.1	4.16	4.16	1.23	3.23	3.23
$T_{R,kn}$ —approach b (years)	101	55	692	692	63	383	383
$I_{S,kn}$ —approach b	0.27	0.46	1.21	1.46	0.73	0.67	0.81

capacity equal to $0.4d_0$, while, in case of those analyzed by the SEM model, through the check of the attainment of 60 % base shear decay (macroelement scale) and of the $\Delta_{P,DL5}$ equal to 3 % (element scale). Figure 12 illustrates the fragility curves obtained for two directions examined and the evaluation of $IM_{kn,G}$ according to the approach b.

Finally, Table 6 summarizes the values of $IM_{kn,G}$ resulting from the application of two approaches aforementioned and, in case of approach b, the corresponding values of the return period compatible with the fulfillment of PLs and the safety indexes ($I_{S,kn} = 50T_{R,kn}/\bar{T}_{R,kn}$), where values less than 1 indicate that the PL is not satisfied. In general, it can be observed that Y direction is the most vulnerable one, as no one of considered PL is verified, while, in X direction, only the 2U fails the check. Also the performance level related to the artistic asset (3A) is not satisfied; indeed, from Fig. 11, it is possible to observe as the structural element which is linked to reaches significant drift values for low values of displacement u_{wallSE} , when the pushover curve of the whole macroelement is still in the elastic phase.

6 Rehabilitation decisions

In order to improve the seismic response of the Great Mosque and satisfy the fulfillment of all PLs, the effect of a stiffening intervention of the roof has been investigated. It aims to pursue two main objectives: (i) to guarantee a better redistribution of seismic forces among

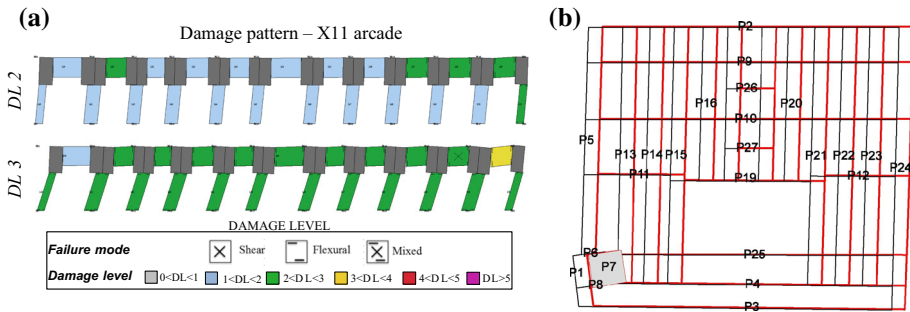


Fig. 13 **a** Example of damage pattern for the X11 arcade at DL2 and DL3; **b** plan deformed shape obtained from the pushover analysis on X direction (the minaret is marked in light grey)

macroelements; (ii) to limit the occurrence of great displacements in certain parts of the building, as those of elements linked to the artistic asset. As mentioned also in Sect. 4, this intervention determines a seismic behavior of the structure that passes from that described by Class B to that by Class A—*assets with a box behaviour*. This latter implies the adoption of a global 3D model and the PBA carried out on the single capacity curve aimed to describe the response of the structure as a whole.

In the 3D global SEM model adopted, diaphragms are modelled as orthotropic membrane finite elements, where in particular: normal stiffness provides a link between piers of the wall, influencing the axial force on spandrels; shear stiffness influences the horizontal force transferred among the walls. The values of elastic parameters $E_{1,roof}$ and G_{roof} have been assumed equal to 500 and 1,000 GPa, respectively: they correspond to the insertion of a double flooring board.

For the simulation of the arcade system response, the use of the equivalent frame model has been validated by calibrating the mechanical parameters and the geometry of masonry panels (see Sect. 4). In Fig. 10a the comparison between the capacity curves of arcade X11 obtained by MBM and SEM models is illustrated: a good agreement, at least in terms of elastic branch and overall strength, may be observed. The sudden loss of strength in the SEM model is associated to the achievement of the drift thresholds related to the rocking response of piers elements (as summarized in Table 4); moreover the ultimate displacement capacity of SEM model is not so far from that associated to the 3B performance level obtained through the MBM and, in any case, conservative. Also the damage pattern simulated by the SEM model illustrated in Fig. 13a shows a good agreement with that of the CCLM model (see Fig. 9c), in particular in terms of portions characterized by the concentration of higher damage levels and prevailing type of failure mode (flexural).

Finally, Fig. 13b shows the deformed shape in plan, resulting from the nonlinear static analysis, for the earthquake in X direction, which shows that a quite uniform global response has been achieved.

Figure 14 shows the final pushover curves in X and Y directions and the final position of the PLs that have to be checked on them. These latter have been defined according to the multiscale approach proposed in Lagomarsino and Cattari (2014) where, with respect to what already described in Sect. 5 in the case of the single external walls of Great Mosque, also the checks at global scale have been added. These are related to the achievement of fixed rates of the overall base shear on the global pushover curve (in particular, the values equal to 1 and 0.8 have been assumed for the 2U and 3U/3B performance levels, respectively).

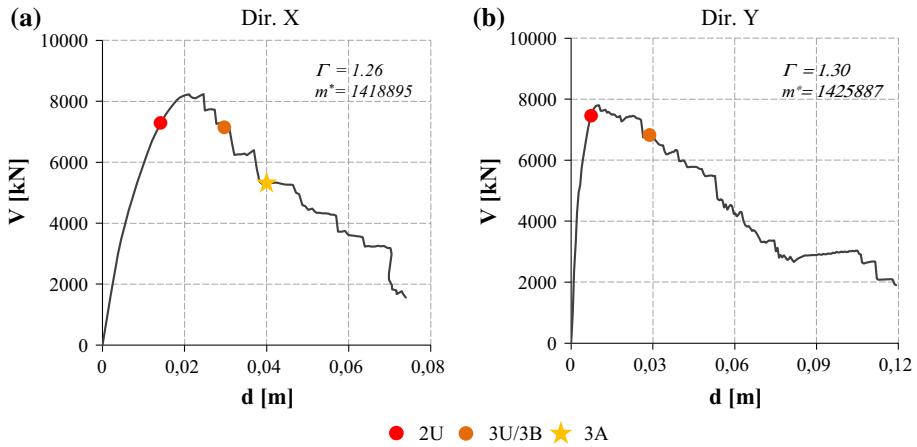


Fig. 14 Comparison between the pushover curves obtained by considering rigid floors: **a** X direction; **b** Y direction

Table 7 Values of $IM_{kn,G}$ and $I_{s,kn}$ in case of strengthened state of the Great Mosque

	Dir.—X				Dir.—Y		
	3A	2U	3U	3B	2U	3U	3B
$IM_{kn,G}$	4.32	2.85	3.99	3.99	2.65	3.96	3.96
$I_{s,kn}$	1.34	2.37	1.09	1.32	2.02	1.08	1.29

It is worth noting how the 3A position is brought farther the elastic branch, demonstrating as the roof bracing revealed to be effective in limiting the occurrence of local peaks of displacements.

Then, the PBA has been evaluated by adopting the same approach of nonlinear static analyses already introduced in Sect. 5 (the values of Γ and m^* factors are shown in Fig. 14): in this case the capacity curve obtained from the SEM model is representative of the whole response of the asset and no additional combination of results is required.

Table 7 summarizes the PBA results in terms of $IM_{kn,G}$ and $I_{s,kn}$: in Y direction, all the safety factors have a significant increase, while in X direction this is true only for the 2U and 3A performance levels. The 3B and 3U PLs exhibit a small decrease (even if they are still greater than 1): this could be related to the underestimation of the position of such PLs for the arcade system in case of SEM model.

In general, it may be stated the intervention analysed provides a significant improvement of seismic response of the Great Mosque.

7 Conclusions

In the paper the integrated use of different modeling strategies to provide a more reliable seismic performance based assessment of the Great Mosque in Algiers has been discussed. In particular, the use of a detailed model (finite element analysis with nonlinear constitutive laws for masonry) allowed, on the one hand, to improve the evaluation carried out through

the nonlinear kinematic analysis (rigid macro-block model) and, on the other one, to calibrate the use of the 3D macroelement model (equivalent frame approach) for catching the response of the arcade structural system, one of main representative feature of this mosque.

The complete methodological path followed for the PBA has been described in the paper, including also an example concerning the assessment of an artistic asset (the *mihrâb*).

Results highlighted in general a quite good seismic response of the Great Mosque but also some deficiencies related to its capability to fulfill the performance levels related to the conservation of the *mihrâb* and the safety of people who daily use the mosque (*2U—Immediate occupancy*).

The analyses performed in a possible strengthened condition, after the roof bracings, proved this intervention is effective in improving its seismic response, in particular by limiting the occurrence of too high deformations in local portions of the building (main cause of the deficiency in fulfill the performance level of the artistic asset) and exploiting the contribution of all macroelements of the structure in nonlinear phase.

Acknowledgments The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007–2013) under Grant agreement No. 244229 (www.perpetuate.eu). The authors are grateful to Zekagh Abdelwahab (Ecole Polytechnique d'Architecture et d'Urbanisme, Algeria) and to Prof. Djillali Benouar (University of Bab Ezzouar, Algeria) and Chergui Samia (University Said Dahleb, Algeria) for the data and support provided.

References

- Anthoine A, Magonette G, Magenes G (1995) Shear compression testing and analysis of brick masonry walls. In: Proceedings of the 10th European conference on earthquake engineering, Vienna, 1995, pp 1657–1662
- Beyer K, Mangalathu S (2013) Review of strength models for masonry spandrels. Bull Earthq Eng 11:521–542. doi:[10.1007/s10518-012-9394-3](https://doi.org/10.1007/s10518-012-9394-3)
- Binda L, Tiraboschi C, Abbaneo S (1997) Experimental research to characterize masonry materials. Mason Int 10(3):92–101
- Calderini C, Lagomarsino S (2008) Continuum model for in-plane anisotropic inelastic behavior of masonry. J Struct Eng 134(2):209–220
- Calderini C, Cattari S, Lagomarsino S (2009) In-plane strength of unreinforced masonry piers. Earthq Eng Struct Dyn 38(2):243–267
- Calderini C, Cattari S, Lagomarsino S, Rossi M (2010) Review of existing models for global response and local mechanisms, PERPETUATE (EU-FP7 Research Project), Deliverable D7. www.perpetuate.eu/final-results/reports/
- Calderini C, Degli Abbatì S, Cotić P, Kržan M, Bosiljkov V (2014) In-plane shear tests on masonry panels with plaster: correlation of structural damage and damage on artistic assets. Bull Earthq Eng. doi:[10.1007/s10518-014-9632-y](https://doi.org/10.1007/s10518-014-9632-y)
- Cattari S, Lagomarsino S (2008) A strength criterion for the flexural behaviour of spandrel in unreinforced masonry walls. In: Proceedings of the 14th World Conference on Earthquake Engineering, Beijing, China, 12–17, Oct 2008
- Cattari S, Lagomarsino S (2013) Masonry structures. In: Sullivan T, Calvi GM (eds) Developments in the field of displacement based seismic assessment, IUSS Press (PAVIA) and EUCENTRE, pp. 524, ISBN 978-88-6198-090-7
- Cattari S, Lagomarsino S, Bosiljkov V, D'Ayala D (2014) Sensitivity analysis for setting up the investigation protocol and defining proper confidence factors for masonry buildings. Bull Earthq Eng. doi:[10.1007/s10518-014-9648-3](https://doi.org/10.1007/s10518-014-9648-3)
- CEN (2005) Eurocode 8: design of structures for earthquake resistance—part 3: assessment and retrofitting of buildings. EN1998-3:2005. Comité Européen de Normalisation, Brussels
- De Canio G, Clemente P, Mongelli M, Rinaldis D, Roselli I, Calderini C, Rossi M (2012) Results of experimental test on damage measures and reference values to be considered, PERPETUATE (EU-FP7 Research Project), Deliverable D12. www.perpetuate.eu/final-results/reports/
- Fajfar P (2000) A non linear analysis method for performance-based seismic design. Earthq Spectra 16(3):573–591

- Faouzi G, Nasser L (2013) Scalar and vector probabilistic seismic hazard analysis: application for Algiers City. *J Seismol*. doi:10.1007/s10950-013-9380-5
- Heyman J (1966) The stone skeleton. *Int J Solids Struct* 2:249–279
- Gambarotta L, Lagomarsino S (1997) Damage models for the seismic response of brick masonry shear walls. Part I: the mortar joint model and its applications. *Earthq Eng Struct Dyn* 26:423–439
- Lagomarsino S (2014) Seismic assessment of rocking masonry structures. *Bull Earthq Eng*. doi:10.1007/s10518-014-9609-x
- Lagomarsino S, Modaressi H, Pitilakis K, Bosjlikov V, Calderini C, D'Ayala D, Benouar D, Cattari S (2010) PERPETUATE Project: the proposal of a performance-based approach to earthquake protection of cultural heritage. *Adv Mater Res* 133–134:1119–1124
- Lagomarsino S, Abbas N, Calderini C, Cattari S, Rossi M, Ginanni Corradini R, Marghella G, Mattolin F, Piovanello V (2011) Classification of cultural heritage assets and seismic damage variables for the identification of performance levels. In: Proceedings of structural repairs and maintenance of heritage architecture conference, WIT Press, pp 697–708. doi:10.2495/STR110581
- Lagomarsino S, Penna A, Galasco A, Cattari S (2012) TREMURI program: seismic analyses of 3D masonry buildings, Release 2.0, University of Genoa, Italy, <mailto:tremuri@gmail.com>
- Lagomarsino S, Penna A, Galasco A, Cattari S (2013) TREMURI program: an equivalent frame model for the nonlinear seismic analysis of masonry buildings. *Eng Struct* 56:1787–1799. doi:10.1016/j.engstruct.2013.08.002
- Lagomarsino S, Ottonelli D (2012) A Macro-Block program for the seismic assessment (MB-PERPETUATE). PERPETUATE (EU-FP7 Research Project), Deliverable D29. www.perpetuate.eu/final-results/reports/
- Lagomarsino S, Cattari S (2014) PERPETUATE guidelines for seismic performance-based assessment of cultural heritage masonry structures. *Bull Earthq Eng*. doi:10.1007/s10518-014-9674-1
- Magenes G, Calvi GM (1997) In-plane seismic response of brick masonry walls. *Earthq Eng Struct Dyn* 26:1091–1112
- Mann W, Müller H (1980) Failure of shear-stressed masonry—an enlarged theory, tests and application to shear-walls. In: Proceedings of international symposium on load-bearing Brickwork, London, UK, pp 1–13
- Milani G, Lourenço P, Tralli A (2007) 3D homogenized limit analysis of masonry buildings under horizontal loads. *Eng Struct* 29:3134–3148
- NTC (2008) Decreto Ministeriale 14/1/2008. Norme tecniche per le costruzioni. Ministry of Infrastructures and Transportations. G.U. S.O. n.30 on 4/2/2008; 2008 (in Italian)
- Page AW (1983) The strength of brick masonry under biaxial tension-compression. *Int J Mason Constr* 3(1):26–31
- Roca P, Cervera M, Gariup G, Pelà L (2010) Structural analysis of masonry historical constructions. Classical and advanced approaches. *Arch Comput Methods Eng* 17:299–325. doi:10.1007/s11831-010-9046-1
- Rossi M, Cattari S, Lagomarsino S, Benouar D, Abdelwahab Z, Samia C (2014) Performance-based assessment of the Great Mosque of Algiers, Proc. of the 2nd International Conference on Protection of Historical Constructions, F.M. Mazzolani and G. Altay Eds, pp. 347–353, ISBN 978-975-518-361-9