Bull Earthquake Eng (2012) 10:967–983
DOI 10.1007/s10518-012-9340-4

ORIGINAL RESEARCH PAPER

Capacity models for shear strength of exterior joints in RC frames: state-of-the-art and synoptic examination

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Received: 2 February 2011 / Accepted: 22 January 2012 / Published online: 14 February 2012

Abstract Damage observed in existing structures after recent earthquake events pointed out the key importance of beam-to-column joints in influencing the global response of reinforced concrete structures. In the last two decades several theoretical and empirical models have been proposed for evaluating shear strength of beam-to-column joints. The present paper reports an overview of the models currently available in the scientific literature for evaluating shear capacity of exterior beam-to-column joints. The present study is the first step of a wide analysis aimed at assessing such models and improving them. Moreover, the uncertainties deriving by applying the mentioned models will be also quantified therein, by means of well-established procedures for probabilistic seismic analysis of structures. The final results of that study are reported within a companion paper.

Keywords Reinforced concrete · Joints · Shear strength · Capacity model · Seismic behaviour

Notations

A_c	Cross-sectional area of column
A_g	Cross-sectional area of joint
$A_{sb,inf}$	Bottom flexural reinforcement in the beam
$A_{sb,sup}$	Top flexural reinforcement in the beam
A_{sc}	Area of reinforcement in column

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$A_{sc,0}$	Area of the layer of steel farthest from the maximum compression face in a column
A_{sjh}	Area of horizontal stirrups in the joint panel
A_{str}	Effective area of the diagonal strut
a_s	Strut depth
a_v	Effective shear span
b_b	Beam width
d_b	Beam depth
d_b'	Thickness of the cover concrete in beam
b_c	Column width
BI	Beam reinforcement index
b_j	Effective joint width
c	Depth of the flexural compression zone of the elastic column
C_d	Diagonal compressive strength in the concrete strut
$C_{d,n}$	Nominal diagonal compressive strength in the concrete strut
D	Force of the diagonal strut
d_c	Column effective depth
d_c'	Thickness of the cover concrete in column
e_b	Eccentricity between the beam centreline and the column centroid
f_c	Compressive concrete strength
F_h and F_v	Horizontal and vertical forces components of the joint mechanism
f_{yb}	Yielding stress of beam reinforcement
f_{yc}	Yielding stress of column reinforcement
f_{yj}	Yielding stress of joint stirrups
h_b	Beam depth
h_c	Column depth
JI	Joint transverse reinforcement index
λ	Overstrength factor of the steel
N_c	Axial force in column
ν	No dimensional compressive stress at the bottom of the top column
θ	Inclination of the concrete strut
θ_{crit}	Critical inclination of the concrete strut
R	Radius of the centreline of the beam bars bent into the joint
ρ_b	beam reinforcement ratio
R_c	Concrete cube strength
ρ_c	Column longitudinal reinforcement ratio
ρ_j	Volumetric joint transverse reinforcement ratio
ρ_{jh}	Horizontal stirrup ratio
s	Spacing of the stirrups
σ_c	Column normal stress
$\sigma_{d,\max}$	Maximum stress of the diagonal strut
T	Tensile strength of the top beam reinforcement
V_c	Shear force acting the bottom of the top column
V_{ch}	Shear strength provided by concrete
v_{jh}	Shear stress of joint
V_{jh}	Shear strength of joint
$V_{jh,E}$	Shear force acting the joint
V_{sh}	Shear strength provided by stirrups
z_b	Distance between the bars in tension and the resultant of the compression

stresses in the sectional analysis of the beam

z_c Distance between the two outer bars in the column

χ Ratio between the effective shear span and the column effective depth

ζ Stiffness coefficient

1 Introduction

Beam-to-columns joints are more and more recognised as critical regions of reinforced concrete (RC) structures, as they often control the seismic behaviour of frames under seismic actions. The cyclic response of RC joints is of concern especially in cases of structures designed for gravitational loads only (Favvata et al. 2008). Since no particular design criteria or detailing rules are applied to RC joints (Pampanin et al. 2003), their dynamic response is deeply affected by significant damage phenomena resulting in brittle failures of joint regions. The last generation of seismic codes (EN 1998-3 2005) as well as the design methods inspired to the so-called "capacity design approach" (Paulay and Priestley 1992), provide engineers with stricter rules for designing joint stirrups and detailing the anchorage of the crossing rebars. Those rules generally derive by theoretical models for evaluating the shear capacity of RC joints.

Since plenty of models are currently available in the scientific literature, the present paper is primarily aimed at reporting a short, though reasonably wide, overview of those models, by outlining their formulation and emphasising the key geometric and mechanical parameters which actually affect them. In particular, the present study only deals with the so-called "exterior" RC joints in which only one beam is connected to the column.

The key features of the shear strength of RC joints are described in Sect. 2 which explains the basic relationships defining the shear force applied on the joint which depends on the stresses in both column and beam.

An overview of the main capacity models for shear strength of RC joints is reported in Sect. 3. As a matter of principle, they can be classified in "theoretical" and "empirical" models, as they derive by either mechanically consistent equations or simplified formulations calibrated on a set of experimental results.

The relevant geometric and mechanical parameters taken into account by the above mentioned models are briefly discussed in Sect. 4, pointing out the different behaviour of some of those models with respect to parameters of even primary importance, such as the amount of shear reinforcement in the joint panel, the geometric aspect ratio or the mechanical properties of concrete.

The results reported in this paper represent a preliminary step toward a thorough assessment of both accuracy and reliability of capacity models for RC joints, with special emphasis on those belonging to existing structures. The models outlined and discussed in this paper will be assessed and compared in a companion work (Lima et al. 2012) which collects a wide database of experimental results of tests on exterior beam-to-column joints. Thus, a complete evaluation of the relevant error and dispersion factors of those models will be presented therein.

2 Capacity models for RC joints

A state-of-art report about the models available for evaluating shear capacity of beam-to-column joints in RC structures is outlined in this section.

2.1 Outline of the main contributions

The first studies on beam-to-column joints were carried out by Hanson and Connor (1967) that performed a series of tests on full size specimens of exterior RC subassemblages with the aim of estimating the minimum joint reinforcement to achieve the required ultimate capacity. However, one of the first studies aimed at evaluating shear strength in exterior RC joints was developed by Zhang and Jirsa (1982). Moreover, plenty of studies have been carried out in the last 25 years on the same topic. Sarsam and Phillips (1985) proposed a semi-theoretical model calibrated on the basis of a small experimental database available in the scientific literature at that time. As a matter of principle, they supposed that the shear strength of joints depends upon concrete strength, geometric properties of the joint panel, the amount of transverse reinforcement into the panel zone and axial load in the top column. Pantazopoulo and Bonacci (1992) provided a completely theoretical model based on simulating the fundamental mechanisms controlling the joint behaviour. Paulay and Priestley (1992) proposed a very well-known strut-and-tie model, which is considered one of the most important contributions on this topic. Ortiz (1993) developed a strut-and-tie model for exterior beam-to-column joints which covers both reinforced and unreinforced joints. In the following years, two simple models were developed by Scott et al. (1994) and Parker and Bullman (1997).

In the last 10 years, the research activities about beam-to-column joints gained further momentum. Several theoretical and empirical models have been developed due to both the increasing awareness about the role of joints in influencing the structural response under seismic actions and the growing number of experimental results recently made available in the scientific literature. In particular, the two models by Vollum and Newman (1999) and Hwang and Lee (1999) have been proposed in the same year. The last one is a rather complex theoretical model based on a softened strut-and-tie mechanism and the same authors (Hwang and Lee 2002) provided a simplified procedure useful for both interior and exterior joints. The theoretical model by Parra-Montesinos and Wight (2002) and the semi-theoretical model by Bakir and Boduroglu (2002) were developed just few years later. Then, they have been followed by the empirical model by Hegger et al. (2003) based on several experimental results.

Attaalla (2004) proposed a theoretical model as an evolution of a previous one (Attalla 1997) which requires a complex iterative procedure. Russo and Somma (2006) proposed a theoretical model taking into account the three following contributions to joint shear strength: the vertical stresses transmitted by the column to the concrete, the longitudinal beam reinforcement and the possible passive confinement due to the stirrups into the joint panel. The commonly used strut-and-tie model was revised by Tsonos (2007), while Vollum and Parker (2008) adopted a rotational strut-and-tie model for designing exterior beam-to-column joints. A very recent empirical model useful for evaluating the shear capacity of both unreinforced and reinforced exterior joints has been developed by Kim et al. (2009) as an evolution of a previous one proposed by the same authors (Kim et al. 2007), in which the ultimate shear strength was evaluated on the basis of the results of a wide experimental database including only reinforced joints.

2.2 Basic concepts

The development of plastic hinges in the beams immediately connected to the column faces is expected under seismic actions. Then, the joint panel is subjected to high shear stresses

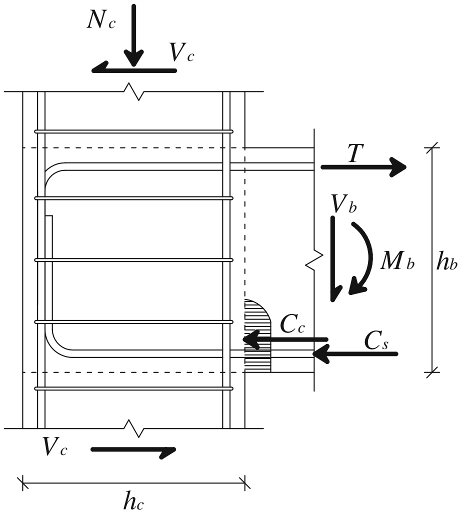

Fig. 1 Strut-and-tie mechanism for exterior joint

induced by both the shear force in the upper column and the rebars of the beam yielded in tension.

Consequently, the horizontal shear force across the joint region can be evaluated as follows (Fig. 1):

$$V_{jh,E} = T - V_c \tag{1}$$

in which T is the tensile stress of the top beam reinforcement (assuming, as a reference, negative bending moment on the beam) and V_c is the shear force of the top column (Fig. 1). Thus, the following design expression can be easily derived for $V_{jh,E}$

$$V_{jh,E} = A_{sb,sup} \cdot \lambda \cdot f_{yb} - V_c \tag{2}$$

where λ is the overstrength ratio of steel rebars, f_{yb} their yielding stress and $A_{sb,sup}$ the area of the upper layer of steel rebars in the beam Eq. (2) derives by assuming that the ultimate bending moment is attained at the end of the beam, as usually accepted within the capacity design approach (Paulay and Priestley 1992).

3 An overview of available capacity models for RC joints

Several proposals are currently available for evaluating the shear strength V_{jh} of RC joints. Those formulae are generally based on the sum of two basic contributions V_{ch} and V_{sh} related to concrete and steel stirrups, respectively:

$$V_{jh} = V_{ch} + V_{sh}. \tag{3}$$

In other cases alternative expressions, often derived by empirical formulations, are considered for evaluating the shear strength V_{jh} of RC joints. However, the design of stirrups in joints can be generally carried out by imposing V_{jh} to be equal to $V_{jh,E}$ derived by Eq. (2).

The following subsections summarise the key aspects of the most important and well known capacity models of RC joints with the aim of pointing out the relevant geometric and mechanical parameters which are supposed to control shear strength in RC joints.

The symbols adopted in the following are completely defined in the section titled "Notation"; thus, they are not redefined in the following subsections.

3.1 The model by Sarsam and Phillips (1985)

The empirical model developed by Sarsam and Phillips (1985) defines the total shear strength of RC joints V_{jh} defined in Eq. (3) as sum of the two basic contributions evaluated as follows:

$$V_{ch} = 5.08\,(R_c \cdot \rho_c)^{0.33} \left(\frac{d_c}{d_b}\right)^{1.33} b_c \cdot d_c \cdot \sqrt{1 + 0.29\frac{N_c}{A_c}} \tag{4}$$

$$V_{sh} = 0.87 \cdot A_{sjh} \cdot f_{yj} \tag{5}$$

where the concrete cube strength R_c is expressed in MPa, ρ_c is the column longitudinal reinforcement ratio $\left[\rho_c = A_{sc,0}/(b_c \cdot d_c)\right]$ and $A_{sc,0}$ is the area of the layer of steel rebars in tension of the column. The following upper limits were also proposed for both shear strength of the joint and some relevant parameters involved in its definition:

$$V_{jh} \leq 2.4\,(R_c)^{0.33}\, b_c \cdot d_c \qquad with \quad R_c \leq 70\,\text{MPa}, \tag{6}$$

$$\frac{N_c}{A_c} \leq \frac{R_c}{3}. \tag{7}$$

3.2 The model by Paulay and Priestley (1992)

This theoretical model developed by Paulay and Priestley (1992) is based on the sum of the two basic contributions defined in Eq. (3). In particular, the contribution of concrete strut for an exterior joint can be obtained as follows:

$$V_{ch} = \left(1 - \frac{\beta}{\lambda}\right) T + \frac{1.4 \cdot c \cdot \beta}{\lambda \cdot h_c} T - V_c \tag{8}$$

where $\beta = A_{sb,\inf}/A_{sb,sup}$ is the ratio between the longitudinal beam rebars in compression and the longitudinal reinforcement in tension in the beam and c is the depth of the flexural compressed zone of the elastic column depending by the axial load N_c. This parameter can be approximated by:

$$c = \left(0.25 + 0.85\frac{N_c}{f_c \cdot A_g}\right) h_c \tag{9}$$

with the following definition of the effective joint width b_j

$$b_j = \min\left[b_c\,;\ \frac{(b_c + b_b)}{2}\right]. \tag{10}$$

Since the Authors originally formulated this model within the framework of a "Capacity-Design" approach, the tensile force T in the steel rebars of the beam needed in Eq. 8 can be evaluated as follows:

$$T = \lambda \cdot f_{yb} \cdot A_{sb,sup}. \tag{11}$$

The contribution V_{sh} of steel stirrups to the shear strength of joints can be easily evaluated as follows:

$$V_{sh} = A_{sjh} \cdot f_{yj}, \tag{12}$$

thus, the shear strength V_{jh} of beam-to-column joints can be determined according to Eq. (3) as the sum of V_{ch} in Eq. (8) and V_{sh} in Eq. (12).

Finally, the following upper bound $V_{jh,lim}$ is proposed for V_{jh} to avoid the possible brittle failure of the diagonal strut:

$$\frac{V_{jh,lim}}{A_g} \leq 0.25 \cdot f_c \leq 9 \text{ MPa}. \tag{13}$$

3.3 The model by Scott et al. (1994)

Scott et al. (1994) proposed a simple theoretical model for estimating shear strength V_{jh} in exterior beam-to-column joints based on a single diagonal strut, neglecting both the vertical and horizontal contributions of stirrups:

$$V_{jh} = \frac{2 \cdot \sqrt{f_c}}{\left(\frac{z_c}{z_b} + \frac{z_b}{z_c}\right)} \cdot b_c \cdot d_c. \tag{14}$$

The ultimate shear strength V_{jh} depends on the concrete strength f_c and geometric properties of both column and beam; in particular, V_{jh} depends on the distance between the lever arms z_c and z_b in the cross section of the column and the beam, respectively.

3.4 The model by Parker and Bullman (1997)

Parker and Bullman (1997) developed a theoretical model for predicting shear strength of beam-to-column joints based on the evolution of a previous one originally developed for evaluating the shear strength of RC beams (Parker and Bullman 1995). The shear strength of beam-to-column joints depends upon the four values V_1, V_2, V_3 and V_4 defined in the following.

The value V_1 of shear strength is given below:

$$V_1 = \left(A_{sc} \cdot f_{yc} + N_c\right) \cdot \tan \theta_{crit} \tag{15}$$

where A_{sc} is the total area of reinforcement in column and the angle of inclination θ_{crit} of the concrete strut depends on the following factor χ:

$$\chi = \frac{a_v}{d_c} \tag{16}$$

in which the parameter a_v is the effective shear span of the connection depending by the radius R of the curved anchorage of rebars (taken either positive if they are bent downward into the joint or negative otherwise) and defined as follows:

$$a_v = 0.8 \cdot \left(h_b - d_b'\right) - 0.8 \cdot R \tag{17}$$

Based on the values of χ derived by Eq. (16), the critical angle of inclination θ_{crit} of the direct strut is evaluated as follows:

$$\tan \theta_{crit} = \begin{cases} 1 - \frac{\chi}{2} \quad if \quad \chi < 0.5 \\ \frac{1}{2\chi} + \frac{(\chi^2 - 0.75)}{(6 \cdot \chi^3 + 2.5 \cdot \chi)} \quad if \quad \chi \geq 0.5 \end{cases} \tag{18}$$

The second value of shear strength V_2 represents the contribution to the shear capacity provided by the concrete strut and is evaluated as follows:

$$V_2 = \alpha \cdot \nu \cdot R_c \cdot b_c \cdot \left(h_c - d_c'\right) \tag{19}$$

where:

$$\alpha = \frac{(1 - \chi \cdot \tan\theta_{crit})}{(\tan\theta_{crit} + \cot\theta_{crit})} \tag{20}$$

$$\nu = 0.56 - \frac{R_c}{310} \geq 0.4 \tag{21}$$

in which R_c is the concrete compressive cube strength expressed in MPa.

The shear strength V_{jh} of unreinforced joints can be evaluated as the minimum value between V_1 and V_2 defined in Eqs. (15) and (19). On the contrary, two more strength parameters are defined for reinforced joints:

$$V_3 = V_2 + A_{sjh} \cdot f_{yj} \cdot \left(\frac{a_v}{s} - 1\right), \tag{22}$$

$$V_4 = \beta \cdot V_2, \tag{23}$$

where A_{sjh} is the area of stirrups and f_{yj} and s are the yield stress and the spacing of the stirrups, respectively, while in Eq. (23) the terms β and d_v are derived as follows:

$$\beta = \frac{(d_v - s \cdot \tan\theta_{crit})}{\left(h_c - d'_c - a_v \cdot \tan\theta_{crit}\right)} \geq 1, \tag{24}$$

$$d_v = 0.9 \cdot \left(h_c - d'_c\right). \tag{25}$$

Consequently, the shear strength of reinforced joints is determined as the minimum among V_1, V_3 and V_4 defined in Eqs. (15), (22) and (23).

3.5 The model by Vollum and Newman (1999)

Vollum and Newman (1999) analysed several results of experimental tests on RC joints under monotonic actions, available in the scientific literature. Their research was aimed at determining the influence of the relevant geometric and mechanical parameters (i.e. concrete strength, column load, joint aspect ratio, reinforcement detailing and stirrups) on the mechanical behaviour of RC joints.

After a parametric study, a simple empirical design equation was proposed for evaluating the shear strength of exterior beam-to-column joints:

$$V_{jh} = V_{ch} + \left(A_{sjh} \cdot f_{yj} - \alpha \cdot b_j \cdot h_c \cdot \sqrt{f_c}\right) \tag{26}$$

in which A_{sjh} is the cross-sectional area of the joint stirrups within the top five-eighths of the beam depth, α is a coefficient including the effects of the column axial load, the concrete strength, the amount of stirrup into the panel zone and the joint aspect ratio (conservatively, the authors suggest $\alpha = 0.2\left[\text{MPa}^{0.5}\right]$) and b_j is the effective joint width defined as follows:

$$b_j = \begin{cases} \min\left(\frac{b_c+b_b}{2}; b_b + \frac{h_c}{2}\right) & if \quad b_b \leq b_c \\ \min\left(b_b; b_c + \frac{h_c}{2}\right) & if \quad b_b > b_c \end{cases} \tag{27}$$

The shear strength of joints without stirrups V_{ch} can be estimated by:

$$V_{ch} = 0.642 \cdot \beta \cdot \left[1 + 0.555 \cdot \left(2 - \frac{h_b}{h_c}\right)\right] \cdot b_j \cdot h_c \cdot \sqrt{f_c} \tag{28}$$

where β is 1.00 for connection with L end-bars and 0.90 for connections with U end-bars bent into the panel zone.

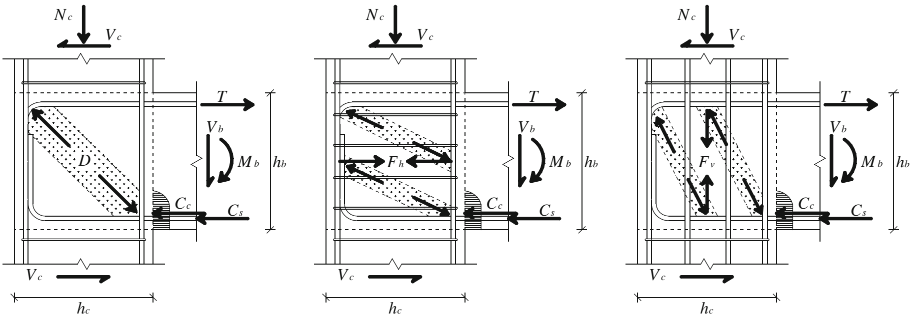

Fig. 2 Joint shear resisting mechanisms

The joint shear strength should be limited to the following upper bound:

$$V_{jh} \leq 0.97 \cdot b_j \cdot h_c \cdot \sqrt{f_c} \cdot \left[1 + 0.555 \cdot \left(2 - \frac{h_b}{h_c}\right)\right] \leq 1.33 \cdot b_j \cdot h_c \cdot \sqrt{f_c} \qquad (29)$$

based on the assumption that the higher the joint aspect ratio, the lower its shear strength V_{jh}.

3.6 The model by Hwang and Lee (2002)

In the theoretical model developed by Hwang and Lee (2002) the mechanical behaviour of joints is based on the action of three force components: diagonal strut D, horizontal F_h and vertical F_v mechanisms (Fig. 2).

The authors (Hwang and Lee 1999) developed an iterative procedure for evaluating the three force components affecting the shear strength of exterior beam-to-column joint. As the iterative procedure was very complex, the same authors (Hwang and Lee 2002) proposed a simplified method useful for both interior and exterior joints which is reported below.

The horizontal shear strength of beam-to-column joints is provided by the projection of the diagonal nominal strength $C_{d,n}$ on the horizontal axis:

$$V_{jh} = C_{d,n} \cdot \cos\theta \qquad (30)$$

in which nominal diagonal compressive strength $C_{d,n}$ is estimated as:

$$C_{d,n} = K \cdot \zeta \cdot f_c \cdot A_{str} \qquad (31)$$

and the stiffness coefficient ζ can be approximated as follows:

$$\zeta = \frac{3.35}{\sqrt{f_c}} \leq 0.52. \qquad (32)$$

The effective area of the diagonal strut A_{str} is defined as a function of the strut depth a_s and the effective width b_j of the joint:

$$A_{str} = a_s \cdot b_j \qquad (33)$$

$$a_s = \left(0.25 + 0.85\frac{N_c}{f_c \cdot A_c}\right) h_c \qquad (34)$$

in which A_c is the cross-sectional area of the column.

The K factor in eq. (31) represents the beneficial effects of the tie force on the shear strength and in the model by Hwang and Lee (2002) is developed in simplified way as follow:

$$K = K_h + K_v \tag{35}$$

in which

$$K_h = 1 + \left(\overline{K_h} - 1\right) \cdot \frac{F_{yh}}{\overline{F_h}} \le \overline{K_h}, \tag{36}$$

$$K_v = 1 + \left(\overline{K_v} - 1\right) \cdot \frac{F_{yh}}{\overline{F_h}} \le \overline{K_v}. \tag{37}$$

The additional contributions of the sufficient horizontal and vertical tie for the diagonal compressive strength are provided by the below simplified equations:

$$\overline{K_h} = \frac{1}{1 - 0.2\left(\gamma_h - \gamma_h^2\right)}, \tag{38}$$

$$\overline{K_v} = \frac{1}{1 - 0.2\left(\gamma_v - \gamma_v^2\right)}, \tag{39}$$

Consequently, the tie forces corresponding to the additional horizontal and vertical contributions are evaluated as follows:

$$\overline{F_h} = \gamma_h \cdot \left(\overline{K_h} \cdot \zeta \cdot f_c \cdot A_{str}\right) \cdot \cos\theta \tag{40}$$

$$\overline{F_v} = \gamma_v \cdot \left(\overline{K_v} \cdot \zeta \cdot f_c \cdot A_{str}\right) \cdot \sin\theta \tag{41}$$

The strain parameters γ_h and γ_v which control the relative importance of the two contributions defined in Eqs. (40) and (41) can be evaluated through the following simplified expressions:

$$\gamma_h = \frac{2\tan\theta - 1}{3} \qquad 0 \le \gamma_h \le 1, \tag{42}$$

$$\gamma_v = \frac{2\cot\theta - 1}{3} \qquad 0 \le \gamma_v \le 1. \tag{43}$$

Finally, the tensile forces of horizontal and vertical stirrups at yielding employed in Eqs. (36) and (37) are evaluated as follows:

$$F_{yh} = A_{sjh} \cdot f_{yj}, \tag{44}$$

$$F_{yv} = A_{sjv} \cdot f_{yj}. \tag{45}$$

Further details about this model are omitted herein for the sake of brevity and can be found in the original work of the authors (Hwang and Lee 2002).

3.7 The model by Bakir and Boduroglu (2002)

Bakir and Boduroglu (2002) proposed a new empirical design equation for exterior joints, based on a parametric study involving 58 tests conducted in Europe. According to experimental results, the shear strength of exterior beam-to-column joints depends by the concrete cylinder strength, the column and beam reinforcement ratios, the beam reinforcement detailing (U or L bars anchored into the connection), the joint aspect ratio and the amount of stirrups into the joint panel. The shear strength of RC joints can be derived as sum of the concrete strut and tie steel contributions by means of Eq. (3).

The contribution V_{sh} of joint stirrups is defined as follows:

$$V_{sh} = \alpha \cdot A_{sjh} \cdot f_{yj} \tag{46}$$

in which α depends on the amount of stirrups:

$$\rho_{jh} = \frac{A_{sjh}}{b_j \cdot h_c} \tag{47}$$

where b_j is taken as the minimum between the column width and the average of the column and the beam width. In particular, the parameter α is defined as follows:

$$\alpha = \begin{cases} 0.664 & \text{if} \quad \rho_{jh} < 0.0030 \\ 0.600 & \text{if} \quad 0.0030 \le \rho_{jh} \le 0.0055 \\ 0.370 & \text{if} \quad \rho_{jh} > 0.0055 \end{cases} \tag{48}$$

The contribution V_{ch} of concrete is evaluated through the following equation:

$$V_{ch} = \frac{0.71 \cdot \beta \cdot \gamma \cdot \left(100 \frac{A_{sb,sup}}{b_b \cdot d_b}\right)^{0.4289}}{\left(\frac{h_b}{h_c}\right)^{0.61}} b_j \cdot h_c \cdot \sqrt{f_c} \tag{49}$$

in which $\beta = 0.85$ for joints detailed with U bars and $\beta = 1.00$ for joints detailed with L bars, $\gamma = 1.37$ for inclined bars into the joint and $\gamma = 1.00$ for other cases.

3.8 The model by Parra-Montesinos and Wight (2002)

The semi-theoretical capacity model proposed by Parra-Montesinos and Wight (2002) is based on assuming plane strains in the joint panel region and defining a ratio between the principal axial strains which develop therein. This ratio has been calibrated on experimental results (Parra-Montesinos and Wight 2002). The shear strength of RC beam-to-column joints is controlled by two mechanisms:

- a strut mechanism activated by direct bearing on the concrete from the adjoining compression zone of beam and column;
- truss mechanism depending on the amount of force transferred to the joint by bond between the beam and column bars passing through the joint core.

As a result of a parametric study, the authors proposed the following semi-analytical expression for evaluating the shear strength of exterior and interior beam-to-column joints:

$$V_{jh} = \nu_{jh} \cdot b_j \cdot h_c \tag{50}$$

where υ_{jh} is the shear stress capacity estimated as follows:

$$\nu_{jh} = \alpha_1 \cdot \alpha_2 \cdot f_c. \tag{51}$$

The parameters α_1 and α_2 take into account the influence of a factor k_s and of the concrete compressive strength f_c and can be defined by the following equations:

$$\alpha_1 = 0.34 - 0,00018 \cdot k_s \tag{52}$$

$$\alpha_2 = 0.00018 \cdot f_c^2 - 0.03 \cdot f_c + 1.7 \tag{53}$$

in which f_c is expressed in MPa. The effective joint width b_j, needed in Eq. (50), is evaluated according to ACI 352 (2002) recommendations, by assuming that e_b is the eccentricity between the beam centreline and the column centroid:

$$b_j = \min\left(\frac{b_b + b_c}{2};\ b_b + \frac{h_c}{2}\right) \qquad if\ e_b = 0 \tag{54}$$

$$b_j \leq b_b + \frac{0.3 \cdot h_c}{2} \qquad if\ e_b \neq 0 \tag{55}$$

A single value of $k_s = 500$ was proposed for exterior joints (Parra-Montesinos and Wight 2002).

3.9 The model by Hegger et al. (2003)

Hegger et al. (2003) proposed an empirical model based on various experimental results. The model is described by a relationship for V_{jh} formally similar to the sum of the concrete and the steel contributions according to Eq. (3). The concrete contribution V_{ch} may be expressed in the following form:

$$V_{ch} = \alpha_1 \cdot A \cdot B \cdot C \cdot b_j \cdot h_c \tag{56}$$

where α_1 is an anchorage factor reflecting the efficiency of the anchorage of the beam reinforcement, A takes into account the joint slenderness, B accounts for the column reinforcement ratio ρ_c, C depends on the concrete compressive strength fc, and b_j is evaluated according to ACI 352 (2002) recommendations. The shear resistance V_{sh} due to stirrups in the joint can be expressed as:

$$V_{sh} = \alpha_2 \cdot A_{sjh} \cdot f_{yj} \tag{57}$$

in which α_2 is the efficiency factor for the shear reinforcement and A_{sjh} is the area of the shear reinforcement within the joint.

A regression analysis was performed by the authors (Hegger et al. 2003), for evaluating the factors controlling the two contributions to the shear strength V_{jh}. The effect of the beam reinforcement anchorage on the shear resistance of exterior beam-to-column connections is taken into account by assigning $\alpha_1 = 0.85$ or $\alpha_1 = 0.95$ for the case of 180 degree bend bars and 90-degree bend, respectively. Furthermore, from experimental tests was observed that the shear resistance decreases as the joint slenderness increases. So, through a regression analysis the following expression was provided for A:

$$A = 1.2 - 0.3\frac{h_b}{h_c} \tag{58}$$

It was observed that the anchorage efficiency inside the joint, the stiffness and the depth of the compression zone of the column increased by increasing the longitudinal column reinforcement ratio ρ_c; the following equation is suggested for the factor B:

$$B = 1.0 - \frac{\rho_c - 0.5}{7.5} \tag{59}$$

The concrete strength factor C is assumed to be provided by the following equation:

$$C = 2 \cdot (f_c)^{1/3} \tag{60}$$

with $20 \leq f_c \leq 100$ MPa.

The shear reinforcement efficiency factor α_2 was evaluated using the short beam analogy; the authors suggested different values for α_2 for different detailing as shown in Table 1.

An upper limit of the shear force $V_{\max}$ was provided according experimental results:

$$V_{\max} = \gamma_1 \cdot \gamma_2 \cdot \gamma_3 \cdot 0.25 \cdot f_c \cdot b_j \cdot h_c \leq 2 \cdot V_{ch} \tag{61}$$

Table 1 Shear reinforcement coefficient factor α_2

Anchorage type	Hairpins	Closed stirrups
90-degree bent or headed bars	0.7	0.6
180-degree bent bars	0.6	0.5

where γ_1, γ_2 and γ_3 are coefficients accounting for the anchorage efficiency of the beam reinforcement, the column normal force and the slenderness of the connection, respectively. Hegger et al. (2003) suggested a value of 1.0 for γ_1 for bend bars and 1.2 for headed ones, while the following two equations were proposed for the other two coefficients:

$$\gamma_2 = 1.5 - 1.2\frac{\sigma_c}{f_c} \leq 1.0 \tag{62}$$

$$\gamma_3 = 1.9 - 0.6\frac{h_b}{h_c} \leq 1.0 \tag{63}$$

in which σ_c is the column normal stress $\sigma_c = N_c/A_c$.

3.10 The model by Kim et al. (2009)

The empirical model by Kim et al. (2009) is based on the results of a wide experimental database only including reinforced joints. This model has been developed after having observed that a previous one proposed by the same authors (Kim et al. 2007) was not suitable for evaluating shear strength in unreinforced beam-to-column joints collected in a database wider than the one collected in 2007 (Kim et al. 2007). The empirical model developed in 2009 (Kim et al. 2009) and reported herein has been obtained as an evolution of the model by Kim et al. (2007) obtaining a model suitable for interior and exterior joints both reinforced and unreinforced.

As a result of the probabilistic method used by the authors (Kim et al. 2009), the shear strength is evaluated as the product of the specific shear strength v_{jh} and the dimensions of the joint panel:

$$V_{jh} = v_{jh} \cdot b_j \cdot h_c \tag{64}$$

where the effective joint width is provided by the minimum value between the column width and $(b_b + b_c)/2$. The specific shear strength v_{jh} can be evaluated as follows:

$$v_{jh} = \alpha_t \cdot \beta_t \cdot \eta_t \cdot \lambda_t \cdot (JI)^{0.15}\,(BI)^{0.30}\,(f_c)^{0.75} \qquad [\text{MPa}] \tag{65}$$

in which α_t is a parameter for describing the in-plane geometry assumed to be equal to 1.0 for interior joints, 0.7 for exterior connections and 0.4 for knee ones; β_t is a parameter describing the out-of-plane geometry assumed equal to 1.0 for joints without or with one transverse beam and 1.18 for connections with two opposite transverse beams; λ_t is a coefficient equal to 1.31 for setting the overall average of the ratios of Eq. 65 at 1.0 and η_t is a parameter that accounts for the beam eccentricity e_b and is defined as follows:

$$\eta_t = \left(1 - \frac{e_b}{b_c}\right)^{0.67}. \tag{66}$$

The parameter BI is the so-called beam reinforcement index and is provided by the following equation:

$$BI = \frac{\rho_b \cdot f_{yb}}{f_c}, \tag{67}$$

while JI is the joint transverse reinforcement index:

$$JI = \frac{\rho_j \cdot f_{yj}}{f_c} \geq 0.0139, \tag{68}$$

where the limitation above to 0.0139 was introduced later by the authors (Kim et al. 2009) as a difference with respect to a previous proposal (Kim et al. 2007) and with the aim of taking into account the shear strength of joints without transverse reinforcement.

In particular, the beam reinforcement ratio ρ_b and the volumetric joint transverse reinforcement ratio ρ_j needed for evaluating the beam and transverse reinforcement index, respectively, can be evaluated as follows:

$$\rho_b = \frac{A_{sb,sup} + A_{sb,\inf}}{b_b \cdot h_b} \tag{69}$$

$$\rho_j = \frac{A_{sjh} \cdot h_c}{h_c \cdot b_c \cdot \left(h_b - 2 \cdot d_b'\right)} \tag{70}$$

4 Preliminary comments about the capacity models considered in this study

The capacity models reported and briefly commented in Sect. 3 represent a wide and comprehensive selection of the even wider number of scientific contributions about the topic under consideration. They have been presented in chronological order for emphasising the progressive evolution of knowledge about shear strength of RC joints and the general growing complexity of the analytical expressions proposed by various authors in the last two decades. The predictive capacity of those models is thoroughly assessed in a companion paper (Lima et al. 2012) against a huge number of experimental results. However, this section proposes a synoptic comparison of the analytical expressions of the models under consideration with the aim of pointing out preliminary comments on them. Table 2 highlights the key parameters actually involved within the above mentioned analytical expressions. In particular, the various capacity models mentioned in Sect. 3 are listed in the first column of the table, while the key geometric and mechanical parameters are reported on its first row. A cross ("X") is drawn in the generic cell of Table 2 if the parameter reported in its generic column is relevant for the model mentioned in the current row.

Almost all considered models are influenced by the geometric properties of the joint panel, such as width and depth of the column cross section. In particular, the aspect ratio h_b/h_c is generally considered as a relevant parameter for determining the shear strength of RC joints (Fig. 3). Regarding the mechanical parameters, all models take into account the concrete strength f_c as the key property which controls shear capacity in exterior joints. As a matter of fact, all analytical expressions are based on a rising relationships between f_c and V_{jh} (Fig. 4). Moreover, the amount of horizontal reinforcement A_{sjh} in the panel zone is always obviously a key parameter for evaluating the (steel stirrups contribution to) shear strength. However, only the models by Scott et al. (1994) and by Parra-Montesinos and Wight (2002) neglect the latter parameter. Finally, only few models take into account the influence of the longitudinal rebars in beam and column denoted by A_{sb} and A_{sc}, respectively.

Table 2 Geometric and mechanical parameters taken into account by the capacity models

Models \ parameters	b_c	b_b	h_c	h_b	N_c	f_c	A_{sjh}	f_{yj}	A_{sb}	f_{yb}	A_{sc}
Sarsam and Phillips (1985)	X	–	X	X	X	X	X	X	–	–	X
Paulay and Priestley (1992)	X	–	X	–	X	X	X	X	X	X	–
Scott et al. (1994)	X	–	X	X	–	X	–	–	–	–	–
Parker and Bullman (1997)	X	–	X	X	X	X	X	X	–	–	X
Vollum and Newman (1999)	X	X	X	X	–	X	X	X	–	–	–
Hwang and Lee (2002)	X	X	X	X	X	X	X	X	–	–	X
Bakir and Boduroglu (2002)	X	X	X	X	–	X	X	X	X	–	–
Parra-Montesinos and Wight (2002)	X	X	X	–	–	X	–	–	–	–	–
Hegger et al. (2003)	X	X	X	X	X	X	X	X	–	–	X
Kim et al. (2009)	X	X	X	X	–	X	X	X	X	X	–

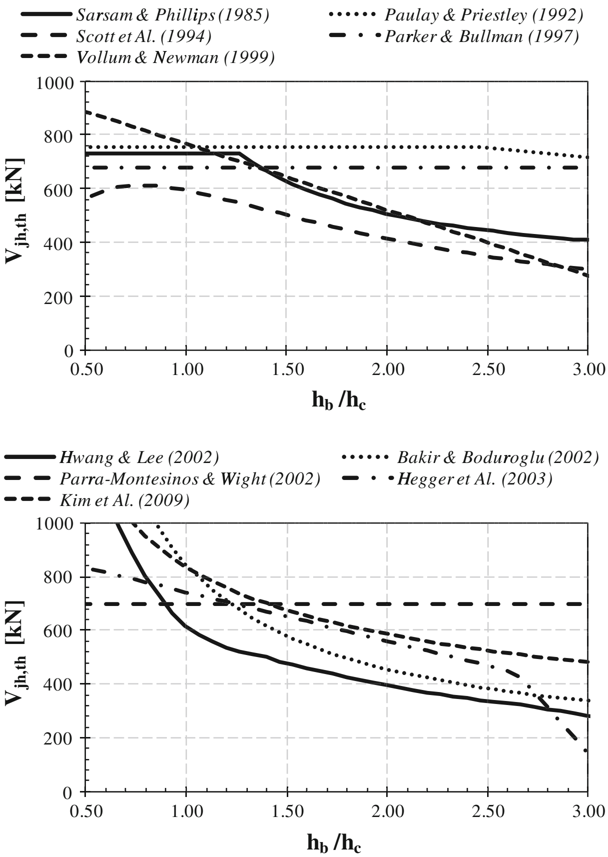

Fig. 3 Relationship between aspect ratio h_b/h_c and V_{jh} according to the considered capacity models

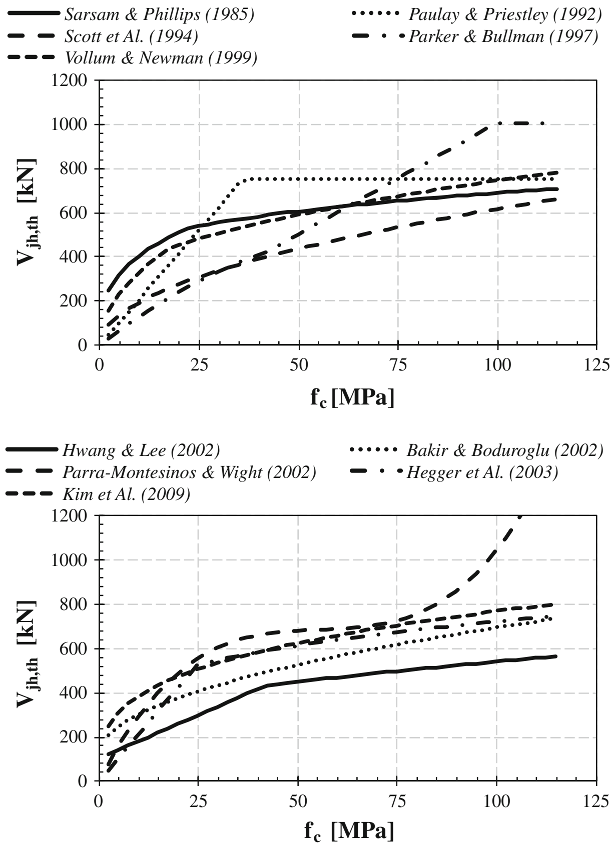

Fig. 4 Relationship between concrete strength f_c and V_{jh} according to the considered capacity models

5 Concluding remarks

This paper presents an overview of the capacity models currently available in the scientific literature for evaluating the shear strength of exterior beam-to-column joints in RC frames. A synoptic comparison among the analytical expressions of those models point out the key geometric and mechanical parameters which are actually supposed to control shear strength in exterior beam-to-column joints. Table 2 summarises the key aspects of this comparison.

The comments reported herein represent a preliminary exam of the models considered in this study whose predictive capacity is thoroughly assessed in a companion paper (Lima et al. 2012) aimed at measuring their accuracy and reliability on the basis of a wide number of experimental results collected from the scientific literature.

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