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ORIGINAL RESEARCH PAPER

Understanding the trends in torsional effects in asymmetric-plan buildings

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Abstract This paper studied the reasons behind the four trends in torsional effects in asymmetric-plan buildings observed in the current literature. It was found that the modal eccentricities and the non-proportionality between the modal translations and the modal rotation are key to understanding these trends in torsional effects in asymmetric-plan buildings. These key points were obtained from the three-degree-of-freedom modal systems, which represent the single vibration mode of a two-way asymmetric-plan building. This paper showed that the modal eccentricities, rather than the overall structural eccentricities, are the critical parameters for deciding the trend of the unequal displacement demand on the floor diaphragm. In addition, the non-proportionality between the modal translations and the modal rotation leads to the trend that the torsional effects generally decrease when plastic deformations increase.

Keywords Asymmetric-plan buildings · Torsional effects · Modal eccentricity · Inelastic deformation

1 Introduction

Due to architectural and functional requirements, most practical buildings are asymmetrical. Such type of buildings is one of the most frequently damaged structures under the exertion of earthquake loads (Stefano and Pintucchi 2008). The rotational response of asymmetric-plan buildings leads to unequal displacement demands on the floor diaphragm.

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The torsional effects, generally represented as the ratio between the displacements of the floor edges to the displacement of the center of mass (CM), are unique to asymmetric-plan buildings compared with symmetrical buildings. Therefore, identifying torsional effects in a building is a crucial step in several nonlinear static (pushover) analysis procedures for asymmetric-plan buildings (Tso and Moghadam 1997; Moghadam and Tso 2000; Fajfar 2000; Fajfar et al. 2002, 2005; Chopra and Goel 2004). Erduran (2008) evaluated the capabilities of the extension of the N2 method (Fajfar et al. 2002, 2005) and the modal pushover analysis (MPA) procedure (Chopra and Goel 2004) on assessing the torsional effects of low-rise asymmetric-plan buildings. He concluded that the MPA procedure provides overestimates for the torsional effects which, on the flexible side (*FS*), are beneficial because the displacement demands are conservatively estimated. However, on the stiff side (*SS*), an overestimation of torsional effect results in un-conservative estimates of the displacement demands. The extension of the N2 method, discounting the reduction of displacement demands due to torsional effects, remains conservative for the estimation of the displacement demands on the *SS* as well as the *FS* (Erduran 2008). Recently, Stefano and Pintucchi (2010) pointed out that the extension of the N2 method is generally conservative for frame structures. Nevertheless, the estimation of the torsional effects calculated from this method becomes unsuitable for asymmetric buildings with few lateral resisting elements primarily located at the outer portions of the floor diaphragm. They explained the phenomenon to be due to the concentration of stiffness and strength at the floor perimeter and that the yielding of only one resisting element resulted in a significant shift in the instantaneous center of stiffness accompanied by a contemporaneous sudden loss of torsional stiffness (Stefano and Pintucchi 2010).

Fajfar et al. (2005) investigated the general trends in the torsional effects in two-way asymmetric-plan buildings under bi-directional ground excitations. Some trends found in the torsional effects reported in the original paper (Fajfar et al. 2005) are re-stated here: (1) torsional effects generally decrease when plastic deformations increase; (2) between the two horizontal directions, the torsional effect on displacements in the more flexible direction is smaller than that in the stiffer direction; (3) for the *SS*, it is difficult to make general conclusions. The response of the *SS* generally has a strong dependence on the effects of several modes of vibration, and on the influence of the ground motion in the transverse direction. The structural and ground motion characteristics in both directions are influential; (4) transitions from de-amplification to amplification of the displacement demand may occur on the *SS* in some cases.

The aforementioned four trends in torsional effects in asymmetric-plan buildings seem difficult to be completely understood by using the general intuition. For example, the more plastic deformations occur, which unevenly concentrate on one of the two sides of the floor diaphragm, the more extent of the structural eccentricity becomes. Therefore, the torsional effects should increase when plastic deformations increase, which is against the aforementioned first trend in torsional effects. In addition, the displacement demand induced from the rotational response should be more substantial in the flexible direction than that in the stiff direction. Therefore, the torsional effects on displacements in the more flexible direction should be larger than that in the stiffer direction, which is against the aforementioned second trend in torsional effects. In a word, the aforementioned four trends in torsional effects (Fajfar et al. 2005) can not be intuitively explained. The overall structural parameters, not in the modal level, that are related to the rotational responses, include the radius of gyration for the mass moment of inertia, the normalized eccentricity and the frequency ratio, etc. These parameters have been intensively studied (Paulay 1998; De-La-Colina 1999). Nevertheless, it also appears that there exists no satisfactory explanation for the aforementioned trends in the torsional effects solely using these overall structural parameters. There is no doubt that the rationale of the torsional effects is an issue deserving of more emphasis. Investigating the reasons behind these trends in torsional effects is helpful not only to understand the seismic response of asymmetric-plan buildings but also for the development of suitable seismic assessment procedures.

Based on the ways of constructing the single-degree-of-freedom (SDOF) “modal” systems for inelastic buildings proposed by Chopra and Goel (2004), it has been found that the modal translation and the modal rotation are very likely to be non-proportional for inelastic multi-story asymmetric-plan buildings (Lin and Tsai 2007, 2008). The authors Lin and Tsai (2007, 2008) therefore developed the two-degree-of-freedom (2DOF) and the three-degree-of-freedom (3DOF) modal systems to represent the “vibration modes” of inelastic multi-story one-way and two-way asymmetric-plan buildings, respectively. The force-deformation relationships of these modal systems represent the roof translations against the base shears and the roof rotation against the base torque relationships of the original asymmetric-plan multi-story buildings pushed by using the corresponding modal inertia force vector. It was found that only one of the vibration modes in the elastic 3DOF modal system itself is active, and the other two vibration modes are spurious, i.e. do not contribute to the elastic vibration of the modal system itself. Thus, each 3DOF modal system exactly represents only one vibration mode of the original two-way asymmetric-plan multi-story building. In addition, the 3DOF modal system is capable of capturing the non-proportionality between the modal translation and the modal rotation in an inelastic asymmetrical building. The modal parameters related to the rotational responses are also explicitly available in the 3DOF modal system. Thus, the 3DOF modal system provides an extremely convenient vehicle to study the trends in torsional effects in terms of the parameters in the modal level rather than those in the overall structural level. The present study first used the elastic and inelastic properties of the 3DOF modal systems to explain the trends in the torsional effects in asymmetric-plan buildings, which were found in the literature (Fajfar et al. 2005; Erduran 2008). It is expected to improve the understanding of the characteristics of the seismic behavior of asymmetric-plan buildings.

2 An explanation of the trends in torsional effects

The X- and the Z-axes are the two horizontal axes in the coordinate system used in this study. The direction of the Y-axis is opposite to the direction of gravity. The subscripts x, z and θ used in this study represent the corresponding quantities associated with the X- and Z-translational and Y-rotational components, respectively. For the purpose of discussions, the 3DOF modal parameters are briefly presented herein. More details can be found in the reference (Lin and Tsai 2008).

2.1 The 3DOF modal parameters

Keeping in mind that the modal vibrations of two-way asymmetric-plan multi-story buildings are translation-rotation coupled, there are three force-deformation relationships that can be simultaneously obtained from the original building pushed with its modal inertia force vector (Lin and Tsai 2008). The three force-deformation relationships, representing two roof translation-base shear relationships and one roof rotation-base torque relationship can be transformed into the acceleration-displacement-response-spectra (ADRS) format. The results are schematically shown in Fig. 1a. Figure 1 shows that these three pushover curves overlap during the initial elastic state, but bifurcate after yielding. These three pushover curves can be idealized as three bilinear curves (Fig. 1b), and the corresponding 3DOF modal system

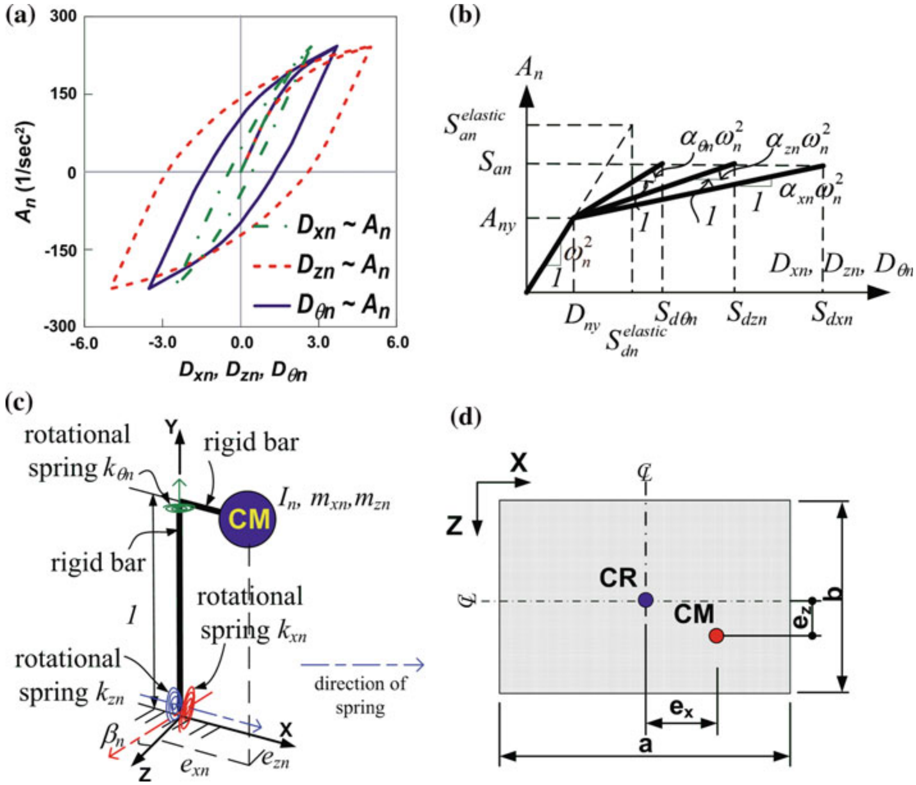


Fig. 1 **a** The typical one-cycle push-pull curves representing the roof translations versus the base shears and the roof rotation versus the base torque relationships in the ADRS format; **b** the typical three bilinear pushover curves for a two-way asymmetric-plan building; **c** the 3DOF modal system; **d** the floor diaphragm of a two-way asymmetric-plan building

is constructed accordingly (Fig. 1c) (Lin and Tsai 2008). The displacement vector, **u,** the nth mode shape, $\boldsymbol{\varphi}_n$, mass matrix, **M**, stiffness matrix, **K**, of the original N-story two-way asymmetric-plan building are expressed as:

$$\mathbf{u}=\begin{bmatrix}\mathbf{u}_x\\ \mathbf{u}_z\\ \mathbf{u}_\theta\end{bmatrix}_{3N\times 1},\ \boldsymbol{\varphi}_n=\begin{bmatrix}\boldsymbol{\varphi}_{xn}\\ \boldsymbol{\varphi}_{zn}\\ \boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3N\times 1},\ \mathbf{M}=\begin{bmatrix}\mathbf{m}_x & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{m}_z & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}_0\end{bmatrix}_{3N\times 3N},\ \mathbf{K}=\begin{bmatrix}\mathbf{k}_{xx} & \mathbf{k}_{xz} & \mathbf{k}_{x\theta}\\ \mathbf{k}_{zx} & \mathbf{k}_{zz} & \mathbf{k}_{z\theta}\\ \mathbf{k}_{\theta x} & \mathbf{k}_{\theta z} & \mathbf{k}_{\theta\theta}\end{bmatrix}_{3N\times 3N} \tag{1}$$

The displacement vector defined at the lumped mass, the mass matrix and the stiffness matrix of the nth 3DOF modal system, representing the nth vibration mode of the original building, are then expressed as:

$$\mathbf{D}_n=\begin{bmatrix}D_{xn}\\ D_{zn}\\ D_{\theta n}\end{bmatrix}_{3\times 1},\quad \mathbf{M}_n=\begin{bmatrix}\boldsymbol{\varphi}_{xn}^T\mathbf{m}_x\boldsymbol{\varphi}_{xn} & 0 & 0\\ 0 & \boldsymbol{\varphi}_{zn}^T\mathbf{m}_z\boldsymbol{\varphi}_{zn} & 0\\ 0 & 0 & \boldsymbol{\varphi}_{\theta n}^T\mathbf{I}_0\boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3\times 3}$$

$$\mathbf{K}_n=\begin{bmatrix}\boldsymbol{\varphi}_{xn}^T\mathbf{k}_{xx}\boldsymbol{\varphi}_{xn} & \boldsymbol{\varphi}_{xn}^T\mathbf{k}_{xz}\boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{xn}^T\mathbf{k}_{x\theta}\boldsymbol{\varphi}_{\theta n}\\ \boldsymbol{\varphi}_{zn}^T\mathbf{k}_{zx}\boldsymbol{\varphi}_{xn} & \boldsymbol{\varphi}_{zn}^T\mathbf{k}_{zz}\boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{zn}^T\mathbf{k}_{z\theta}\boldsymbol{\varphi}_{\theta n}\\ \boldsymbol{\varphi}_{\theta n}^T\mathbf{k}_{\theta x}\boldsymbol{\varphi}_{xn} & \boldsymbol{\varphi}_{\theta n}^T\mathbf{k}_{\theta z}\boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{\theta n}^T\mathbf{k}_{\theta\theta}\boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3\times 3} \tag{2}$$

In an elastic state, $D_{xn} = D_{zn} = D_{\theta n}$; in an inelastic state, D_{xn}, D_{zn} and $D_{\theta n}$ are very likely to be not equal to each other (Fig. 1a, b). The modal eccentricities of the nth vibration mode are as follows:

$$e_{xn} = \frac{\boldsymbol{\varphi}_{zn}^T \mathbf{k}_{z\theta} \boldsymbol{\varphi}_{\theta n}}{\boldsymbol{\varphi}_{zn}^T \mathbf{k}_{zz} \boldsymbol{\varphi}_{zn}}, \quad e_{zn} = \frac{-\boldsymbol{\varphi}_{xn}^T \mathbf{k}_{x\theta} \boldsymbol{\varphi}_{\theta n}}{\boldsymbol{\varphi}_{xn}^T \mathbf{k}_{xx} \boldsymbol{\varphi}_{xn}} \tag{3}$$

The α_{xn}, α_{zn} and $\alpha_{\theta n}$ shown in Fig. 1b represent the post-yielding stiffness ratios of the nth vibration mode in the three directions. When the modal translations and the modal rotation are non-proportional, α_{xn}, α_{zn} and $\alpha_{\theta n}$ would not be equal. Furthermore, from previous studies (Lin and Tsai 2008, 2009, 2011), it has been found from the typical building structures that the smallest value among α_{xn}, α_{zn} and $\alpha_{\theta n}$ was usually either α_{xn} or α_{zn}; and that $\alpha_{\theta n}$ may be in the middle or be the largest. Thus, for the purposes of discussion in this study, α_{xn} is assumed to be the smallest among this set (Fig. 1b).

2.2 The first trend of torsional effects

The first trend of torsional effects is that torsional effects generally decrease when plastic deformations increase. Taking the contribution of the nth vibration mode of an elastic two-way asymmetric-plan building subjected to a X-directional ground motion into consideration, the equations for the torsional effect, defined by the displacements on the FS and SS of the jth floor divided by the corresponding displacement at the CM (Fig. 1d), are:

$$\frac{u_{FS,nj}^{elastic}}{u_{CM,nj}^{elastic}} = \frac{u_{xn,j} + \left(\frac{b}{2} - e_z\right) u_{\theta n,j}}{u_{xn,j}} = \frac{\phi_{xn,j} D_{xn} + \left(\frac{b}{2} - e_z\right) \phi_{\theta n,j} D_{\theta n}}{\phi_{xn,j} D_{xn}}$$
$$= 1 + \left(\frac{b}{2} - e_z\right) \frac{\phi_{\theta n,j}}{\phi_{xn,j}} = 1 + \eta_{FS,nj}^{elastic} \tag{4a}$$

$$\frac{u_{SS,nj}^{elastic}}{u_{CM,nj}^{elastic}} = \frac{u_{xn,j} - \left(\frac{b}{2} + e_z\right) u_{\theta n,j}}{u_{xn,j}} = \frac{\phi_{xn,j} D_{xn} - \left(\frac{b}{2} + e_z\right) \phi_{\theta n,j} D_{\theta n}}{\phi_{xn,j} D_{xn}}$$
$$= 1 - \left(\frac{b}{2} + e_z\right) \frac{\phi_{\theta n,j}}{\phi_{xn,j}} = 1 + \eta_{SS,nj}^{elastic} \tag{4b}$$

where b and e_z are the floor diaphragm dimension and the overall structural eccentricity shown in Fig. 1d respectively; $\phi_{xn,j}$ and $\phi_{\theta n,j}$ are the jth-floor components of the nth mode shape in the X-translational and Y-rotational directions respectively. The $\eta_{FS,nj}^{elastic}$ and $\eta_{SS,nj}^{elastic}$ are referred to as the elastic torsional indices for the nth vibration mode on the FS and SS of the jth floor, respectively. The larger the absolute value of the torsional index, the more substantial the torsional effect. In an inelastic state, using the concept of "weak coupled vibration modes" (Chopra and Goel 2004), the torsional effects on the FS and SS of the jth floor resulting from the nth "vibration mode" are approximated as:

$$\frac{u_{FS,nj}^{inelastic}}{u_{CM,nj}^{inelastic}} \approx \frac{u_{xn,j}+\left(\frac{b}{2}-e_z\right)u_{\theta n,j}}{u_{xn,j}} = \frac{\phi_{xn,j}\left(D_{ny}+\Delta D_{xn}\right)+\left(\frac{b}{2}-e_z\right)\phi_{\theta n,j}\left(D_{ny}+\Delta D_{\theta n}\right)}{\phi_{xn,j}\left(D_{ny}+\Delta D_{xn}\right)}$$
$$= \frac{\phi_{xn,j}+\left(\frac{b}{2}-e_z\right)\phi_{\theta n,j}\frac{D_{ny}+\Delta D_{\theta n}}{D_{ny}+\Delta D_{xn}}}{\phi_{xn,j}} = 1 + \left(\frac{b}{2} - e_z\right) \frac{\phi_{\theta n,j}}{\phi_{xn,j}} \frac{\mu_{\theta n}}{\mu_{xn}} = 1 + \eta_{FS,nj}^{inelastic} \tag{5a}$$

$$\frac{u_{SS,j}^{inelastic}}{u_{CM,j}^{inelastic}} \approx \frac{u_{xn,j}-\left(\frac{b}{2}+e_z\right)u_{\theta n,j}}{u_{xn,j}} = \frac{\phi_{xn,j}\left(D_{ny}+\Delta D_{xn}\right)-\left(\frac{b}{2}+e_z\right)\phi_{\theta n,j}\left(D_{ny}+\Delta D_{\theta n}\right)}{\phi_{xn,j}\left(D_{ny}+\Delta D_{xn}\right)}$$
$$= \frac{\phi_{xn,j}-\left(\frac{b}{2}+e_z\right)\phi_{\theta n,j}\frac{D_{ny}+\Delta D_{\theta n}}{D_{ny}+\Delta D_{xn}}}{\phi_{xn,j}} = 1 - \left(\frac{b}{2} + e_z\right) \frac{\phi_{\theta n,j}}{\phi_{xn,j}} \frac{\mu_{\theta n}}{\mu_{xn}} = 1 + \eta_{SS,nj}^{inelastic} \tag{5b}$$

where D_{ny} is the yielding displacement (Fig. 1b); ΔD_{xn} and $\Delta D_{\theta n}$ represent the inelastic parts of the X-translation and Y-rotation, respectively. The $\eta_{FS,nj}^{inelastic}$ and $\eta_{SS,nj}^{inelastic}$ are referred to as the inelastic torsional indices for the nth vibration mode on the *FS* and *SS* of the jth floor. Comparing the elastic and inelastic torsional indices shows that the inelastic torsional indices are equal to the counterpart elastic torsional indices multiplied by $\mu_{\theta n}/\mu_{xn}$. As a reminder, the modal acceleration A_n and the modal displacements D_{xn}, D_{zn} and $D_{\theta n}$ shown in Fig. 1b are computed as (Lin and Tsai 2008):

$$A_n = \frac{V_{bxn}}{\Gamma_{xn} M_n} = \frac{V_{bzn}}{\Gamma_{zn} M_n} = \frac{T_{bn}}{\Gamma_{\theta n} M_n} \tag{6a}$$

$$D_{xn} = \frac{u_{xn,r}}{\phi_{xn,r}},\ D_{zn} = \frac{u_{zn,r}}{\phi_{zn,r}},\ D_{\theta n} = \frac{u_{\theta n,r}}{\phi_{\theta n,r}} \tag{6b}$$

where $\phi_{xn,r}$, $\phi_{zn,r}$ and $\phi_{\theta n,r}$ are the roof components of the nth mode shape in the three directions; $u_{xn,r}$, $u_{zn,r}$ and $u_{\theta n,r}$ are the roof displacements in the three directions; V_{bxn}, V_{bzn} and T_{bn} are the base shears and base torque of the original multi-story building pushed by using the nth modal inertia force vector; M_n is the modal mass; Γ_{xn}, Γ_{zn} and $\Gamma_{\theta n}$ are the modal participation factors in the three directions. Because the pushover force vector keeps proportionally increasing in the three directions, the modal acceleration A_n also proportionally increases in the three directions (Eq. 6a), but the modal displacements D_{xn}, D_{zn} and $D_{\theta n}$ are non-proportionally increased (Eq. 6b). Since α_{xn} is less than $\alpha_{\theta n}$ (Fig. 1b), the value of $\mu_{\theta n}/\mu_{xn}$ is less than one and decreases as A_n increases. For example, when $\alpha_{xn} = 5\%$, $\alpha_{\theta n} = 25\%$ and $\mu_{xn} = 2$, $\mu_{\theta n}$ is then equal to 1.2 and $\mu_{\theta n}/\mu_{xn} = 0.6$. When A_n increases and μ_{xn} is up to 4, $\mu_{\theta n}$ is then equal to 1.6 and $\mu_{\theta n}/\mu_{xn} = 0.4$. Thus, the absolute values of the inelastic torsional indices, $\eta_{FS,nj}^{inelastic}$ and $\eta_{SS,nj}^{inelastic}$, decrease as A_n increases. That is to say, the torsional effect decreases when plastic deformations increase, which explains the first trend in torsional effects (Fajfar et al. 2005). Because the conventional SDOF modal system is not able to take the non-proportionality between the modal translations and the modal rotation into accounts, $\mu_{\theta n}/\mu_{xn}$ equals to one and leads to an overestimate of the torsional effects in an inelastic state. Consequently, it also explains Erduran's conclusion (2008) that the MPA procedure (Chopra and Goel 2004), adopting the conventional SDOF modal system to compute the inelastic "modal" responses in all three directions, overestimates the torsional effects.

2.3 The second trend of torsional effects

The second trend in torsional effects is that the torsional effect on displacements in the more flexible direction, i.e. the weaker direction, is smaller than that in the stiffer direction (Fajfar et al. 2005). Fajfar et al. (2005) defined the weaker direction to be the direction in which a building experiences larger plastic deformation than in the other direction. Using Fig. 1b as an example, the X-direction is the "weaker direction" because α_{zn} is larger then α_{xn}. From Eq. 5, the inelastic torsional effects in both the stiff and the flexible directions are equal to the counterpart elastic torsional effects multiplied by $\mu_{\theta n}/\mu_{zn}$ and $\mu_{\theta n}/\mu_{xn}$, respectively. Because α_{zn} is larger than α_{xn}, $\mu_{\theta n}/\mu_{zn}$ is larger than $\mu_{\theta n}/\mu_{xn}$. It consequently explains the second trend of torsional effects.

2.4 The third trend of torsional effects

The third trend of torsional effect is that the seismic responses on the *SS* generally depend on the influences of several modes of vibration, and the ground motion in the transverse

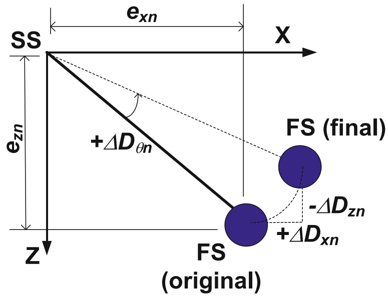


Fig. 2 The bird's-eye view of the displacement increments of the 3DOF modal system with positive eccentricities

direction. Thus, it is difficult to make general conclusions about the torsional effects on the *SS*. In order to explain the third trend of torsional effects, the relationships between the directions of the modal eccentricities and the trends in unequal modal displacement demands are first discussed as follows:

It is clear that the rotational response resulting from the structural eccentricity leads to unequal displacement demands on the *FS* and the *SS* of the floor diaphragm in asymmetrical buildings. This suggests that the directions of modal eccentricities, i.e. e_{xn} and e_{zn} (Eq. 3) shown in Fig. 1c, are influential on the trend in the unequal modal displacement demands on floor diaphragm edges. The beam end with the lumped mass in the 3DOF modal system (Fig. 1c) is regarded as the *FS* and the other beam end is regarded as the *SS*. When a 3DOF modal system with positive e_{zn} and e_{xn} has a positive Y-rotational increment, the displacements on the *FS* in the X- and the Z-directions are increased and decreased, respectively (Fig. 2). That is to say, when the modal eccentricities e_{zn} and e_{xn} are positive, the *FS* in the X-direction and the *SS* in the Z-direction face a larger displacement demand than the other side of the floor diaphragm in the same direction. Conversely, when the modal eccentricities e_{zn} and e_{xn} are negative, the *SS* in the X-direction and the *FS* in the Z-direction face a larger displacement demand than the other side of the floor diaphragm in the same direction.

The aforementioned relationships between the directions of the modal eccentricities and the trends in unequal modal displacement demands were validated by investigating a 20-story two-way asymmetric-plan building shown in Fig. 3. The 20-story example building was a variation of the symmetrical 20-story SAC building located in Los Angeles (Krawinkler 2000). The variation was in the position of the CM of the 20-story symmetrical building where it was moved away from the CR with eccentricity ratios equal to 20% in both the X- and Z-directions (Fig. 3). Table 1a shows the vibration periods and the modal eccentricities e_{xn} and e_{zn} for the first three vibration modes of the 20-story example building. According to the previously mentioned relationship between the directions of the modal eccentricities and the trend in unequal modal displacement demands, Table 1b shows the predicted floor-diaphragm side as having a larger displacement demand than that in the other side. Figure 4 shows the bird's-eye view and the elevations of the first three mode shapes obtained from the eigenvalue analysis for the 20-story example building. The tools for the eigenvalue analysis and the visualization of the analytical results are the PISA3D and GISA3D computer programs (Lin et al. 2009), respectively. Note that the rotational component $\phi_{\theta n}$ shown in the elevation of the mode shapes (Fig. 4) is multiplied by a or $0.1a$, where a is the X-directional floor-diaphragm dimension equal to 30,480 mm (Fig. 3). The bird's-eye view of the first three mode shapes (Fig. 4) shows that in each direction the floor-diaphragm side having a larger displacement demand than the other side is the same as those shown in Table 1b. For example, Table 1b indicates that *SS* has a larger displacement between the two floor-diaphragm sides

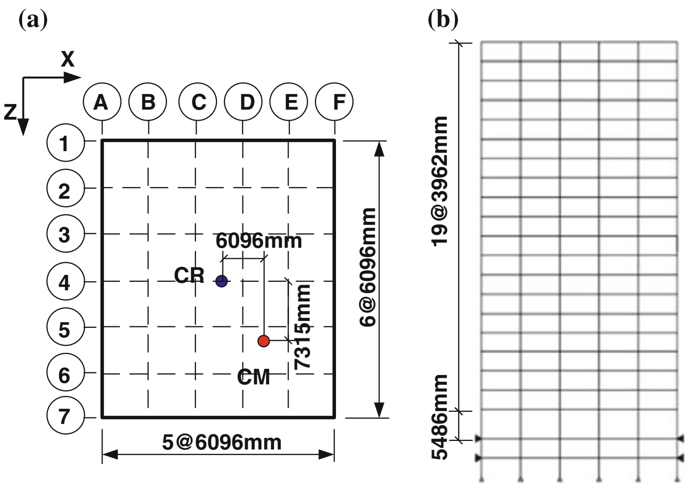


Fig. 3 **a** The typical floor diaphragm and **b** the elevation of the 20-story example building

Table 1 (a) The vibration periods and modal eccentricities of the 20-story example building (b) the floor-diaphragm side having a larger displacement demand between the two sides in each direction for the first three vibration modes of the example building

Mode	1	2	3			
(a)						
Period (sec.)	4.11	3.57	1.87			
Dominant motion	X	Z	R			
e_{xn}	−0.283	−0.052	2.154			
e_{zn}	0.122	−0.161	−2.220			
Mode	1		2		3	
(b)						
Direction	X	Z	X	Z	X	Z
Floor-diaphragm side	*FS*	*FS*	*SS*	*FS*	*SS*	*SS*

in each of the X- and Z-directions in the 3rd mode. Using the corresponding bird's-eye view and the mode shape directions (Fig. 4c), it can be observed that the *SS* of the floor-diaphragm in both the X- and Z-direction indeed has a larger displacement than that in the corresponding *FS*. This confirms the previously mentioned relationship between the directions of the modal eccentricities and the trends in unequal modal displacement demands.

Alternatively when the bird's-eye view of the modal deformation is not available, the following procedure can be used to verify the relationships between the directions of the modal eccentricities and the trends in unequal modal displacement demands. This procedure assumes Table 1b is accurate for the 20-story example building, then the direction of the floor-diaphragm rotation of the first mode can be obtained from using Table 1b and the directions of the translational components of the mode shape (Fig. 4a). If the resulting direction of floor-diaphragm rotation is indeed correct, then it can be confirmed that Table 1b must be accurate. The following two steps are schematically shown in the two left drawings in Fig. 5a. First, as the direction of the X-translational component of the first

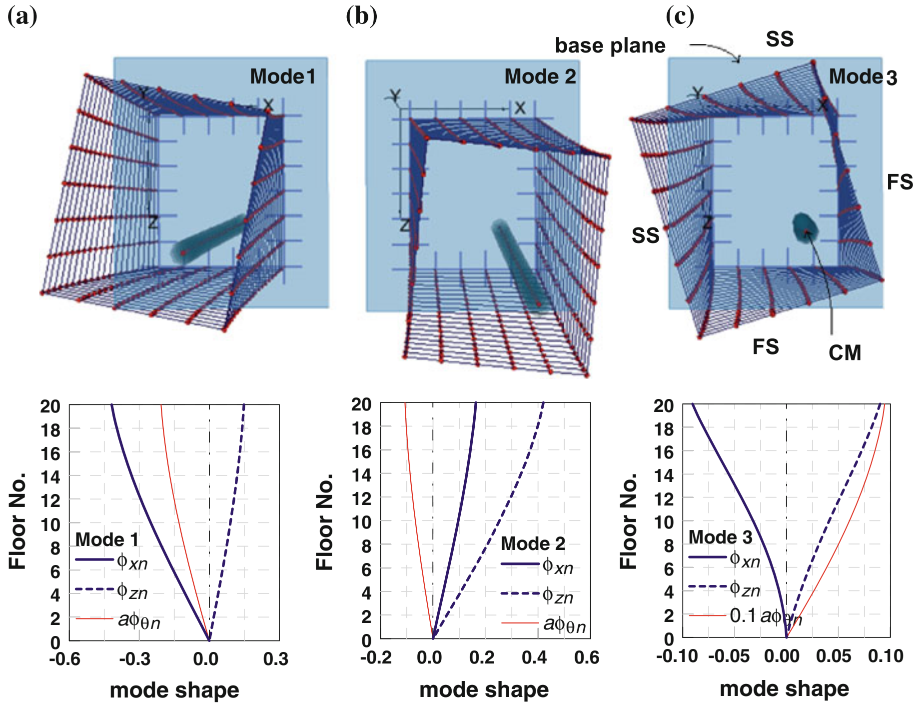
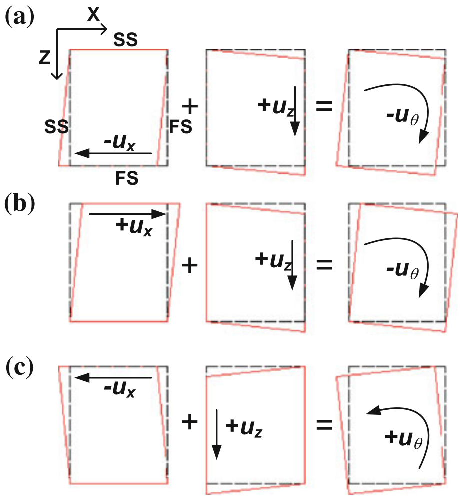


Fig. 4 The bird's-eye view and elevation of the mode shapes of the 20-story example building: **a** the first mode **b** the second mode **c** the third mode

Fig. 5 Synthesis of the rotational deformation of the 20-story example building: **a** the first mode **b** the second mode **c** the third mode



mode shape is negative (Fig. 4a) and the X-directional *FS* has a larger displacement demand than that on the *SS* (Table 1b), the X-directional *FS* was moved in the negative X-direction. Second, as the direction of the Z-translational component of the first mode shape is positive (Fig. 4a) and the Z-directional *FS* has a larger displacement demand than that on the *SS* (Table 1b), the Z-directional *FS* was moved in the positive Z-direction. Third, adding the X and Z displacement components of the four corners in the two parallelograms on the left of Fig. 5a results in a rectangle shown on the right of Fig. 5a. Figure 5a shows a clockwise (negative) rotation of the floor-diaphragm and is consistent with the direction (negative) of the Y-rotational component of the first mode shape obtained from the modal analysis (Fig. 4a). This approach is also validated for the second and third vibration modes shown as Fig. 5b, c, respectively. Thus, in other words, the proposed procedure (Fig. 5) confirmed the initial assumption (Table 1b), which describes the relationship between the direction of modal eccentricities and the trends in unequal modal displacement demands on the floor-diaphragm sides. In addition, the proposed procedure shows that the direction of the mode shape in each of the three directions is related to the directions of modal eccentricities.

It is clear that all structural vibration modes contribute to the structural seismic response of the building. The extent of each mode's contribution depends on the characteristics of the vibration mode and the seismic ground motions, e.g. the frequency content of the ground motions. Table 1a shows that the first and second vibration modes are the fundamental modes of the 20-story example building in the X- and Z-directions, respectively. Considering solely the fundamental modes in these two horizontal directions, Table 1b indicates that both these two modes have the *FS* in the Z-direction facing larger displacement demands than the *SS* in the same direction. This shows that, in the Z-direction, it is certain that the total displacement on the *FS* is larger than that in the *SS* when only the contributions of the two fundamental modes are considered. However, these two modes respectively have the *FS* and the *SS* in the X-direction facing larger displacement demands than on the other side in the same direction. Therefore, in the X-direction, it is uncertain whether the *FS* or the *SS* has larger displacement demand than the other side when only the contributions of the two fundamental modes are considered. For example, when the fundamental mode in the Z-direction is considerably excited by the Z-directional component of a bi-directional ground motion, the *SS* in the X-direction of the 20-story example building may have a larger displacement demand than its counterpart *FS*. It shows that the modal eccentricities, rather than the overall structural eccentricities, should be used to judge the unequal displacement demands on the floor-diaphragm sides of asymmetric-plan buildings. Consequently, it was difficult to estimate the torsional effects for the X-directional *SS* of the 20-story example building which depend upon the characteristics of the first two modes and the contents of the Z-directional component of bi-directional ground motions. This brings to light an explanation for the third trend in torsional effects stating that it is difficult to reach general conclusions about the torsional effects on the *SS*, which strongly depend on the effects of several vibration modes, and on the influence of the ground motion in the transversal direction (Fajfar et al. 2005).

2.5 The fourth trend of torsional effects

The fourth trend found in torsional effects is that, in some cases, transitions from de-amplification to amplification of the displacement demand may occur on the *SS* (Fajfar et al. 2005). From the discussion for the third trend of torsional effects, it was clear that the displacement demand on the *SS* is possibly larger than the displacement demand on the *FS* in the same direction. In addition, the torsional effects on the *SS* are on the influence of the ground motion in the transversal direction. Due to the time-to-time varied characteristics of the seismic ground motions, the torsional effect on the *SS* may be changed from de-amplification to amplification of the displacement demand in some cases. Consequently, it explains the fourth trend of torsional effects.

3 Summary and conclusions

The trends in torsional effects in two-way asymmetric-plan buildings excited by bi-directional ground motions are complex and practical. These trends in torsional effects seem difficult to be completely explained by using the overall structural parameters, e.g. the overall structural eccentricities, or only examining the SDOF modal systems for inelastic asymmetrical buildings. This study provided an alternative of effectively explaining these trends in torsional effects by using the characteristics of the 3DOF modal systems. Among the characteristics of the 3DOF modal systems, the modal eccentricities and the non-proportionality between the modal translations and the modal rotation are the keys to reaching the explanations behind the trends in torsional effects.

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