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Fundamental periods of vibration of RC buildings in Portugal from in-situ experimental and numerical techniques

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Abstract Since the early nineteen seventies we have been measuring the in-situ dynamic characteristics of the different structures built in Portugal, essentially based on ambient vibration and using expedite techniques. A *data-base* containing not only the fundamental dynamic characteristics of those structures but also their most important geometric and constructive properties has been created with the aim of setting correlations between construction typologies and fundamental periods or frequencies, and damping characteristics, and calibrate numerical modelling of those structures. This paper presents the main results for circa 197 reinforced concrete (RC) buildings, obtaining the fundamental period as a linear function of height or number of storeys for different typologies and situations, and showing that numerical models, made for a number of illustrative cases, can reproduce with great accuracy the in-situ measurements. The main parameters having remarkable influence on the overall correlation laws are identified and a measure of uncertainty deduced. Comparisons with published formulae for other regions of the world show that we can group these laws by regions with similar expression within each group but with large variations from group to group. Discussion on how to deal with the elongation of the periods of vibration due to moderate and large amplitude motion, causing changes in the seismic behaviour and on appearance of damage, will also be briefly introduced, keeping in mind current code practices.

Keywords Ambient vibration · Fundamental period · RC buildings · Infill walls · Non-linear behaviour

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1 Introduction

It is well known that periods of vibration and damping ratios of buildings are important characteristics in determining their seismic behaviour. Local site effects, based on subsurface ground conditions, also have remarkable influence on the level of building damage and on damage distribution during an earthquake. Since the 1985 Michoacán earthquake (México), building damage has been studied analysing the contribution of site response (Kobayashi et al. 1986), in particular when resonant phenomena between soil and structure appears. More recent destructive earthquakes (e.g., Northridge, California 1994; Kobe, Japan 1995; Izmit, Turkey 1999 or Chi-Chi, Taiwan 1999; Colima, Mexico 2003; Al Hoceimas, Morroco, 2004) have shown how unconsolidated soil and sediment deposits were responsible for important modifications in ground motion amplitude in a range of periods and how building damage increases when the fundamental vibration period of the building is close to the predominant period of the soil motion.

The relationship between soil amplification and the level of building damage has been recently confirmed for several moderate earthquakes in Europe from a large number of observational studies (Mucciarelli and Monachesi 1998; Navarro et al. 2000, 2002). The results show that local amplification effect has played a role in the seismic damage distribution in urban areas, especially when the fundamental vibration period of the building is of the same value as that of ground motion. Buildings with the same typology placed in areas underlain by similar surface geology, showed significant damage differentiation from place to place (some with moderate damages and others undamaged) and the only appreciable difference amongst them was the number of stories of the buildings. Therefore, the determination of seismic hazard oriented to seismic risk management in urban areas forces us to know the resonance “range” between ground motion input and building dynamic characteristics. This data is of great importance for early evaluations of seismic damage, quantification of the seismic risk and for planning of scenarios of seismic damage in urban areas.

As referred to above, the seismic response of a building is mainly dominated by the natural period of vibration and the damping coefficient. To measure them different exciting sources have been used: induced vibration, record of earthquakes or explosions and ambient vibration or microtremors. The technique of induced vibration (Jeary 1986) is, in general, applicable to singular structures and its application to a great number of buildings supposes a very high cost. To obtain records of earthquakes and/or explosions it is necessary to maintain a permanent monitoring of the building, which implies a high cost and a low yield in areas with a moderate seismicity. The application of the microtremor measurements has been proved as a quick, efficient and economic method (e.g., Kobayashi et al. 1986, 1987, 1996; Midorikawa 1990; Oliveira 1997; Navarro et al. 2002; Dunand et al. 2002; Brownjohn 2003; Satake et al. 2003; Navarro and Oliveira 2004) and it is based on the principle that the microtremor is the input to the structure of the building and it is amplified at different periods depending on the dynamic response of the building.

This paper shows the potentialities of the experimental technique based on in-situ ambient vibration records with a single sensor at the top of the building as a simple method to determine the natural period and damping of buildings for low amplitude input motions, and that these determinations are very much dependent on the construction types practiced in different regions around the world. We will present results from the construction types in Portugal which may also be extensive to other areas of the world with similar construction. The paper also refers to changes in natural periods and damping for larger input motion.

This technique proposed in early 1930s (Carder 1936), and much applied in the 1960s and 1970s in the context of Forced Vibrations (example: Trifunac 1972). It has great advantages due to the simple technicality required and to the accuracy of obtained results and is quite adequate to complement the complex analytical modelling of soil and structural systems, calibrating mechanical parameter values. However, the method proposed considers soil-structure uncoupling and cannot follow with accuracy the non-linearity of both soil and structure. These two issues are important and can only be analysed on an individual basis.

A few examples of numerical and experimental techniques are presented in order to show that it is possible to calibrate the models to predict very approximately the experimental in-situ results, reducing the great gap existing in the literature concerning these problems. Another point of great interest is to understand the uncertainties in the evaluations and what their influences on the results of a dynamic analysis are.

2 Seismic response and techniques for identification of natural periods and evaluation of damping ratio

The response of a structure to seismic loading depends on its dynamic characteristics, and on the input ground motion. For low amplitude motion, materials behave linearly but as amplitude increases, non-linear behaviour starts taking place altering the dynamic properties of the structures.

The linear MDOF (multi-degree of freedom system), representing the building under seismic action, presents a set of N orthogonal mode shapes Φ_i with natural frequencies ω_{oi}, and damping ratios ξ_i. In most cases, the response of the system can be described to a certain extend just by the first modes and, consequently, the knowledge of the fundamental frequency (first mode in X, Y—horizontal directions) becomes of great importance.

The linear behaviour holds only for low amplitude motion when all elements are in the linear range. As long as input motion increases, material and connecting parts of a structure initiate the non-linear regime and frequencies tend to decrease while damping tends to increase. And the response of the structure will vary as frequency varies continuously in time.

Essentially there are two ways to obtain the dynamic characteristics of a building: (1) using numerical modelling based on the mechanical properties of the components; (2) by experimental monitorization of the real building for different input motion. Both are important and complementary, the second being a way to calibrate the first.

Event though the exact behaviour of a building depends on the evolution of the frequencies along the input time-history, the knowledge of its fundamental frequency at low amplitude values is very important due to the following reasons:

- Numerical models in the linear regime should reproduce the measured in-situ determinations;
- Response of a building starts with its initial higher frequencies;
- Performance-based analysis requires initial frequency;
- Tuned-mass damping requires exact frequency values;
- Ground—RC building structure resonance phenomena can occur;
- Identify damages in a structure;
- Too low frequency might be on the unsafe side due to the pronounced drop-off of present-day design spectra.

On the other hand, in many applications for vulnerability assessment of building behaviour at a macro level (stock of buildings) it is very important to have an expedite form of determining the fundamental frequencies of different types of buildings.

For these reasons many authors have researched this area and there are a large number of formulae to obtain the fundamental frequency of a building, based on very simple parameters: Number of storeys (e.g., Kobayashi et al. 1987), building height (e.g., Dunand et al. 2002) and damping predictor (e.g., Lagomarsino 1993). Code regulations, even the most modern ones, give options to the structural analyst for a first hand evaluation of the fundamental frequency.

The experience gained with measuring frequencies of a large group of RC buildings in Portugal is brought together here with some of the results obtained.

Frequencies and mode shapes of a building are robust characteristics of a building. In fact, despite depending on many factors of structural type (structural system, material of the structural system, mass distribution, geometry, presence of the so-called "non-structural elements", type of foundation), but also on other factors such as amplitude of vibration, aging, state of conservation, etc.), frequencies and damping are very stable characteristics. In a more refined way, they may also depend on temperature, moisture of soil and peripheral walls, etc.

However, this long and complex process to characterize the frequencies, in many cases gives rise to a simple formula which relates frequency or period with height or number of storeys of the building.

2.1 Influence of amplitude of motion: non-linear behaviour

Increasing the amplitude of motion will change the dynamic properties of building structures due to the occurrence of non-linear effects originated in the resisting properties of the elements and of its foundation. Friction between adjacent elements and cracking of important elements are the first causes of non-linear behaviour. The simplest case can be summarized in Fig. 1 for the SDOF, where in a force-displacement diagram, departure from linearity is present when a certain limit is exceeded. In the continuation of the experiment with changes of the direction of application of forces, hysteretic loops are formed with a tendency to aggravate the non-linearity with the increased number of cycles. Hashemi and Mosalam (2006) in a recent study conclude that the non-linear behaviour of the infill panels cause great changes not only in the frequencies but also in the force distributions in the confining elements.

The consequence of the non-linear behaviour in a SDOF is the elongation of the natural period of vibration with a decrease of stiffness (given by the slope) and the increase of damping ratio (given by the area of the hysteresis loop). These effects are more pronounced as the amplitude of motion (here translated into forces-displacements) increases.

The non-linear effects are very much dependent on structural type and material, and become much more complex in MDOF systems.

2.2 Identification of natural periods

The determination of the natural period of a building can be obtained by experimental methods with observation of the dynamic in-situ behaviour of the structure or using analytical modelling of its entire structure, including all elements contributing either to the mass or stiffness of the system as well as the foundation. Experimental methods constitute a large variety of techniques essentially by measuring the vibration of the structure by means of highly sensitive dynamic transducers, which can capture with great accuracy the main trends of the way the structure is vibrating. We want to emphasize that one can easily extract and at a small cost the longest period of vibration, the so-called fundamental period or its inverse, the fundamental frequency.

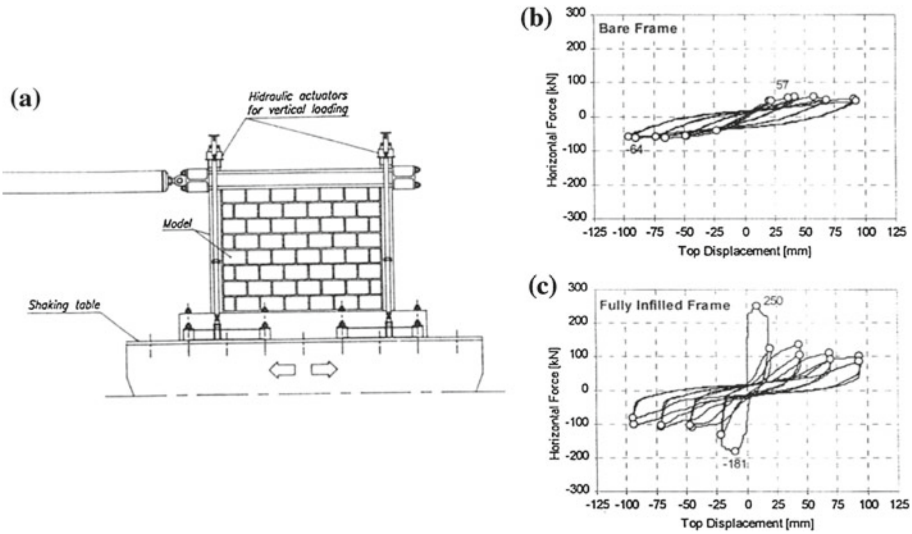

Fig. 1 **a** Schematic non-linear representation (force-displacement) for a SDOF: **b** MRF without and **c** with infill brick walls (from Pires et al. 1997)

As we are in the presence of a building which is a complex system with so many elements (beams, columns, slabs, shear walls, infill walls, stairways, piles, caves, etc.) one might question the accuracy of such measurement and what is the minimum amount of information one needs to obtain reliable results.

The best technique for both the determination of period and associated damping is to identify the vibration mode shapes which can only be achieved if two or more synchronized sensors can record the motion at different locations in the building. However, for determining just the fundamental period, a single instrument located at the top of the building can, in most cases, accurately determine that period.

Several sources of excitation can be used for inducing vibration in a building, some of natural origin such as earthquakes and winds, others provoked by traffic and other human activity—so called cultural or ambient noise or microtremors—and, finally, the forced vibration as the one caused by dynamic shakers, "man-induced" vibrations, explosions or sudden dropping masses in the immediate neighbourhood, free vibration originated from the rupture of pulling cables, and other types of human activity, such as internal noise as the case of elevators, moving shafts, workmanship, etc. These sources are of different quality in respect to amplitude of motion, frequency content and local of application within the structure. With the sole exception of the presence of a strong harmonic source in the neighbourhood of the instrument, all other sources of noise contain energy with a wide spectrum and the building acts as a filter enhancing its dynamic properties.

In brief we summarised the sources of excitation we have been using throughout these years.

2.2.1 Forced vibration

Forced vibration testing in buildings is not usually done due to its cost, preparation period, and because the results obtained have not differed essentially from microtremor for the case of no damage inflicted to the structure. In fact, the forced vibration may be of great value to other types of structures such as large dams for which large forces are needed to excite those structures, but the few examples carried out with buildings show a remarkable coincidence of results with microtremor, both in frequency estimation as well as for damping. Of course mechanical shakers using a controlled harmonic force applied can be used to trace transfer functions and consequently derive the dynamic properties of the building. In some cases, if forces are significant, the influence of amplitude is also captured. The classical shaker works as rotating masses, thus forces are proportional to the square of the rotating velocity and very small for low frequencies. For low frequencies (<3 Hz) other techniques have been implemented. Among them one should be mentioned the "man-induced" vibration produced by a few persons pulling a wall at the top floors of a building in phase with the movement of the building (resonant condition). Very good results (with amplitudes of ∼10 mg at the top of 10 storey buildings) are obtained with this rudimentary technique not only for frequency identification but also for damping evaluation (Oliveira 1997, 2004). The use of a suspended mass working as a pendulum can also be done (Ravara 1973).

2.2.2 Earthquake records

Records obtained in a building during earthquakes are of most importance because they can depict a great deal of information on how the building has performed. We can classify these records (obtained at the top) into three main categories on PGA value ranges: very weak or microtremors (<1 mg), weak (1–50 mg) and strong motion (>50 mg) as the amplitude of incoming wave is capable of inducing significant non-linear effects or not. For a modern reinforced concrete (RC) building, built in a moderate seismic environment, this threshold might be 100 mg. Figure 2 presents weak motion records from distant earthquakes at upper floors of two building described in Sect. 4 (Solmar and Portela), and the corresponding Fourier spectrum.

Recent monitoring of buildings, some with online recording (Celebi et al. 2004), will inform on the alterations suffered by the building through its life, in particular, during earthquakes. But, so far, the data on earthquake recordings are essentially from buildings which recorded the Californian, Mexican and Japanese earthquakes, with one station at the top and another on ground floor. There are a few other cases some of which will be reported here.

2.2.3 Microtremor measurement

The microtremors are vibrations on the order of several micrometers, caused by natural phenomena (atmospheric front; geothermal reactions, marine waves etc.) and/or artificial sources (traffic vehicles, heavy machinery, human activities, etc.). The range of vibration period is between 0.1 and 10 s. The vibrations that have periods smaller than 1 s are usually called microtremors and those with a larger period range are called microseisms. Kanai (1957) originally introduced a theoretical interpretation and practical engineering applications of microtremors. An application includes the natural period and damping characteristics of a structure (e.g., Kobayashi et al. 1987, 1996). Generally, a building is excited by natural phenomena (wind conditions), microtremor and artificial sources. Figure 3 presents a comparison of Fourier spectra obtained for microtremor and weak motion at the Portela building and it is very clear that both sources of excitation for PGA′s up to 10 mg (at 7th floor) produce exactly the same fundamental periods of vibration.

Experience has shown that the best location to obtain the fundamental period in each orthogonal direction is to place the transducer at the top floor, where the highest noise level is

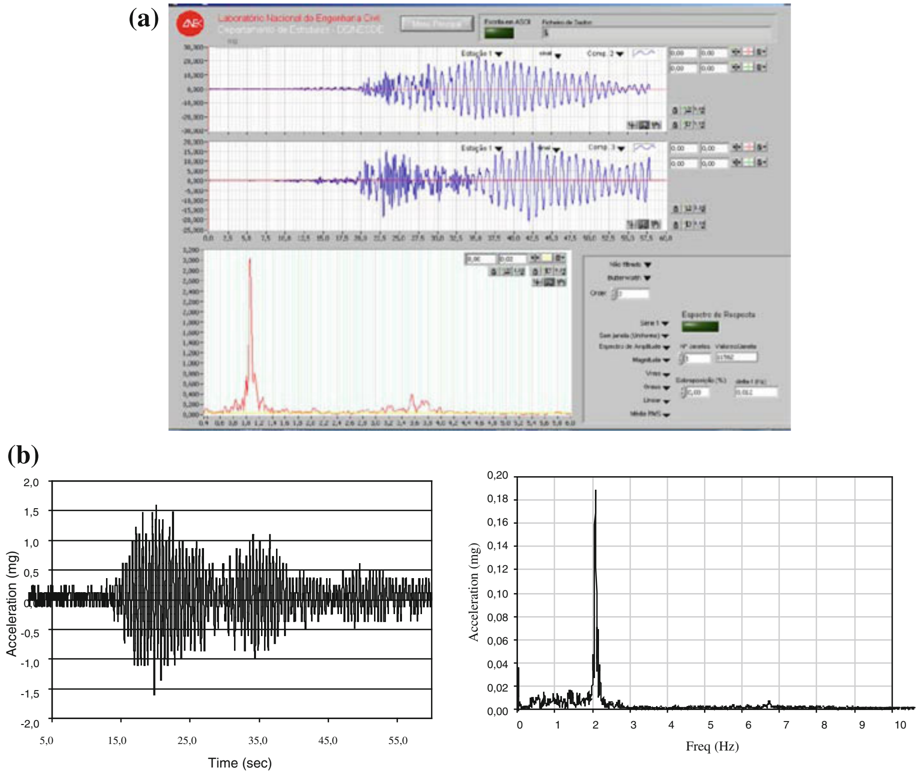

Fig. 2 Weak motion recorded at upper floors: **a** Solmar (courtesy LNEC) and **b** Portela buildings for $M = 4.0/5$ at 250/500 km from the epicentre (time history and corresponding Fourier spectrum)

expected and the recording should be taken for a long period of time. The Fourier amplitude spectrum will then show a pronounced peak, centred at the fundamental period/frequency of the building. This peak is more pronounced for the case of longer periods, that is, for tall buildings (Fig. 4), due to the highest amplitude for higher buildings. In the case of smaller buildings the peaks might be more difficult to identify. There are other cases for which caution should be taken to exclude spurious peaks which can be observed due to interaction of the study building with adjacent ones, or due to complex configurations of the building for which the two orthogonal modes and the torsion mode might stand close to each other.

For identification of modal shapes, which is not the case of the present paper and do not diminish the objectives persuaded, we should use various synchronized sensors and the modal identification technique to make sure which mode a given frequency belongs to. Much work has been made in this direction and the there are various techniques for identification of modal shapes with great accuracy. Brincker et al. (2003) and Cunha and Caetano (2005) are among the published literature where identification algorithms were used with great success.

In conclusion, expedite in-situ measurement with a single sensor at the top of the building is good for determining the fundamental frequencies of buildings; usually the X and Y horizontal and the torsional modes are obtained. Sometimes the second modes are also identified. In isolated buildings torsional modes are easily identified if the sensor is placed in a position away from the centre of rotation, and considering that higher modes have much higher values, there is no doubt on that identification.

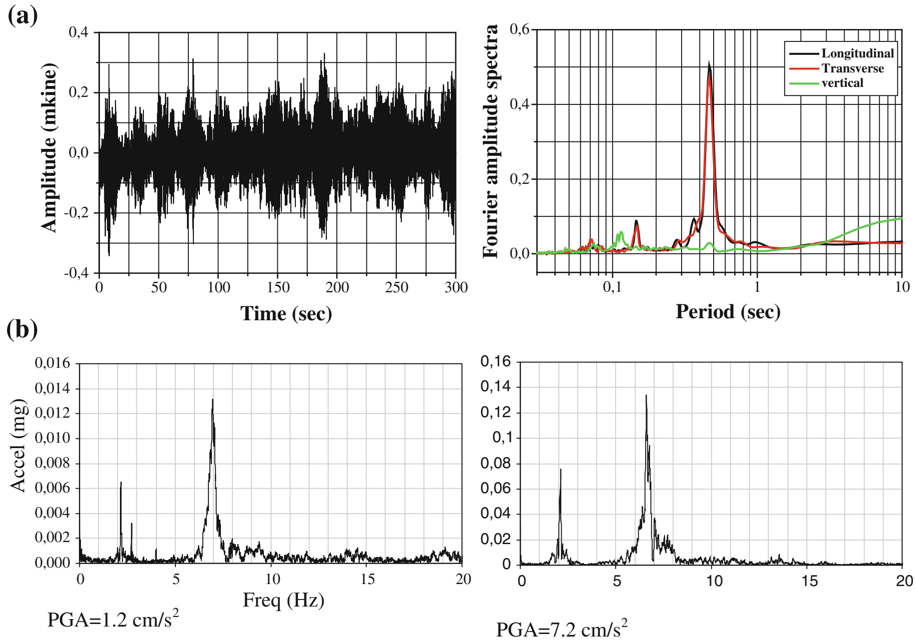

Fig. 3 Comparison of Fourie spectrum of **a** microtremor record at the center of plan on the roof floor of the Portela building with **b** weak motion for low amplitude values

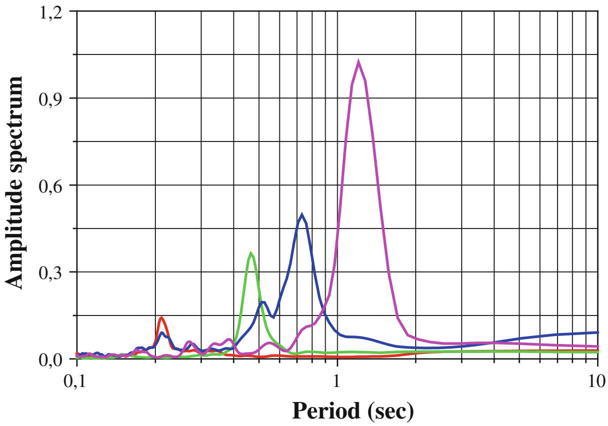

Fig. 4 Examples of amplitude Fourier spectrum of longitudinal component for several buildings with different numbers of storeys in Lisbon city (Portugal). (a) 5 storeys (*red colour*); (b) 10 storeys (*green colour*); (c) 15 storeys (*blue colour*); (d) 27 storeys (*mangenta colour*)

As far as the gathering and processing of records of ambient noise, besides the sensitivity of the instrument used, we would like to draw attention to two problems; one related to the duration of record which will determine the degree of accuracy on the frequencies values, and the other related to the sampling rate which determines the highest frequency to be detected. The presence of spurious frequencies has to be checked carefully in order to be able to delete them. As we are dealing with general behaviour of buildings we only care about two digit information on frequency. In this case we need a few samples with 60 s durations. If we were interested in obtaining more accurate information on frequencies as the case of identification of higher modes, larger durations were required.

2.3 Identification of damping ratio

The damping ratio is not an intrinsic parameter of the building, thus its determination constitutes an extremely complex problem, as much from a theoretical point of view as empirically, because many factors take part (structure and soil characteristic and soil-structure interaction). The time variation of these factors due to, for example, frictions in the connections, dissipation of heat, plastic deformations of the soil, etc., modifies the damping ratio of the structure, so damping values should only be considered as information of general character.

In order to estimate the damping ratio of a singular building structure, several classical methods using induced vibration techniques have been applied based on free-vibration response (Free-vibration Decay method), frequency-response curve (Half-Power method) or damping-energy loss curve (Energy Loss per cycle method). Recently, the Random Decrement Technique (Randomdec) has been applied to determine the damping ratio of RC buildings using ambient noise measurements. This technique was first developed by Cole (1968) as a structural monitoring technique for space craft and it has been proved as an effective crack detection method. This method has been shown to be applicable to determine the damping of dynamic systems subject to unknown random excitation, such as microtremor, and it has been applied to detect damages in structures (Yang and Caldwell 1976; Yang and Dagalakis 1980) and to determine damping ratio of soil (Yang et al. 1982) and RC buildings (e.g., Navarro et al. 2002; Dunand et al. 2002). The Randomdec analysis requires only the measured output of the dynamic response of a structure, and not the random excitation input and permits one to obtain the response due to initial displacement, representative of free vibration decay curve of the system from the ensemble averaging N segments of the length τ of the response of a linear system (Yang and Caldwell 1976). Also, we do not discuss this item in any detail and we do not examine the influence of amplitude (x) on damping ratios, most of them using a linear dependence (Jeary 1986; Satake et al. 2003). After the analysis we obtain the damped free vibration response of the system and standard techniques (e.g., Free-vibration Decay) can be used to obtain the damping ratio. This technique has been compared with the spectral damping estimation approach using the half-power bandwidth method (Kareem and Gurley 1996) and results are of the same order of magnitude if frequency of the mode under analysis is well constrained. Several examples of random decrement signatures are illustrated in Fig. 5.

3 Brief characterisation of structural layout of the RC buildings under analysis

Reinforced concrete (RC) in Portugal can be considered as an extension of the masonry buildings constructed until 1930–1940 with the progressive introduction of RC elements in various parts of the building.[1] In a very summarized way, the first elements in RC were the slabs, then the use of beams in the lower storeys to permit the opening of spaces for commerce on the ground floor, and then the use of moment resisting frames (MRF) at a later stage (∼1960). Figure 6 presents the typical plan layout of a 1950 RC building with more

[1] In a simplified way, types ***A*** and ***B*** are masonry buildings where ***A*** is the traditional nineteenth century construction and ***B*** is the transition to the RC, in the first half of twentieth century. Type ***C*** are RC buildings especially the MRF and type ***D*** are SW buildings.

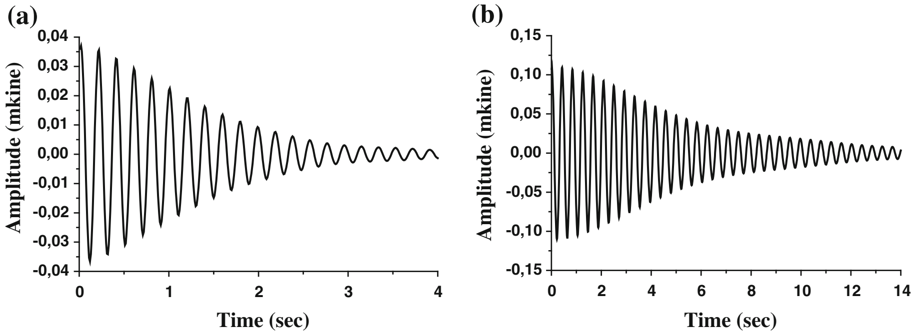

Fig. 5 Examples of random decrement signatures for several RC buildings with different storey number in Lisbon city (Portugal), excited from microtremors. **a** With four storeys ($T = 0.19$ s; $\zeta = 2.9\%$); **b** with 11 storeys ($T = 0.42$ s; $\zeta = 1.5\%$)

density of internal brick masonry, and Fig. 7 a 1960 RC building with larger spans. Buildings of these epochs had large portions of solid brick walls confining the RC frames. The new era of codes permitted the introduction of pure MRF (∼1980) which were designed for lateral forces to withstand seismic loads. In most cases, stairways in the centre plan had walls of solid brick or of concrete, Fig. 8. This concept has been kept as a design detail practiced almost always. Partitions and peripheral walls were until nowadays made of hollow brick walls cast against the RC members. The partition with a thickness of 15 cm and the peripheral with two layers and an empty space in the middle making a total of ∼30 cm. RC shear walls (SW) only appear in buildings higher than 8–10 storeys in the high seismicity zones. They become more important in the case of taller buildings, and together with the central core for elevators and stairways, they are supposed to be the structural system capable of supporting the lateral seismic loads.

In the late 1980s flat slab buildings become very numerous and were used essentially for open space offices, Fig. 9.

There is a large group of buildings in the 1960–1990 which were built with a mixture of "pilotis" and infill's in part of the ground floor to allow for car parking, commercial activities, shops, or just for architectural reasons. These structures usually concentrate the infills in the centre of the plan, Fig. 10.

According to the above description we should separate the MRF (type C) from the MRF + SW (type C1) and from pure SW (type C2). Almost all of them have hollow clay brick infills. The percent of SW in relation to the overall seismic resistance is an important parameter measuring the overall behaviour of the structure. Frequencies of vibration depend on this relation as well as in the percent of total infills.

4 Comparison of methods. Calibration with analytical techniques

The dynamic characteristics of three prototype RC buildings of different typologies were evaluated through the development of structural analytical models, which were also subjected to weak motion records, ambient noise and microtremors.

The first example consists of a 12 storey MRF isolated building (Portela); the second is a 23 storey MRF building with three shear walls cores (SW) (Solmar), connected at the first 6 storeys with adjacent structures. These examples, built in the 1970s have partitions and

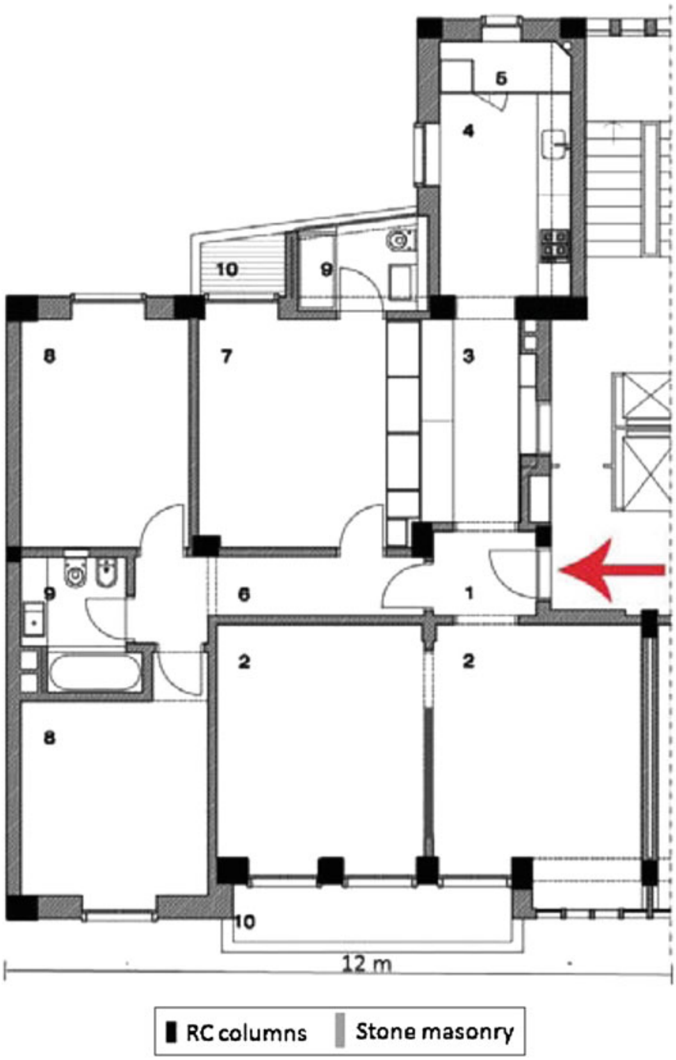

Fig. 6 Architecture and structural layout for RC buildings in the 1950s

peripheral brick wall infills. The third example is an isolated building with 19 storeys, the first one constituting a high mezzanine. The structural system has a peripheral SW and two SW cores. The interior connecting to the SW sections is a MRF (Monsanto Tower).

A 3-D model structure was made using the most modern commercial technologies to simulate the linear dynamic behaviour (SAP2000(®) 2000). Columns, beams and shear walls were considered as RC frame elements with geometrical characteristics taken from the design and mechanical properties from statistical studies and slabs were considered as rigid diaphragms. Infill brick walls were simulated as diagonal struts with geometrical and mechanical properties taken from analytical and laboratorial testing (Baptista and Oliveira 2004).

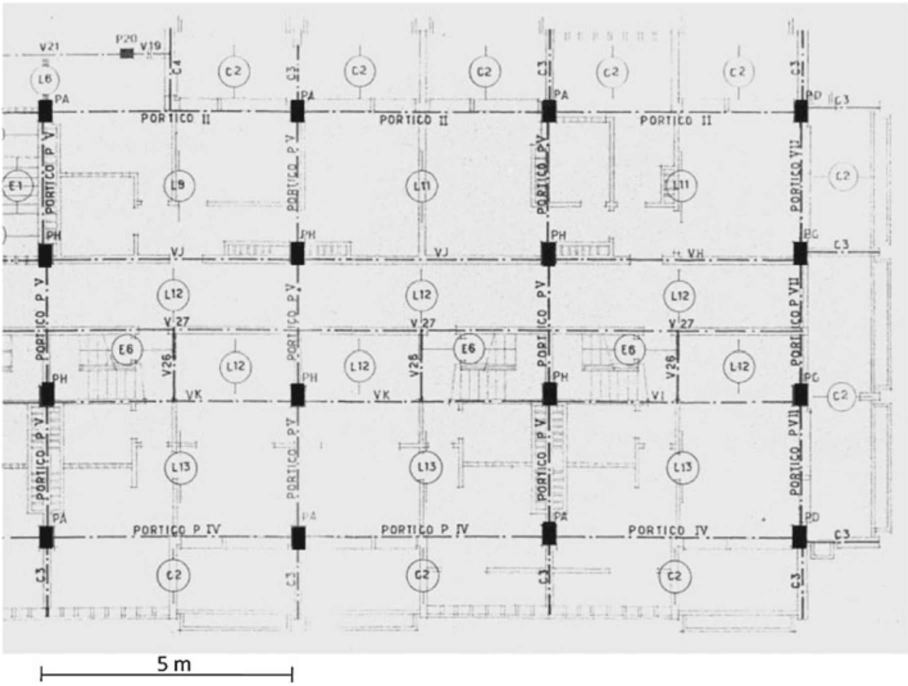

Fig. 7 Plan of typical RC building in the 1960s

Also a few "academic studies" for typical RC buildings in Portugal are presented to show the remarkable differences in frequencies values encountered in the common engineering design practice (offices) and the in-situ measured values.

These examples illustrate the potentialities of the experimental techniques versus the analytical modelling. Even though, the results suggest a good confidence on the experimental methods, considering low amplitude range. More examples are needed for a more robust calibration.

4.1 Portela: 12 storey building (Type ***C***)

The structure under analysis (Baptista 2006, Personal Communication) is a typical reinforced concrete building constructed in Lisbon in the 1970s, prior to the present Portuguese seismic code (RSA 1983). It is an isolated 14 storey building, founded on a thick gravel soil, the first two floors with storage rooms and the others with four housing units per storey (Fig. 11a). The plan is almost square with $21 \times 21\,\mathrm{m}^2$, (Fig. 11b) with columns spanning 4–6 m and beams connecting them forming a quite regular grid (Fig. 11b). The larger columns are about 1.40×0.40 on the first floor and reducing their dimensions to the upper floors. Slabs are 17 cm thick, cast in-situ concrete. The central part of the building, which gives access to each apartment, is an open space with $6 \times 6\,\mathrm{m}^2$ where a full stair and two elevators are located (Fig. 11b). The room space is divided by the current traditional partitions of hollow clay brick walls, 15 cm thick in the interior plan and 30 cm in the outer façades.

The masses were estimated from material weights and existing live loads (Fig. 11). The model was assembled and run for two hypotheses, one assuming that the infill walls were not

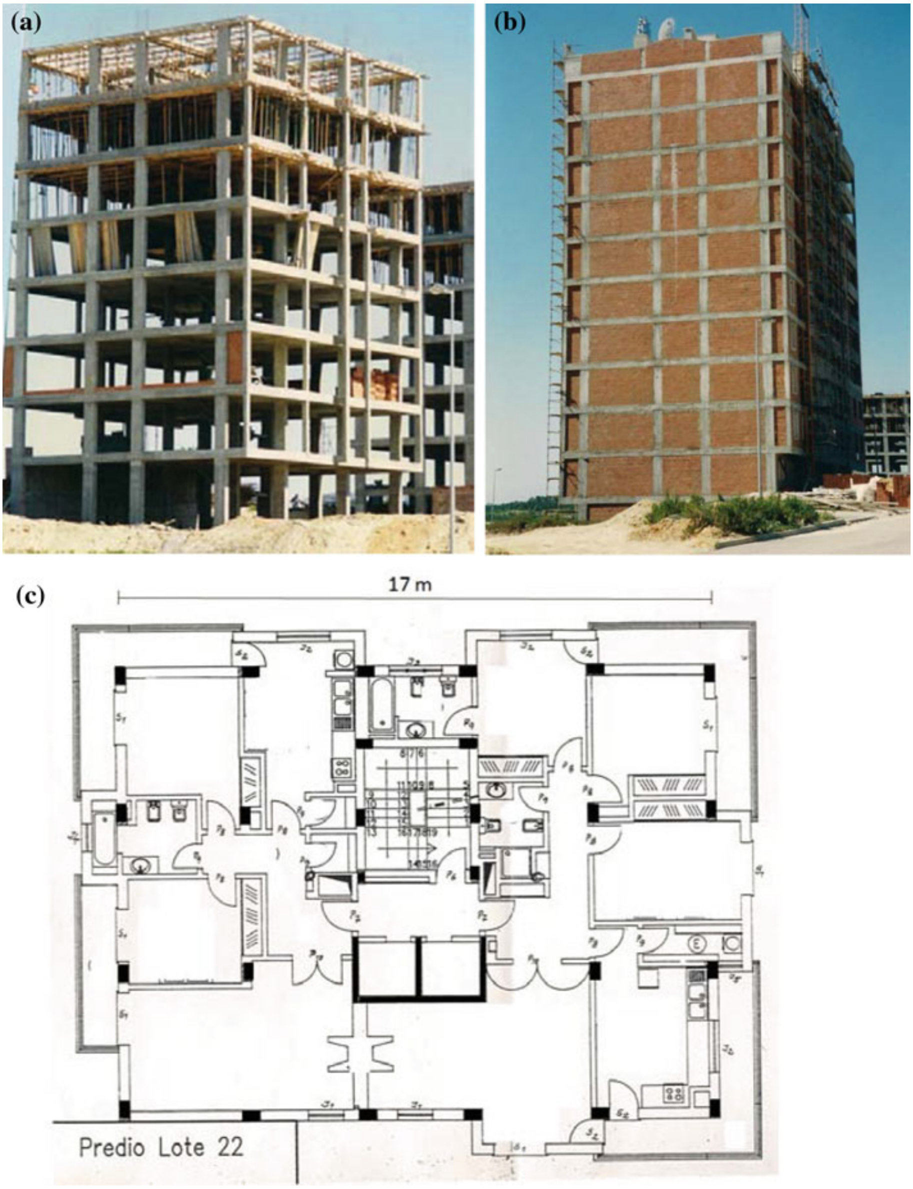

Fig. 8 Recent MRF buildings (1990s): **a** without infill brick walls; **b** with infill brick walls; **c** typical plan view

participating in the stiffness of the wall structure, and the second, corresponding to the "real" structure placing the infill walls at their locations. Frequencies and corresponding modes were computed for those two situations and results are presented in Table 1 together with the experimental values obtained by in-situ measurements. As can be observed, the results are quite similar when comparing the experimental values with the second model with infill brick walls.

Fig. 9 Recent MRF + SW structures without interior infill brick walls (*open spaces*): **a** exterior view; **b** interior view

Fig. 10 Typical soft-storeys buildings in the 1960s ("pilotis")

The microtremor measurements were performed at the center of plan on the top floor of the building, using a data acquisition system composed by a three component short period seismometer with natural period of 1 s, amplifier and notebook computer with A/D converter. The system was used to record the horizontal and vertical components of microtremor. The first channel was adjusted to the longitudinal direction, the second one to the transverse direction and the third to the vertical direction, respectively. A time history 300 s long of microtremor signal was recorded, sampled at a rate of 100 samples per sec (Fig. 3a). Fast Fourier Transformation was applied to a record in order to compute Fourier spectrum, smoothening with a Parzen's window of 0.3 Hz width. The longitudinal and transversal components show 0.47 s natural periods.

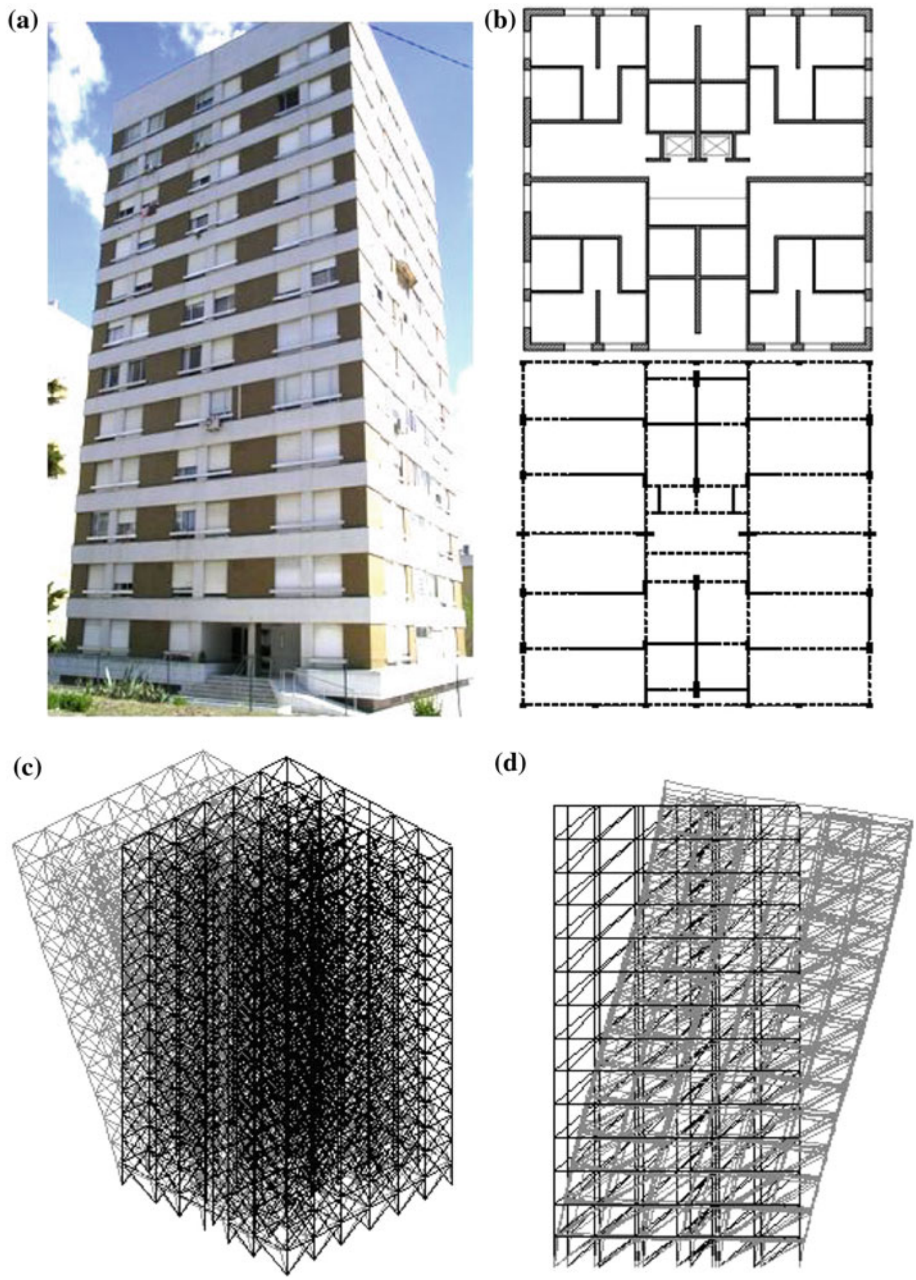

Fig. 11 Portela building: **a** view; **b** plan and **c** and **d** 3-D analytical model with fundamental modes

The random decrement signature for each horizontal component was obtained applying the random decrement technique and the damping ratio has been calculated using the free vibration decay method. The damping ratio value for longitudinal component is 0.85 and 1.19% for the transverse component, respectively.

Table 1 Portela building: comparison of frequencies measured in-situ and computed with an analytical model (units: Hz)

Vibration mode	In-situ testing	Analytical model	
		Without infill walls	With infill walls
1X direction	2.15	0.72	2.11
1Y direction	2.15	0.81	2.14
1Torsion	2.61	0.86	2.98
2X direction	7.03	1.94	6.36
2Y direction	7.03	2.23	6.61
XY direction	13.6	3.53	11.4

We have recorded in two cases (Fig. 3b) with weak motion exhibiting amplitudes 5 to 10 times the values recorded during microtremor testing. In these circumstances, the frequencies of lower modes are similar.

4.2 Solmar: 23 storey building

The Solmar building presents particular characteristics like (Marques 2007): (1) 23 storeys and (2) three cores which practically define the building behaviour to flexion and torsion. The calibration of the numerical model through the results of experimental tests is the aim of this linear dynamic study. The influence of the overload is taking into account, as well as the influence of the infill masonry walls and the modulation of the three cores.

The building of reinforced concrete and walls composed by blocks of concrete masonry has 23 storeys, and the plan view consists in three rectangular bodies with approximately 13 m aside. The two side bodies make 30° with the x–x axis. The Solmar building stands out from the other constructions around for its dimensions in height, with ∼73 m.

The inter-storey height is 3.6 m for the underground floor and for the first 6 storeys, and then reduces to 3.35 m for the above floors. The 21st storey has a set-back occupying ∼50% of the overall plan area.

The plan view and the referential are presents at the Fig. 12a, the view in Fig. 12b and the model in Fig. 12c.

Table 2 compares the results of frequencies obtained with the computed and in-situ techniques for low amplitude motion for several modes.

Experimental testing was done using three different types of input: (1) ambient vibration (2 mg); (2) weak motion from an earthquake 5 April 2007, magnitude 4 at 80 km distant (20–30 mg); (3) human excitation. For the latter $\zeta = 1.3\%$.

The “inter-test” error was smaller than 2%. A HVSR (spectral ratio for horizontal to vertical components) test presented a large peak at 1.43 Hz and almost no indication at smaller frequencies, missing the fundamental modes. A similar situation was reported by Herak and Herak (2008).

In this case even using all available numerical tools the larger errors are observed in the modes of first order, mostly in the bending ones. The biggest error is ∼12.7%. That means the numerical model is a “reasonable” representation of the real building. The influence of infills herein is almost negligible (1% error) due to the large presence of shear wall cores. However, the larger source of error comes from the representation of the cores and shear walls. The first solution, made with bars representing plane surfaces rigidly linked to each other, would give a first frequency 30% below the value presented in Table 2, which was

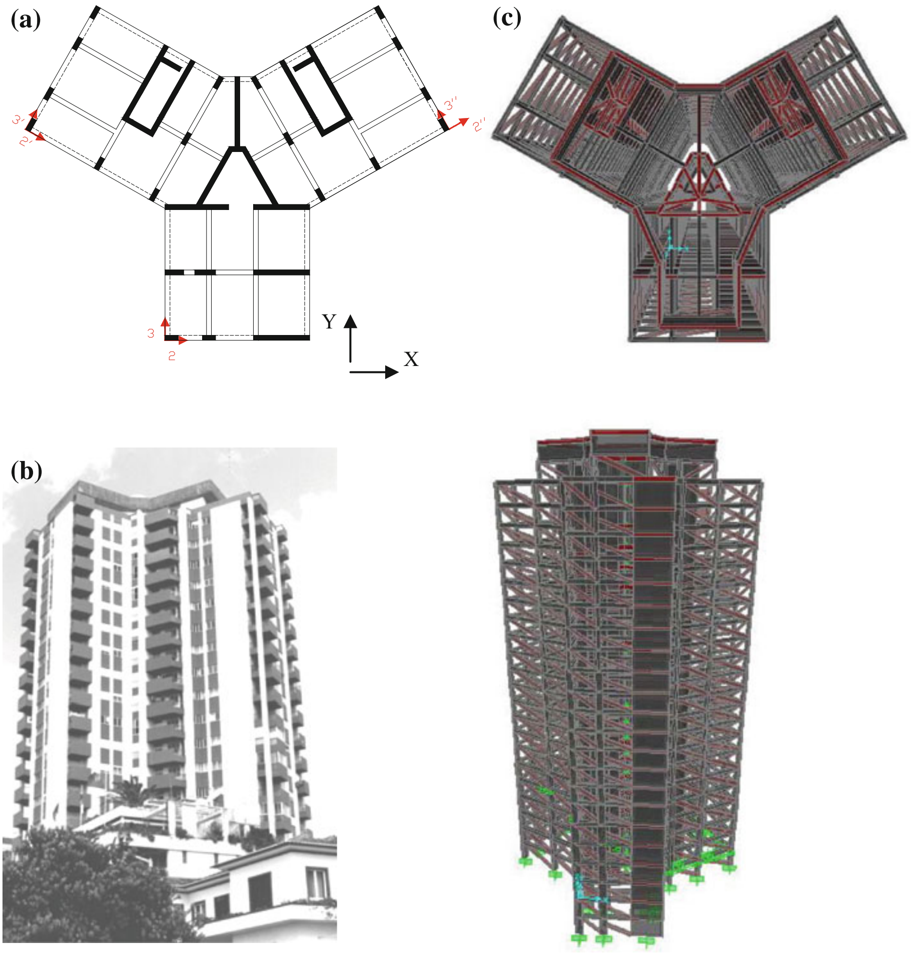

Fig. 12 Solmar building: **a** plan; **b** view and **c** 3-D analytical model

obtained when cores were represented by a single vertical bar, two with the properties of a cross-section of a hollow rectangle and one of a hollow triangle.

The errors for the Solmar building are larger than in the case of the Portela building, but we think they can be reduced if better knowledge of a few items could be made, such as thickness of slabs and the influence of adjacent buildings.

4.3 Monsanto tower building

The Monsanto Tower is a recently built isolated building with approximately 100 m high, with 19 storeys above ground, and two below for car parking. The first storey is a mezzanine with 13 m high. The building Fig. 13 has a paraboloid shape in the periphery made with a SW, opening at the other side. The interior is a MRF with a grid of circular columns linked by strong beams running in an orthogonal layout. At the apex of the paraboloid there are two SW cores for stairways and two smaller for elevators.

Table 2 Solmar building: comparison of frequencies measured in-situ and computed with an analytical model (units: Hz)

Vibration mode	In-situ testing	Analytical model	Error (%)
1Y direction	1.07	0.934	−12.7
1X direction	1.08	0.970	−10.1
1Torsion	1.43	1.521	6.4
2Y direction	3.6	3.401	−5.5
XY direction	3.6	3.642	1.2
2Torsion	4.6	4.758	3.4

Error = (analytical − in-situ)/(in-situ)

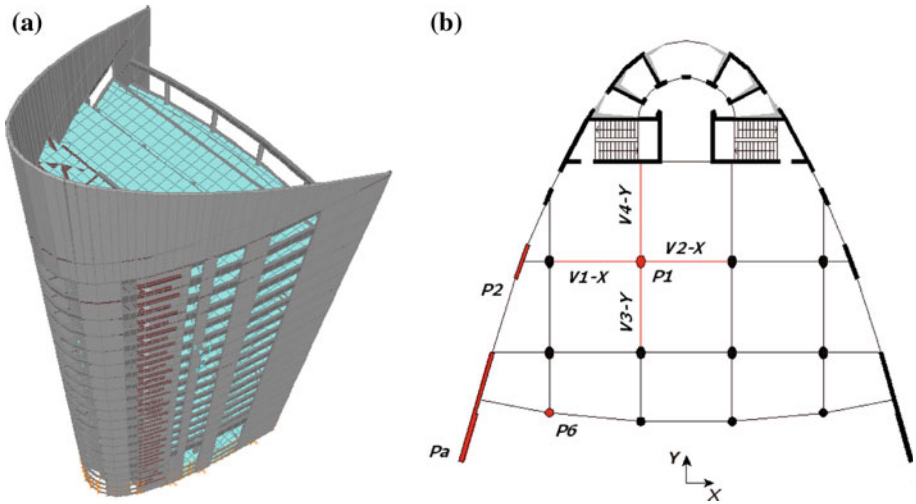

Fig. 13 Monsanto Tower building: **a** view and **b** 3-D analytical model

Table 3 Monsanto Tower building: comparison of frequencies measured in-situ and computed with an analytical model (units: Hz)

Vibration mode	In-situ testing	Analytical model	Error (%)
1X direction	0.55	0.479	−12.9
1Y direction	0.82	1.013	23.5
1Torsion	1.45	1.359	−6.3

Only the first slab has full beam support, all the others are flat slab with beams only at the periphery. The building is for offices with total open space, the front with huge glass windows. The in-situ test was done for ambient vibration with no furniture.

Table 3 compares the results of frequencies obtained with the computed and in-situ techniques for just the two fundamental modes. The errors are of the same order of magnitude as the ones obtained at the Solmar building. No special effort was made in order to get a better match because for the objective to perform push-over analysis, the errors were in a range of no great importance (Peixoto 2007).

In any case, we can attest that a first order evaluation of the dynamic characteristics through numerical modelling is similar to the in-situ determinations.

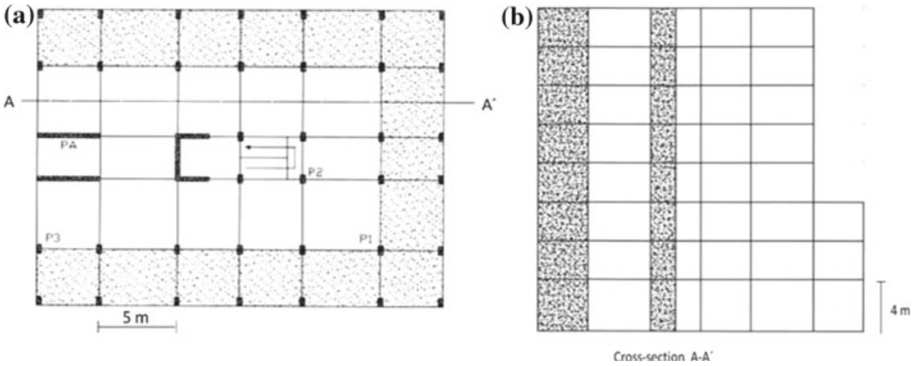

Fig. 14 Academic study of a MRF + SW: **a** plan; **b** vertical cross-section

4.4 Numerical models (academic examples)

In a final academic exercise we tried to compute with simple numerical models, the ones used in the design offices, the frequencies of the set of buildings representative of present-day construction in Portugal. We selected a group of a dozen structures with 8 and 10 storeys having a similar plan layout (Fig. 14), with MRF and SW structures without infill walls. They differ in the first 3 storeys where they may have a wider area exterior to the main body of the building. The plan layout is not balanced due to a large non-symmetry of the SW, but this irregularity emphasizes the torsional capacity of the building. The buildings have columns with 0.60×0.30 m in the lower storeys and 0.50×0.30 in the upper, beams with sections 0.30×0.50 m, and shear walls 0.30 cm thick. Stairways were considered in the analysis and slabs act as diaphragms. The results are as follows: f_x varies from 0.91 to 0.79 Hz for $N = 8$ and $N = 10$; f_y varies from 1.04 to 0.88 Hz for $N = 8$ and $N = 10$, with a standard deviation in the range 5–10%.

4.5 Discussion on the influence of infill brick walls

The technique of in-situ measurement on the top floor, for obtaining the first natural frequency of a building in each orthogonal horizontal direction, as well as the corresponding damping ratio, as described in the above sections, can be considered an efficient and cost-effective tool. Efficient because it only requires a single instrument and presents the best values for high level noise. Some caution might be exercised for the identification of modes, especially for the cases of low rise buildings, for complex buildings or in cases of interaction either with adjacent buildings or with soil layers. The last topic is analysed in detail in Sect. 6 under the soil point of view, for which case microzonation may lead to a soil spurious frequency (frequency attributed to the building).

The classic HVSR (spectral ratio for horizontal to vertical components), which can lead to very good results in identification of predominant soil frequencies, does not add much value to the identification of frequencies in buildings, contrarily to the cases of microzoning. This technique (Mucciarelli et al. 2004), which is not supported by any theoretical building behaviour, presents the same results as the simple analysis of peaks in the Fourier spectra. As already mentioned, further research in this area with various examples with different types of buildings is needed to set more sound conclusions.

Table 4 Influence of infill brick walls for RC buildings with $N = 9$ storeys as a function of % of "shear walls" in the frequencies of first mode (Hz)

% Of "shear walls" (SW)	Direction mode	Without walls	With walls	Increase[a]
9 Storeys	Transversal	0.88	1.73	1.96
No "SW" (Seixal-Sta. Marta)	Longitudinal	1.27	2.15	1.69
9 Storeys	Transversal	1.27	1.86	1.49
Some % of "SW" (Faro)	Longitudinal	1.17	2.05	1.75
	Torsion	1.18	2.64	2.24
9 Storeys	Transversal	2.15	2.64	1.25
Large % of "SW" (Lisbon)	Longitudinal	1.76	1.86	1.07

[a] If the additional mass is accounted for in the measurements (without walls) the increase in frequencies would be 20% higher, approximately.

The influence of infill "brick walls" inside RC structures is another issue of great importance at several levels: First of all, these "non-structural" elements are part of a large majority of buildings in southern Europe and Latin-American countries. Secondly, these elements are usually built against the structural frames and no engineering design office considers their stiffness, which is of particular relevance especially in the in-plane direction. And, in third place, the dynamic behaviour of the buildings is greatly influenced by the interaction between the infill walls and the frame, dictating in several instances the entire behaviour of the building for large input motions.

Noise measurements made in several buildings with different percents of "infill walls" (Table 4) revealed that the frequencies obtained are much higher (up to about five times depending on the number of storeys and the percent of infills, and the presence of SW), than what would be predicted by the analytical model which does not takes the "infills" into consideration.

Baptista and Oliveira (1999, 2004) have shown that it is possible, using a detailed analytical model which considers the "infill walls" as diagonal struts to arrive to a model leading to frequency values differing no more than 5% from the in-situ values.

It is recognized that the influence of the "infill walls" on the percent of increase of the frequencies relatively to the values not considering these walls depends essentially on the "quantity" of "infill walls" as a percent of the total horizontal area and also on the amount of "shear walls" present in the building. The increase is larger the higher the presence of "infill walls" and lower the presence of "shear walls". By using only the noise technique, see Table 4, the frequency suffers an increase of about 180% for the case of no "shear walls", 150% for the case of a few "shear walls" and 120% when a large portion of "shear walls" are present.

The same conclusion was obtained by Kobayashi et al. (1996) for the case of a 7 storey RC building tested under construction when infill walls were not present and after the completion of the building. In this case, the microtremor information gave a shifting in period from 0.56 to 0.35 s, which means an increase in frequency of 50%.

A way to quantify the influence of the % of SW in a building and then correlate it with frequency increase due to brick infill walls may be by measuring the percent of the sum of inertia moment of all SW elements (computed as entire structure) in a given direction, in relation to the sum of inertia moment of all columns. Also the proposal by Boutin et al. (2005) introducing a dimensionless parameter $C = \mathrm{EI}/\mathrm{KL}^2$ in relation to the Timoshenko beam (E, K: bending and shear elastic modulus; I: gross cross-section inertia; L:$H/2\pi$; H: total height) could be adapted to this situation.

4.6 Numerical model encountered problems

Numerical modelling of a building poses a few options when we think of structural analysis under earthquake loading. If we just concentrate on modelling the building to match the in-situ frequencies we have to look in detail to a few items which are not considered in the design phase. The most important item is the presence on the infill walls which are very stiff in the plan direction and can quite considerably increase the overall stiffness of the building. Even thin brick walls or walls with openings are important. The second item has to do modelling of structural cores in L, U, H or shape or with some other geometric shape which are still difficult to model, especially if there are openings. Also the presence of stairways is important to include for a correct modelling. A proper mechanical characterization of the materials, i.e., the modulus of elasticity of concrete, steel and wall bricks are essential for the modelling. Another item not to overlook is the difficulty in assessing the exact mass at the time of testing.

Two more items dealing with "boundary conditions" should be looked at with care: (1) the soil influence normally solved by the introduction of rocking springs and (2) the presence of adjacent buildings, which may interact with the building under analysis, especially if we talk about buildings in urban blocks without proper separation width.

Detailed identification analysis from ambient vibration can be used with success for simultaneous calibration of several parameters of the numerical model of a building (Ventura et al. 2005) using optimization techniques, but in this study we are concerned with the most important uncertainties controlling the numerical process and, consequently, we only considered the variation of a few parameter values.

5 Correlation of natural frequencies and damping with geometry of buildings

5.1 Generalities

The formulas used for predicting the fundamental period (T) as a function of geometry of buildings are usually proportional to the power of height (H) or number of storeys (N),

$$T = \alpha H^{-\beta} \tag{1}$$

This expression is sometimes corrected by using the depth (B) in plan or the percent of area of walls in relation to total in-plan area. (α and β are constants to fit data)

$$T = \alpha H^{-\beta} f(B) \tag{2}$$

β is under 1, but in many cases very close to 1, specially for the cases of a large number of shear walls. General dispersion of data leads to global variance $R^2 = 0.6$–0.7.

For some structural types closed-form solutions can be derived, as is the case of "pure shear walls" (Boutin et al. 2005), taken as a cantilever,

$$T_1 = \alpha_1 H \sqrt{1 + \beta_1 \frac{H^2}{Lt}} \tag{3}$$

where H is the height, L the length and t the thickness of the wall. α_1 and β_1 are constants related to material properties.

The presence of wall partitions increases the frequency in a way proportional to the square root of an equivalent cross section area of these partitions.

Damping ratios present more scatter than natural periods because they are influenced not only by the amplitude of motion but also by the foundation type, soil-structural interaction, density of internal partitions, etc., (Satake et al. 2003; Carydis and Mouzakis 1986). For small amplitude, before clear non-linear behaviour takes place, damping has essentially two plateaus (Jeary 1997), one for very small amplitudes and corresponds to "large imperfections" existing in all connecting surfaces, and the other when "small imperfections" are mobilized at larger amplitudes. In between, damping shows an increased linear trend.

The so-called Rayleigh damping, commonly used to treat damping in structural dynamics

$$C = \alpha M + \frac{\beta}{K} \tag{4}$$

where M and K are the mass and stiffness matrices, respectively, and α and β are values to be adjusted, leads for any mode k to a situation of the type (Lagomarsino 1993)

$$\xi_k = \alpha' T_k + \frac{\beta'}{T_k} \tag{5}$$

where now T_k is the period of k-mode and ξ_k is the corresponding damping ratio. Lagomarsino (1993) has shown that the second term is more important for structures with periods less than 1 s, whereas for larger periods the contribution of the first term becomes more and more important as T increases.

As will be seen next, preference is given, whenever possible, to simple formulas of the type Eq. (4) with $\beta = 1$ and Eq. (5) with $\alpha' = 0$.

5.2 Formulae for different RC structural type buildings in Portugal (low amplitude motion)

As already mentioned, we have been collecting data since early 1970 on the fundamental period of various types of buildings in Portugal (whenever possible we also computed torsion and second modes, always in the X and Y directions or *transversal* and *longitudinal*). In this paper we are only analysing the reinforced concrete (RC) structural system with and without "infill wall" panels. As referred to in previous sections we have classified the RC buildings in several typologies (B, C, C1 and C2) and selected the buildings to have quite a uniform distribution in terms of number of storeys. As in-situ measurements took place along a large period of years, the techniques of testing (source of excitation, equipment and data treatment) changed considerably and for a few buildings we repeat the test a few years later. No important earthquake took place in this period, so these buildings were never subjected to PGA's larger than 20 cm/s^2. We have circa 211 RC buildings out of 235 buildings, and in the following we will present results for different situations, separating several cases and commenting the results observed. This study will consider essentially the C structures.

The *data-base* (see supplementary materials) contains information on the following fields: City, street address, date of construction, number of storeys below and above ground, total height above ground, typology (A–C), infill panels information (no, all storeys above ground, "pilotis"), date of testing, first frequencies in transverse, and longitudinal directions and torsion, other frequencies, and a field for observations. The majority of buildings in the *data-base* are located on firm soil types. The very few located in alluvial materials had several storeys below ground, which were excavated until reaching the firm stratum.

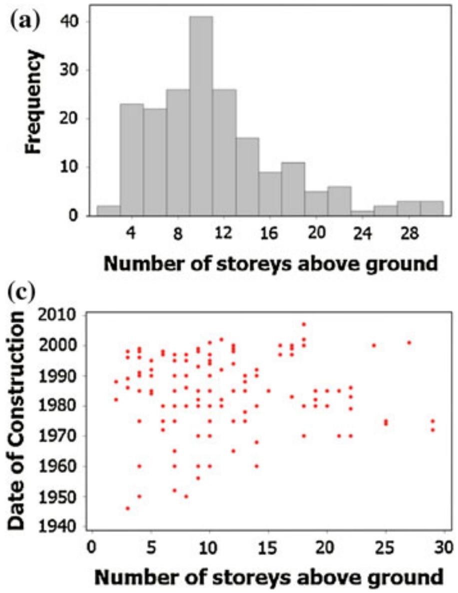

Fig. 15 Brief analysis of the *data-base* contents

A brief analysis of the *data-base* is summarized in Fig. 15. Buildings were selected from cities in southern Portugal with main emphasis in Lisbon. As we can see, the majority are in the range 4–12 storeys high and with construction after the seventies. Frequencies in transversal and longitudinal directions are very similar.

We used Eq. (1) for the correlation between fundamental period (T s) and number of storeys (N) or height (H m), and Eq. (5) for damping (Note: we use α_N if dealing with number of storey; N has the meaning of number of oscillating masses).

The dynamic characteristics of all types of existing buildings in *data-bank* excited by microtremors were analyzed using full-scale experimental measurements on 235 buildings (Oliveira 2004). The average relationship obtained was $T = 0.045\,N$.

For a group of 130 buildings, all with infill brick walls, the data is presented in Fig. 16a and for nine buildings without infill brick walls the data is in Fig. 16b. For the first case we have $T_t = 0.013H$ and $T_l = 0.012H$ for transversal and longitudinal directions with a correlation coefficient $R^2 \sim 0.8$ (Note: if we take the inter-storey height as 3.0 m, the conversion for α_N is $T_t = 0.039N$ and $T_l = 0.037N$). For the buildings without infills, which are only 9, the correlation is quite poor ($R^2 \sim 0.5$) but it is very clear that the trend points to values of α almost double. Unfortunately, the *data-base* in this respect contains only a few examples for which it is almost impossible to assure some degree of structural homogeneity.

If we now separate the different typologies in the *data-base* we obtain the values in Table 5, where variations are not very significative while correlation coefficient decreases slightly.

The value $T_1/T_2 = 2.6$ for the relation of periods of the first 2 modes is indicative of a "pure shear beam" in the concept of Boutin et al. (2005) where this value should be 3.

Besides collecting in-situ data whenever possible, along the decades we have performed separate experiments with the aim of understanding the influence of a few parameters on the fundamental periods of different buildings. Next we refer to two of those experiments which

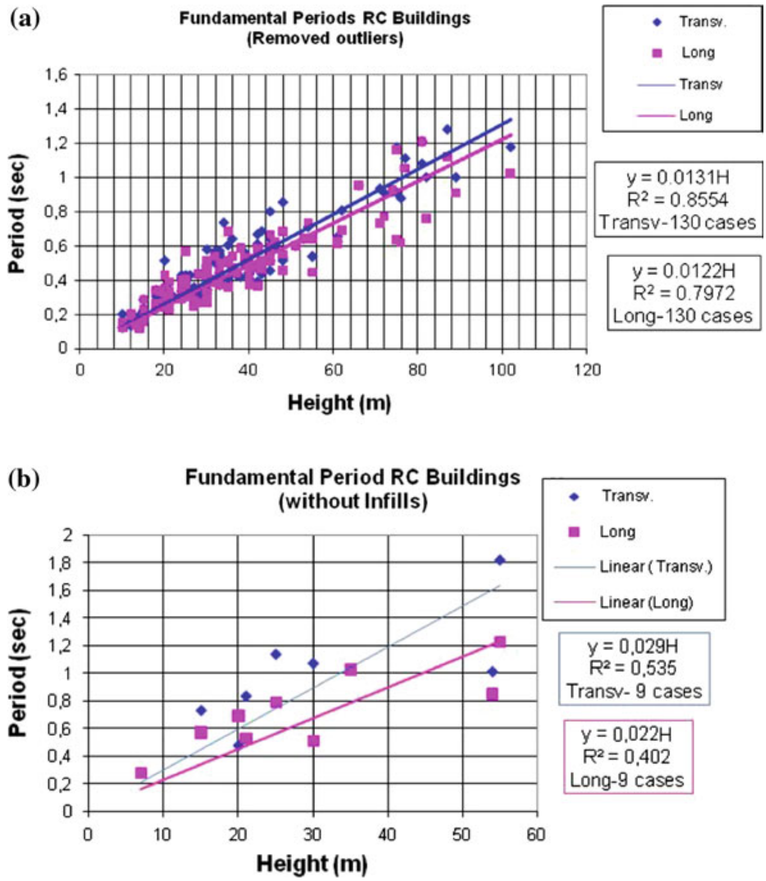

Fig. 16 Correlation of fundamental periods (T) with height (H). **a** RC with brick infills (130 cases); **b** RC without brick infills (9 cases)

Table 5 Influence of infill brick walls for different RC typologies: values of α for the formula $T = \alpha_N N$

Type	Transv	Long	# cases	R^2
B	0.050	0.054	18	0.7
C	0.047	0.042	193	0.6
C_torsion	0.032		56	0.4
C1/C2	0.048		125	0.7
C2	0.048		20	0.8

Other relations:
$T_1/T_2 = 2.6$; $T_1/T_{\text{torsion}} = 1.5$

support the main results obtained with data in *data-base*, emphasizing the common observed overall trend.

In the first experiment we measured periods in two different groups of a subset of buildings with infill wall with similar conditions of construction in the 1990s. From the observation of Fig. 17 we see that they both show a good correlation of fundamental period with number of storeys but they are positioned in a parallel way. The upper $T = 0.037N + 0.20$ corresponds to buildings with less percent of infill walls (upper middle class), whereas the lower $T = 0.035N$ corresponds to buildings with higher percent of infill walls (social care housing

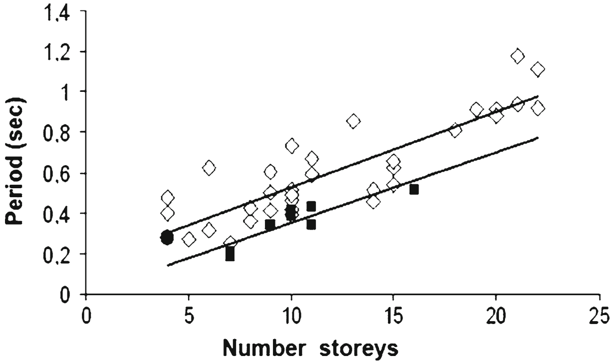

Fig. 17 Correlation of fundamental periods (T) with number of storeys (N) for a subset of buildings which were object of special treatment: in dark squares are buildings of social care; in open diamonds buildings of upper middle class

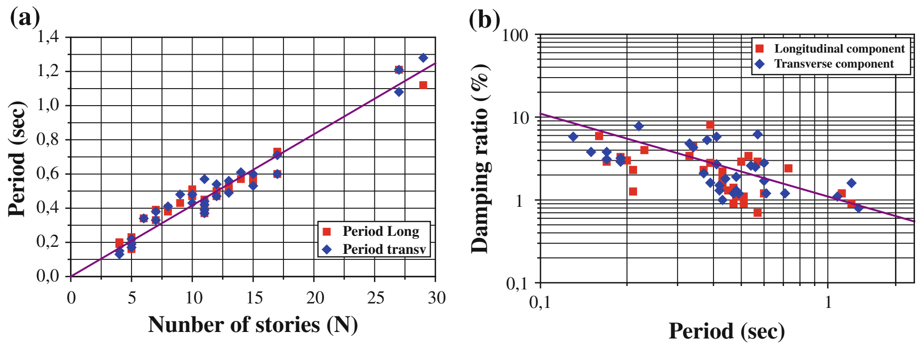

Fig. 18 Subset of a group of 37 recent RC buildings: **a** correlation of fundamental periods (T) with number of storeys (N), $T = (0.043 \pm 0.001)N$; **b** correlation of damping ratio (ζ) with fundamental period (T), $\zeta T = 1.1 \pm 0.6\%$ s

with smaller rooms). A dark circle corresponds to a special structure made essentially of brick masonry walls with lintels. These buildings were later the object of numerical analysis (Baptista and Oliveira 2004) to test the influence not only of infill panels, but also the inclusion of stairways, stiffening of beam-columns connections, etc., in the analytical model.

In a recently carried out second experiment, 37 RC buildings (with infill panels)[2] and with a number of storeys between 4 and 29 were tested under ambient noise with very sensitive sensors (Navarro and Oliveira 2006) (Fig. 18a). The relationship between the natural period and the number storeys N is $T = (0.043 \pm 0.001)N$ for swaying motion. These values are slightly higher than the ones obtained in 2004, and can attest the limits of uncertainty in these estimations, on the order of ± 0.004.

To this set, the Random Decrement Technique was applied. The value range of the damping factor for longitudinal and transverse components is from 0.7 to 8.1% and the average value is 2.6% with a large standard deviation of 1.7%. According to previous research (e.g., Kobayashi et al. 1987; Midorikawa 1990), the damping factor can be considered inversely proportional to the natural period and ξT value can be considered almost constant for buildings located on

[2] These buildings were all tested using the same instrumentation, procedure and within 2 weeks.

soils with same typology. The relationship between the damping factor and the natural period for swaying motion in Lisbon city (Fig. 18b) has been estimated as $\xi T = 1.1 \pm 0.6\%$ s.

In order to compare these α_N values with the results of the simple academic analytic exercise presented in Sect. 4.4, where no infills or any other modelling considerations were included, $\alpha_N = 0.12$ to 0.13 (N) in either direction X or Y with a dispersion of $\sigma^2 \sim 0.80$. We also found $\alpha_N = 0.11$ for torsion and $\alpha_N = 0.036$ for second modes.

5.3 Formulae for different regions and structural types (low amplitude motion)

The natural period and damping predictors of various types of buildings in different regions were analysed based on several collected full-scale data. In common they have a reinforced concrete (RC) structural system with "infill wall" panels. A few cases of steel frames (SF) are also presented. Each set of data is formed by a number of buildings, ranging from 20 to a couple of hundred, with a quite uniform distribution of storeys.

Most results are presented in Table 6 and Fig. 19. Besides these values we can add data observed in Taiwan (Hong and Hwang 2000), in Grenoble (Farsi and Bard 2003), in Nice (Dunand et al. 2003), in Egypt (Sobai et al. 2008), and in Potenza, with and without infills (Masi and Vona 2009). Overall information indicates similar trends on the relationship between natural period and number of storeys for the RC buildings analysed with the exception of Mexico City, where soft soils play a very important role, which shows much larger periods for the same number of storeys. In the USA, natural periods of modern buildings of several types (Chopra and Goel 2000) (RC MRF, RC SW and Steel frames) taken from weak motion monitoring not exceeded 0.15 g are essentially 2.5 times the values obtained elsewhere. The reason for such a difference is due, not to amplitude of motion, but to the geometric characteristics of USA structures, the non-existence of "infill brick walls" and the considerable lower vertical loads commonly practiced there.

6 Amplitude dependence

The seismic behaviour of a building under different input motions depends, in the first place, on both the amplitude of motion and on the proximity of resonance (period of input of motion equal to the period of the building). In the second place it depends on the capacity and integrity of the structural system to support that input. Damping for small amplitudes is also small though important for amplification.

All methods bringing out the possibility to evaluate resonance (periods and damping ratios of buildings) are important tools for anticipating their seismic behaviour. With the increase of magnitude of input motion there is a tendency to increase the period of the content of input motion as well as an increase of the natural period of the building. This phenomenon in the building depends on the amount of damage that might be inflicted. Both tendencies, either for the predominant period of input and of the building, are in the same direction; an initial tuning between them causing resonance might disappear, depending on the rates of change of period in both cases. But also the opposite may appear, e.g., an initial linear de-tuning might turn into tuning at a later stage.

Recent monitoring of buildings, which already recorded a few earthquake inputs, are important elements to quantify the effect of non-linearity and evaluate the reliability of the methods that exclusively use linear techniques. They are also essential keys to eventually allow corrections of the linear methods accounting for the non-linear effects.

Let's give a few examples already available in the literature.

Table 6 Summary of results on periods and damping for different regions

Authors	City (Country)	No. buildings	$T(N)$	$T(H)$	$\xi(T)$
Kobayashi et al. (1987)	Mexico City (Mexico)	20 RC	$T = 0.105N$		$\xi T = 4.0\%$
Midorikawa (1990)	Santiago de Chile (Chile)	107 RC	$T = 0.049N$		$\xi T = 0.8\%$
Midorikawa (1990)	Villa del Mar (Chile)	21 RC	$T = 0.049N$		$\xi T = 1.2\%$
Lagomarsino (1993)		182 RC+SF			$\xi = 0.0073 + 0.007T^{-1}$
Kobayashi et al. (1996)	Granada (Spain)	21 RC	$T = 0.051N$		$\xi T = 2.0\%$
Enomoto et al. (1999)	Almeria (Spain)	34 RC	$T = 0.05N$		$\xi T = 0.8\%$
Espinoza (1999)	Barcelona (Spain)	25 RC	$T = 0.089N + 0.032$		
Enomoto et al. (2000)	Caracas (Venezuela)	57 RC	$T = 0.06N$		
Sánchez et al. (2002)	Adra (Spain)	39 RC	$T = 0.049N$		
Navarro et al. (2002)	Granada (Spain)	89 RC	$T = 0.049N$		$\xi T = 2.1\%$
Messele and Tadese (2002)	Addis Ababa (Ethiopia)	28 RC	$T = 0.057N$	$T = 0.018H$	
Satake et al. (2003)	(Japan)	205 RC+SF		$T = 0.015H$	$\xi T = 1.4\%$
Dunand et al. (2002)	Grenoble (France)	26 RC		$T = 0.015H$	$\xi = 0.7 \times T^{-0.25}$
Oliveira (2004)	Lisbon (Portugal)	193 RC	$T = 0.042N$		
Navarro and Oliveira (2004)	Lisbon (Portugal)	37 RC	$T = 0.045N$		$\xi T = 1.1\%$
Gallipoli et al. (2009)	Potenza, Senigallia (Italy)	65 RC		$T = 0.016H$	

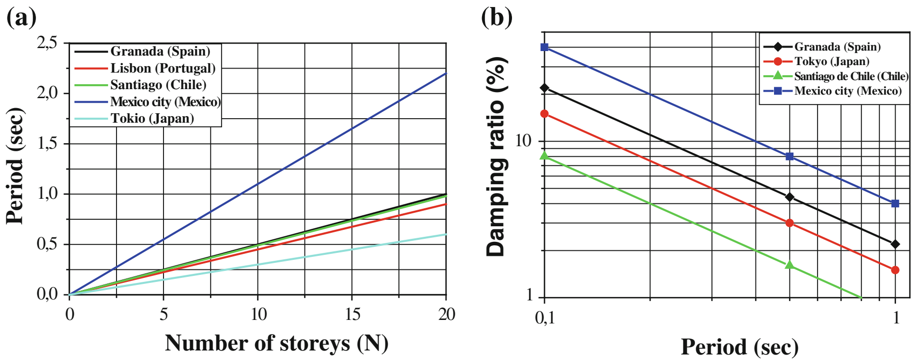

Fig. 19 Comparison of relationships for different regions (Cities around the world): **a** period (T) versus number of storeys (N); **b** damping ratio (ζ) versus period (T)

Table 7 Imperial County Service Building, Imperial County Earthquake, 1979, PGA = 220 cm/s^2. Increase in period for the first modes

Type of mode	1st mode (%)	2nd mode (%)	3rd mode (%)
E-W	45	50	50
N-S	40	45	
Torsion	60	50	

The Imperial County Service Building was subjected to a large event in 1979. The 6 storey RC structure was monitored with several acceleration transducers which recorded the event with a free-field PGA = 220 cm/s^2. The building was severely damaged in the ground floor columns due to poor design and construction, and was demolished. Several modes were identified and their values estimated along the course of the event (Table 7; from Pauschke et al. 1981). For this type of amplitude the increase of period was around 45–50% in all three modes, but after the event the damaged building recovered to a period 28% larger than the initial denoting a "self healing" process.

Trifunac et al. (2001) after the Northridge earthquake of 1994 observed at the damaged Holiday Inn subjected to a PGA = 270 cm/s^2 that an increase in period of 54% for the first mode took place during the event, with a similar recovery to a value of 22% larger than initial. They claim that the "self healing" was caused by settlement and compaction of soil with time due to aftershocks.

With the event of real time monitoring of large important buildings as in California and Japan, where many buildings are becoming instrumented with strong motion networks, it is possible to constitute data-bases with period estimation from moderate to large amplitude motion as a measure to control the structural behaviour of a building along its life-time. It will be possible in a continuous way to understand the evolution of frequencies and detect the presence of non-linearities due to damage suffered in any element (Celebi et al. 2004). There are already some data such as reported by Naeim (1997) which refers to a set of 17 buildings fully instrumented with strong motion accelerometers and that suffered various types of damage. All of them are described in detail, allowing the determination of correlations between damage and change of frequency from the initial linear state.

Todorovska et al. (2006), Mucciarelli et al. (2006), Snæbjornsson et al. (2006), Trifunac et al. (2008), among others have dealt with monitoring of buildings during long periods of time, some of which have undergone strong input motion. Some of the findings are as follows: Frequencies of a building may slightly change (within 5–10%) over the years due to various factors namely weak/moderate motion, weather conditions such as temperature and moister in the walls, occupation of building, changes in nearby buildings, etc. They also change during the course of an event without undergoing important damage (Clinton et al. 2006a). But damage is by far the most important issue contributing to changes in frequency, which may drop 30–50% depending on the degree of damage.

They claim that noise measurements produce the same results (as far as frequency of first modes and mode shapes) as forced vibration by means of eccentric mass vibrators, pullout tests or impulse actions, in spite of the fact that amplitude of induced vibration might attain one order of magnitude above noise. They only differ for the case of very stiff structures on very soft material, for which the soil behaviour controls the overall response. In this case forced vibration tests can excite the building emphasizing its dynamic properties. The decrease of frequency in the building for larger amplitude motions (above the PGA $> 10\,\mathrm{cm/s^2}$) depends on a number of items difficult to express in a simple formula. The non-linearity in the overall behaviour starts with small adjustments of structural and non-structural elements, change of

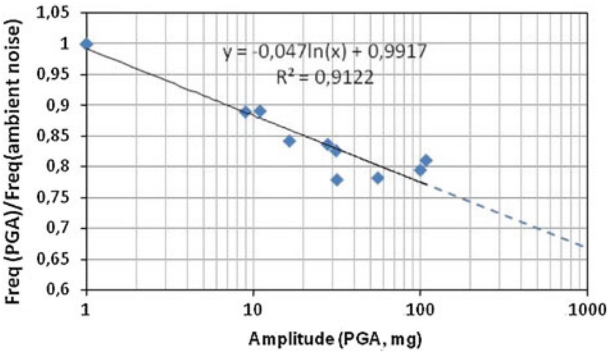

Fig. 20 Decrease in frequency of fundamental mode as a function of amplitude measured at roof level of two buildings in Mexico City (*filled diamond*); extrapolation to PGA > 0.1 g, in agreement with discussions on the 13 ECEE (Clinton et al. 2006b; Dunand et al. 2006)

Table 8 Influence of soil foundation on the values of α_N for the formula T = α N for Mexico (Muriá–Vila and González 1995)

Structural type	Stiff soil (15 buildings)	Soft soil (45 buildings)
RC Frame	0.100	0.126
RC Frame + walls	0.063	0.102
Masonry	0.040	0.073

modulus of elasticity of main structural elements and behaviour of soil. It also depends on the type of structure and on the engineering design level.

Dunkerley's method may be a good approach for including the soil influence in case of shear wall structures (Goel and Chopra 1998). These authors propose upper and lower limits in the equations, to be used in conjunction with the design acceleration and displacement, respectively. Calvi et al. (2006) and Clinton et al. (2006a) present an analysis for the Millikan building subjected to several earthquakes during its life time and concludes that for PGA in the order 0.1–0.3 g the decrease of fundamental frequency can attain 50%.

In the following we present an example illustrating the extension of decrease of frequency with amplitude.

Muriá–Vila and González (1995) studied a group of 60 buildings in Mexico City with different structural and soil foundation types and number of storeys up to 20 and they observe a clear decrease of frequency with increasing PGA (Fig. 20). They also concluded that there are various aspects which control the frequency for microtremors, being the type of structural system, the presence of "infill walls" and the soil (stiff or soft) the most important ones (Table 8).

As a summary of what is known and was a matter of discussion during the 13 ECEE in 2006 (Dunand et al. 2006), and in a very simplified way, we complete Fig. 20 with a proposal for extrapolation for PGA > 100 mg fitting Δf as a linear function of ln(PGA).

As far as code applications are concern, as reflected in Crowley and Pinho (2004, 2006) proposal establishing period-height equations for RC buildings without infills admitting some non-linear behaviour of the structures, or in many recent codes, i.e., EC (CEN 2003), values of α are almost one order of magnitude above the measured ones, even taking into consideration the corrections presented in Fig. 20 for moderate to large amplitude motion. These aspects of code values should be revised in different regions, according to the in-situ observations as the ones presented in this paper, in order to better reflect the real behaviour of buildings for the different limit states of structural safety.

To include uncertainty in the α values, a good solution has been proposed by Chopra and Goel (2000), based on experimental measures in MRF and SW (Goel and Chopra 1997, 1998), with two different formulae for frequencies, one for using with the acceleration spectrum and the other for the displacement spectrum, keeping certain conservatism in both determinations. They play with uncertainties in data measurements, using upper or lower estimates (average $\pm 1\sigma$) to cover this conservatism.

7 Notes and comments on the methodology

Our experience during many years of research in this area with in-situ measurements under the most varied conditions (in construction, full occupancy, repetition of measurements few years past, using various sources of excitation, etc.) and numerical modelling for typical structures, has led to the following comments:

- For structures which are regular to a certain extent, it is relatively easy to identify several modes using a single instrument: the first two modes in each horizontal direction and a torsional mode. Great care should be taken with the HVSR technique which may miss some low frequency modes as has happen in the case of the Solmar building.
- In cases of low rise buildings, irregular plan shapes and when there are adjacent buildings, the identification of the fundamental mode might be possible with a single instrument, but care should be taken with the presence of spurious frequencies that might come from adjacent buildings, predominant soil frequencies, etc.
- The influence of infill brick walls in RC buildings, now possible to represent in the numerical models is larger for more flexible structures. For example, for MRF buildings of 9 storeys, frequency may increase 70–100% in relation to building without infills. This increase reduces to 6–25% for the cases of MRF + SW. The presence of mortar can increase the frequency 2–6%.
- Observations made at the time of the construction and 30 years later may show an increase of frequency of 1–14%.
- Analytical modelling can reach results very close to the measured ones if infills and stairways are modelled.
- Use of rigid elements to stiffen the beam-column connection can go up to 20% whereas the consideration of stairways can account for an increase of 5%.
- Frequencies are altered by the presence of adjacent buildings. In the case of a building ($N = 18$) at a corner of a block this aspect is very clear. The adjacent buildings were of different heights, one with $N = 11$ and the other with $N = 6$. Even though no numerical computation was made for this particular building, structural symmetry would suggest similar frequencies in both directions but a difference of 22% in frequency is in agreement with the presence of those two adjacent buildings. And the overall stiffness has been increased as observed in the correlation equation where this building is slightly stiffer than similar structures of the same height but if considered isolated.
- Push-over techniques and other non-linear analyses should start the computations with a linear structure showing their initial fundamental frequencies, which are the ones measured at in-situ testing for low amplitudes. The first seconds of the response of a building is in many instances in the 2nd or 3rd modes due to higher frequencies of the initial input P-wave vibrations. Only at a later stage, will the fundamental modes be excited and the building is already in its 2nd mode (see Fig. 2a to observe this effect).

- Non-linear analysis may be concentrated in some part of the structure such as an open ground floor or at a level where infills collapsed, and not creating any problem on the above storeys.
- Consideration of cracked beam/column cross-sections (CEN 2003) or the non existence of infills in the cases these are important to the structure, is very critical for frequency assessment and consequently for the design force method (RSA 1983), as forces given by Response Spectra are much lower than what the structure really receives with much higher frequencies. For displacement design (Priestley et al. 2007), as displacement becomes much larger with longer periods, this consideration of longer periods is not so critical. In any case we cannot neglect the first part of the motion where most of the behaviour is settled.

8 Final considerations

The simple method presented is a fine tool for an expedited characterization of in-situ lower frequencies of buildings. It shows great advantages in terms of effectiveness, accuracy and cost. In some cases it may require qualified expertise; moreover, for complex buildings, it might be impossible to obtain those frequencies with confidence.

Besides informing on the lower frequencies of a given building and from this point allowing some indication on the possible resonant effects with soil layers, the knowledge of the frequencies is of great importance to calibrate analytical models (check the value of elasticity modulus of various materials, check the influence of “infill walls” and of other always forgotten elements such as stairways, short columns, or characteristics as depth of columns and beams, border conditions, etc.)

The creation of a *data-base* with frequency and damping information on a large amount of different buildings, where elements dealing with the geometry, in plan and height, type of structure and foundation, soil characteristics, presence of “infill walls”, etc., is of great potential to provide a better knowledge of building performances. The greater the number of entries, the higher the confidence in the results. Dispersion is still very large and difficult to significantly reduce. It depends very much on the type of building and on the type of soil underneath and foundation. Only with a detailed selection of buildings and soils, may this dispersion be reduced.

The equation type $T = \alpha N$ is still the most common and simple way to express the variation of period for the different building types and regional situations, and has been adopted in many countries in Europe and Latin-America to represent their buildings, through clear differences in construction types, materials and existing loads. The great differences existing between numerical and experimental values can be explained if we introduce in the model more detailing than the one presently used in current analyses.

The influence of “infill walls” is clearly depicted from any in-situ measurement for the low amplitude ambient noise levels. The influence of amplitude is also a remarkable property of all these systems and we propose a way to extrapolate the values obtained in in-situ low amplitude motion to moderate amplitudes.

However, this topic needs further research to better understand the structural behaviour of RC buildings under moderate to strong inputs and on how to reflect this knowledge in the code applications for the different limit states of structural safety.

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