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The Influence of Ground-Motion Variability in Earthquake Loss Modelling

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Abstract. Earthquake loss models are subject to many large uncertainties associated with the input parameters that define the seismicity, the ground motion, the exposure and the vulnerability characteristics of the building stock. In order to obtain useful results from a loss model, it is necessary to correctly identify and characterise these uncertainties, incorporate them into the calculations, and then interpret the results taking account of the influence of the uncertainties. An important element of the uncertainty will always be the aleatory variability in the ground-motion prediction. Options for handling this variability include following the traditional approach used in site-specific probabilistic seismic hazard assessment or embedding the variability within the vulnerability calculations at each location. The physical interpretation of both of these approaches, when applied to many sites throughout an urban area to assess the overall effects of single or multiple earthquake events, casts doubts on their validity. The only approach that is consistent with the real nature of ground-motion variability is to model the shaking component of the loss model by triggering large numbers of earthquake scenarios that sample the magnitude and spatial distributions of the seismicity, and also the distribution of ground motions for each event as defined by the aleatory variability.

Key words: earthquake loss models, ground-motion variability, spatial correlation, spatial variability

1. Introduction

The estimation of possible losses due to future earthquakes is vital for emergency planners and for the insurance and reinsurance industries, and potentially also for seismic code drafting committees. Identification, characterisation and appropriate treatment of the uncertainties in the input parameters are amongst the major challenges associated with the development of earthquake loss models. This paper addresses the specific issue of the random variability associated with estimations of the earthquake ground-motion and its impact on earthquake loss calculations for building stock in urban areas, examining approaches that are used in practice and exploring their physical interpretation.

2. Aleatory variability in ground-motion predictions

The ground motions expected from future earthquake events are often estimated using empirical formulae, sometimes referred to as attenuation equations but preferably called ground-motion prediction equations (since they describe both the scaling of ground motions with earthquake magnitude and the attenuation of amplitudes with distance from the earthquake source). These equations generally predict the distribution of ground-motion amplitudes characterised by a logarithmic mean value that is defined as a function of parameters such as magnitude, distance, site classification and fault rupture mechanism, and the standard deviation of the logarithmic residuals, which are assumed to be normally distributed about the mean. This standard deviation is hereafter referred to as sigma, since it is generally represented by $\sigma[\log(Y)]$, where Y is the ground-motion parameter being predicted, such as peak ground acceleration (PGA) or the response spectral ordinate at a particular period. A given ground motion due to a particular earthquake scenario can be characterized by the number of standard deviations that separates the logarithm of its value from the logarithmic mean. This quantity is generally referred to as epsilon (ε) and would take a value of zero for median values, -1 for 16-percentile values and 1 for 84-percentile values.

Sigma is considered to represent the aleatory variability in ground-motion prediction, which can be thought of as the apparent randomness in observed motions with respect to the predictive model, and is interpreted as being inherent variability that cannot be reduced without changing the predictive model. This is contrasted with epistemic uncertainty, which is the component of the ground-motion prediction that results from incomplete knowledge of the earthquake process and which can therefore, in theory, be reduced through the acquisition of additional and better data. An example of epistemic uncertainty is the difference in median values obtained from two equally valid prediction equations derived for the same region, although the total epistemic uncertainty may actually be much larger than this difference if the strong-motion dataset for that region is small and therefore, likely to be biased (which may be the case even in areas with seemingly abundant ground motion records, such as California). The epistemic uncertainty in ground-motion predictions, if considered at all, is generally handled through the use of logic trees, a tool that can be employed for both deterministic and probabilistic approaches to hazard analysis (Bommer *et al.*, 2005). Extending the logic-tree methodology to earthquake loss calculations is simple enough, even if it can result in very large computational effort, but the interpretation of the output may not be so straightforward, since opinions are divided about whether the mean hazard is the most meaningful measure (Abrahamson and Bommer, 2004;

McGuire *et al.*, 2005; Musson, 2005). However, herein the focus is exclusively on the aleatory variability and further discussion of epistemic uncertainty is outside the scope of this paper.

As defined in the random effects model by Abrahamson and Youngs (1992), sigma can be considered to be made up of two components of variability, the first being from one earthquake to another with the same magnitude and rupture mechanism, the second being from one location to another at the same distance and with the same site classification during one earthquake. Herein the former is referred to as inter-event variability and the latter as intra-event variability; the total sigma is the square-root of the sum of the squares of the inter- and intra-event sigma values:

$$\sigma_{\text{total}} = \sqrt{\sigma_{\text{inter}}^2 + \sigma_{\text{intra}}^2} \qquad (1)$$

Whilst sigma cannot, by definition, be reduced for a particular model, refinement of the predictive model can lead to lower values of sigma, although efforts to date have not resulted in large reductions of the aleatory variability (Douglas, 2003). The inclusion of additional source parameters, such as stress drop, could result in lower inter-event variability but at the cost of then having to deal with the epistemic uncertainty associated with this parameter for future earthquakes. Intra-event variability could be reduced by improved site classification parameters and others related to azimuth and directivity.

Bommer *et al.* (2003) present an equation for the prediction of horizontal PGA and spectral acceleration which accounts for the style-of-faulting. The intra-event variability is seen in Figure 1 to be greater than the inter-event variability, which agrees with the findings of other ground-motion prediction equations (e.g. Ambraseys *et al.*, 2005; Boore, 2005). The reason for the higher intra-event variability is likely to be due to the many processes which lead to a spatial variation of ground motions (such as the source-to-site path through the crust, the rupture velocity and the deep and local geological structures beneath the sites) that are not being adequately represented in the ground-motion prediction equation through distance and site-classification parameters alone. Figure 1 shows that the influence in sigma with period, in the short-period range, is mainly due to the increase in inter-event variability, which may reflect the fact that longer period radiation is dependent on gross features of the seismic source.

Leonard and Steinberg (2002) discuss how a major earthquake may affect several different populated areas and thus it is essential to study the ground motions at these areas simultaneously, which is not directly addressed in conventional seismic hazard analysis. For regional earthquake loss modelling, the intra-event (spatial) variability of the ground motion

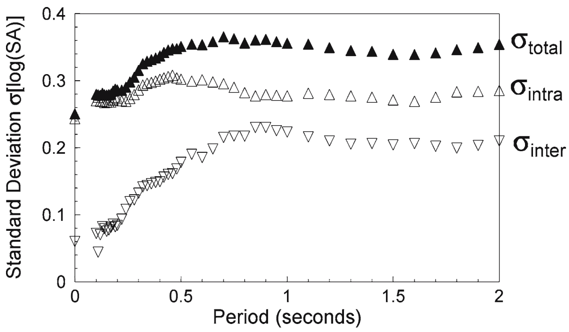

Figure 1. Contributions to the total aleatory variability of the predicted values of PGA and spectral acceleration (defined as the larger horizontal component) from inter- and intra-event variability, as presented in Bommer *et al.* (2003).

is particularly important and this constitutes a significant difference compared with ground-motion predictions for site-specific assessments.

For each earthquake scenario, the inter-event variability will cause the ground motions at all locations to be lower or higher than the median estimates from the predictive equation. For a given earthquake event, the intra-event variability, which Figure 1 shows tends to be larger than the inter-event variability, will lead to large fluctuations of the ground motion from one location to another, with some locations having stronger motion that the median estimates and others weaker motion. Figure 1 indicates that the maximum difference in ground-motion amplitudes at different sites with the same surface geology and located at the same distance from the source of the earthquake, will be appreciably larger than the difference between their median (for the specific earthquake) and the median estimate (from the predictive equation) for the specific combination of magnitude, style-of-faulting, distance and site classification. The distinction between inter- and intra-event variability of ground motion is of obvious relevance in earthquake loss assessment, since the larger intra-event component will tend to lead to greater damage and higher losses in some locations and reduced earthquake impact in others. If all of the variability were treated as inter-event, this would mean ignoring the fact that even for a high-stress drop event, which would have higher than average ground-motions, there would still be appreciable station-to-station variability.

However, the spatial variability of the ground motion, other than due to different distances from the source and different site classifications, is not entirely random: a degree of spatial correlation is generally found whereby the average variability in ground motions between two sites is a function of the distance separating the locations, with values reducing with decreasing distance below a certain threshold separation (e.g. Wesson and Perkins, 2001; Wang and Takada, 2005). Predictions for the distance beyond which correlation can be ignored vary: Boore (1997) suggests spatial correlations are negligible at distances of 10 km, whilst Wesson and Perkins (2001) found that a rapid decrease in correlation occurred at separation distances corresponding to the rupture length of the fault. The importance of the spatial correlation of the ground motion is less apparent: for a lifeline or a network system, the impact of a scenario earthquake at different points is important in assessing the resulting impact on the functionality; Rhoades and McVerry (2001) consider the spatial correlation in the hazard for this very reason. For the assessment of possible losses inflicted to the building stock in a large urban area, where each building can be treated as a separate and independent unit of exposure, the spatial correlation of ground motion will generally not be of sufficient importance to warrant the additional computational effort required for its incorporation. In the remainder of this paper, it is assumed that spatial correlations of ground motion, for the specific case of urban loss assessment models, can be neglected.

3. The influence of sigma in loss estimations

As mentioned previously, the treatment of uncertainties in the input parameters defining both the seismic demand (ground motions) and seismic capacity (structural resistance) is an integral part of earthquake loss modelling. Crowley *et al.* (2005) presented a systematic exploration of the sensitivity of a loss model for the Sea of Marmara region to variations reflecting epistemic uncertainties in the parameters defining the model. The conclusion of that study was that the impact of epistemic uncertainties in the capacity model was greater than that of the epistemic uncertainty in the seismic demand. This finding is actually rather encouraging since although in theory all epistemic uncertainties can be reduced, there is far greater scope for reducing the uncertainty in the capacity parameters (mainly through field investigations) than the epistemic uncertainty in ground-motion predictions, since the latter depends on the accumulation of new data for which it is necessary to wait for future earthquakes. There is even less scope for the reduction of the aleatory variability in ground-motion predictions in the short- or even medium-term, hence it is important to identify appropriate procedures for the treatment of sigma in earthquake loss models.

3.1. Calculating losses from single earthquake scenarios

For many applications, it is desired to estimate the impact of a single earthquake scenario on an urban area. This scenario may be the repetition of a large historical event or an earthquake identified by seismological studies. Such situations, from the perspective of ground-motion estimations, are comparable to deterministic seismic hazard analysis (DSHA), in which the aleatory variability in the ground-motion is either effectively ignored, by using only median values, or else an arbitrary selection of $\varepsilon = 1$ is used (e.g. Krinitzsky, 2002). For earthquake loss modelling in urban areas, the latter option of using the 84-percentile ground motions would be difficult to rationalise since it would imply that all of the ground-motion variability is inter-event and furthermore that a highly unusual (and unfavourable) event had occurred.

In the HAZUS methodology (Kircher *et al.*, 1997; FEMA, 2003), the probability of exceeding damage/limit states is found from a vulnerability curve, given a value of median displacement demand. The vulnerability curves in HAZUS are lognormal functions with a logarithmic standard deviation that models the damage state variability by combining the variability due to the capacity curve, the variability due to the demand spectrum (i.e. sigma) and the variability due to the threshold of the damage state. DBELA, a recently proposed probabilistic displacement-based loss assessment procedure (Crowley *et al.*, 2004), calculates the probability of exceeding a limit state in a similar way to HAZUS. However, vulnerability curves are not explicitly derived in DBELA, but the failure probability is calculated through the integration of the variability in the displacement capacity and in the displacement demand in the classical reliability formula (e.g. Pinto *et al.*, 2004).

Figure 2 shows how the probability of exceeding a limit state is influenced by aleatory variability (sigma) at different levels of spectral displacement, wherein it can be observed that the incorporation of sigma does not always lead to higher probabilities of exceedance: the influence of sigma depends on the level of spectral displacement.

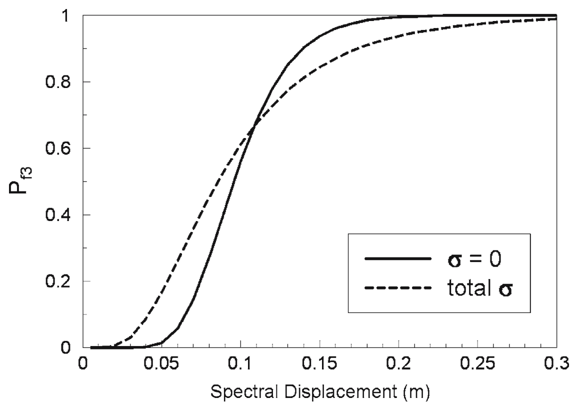

Figure 2. Vulnerability curves calculated with DBELA for a mid-rise frame with zero and total (100%) sigma in the demand displacement (P_{f3} refers to the probability of exceeding the third, collapse, limit state).

In order to explain the effect of aleatory variability on the vulnerability curves in Figure 2, a short example of the integration of the variability in the demand and capacity to obtain the probability of exceedance for different levels of sigma is presented in what follows. In particular two values of the vulnerability curve will be considered in detail: the probabilities of failure for spectral displacement demands of 0.08 and 0.35 m.

Vulnerability curves present the probability of failure, given a level of displacement demand. The formula that is used to calculate the probability of failure comes from the classical reliability formula:

$$P_f = \int [1 - F_D(x)]\, f_C(x)dx \qquad (2)$$

where, F_D gives the cumulative distribution function of the displacement demand (i.e. the lognormal distribution) for a given median displacement demand, and f_C gives the probability density function of the displacement capacity.

The CDF of the demand is shown in Figure 3 for a median demand of 0.08 m, for the cases both with and without aleatory variability. If the displacement demand is deterministic (no aleatory variability) then a step curve is produced for the CDF, whilst a lognormal CDF is obtained when the variability is included.

Figure 4 shows a possible third limit state displacement capacity probability density function, PDF, for a building class, for which the area beneath the curve is unity. In order to calculate the probability of failure for a median displacement demand of 0.08 m, the PDF has to be multiplied by the complementary of a demand CDF (i.e. 1-F_D) with a median displacement demand of 0.08 m (see Equation (2)). The two graphs are plotted together in Figure 4; by multiplying the values of the CDF by the PDF, the distribution that results can be integrated in order to obtain the

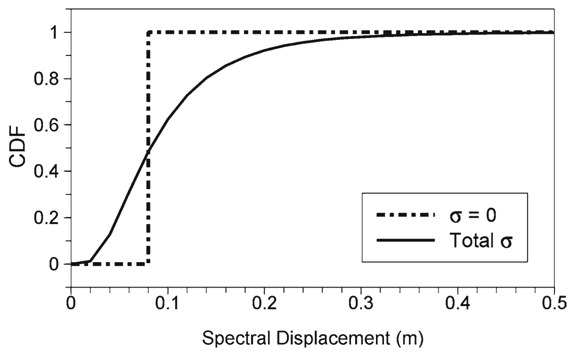

Figure 3. CDF of demand for a median displacement of 0.08 m, both with and without aleatory variability.

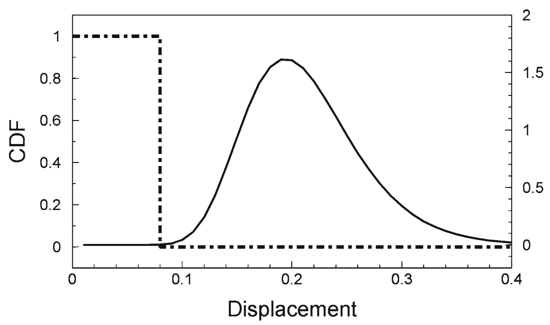

Figure 4. PDF of displacement capacity (solid curve) and (1-CDF) of displacement demand for a median displacement of 0.08 m with no aleatory variability.

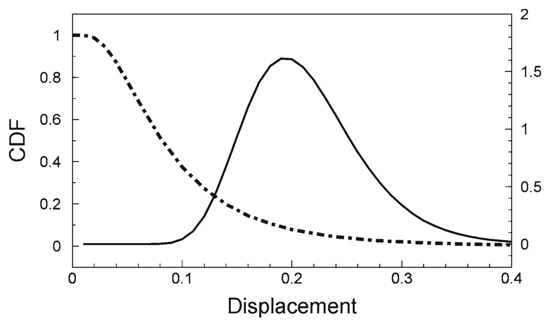

Figure 5. PDF of displacement capacity (solid curve) and (1-CDF) of displacement demand for a median displacement of 0.08 m with 100% aleatory variability.

probability of failure. The same graphs are shown in Figure 5, but this time the aleatory variability has been included and so the (1-CDF) curve is no longer a step curve. The same procedure as before is carried out to calculate the probability of failure, and in this case, the probability of failure could be greater when the aleatory variability is included because the whole of the PDF is multiplied by a value of the complementary CDF, unlike when the aleatory variability is ignored and for values above the median displacement demand, the PDF is multiplied by zero.

The same graphs are shown for a median displacement demand of 0.35 m in Figure 6 (no aleatory variability) and Figure 7 (with aleatory variability). When the displacement demand is defined deterministically, it is understandable that the exceedance probability reaches unity more rapidly because once the median demand is greater than the highest fractiles of displacement capacity (see Figure 6), the area under the combined PDF and (1-CDF) curves will be unity. In this case there is no uncertainty in whether the displacement demand could be lower than the median demand.

However, when the variability in the displacement demand is accounted for (Figure 7), once the median displacement demand is greater than

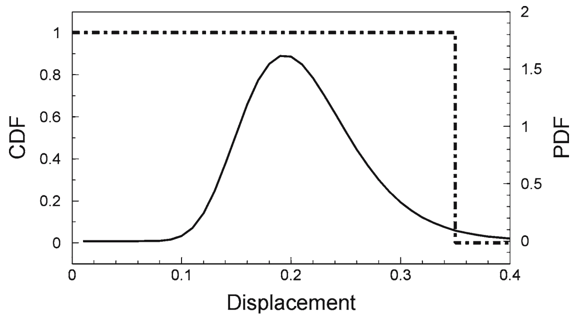

Figure 6. PDF of displacement capacity (solid curve) and (1-CDF) of displacement demand for a median displacement of 0.35 m with no aleatory variability.

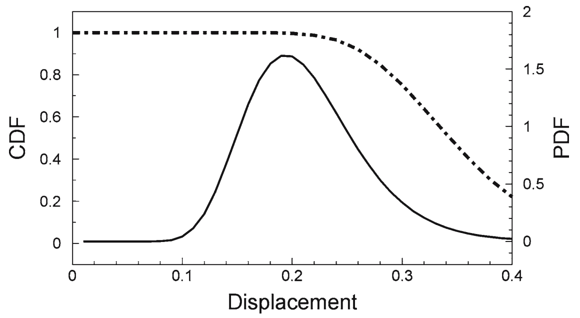

Figure 7. PDF of displacement capacity (solid curve) and (1-CDF) of displacement demand for a median displacement of 0.35 m with 100% aleatory variability.

the highest fractiles of displacement capacity, there is a chance that the displacement demand could be lower than this median demand leading to a probability of failure lower than unity.

3.2. Calculating probability of losses due to earthquakes

Although estimation of the impact of a single earthquake scenario can be very useful, particularly for communicating seismic risk to the public and to decision makers, for many applications, including decision-making processes within the insurance and reinsurance industries and in seismic code drafting committees, it is necessary to estimate the effects of many, or even all, possible future earthquake scenarios that could impact upon the urban areas under consideration. In such cases, the purpose of the loss calculations is to estimate the annual frequency of exceedance (or the return period) of different levels of loss due to earthquakes.

An attractive option for representing the demand in a loss model for multiple earthquakes is to first perform a probabilistic seismic hazard analysis (PSHA) and then perform a convolution of the hazard curves at different locations with the exposure and vulnerability of the building stock. This approach was used, for example, by Cao *et al.* (1999), representing the shaking in the form of macroseismic intensity, and is also employed in FEMA 366 (FEMA, 2000), which uses the capacity spectrum method of HAZUS.

In the FEMA 366 approach the hazard is estimated for each census tract in terms of PGA and SA at 0.3 and 1.0 s, for a number of selected return periods. Smooth acceleration-displacement spectra are then constructed for each return period using the probabilistic hazard estimates. The proportion of buildings in each damage state, for each building class, is predicted using the methodology outlined in HAZUS. In this approach, the ground-motion variability is stripped out from the calculations of vulnerability curves described in the previous subsection, since it is accounted for directly in the PSHA calculations. A mean damage ratio (MDR) can be calculated using the damage ratios defined in HAZUS and this is then multiplied by the value of the building stock to obtain a value of loss at each census tract. The losses at all tracts are then integrated. The aggregated loss is plotted against each annual exceedance probability (obtained from the annual frequency of exceedance using, e.g., a Poisson occurrence distribution) to obtain a loss probability distribution (as exemplified in Figure 8); the expected value of the distribution is calculated to give the annual estimated loss (AEL).

There are several potential problems encountered with the approach of performing a convolution of probabilistic seismic hazard estimates with the exposed building stock, including the fact that it becomes very difficult to correctly model aspects of the over-damped long-period spectral displacements used in the capacity spectrum method. For example, the corner period of the constant displacement plateau is directly dependent on

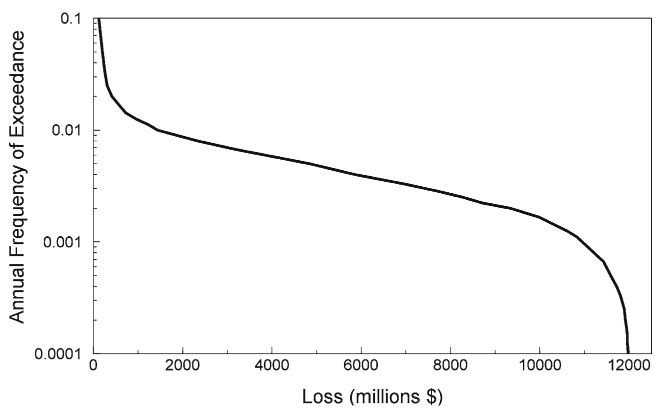

Figure 8. Illustrative example of a loss frequency distribution curve.

earthquake magnitude (Joyner and Boore, 1988) and the scaling of the 5%-damped displacement ordinates for higher levels of damping displays a clear dependence on the duration of the motion, which in turn is dependent upon the magnitude (Bommer and Mendis, 2005). However, within the context of this paper, the main problem with the PSHA approach is the resulting influence of the aleatory variability in the ground motion. In effect, the PSHA at each individual location could be considered as asking the question of how large could the ground motions become if a sufficiently low frequency of exceedance is considered? The answer to this question comes from considering larger magnitude earthquakes (which occur less frequently) at shorter source-to-site distances (and hence are less common because the proportion of the seismic sources in which they can be triggered is reduced) and ground motions with larger epsilon values (i.e. greater exceedance of the median estimates from the predictive equations). For consideration of a single site, this does not present any problems, but when the method is used for loss estimation over a large urban area the same question is being asked simultaneously at all locations; this effectively means that all of the variability is being treated as inter-event variability. Such treatment of spatial (intra-event) variability as temporal (inter-event) variability is the ergodic assumption challenged by Anderson and Brune (1999).

If one accepts that these shortcomings render the FEMA 366 approach unviable, the only option is to model the seismic demand through the triggering of large numbers of earthquake scenarios that are compatible with the seismicity model. One approach, used by Bommer *et al.* (2002) in developing an earthquake loss model for Turkey, is to trigger scenarios based on the earthquake catalogue, assigning frequencies inferred from the recurrence relationships and distributing the events throughout seismic source zones to compensate for spatial incompleteness in the historical and instrumental catalogues. For each scenario, the median ground motions are calculated at each site and used as input to the calculation of the vulnerability following the HAZUS procedure. The same approach could be followed with DBELA, calculating the median ground motions and embedding the ground-motion variability in the calculation of the probability of exceedance of a given limit state. A potential problem with this approach, however, is precisely the treatment of the ground-motion variability. In this approach, the variability is incorporated into the vulnerability calculations as explained in Section 3.1; the physical interpretation of this is that at each location, the ground motions are assumed to be randomly distributed according to the lognormal distribution defined by the sigma value. In a loss model, the exposure will generally be organized in areas, defined by geographical boundaries such as postal codes or municipalities, with the calculations performed for a representative point within each area. In such a framework, the representation of the ground motion as being log-normally distributed within each reference area is not unphysical, but it does make the implicit assumption that the variability is entirely intra-event and also that the spatial distribution is random with respect to the building stock location rather than with respect to geographical location. The approach could be made more rigorous by only using the intra-event component of the variability in the vulnerability calculations and triggering each magnitude-location scenario a few times, adjusting in each case the median ground motions and the associated recurrence frequency in accordance with the inter-event component of the variability.

Another important issue that arises using the approach of modelling multiple scenarios and incorporating ground-motion variability into the vulnerability calculations is the estimation of the exceedance frequency of the losses. In this approach, the exceedance frequency is based only on the recurrence frequency of the earthquake scenarios hence the loss curves cannot be compared with those obtained from the PSHA-based approach. In effect, the scenario-based approach poses a different question, which is what will be the average distribution of losses for different earthquake events with different likelihoods of occurrence, whereas the PSHA-based approach is asking the question of how great could the losses become if these same earthquake scenarios produce uniformly high-ground motions.

The most robust and rigorous approach for calculations based on multiple scenarios may be to use stochastically generated suites of events. Examples of the use of multiple earthquakes scenarios, or stochastic event sets, can be found in the literature (e.g. Grossi, 2000; Liechti *et al.*, 2000; Zolfaghari, 2000); however, it is not always made clear in these papers how and where the aleatory variability in the ground motion is being accounted for: different loss curves will be obtained depending on where the aleatory variability is included in the loss calculations. Bazzurro and Luco (2005) discuss the possibility to use Monte–Carlo simulation to generate multiple scenario earthquakes for loss estimation. Once a seismic source zone model has been prepared, as per conventional PSHA, the spatial and temporal distribution of earthquakes within that region is thus defined. Monte–Carlo simulation is essentially the controlled use of random numbers and can be applied to the generation of stochastic earthquake catalogues (or event sets) covering thousands or millions of years (see, e.g. Musson, 1999; Smith, 2003) by generating random numbers from a uniform distribution between 0 and 1 for each year of the catalogue. For active faults with characteristic earthquakes, if the random number is less than the annual probability of earthquakes on that fault then this implies an earthquake in that year; whilst for seismic source zones, if the number falls between the annual probability of events of the minimum and maximum magnitude thresholds in the Gutenberg–Richter relation, then an earthquake is predicted to occur and its magnitude is therein obtained.

For each generated event in the stochastic earthquake catalogue, ground motions can be simulated at selected sites using the appropriate ground-motion prediction equation and two random draws from the standard normal probability distribution: the first defines the inter-event variability (number of logarithmic standard deviations, $\varepsilon_{\mathrm{inter}}$) for the earthquake and the second defines the intra-event aleatory variability ($\varepsilon_{\mathrm{intra}}$) at each site (see Section 2). Clearly, for a given earthquake, the influence of the inter-event variability will be the same throughout the region of interest, whilst the influence of the intra-event variability will be different at each site. Constraints could be imposed on the sampling of $\varepsilon_{\mathrm{intra}}$ for closely–spaced sites to account for spatial correlation of the ground motion. For each earthquake scenario, defined in terms of (M, D, $\varepsilon_{\mathrm{intra}}$, $\varepsilon_{\mathrm{inter}}$), the displacement demand at each site can be calculated and the loss determined by convolution of the demand and capacity (using the methodologies outlined in previous sections, but with the aleatory variability removed from the vulnerability curves) and then these losses are sorted in order of size. The annual exceedance frequency of each loss is then calculated by finding the number of times a given loss is exceeded over the length of the catalogue. An illustrative example of the generation of stochastic ground motions and their use in loss calculations is given in the companion paper by Crowley and Bommer (2006).

4. Discussion and conclusions

The premise that underlies this article is that the aleatory variability in ground-motion predictions cannot be neglected in the calculation of seismic hazard and consequently in the estimation of seismic risk. This premise then has direct implications for the way in which seismicity and ground motions are modelled in earthquake loss estimation for extended urban areas. The various approaches for the generation of loss-probability curves as discussed in the previous section, and the treatment of sigma in each, are summarised in Table I.

One approach, which is attractive because of the great computational efficiency that it offers, is to use PSHA to obtain hazard curves at many locations and then calculate the exceedance frequency of the resulting losses by convolution of the ground motions at specified return periods with the exposure (building stock) and its vulnerability. However, performing independent PSHA calculations at several locations simultaneously effectively treats the hazard at all sites as being perfectly correlated and the

Table I. A comparison of the different approaches for the derivation of loss probability curves

Seismic demand	Value of σ used to predict site ground motions	Value of σ in vulnerability calculation	Modelling of variability	Annual frequency of exceedance (AFOE)	Comments
PSHA hazard curves	σ_{total}	0	Temporal	Due to recurrence relationship and σ_{total}	Spatial variability ignored; sites assumed to be perfectly correlated; ground motions overestimated
Scenario Earthquakes (Magnitude, Location)	0	σ_{total}	Spatial	Due to recurrence relationship only	Variability assumed to be all intra-event; AFOE not comparable with that from PSHA
	σ_{inter}	σ_{intra}	Temporal and Spatial	Due to recurrence relationship and σ_{inter}	AFOE not comparable with that from PSHA
	σ_{inter} σ_{intra}	0	Temporal and Spatial	Due to recurrence relationship and σ_{total}	Spatial correlation can be accounted for; AFOE comparable with that from PSHA

intra-event component of the aleatory variability, which represents spatial variability, is treated as inter-event variability. Since the intra-event component of the total variability is generally larger than the inter-event component (Figure 1), this approach can be considered to overestimate the ground motions when applied to several sites simultaneously because even in the strongest (high $\varepsilon_{\text{inter}}$) earthquakes, the ground motions would spatially be log-normally distributed about the median motion.

This shortcoming of the PSHA-based approach to earthquake loss estimation makes it necessary to model the seismicity through multiple earthquake scenarios and to generate the ground motions at all sites of interest, and the resulting losses, due to each earthquake event. Inevitably this leads to appreciably larger computational effort than when PSHA is employed. The earthquake scenarios may be defined by the historical earthquake catalogue, perhaps with modifications to account for spatial incompleteness, or through Monte–Carlo simulations based on the seismicity model inferred from the historical catalogue and tectonic considerations. Regardless of how the stochastic earthquake catalogue is obtained, there are fundamentally two options for how the ground-motion variability can be treated: it can either be directly considered in the ground-motion prediction at each location for each event, randomly sampled from the aleatory distribution described by sigma, or else incorporated into the vulnerability calculations at each location, as is done in the HAZUS and DBELA approaches, for example, when considering single earthquake scenarios. The latter option may appear to be more attractive because fewer scenarios need to be triggered and consequently it may appear to be computationally less demanding. However, there is a penalty for including sigma in the vulnerability calculation, which will reduce this advantage: if only the median ground motion at each site is calculated and the aleatory variability is considered in the vulnerability calculations, the implicit assumption is that the variability is entirely intra-event, which for longer response periods (that are of particular relevance to assessment methods based on equivalent linearization) is not realistic (Figure 1). This issue could be addressed by only incorporating the intra-event component of the variability into the vulnerability calculations and then accounting for the inter-event component by multiple triggers of each earthquake scenario. In such a way, any possible advantage in terms of computational effort when compared to random sampling of the ground-motion variability is likely to be entirely lost. Another reason for not adopting the approach of using multiple earthquake scenarios with the aleatory variability embedded within the vulnerability calculations is that the frequency of exceedance of the losses cannot be correctly calculated. For each earthquake scenario the calculations do not consider the probability of exceeding different levels of ground motion but rather assume that the spatial distribution of the ground motions will be exactly lognormal about the median amplitude.

The inevitable conclusion, therefore, is that the only physically meaningful approach is to use Monte–Carlo simulations to trigger multiple occurrences of each magnitude-location scenario, with frequencies assigned according to the recurrence relationships, and then calculate the ground motions for each of these events through random sampling of the intra-event variability and assigning exceedance frequencies on the basis of both the intra- and the inter-event variability. This approach also allows spatial correlation of the ground motion to be incorporated. By triggering very large numbers of magnitude-location-$\varepsilon_{\mathrm{intra}}$–$\varepsilon_{\mathrm{inter}}$ scenarios, the frequency of exceedance of losses can be robustly estimated and, provided the calculations include suitable book-keeping, the disaggregation of the losses can be easily performed. To illustrate the use of multiple scenario earthquakes in an earthquake loss model and to compare the results with the use of the PSHA-based approach, an example case study is presented in a companion paper by Crowley and Bommer (2006).

This study also raises doubts about the feasibility of validating a loss model through comparison of predicted and actual damage statistics for a particular earthquake, since in the absence of a very dense strong-motion network, the actual spatial distribution of the ground shaking will not be known by virtue of the high intra-event variability. However, with a certain number of well-distributed recordings of the earthquake, an estimation of the inter-event variability could be made, i.e. the $\varepsilon_{\mathrm{inter}}$ could be calculated. Furthermore, for a given location where a strong-motion recording of the event is available, if it is assumed that within a limited radius of the accelerograph the variation of the motion is small (which may not be the case) and moreover that the variation of the building stock characteristics approaches that assumed for the whole building class in the region, it may be possible to make meaningful comparisons between damage statistics and damage predictions.

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