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ORIGINAL PAPER

Abolishing Platonism in Multiverse Theories

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Abstract
A debated issue in the mathematical foundations in at least the last two decades is whether one can plausibly argue for the merits of treating undecidable questions of mathematics, e.g., the Continuum Hypothesis (CH), by relying on the existence of a plurality of set-theoretical universes except for a single one, i.e., the well-known set-theoretical universe V associated with the cumulative hierarchy of sets. The multiverse approach has some varying versions of the general concept of multiverse yet my intention is to primarily address ontological multiversism as advocated, for instance, by Hamkins or Väätänen, precisely for the reason that they proclaim, to the one or the other extent, ontological preoccupations for the introduction of respective multiverse theories. Taking also into account Woodin's and Steel's multiverse versions, I take up an argumentation against multiversism, and in a certain sense against platonism in mathematical foundations, mainly on subjectively founded grounds, while keeping an eye on Clarke-Doane's concern with Benacerraf's challenge. I note that even though the paper is rather technically constructed in arguing against multiversism, the non-negligible philosophical part is influenced to a certain extent by a phenomenologically motivated view of the matter.

Keywords Absoluteness · Concept expansion · Constituting subjectivity · Continuum hypothesis · Forcing extension · Multiverse · Multiverse dependent logic · Ontological multiversism · Ordinals · Subjective interpretation

1 Introduction

A key issue in the current debate among set-theorists about the concept of multiverse is the way one may assess the argumentation in favor or against multiversism in set-theory and the mathematical foundations. In a deeper sense this

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debate concerns the content and the terms in which the conflicting views about multiversism or universism are brought out to the fore. Of course the debate on the mathematical universe vs. multiverse approach has almost nothing to do with the corresponding debate among cosmologists regarding the single vs. multiple universes approach insofar as by mathematical universe one normally understands the conventional set-theoretical universe V in terms of which all meaningful mathematics can be done.[1] As to the multiverse approach the question of whether this term stands for a multiplicity of mathematical universes distinct from the conventional one V, the answer depends on the philosophical leanings one might have in terms of a presumed ontological (or other) content of mathematics, something that will be made clear concerning my own philosophical attitude in the next sections.

A key motivation in the setting and elaboration of the pro-multiversism arguments is the fact that some set-theoretical statements of fundamental importance, first and foremost the *Continuum Hypothesis* (**CH**),[2] whose independence in terms of existing mathematical models has puzzled set-theorists and logicians for decades, are thought to be better approached in an 'ontological' sense by multiverse theory. This means that that their independence would not be a de facto result owing perhaps to an insufficiency of the existing axiomatical machinery of the **ZFC+AC** (Zermelo-Fraenkel plus the Axiom of Choice) theory, something that reflected also Gödel's intuitions from the time of *What is Cantor's Continuum Hypothesis?* (1947), but it would rather reflect a situation in which, for instance, the statement of **CH** and its negation are 'ontologically' justified as being true in distinct universes belonging to a multiverse, the latter thought of as a determinate reality at least by ontological multiversism. My intention in this article is to show that key set-theoretical concepts such as absoluteness, countability, generic extension and continuity-as-unity employed with a purely logical content hinge on an irreducible 'affinity' of the multitude of universes inside a multiverse possibly interpretable in subjective-constitutive terms and thus making its conception a rather 'heuristic' tool for handling certain set-theoretical pathologies. Besides, if the set-theoretical quest for maximization and unification in the context of large cardinals theories and the convergence toward an all-encompassing set-theoretical universe V in the sense of Cantor's Absolute can be achieved by means of inner models and forcing extensions within the 'standard' universe V, why bother to enter into the complexities and, after all, the ontological subtleties of multiverse theories?

To be more concrete one may put up an argument against multiversism on account of the claim associated with the ontological multiverse view that "There is no absolute background concept of set and of other set-theoretic notions, such as set, ordinal, cardinal", a consequence of which is that the constructible universe L may not have the same ordinals as the set-theoretical universe V (Ternullo 2019, pp. 60–61). This is due to the fact that the concepts ordinal^L and ordinal^V are not the same

[1] I point out, however, that in adding 'almost' next to 'nothing to do' I reserve a clue to a possible interpretational connection between the mathematical and physical versions of universe - multiplicity of universes in this section (par. 5).

[2] The generalized version of the *Continuum Hypothesis* will be abbreviated in the text as **GCH**.

as there is no absolute background concept of ordinals. It follows in a multiversist sense, contrary to the mainstream view of most logicians and set-theorists including Maddy's account of mathematical naturalism in Maddy (2002), that Gödel's axiom $V = L$ is not inherently restrictive[3] in a context of discussion guided by the drive toward maximization and unification in the foundations of mathematics.

As it happens often with newly appearing trends in the philosophy of mathematics the multiversist approach has branched out in distinct versions depending on the particularities in the formal treatment of the subject matter and possibly on the epistemological or philosophical predilections of the researchers of the field. Far from being of a compelling character a categorization of the multiversist approach proposed by Ternullo could be as follows: **(a)** naive multiversism succinctly put as the idea that "no single model $\mathcal{M}$ of a theory of sets T, should be viewed as 'special', as being the universe of sets, the collection of all sets" **(b)** instrumental multiversism in which multiverses might be an important mathematical tool, yet their ontological as well as epistemological status could be irrelevant in view of their usability *in rem*. Research programmes in this specific category can be regarded Woodin's set-generic multiverse and Friedman's *et al* Hyperuniverse Programme (HP) **(c)** ontological multiversism which is the view that a multiverse is a determinate, independently existing reality "consisting of particular entities, the models of set theory". A major instance of such platonistic conception is Hamkin's version of multiversism (Ternullo 2019, pp. 47–51).

I will mostly address ontological multiversism, even if there is no such thing as a tight compartmentalization of the versions above, for the main reason that except for the elaboration of major questions having been made in line with this version it stands, regarding the multiverse story, as a showcase of the platonistic trend in the philosophy of mathematics. In view of my subjectivist philosophical inclination, more precisely one phenomenologically oriented, my argumentation against ontological multiversism will be accordingly calibrated in the next sections in favor of a subjectively founded approach both at the level of theory and at that of epistemological or ontological content. In terms of the latter it seems that such an approach might give a sense to the question of whether one can draw a parallel between the concept of a plurality of universes in cosmology and that of the mathematical multiverse. For, in the author's view, only on account of the constitutive-eidetic[4] capacities of a subject as invariably applied independently of context can this question be a meaningful one given that in the first place a material, spatiotemporal universe should have no relation whatsoever with the purely formal character of a mathematical universe. But this story sits in a different and rather uncharted territory to be told here in any further detail.

[3] Gödel's axiom $V = L$ essentially identifying the set-theoretic universe V with the constructible universe L is generally thought to be restrictive in the sense of imposing the predicative formation of sets across ordinals in L to the universe V. Notably $V = L$ has been proved consistent with **ZFC** theory if **ZFC** is. See for details Kunen (1982), Ch. VI.

[4] By eidetic laws or eidetic attributes in the world of phenomena one can roughly communicate to a non-phenomenologist what on subjective grounds holds of the existence of objects or states-of-affairs as regularities by essential necessity and not by mere facticity. One may consult a propos E. Husserl's *Ideas I*: Husserl 1983, *Engl. transl.*, pp. 12–15.

If my approach can be regarded as an argumentation against multiversism articulated mostly in subjectively founded terms, one may yet find arguments against (Hamkins') multiversism in purely logical-mathematical terms, even if in essentially metatheoretical ones. This is N. Barton's argumentation in Barton (2016) in which a sound ontology of universes associated with referent models (e.g., mathematical models correlated with set-theoretical concepts) can be refuted on the grounds of Skolem-Löwenheim Theorem. As known, the latter means that if a theory has an infinite model then it can have non-isomorphic models of every infinite cardinality including the countable one. It is known by this theorem that first-order theories are completely unable to uniquely determine their models up to isomorphism, consequently one cannot pick out a model (universe) in a unique fashion and after implementing whatever mathematical construction might choose move to another model-universe and thus vindicate Hamkins' multiversism in the first place. Furthermore, given that choosing a precise universe corresponding to a particular concept of set is impossible as one may only refer to models using concepts expressed as first-order axiomatisations, one can reach an infinite regression of set-theoretical backgrounds, an option out of which would be a genuine selection of particular set-theoretic backgrounds as more privileged than others. This would entail, contrary to ontological multiversism, that some key model-theoretic constructions should be absolute with respect to these models, which would in turn "require a stock of absolutely understood concepts, sufficiently rich in character that we can identify determinately a class of set-theoretic backgrounds" (ibid. pp. 197–198 and pp. 203–204).

As most set-theorists consider the *Continuum Hypothesis* (**CH**) an absolutely undecidable proposition, a major theme of discussion could be whether we may reasonably argue for an ontological foundation of **CH**, an assumption that might help impose a de facto plurality of universes and thus vindicate the strong ontological multiversism for which a multiverse is a determined reality consisting of actually existing distinct entities in the respective set-theoretical constructions. Indicative of the key influence absolutely undecidable statements like **CH** bear on the conception of the multiverse is Steel's attempt through his *MV* axioms to prescribe a collection of universes (sharing anyway **ZFC** and large cardinals theories) which would agree with each other on **CH**, resulting in a (presumably provisional) failure that has pushed him into searching the limits of expressibility of the multiverse language. Even if by an elementary forcing argument it can be shown that the multiverse may have a uniquely definable world included in all others, called the core of the multiverse, it is still true by Steel's confession that "Neither *MV* (axioms) nor its extensions by large cardinal hypotheses of the sort we currently understand decides whether there is a core to the multiverse, or the basic theory of this core if it exists" (Steel 2014, pp. 168–169).

It happens that **CH**, being a $\sum_1^2$ statement expressible in terms of second-order predicate calculus, presupposes a firm grasp of the concept of all subsets of an infinite countable set which is a persisting source of controversy over the epistemological aspects of the **CH** undecidability. Moreover it not only raises the question of whether second-order logic is in fact logic as it appeals to the concept of all subsets of an infinite set on which we do not have a firm grasp but also of "whether full second-order logical truth is a sufficiently determinate notion to be of interpretational use" (Horsten 2019, p. 87). On this account the **CH** undecidability and the general context in which it is brought about may be further seen as pointing to the subtleties of an extra-theoretical, indeed subjectively founded level of discourse for which more will be said in the epistemologically-philosophically oriented Sect. 6.

It should be made clear at this point that platonistic tendencies in multiverse theories and more generally in mathematical foundations are to be construed in the general sense conveyed by the term mathematical platonism which can be succinctly said to be the view that mathematics 'exists' independently of human beings 'out there somewhere' hovering in the realm of platonic eternal ideas. In this view the existence of mathematical objects is objective independently of our knowledge of them so that even the most abstruse of mathematical objects, for instance uncountably infinite sets, sets with larger than $\aleph_1$ cardinalities, space-filling curves and so on, lie 'somewhere' as definite objects, with definite properties available to get to be known if they are not already, in a way that even undecided (e.g., the *Continuum Hypothesis*) mathematical hypotheses have a definite answer waiting for the humans to find it out (Davis and Hersh 1982, p. 68, 318).[5] My own general view being at odds with mathematical platonism, I'll try to show in the next how the platonistic sense of multiversism can be weakened in intra-theoretical terms in favor of an approach that takes into account the constitutive capacities of a subjective factor and the kind of mathematical intuitions it implies. After all, even though the ordinary working mathematician would shun all talk about the epistemological or ontological content his purely mathematical work might have, and the same would go for a multiversist doing the hard mathematics of the subject, he would still hardly avoid the question of the epistemological-ontological implications of his results and the philosophical attitude that serves them better. For instance, Feferman's claim that the **CH** hypothesis "has ceased to exist as a definite problem in the ordinary sense and that even its status in the logical sense is seriously in question"[6] is indicative of the fact that specific philosophical preoccupations and their interpretational agility may underlie even *prima facie* pure mathematical stuff.

An option could be to deny the outright adoption of universes within the multiverse as really existing in a platonic sense and instead construe the ontological multiversism's use of concepts within the framework of a concept expansion, a view of M. Buzaglo in Buzaglo (2002). The main principles of this approach are the following:

[5] A kind of mathematical platonism is often attributed to Gödel, yet even if this has a solid base it is also true that Gödel especially in his later years was allured by Husserl's phenomenology and this was reflected in a certain sense in the view he held of mathematical objects as implying a special kind of intuition we have of them forced by mathematical objects upon us. For more details on this matter the reader may see Livadas (2019).

[6] See Feferman, S: 'The Continuum Hypothesis is neither a definite mathematical problem nor a definite logical problem', p. 2, a revised version of Feferman (2011).

"(1) concepts are flexible constructs; (2) the expansion of a concept is a law-like, forced process, that is, it is guided by the 'stretching' of some pre-established laws (axioms) which force the concept to evolve in a way which is unavoidable and, above all, (3) concept expansion gives rise to new objects.

I set out to comment on these principles, especially on (3), in view of the mathematical experience of the introduction of nonstandard entities within a standard system and further from a more philosophically oriented position with regard to the introduction of absoluteness in the context of formal-mathematical theories in the sense of a concept having a metatheoretical origin and having accordingly such kind of consequences.

2 Concept Expansion and Inter-Theoretical Constraints

It is true that after the Greeks' use of incommensurate magnitudes as a direct product of concrete mathematical practice, talk about nonstandard quantities took a more formal shape through the introduction and use of infinitesimals by Newton and Leibniz in the course of the systematic development of mathematical calculus. My attention will be primarily drawn to the formal introduction of nonstandard elements by axiomatical means, e.g., to A. Robinson's introduction of nonstandard numbers.

In this sense the extensional part of nonstandard analysis whose significant parts can be considered Robinson's axiomatical construction in Robinson (1966), and Zakon's nonstandard ultrapower constructs in Zakon and Robinson (1969), is thought to be fundamentally based on extensions of the classical Cantorian objects of mathematics, whereas the intensional part of non-standard analysis is based on the subjective observations of a potential 'observer' implemented in a local and non-Cantorian way inside an intersubjective universe.

Robinson's quest of ideal elements, in a model theorist's saturation approach, is implemented inside the domain of consistent enlargements of standard axiomatical structures in a way that is conceptually in accordance with both Leibniz's idea of an extended mathematical universe in the sense of the preservation of standard properties in the extended one and Husserl's idea of consistent enlargements of (relatively) definite formal-deductive systems developed in his Göttingen lectures of 1901 (Livadas (2005), p. 118–119). This means no proposition can be proved inside a B-model of the B-enlargement $H_B = K \cup K_B$ of a stratified set of sentences K which when restricted in all its variables to the domain of K cannot be decided in the model M of K (Robinson 1966, pp. 33–34). In fact from an ontological viewpoint Robinson's nonstandard numbers are by-products of theoretical constructions involving universal quantification formulas inside an indefinite horizon of finite sets of constants occurring in a stratified set of sentences K and may be accordingly intuited as exceeding any standard entity of common intuition.

However as a reminder that an extended theory generated by the 'stretching' of existing standard axioms may be essentially constrained by principles on the level of ground theory proven themselves undecidable, nonstandard theories rely in a substantial way on the *Axiom of Choice* or its logical equivalents or on certain *ad hoc* extension principles in other alternative nonstandard theories. For instance, the

Axiom of Choice or its logically equivalent Zorn's lemma are applied both in Robinson's introduction of nonstandard elements by the construction of B-enlargements of standard models and in Zakon's non-constructive version of equivalence classes of infinite sequences modulo an ultrafilter over the set of natural numbers (Livadas 2005, p. 125). In fact there is no concept or principle 'embedded' in a nonstandard theory that outright contradicts, at least on the ground level, the standard intuition of sets as well as the intuitions of well-ordering, of ordinals, of global choice, etc.

In the formal-deductive level, as Robinson conceded, nonstandard models are constructed within the framework of contemporary (classical) mathematics and "thus affirm the existence of all sorts of infinitary entities" (Robinson 1966, p. 282). Yet one is compelled to adjoin to their axiomatical system actual infinity principles, for instance the *Axiom of Choice* or its stronger forms, either in a direct fashion as, e.g. in Zermelo-Fraenkel-Boffa Set Theory with Choice (ZFBC), or indirectly in the details of proofs in the construction of ultraproducts and ultrapowers. The fact is that these axioms or principles presuppose a notion of actual, complete infinity non-eliminable in analytical terms in a sense that fits with natural intuition (Livadas 2005, p. 126). If this brief discussion on nonstandard theories has something to offer to our present reflection on multiversism it is precisely the view that the introduction of new (nonstandard) objects by concept expansion is not unconstrained with regard to standard concepts that eventually appeal to 'finitistic' subjective intuitions.

If Boolean modelization in the formal treatment of foundational questions of mathematics can be considered a concept expansion, then a special place in the current debate has Woodin's generic multiverse which is in fact the Boolean-valued multiverse V_α^B, and the way it is associated with Ω-logic (Woodin 2011, pp. 14–24). The use of the Boolean-valued multiverse or of what would become the set-generic multiverse seems to be an incisive method to elucidate statements of the complexity of **CH** by pointing to what such statements require in terms of solving resources. Even in disregarding that Woodin's assumptions are generally too strong in contradistinction to the naturalness of the *Continuum Hypothesis*, the fact that Ω-provability and consequently Ω-conjecture are conditioned on the introduction of a universally Baire set $A \subseteq R$ to account for the validity of a sentence φ, definitely weakens Woodin's multiverse approach to the truth or falsity of **CH** as it becomes bound to the specific topological structure of the subspace A of the space of reals R. In fact, Woodin's topological constraint put on Ω-provability is not a sole exception in terms of weakening a viable multiverse perspective by such metatheoretical ambivalences. One can even point to 'inner' contradictions in the proof-theoretic structure of multiverse arguments. Take, for instance, Hamkins' assertion that any transitive model M can in principle be 'continued' so that it may ultimately become a model of **ZFC**+ $V = L$, in support of the claim that the principle $V = L$ is not inherently restrictive; "More generally, for any transitive model, we may first collapse it to be countable by forcing, and then carry out the previous argument in the forcing extension. In this way, any transitive set M can in principle exist as a transitive set inside a model of **ZFC** + $V = L$", (Hamkins 2012, p. 435). The assertion hinges on the conviction that the ontological multiverse is associated with a refutation of the concept of absoluteness for such fundamental set-theoretic notion as it is the concept of ordinals.[7] Yet absoluteness criteria are necessarily applied in all forcing methods including of course the one leading to the statement above.

A particular case against multiversism may be found in Steel's adoption of Weak Absolutism (WA), a compromising statement stating that a reference universe $\dot{V}$ not captured by MV axioms,[8] can still make sense as a definable world in its own right included in all other worlds of the multiverse. This means that the multiverse may be reducible to one of its members enjoying a 'preferred' reference status and thus possible to be termed as the core. However WA is dependable on other mathematical conjectures, in particular on the Woodin axiom H which is part of an argument in favor of the concept of a core of multiverse that leads in fact to certain inconveniences. More specifically the Woodin-proposed axiom (i.e., axiom H) roughly stating that the set-theoretical universe V looks like HOD^M for models M of the axiom of determinacy,[9] conducive in the multiverse language to the statement 'the multiverse has a core, and it satisfies Axiom H', can neither be proved as consistent with all large cardinal hypotheses nor as implying **GCH** (while implying **CH**) (Steel 2014, p. 171).

In the Appendix,[10] counting on axiom's H pedagogical value on the matter, I set out to show that the proof-theoretical machinery leading to the axiom $V = \mathrm{HOD}$, and *a fortiori* to its stronger version $V \subseteq \mathrm{HOD}^{V[G]}$, applies certain 'standard'[11] notions developed in the context of large cardinals and inner model theories without the need of taking recourse to radically different concepts which would in turn imply the need for alternative universes.

This will help further strengthen my claim that if 'standard' notions inherently linked with a sense of absoluteness are the gold standard of any meaningful and *a fortiori* insightful mathematical practice, then at least ontological multiversism to the extent that aspires to their 'undermining' may give credit to Ternullo's words that ontological multiversism, for all its professed merits, "is also the most controversial and problematic conception among those examined" (Ternullo (2019, p. 66). The conclusion can sound quite pessimistic having in mind that the other versions of multiversism (the naive and instrumental ones) fail to provide an alternative and supported 'in things themselves' ontological standing.

7 Gödel had noted that there is no element of randomness in the definition of ordinals and hence neither in sets defined in terms of them. He found this particularly clear in von Neumann's definition of ordinals insofar as it is not based on any well-ordering relations of sets which may well involve some random element as applied to various ranks of infinity. See (Gödel 1965, p. 87).

8 Steel's multiverse (MV) axioms in a two-sorted multiverse language are found in Steel (2014, p. 165).

9 A set is ordinal definable, **OD**, if and only if it is definable over the universe of sets from ordinal parameters, and is hereditarily ordinal definable, **HOD**, in the case that itself and all members of its transitive closure are ordinal definable. The precise statement of a version of axiom H is found in Steel (2014, p. 171).

10 The reader who wants to avoid Woodin's extremely technical proofs may well skip the Appendix without losing anything of the general picture.

11 I use the term standard in quotation marks to refer to certain notions in set-theoretical constructions, like the transitivity of $\in$-inclusion or the well-foundedness, having some direct or indirect relation to the concept of absoluteness or to other concepts linked with natural intuition, to distinguish from the conventional term standard as used in non-standard mathematics.

3 Refuting Hamkins' Argumentation for Multiversism

I set off by drawing attention to Barton's claims that if the intent of Hamkinsian multiversism is to study the multiverse through analyzing models of **ZFC**, he has to stick to a determinateness of **ZFC** which is naturally preconditioned on a determinateness and a well-understanding of the notion of proofs and well-formed formulas, "indicative of the fact that certain notions need to be taken to be absolute. It has long been noted that certain mathematical concepts are necessary for the expression of metalogical definitions. By adhering to a very strong form of relativism, the Multiversist undercuts the very concepts required to properly express her own view" (Barton 2016, pp. 16–17). It turns out that Barton's view may be justly thought compatible with a key position of this article, namely the underlying necessity of absoluteness of certain notions, even though my own argumentation on the matter is justified primarily on subjectively founded grounds.

In the next my position on formal-mathematical grounds against Hamkins' multiversism will be primarily presented by a two-fold argumentation:

First, the refutation of Hamkins' (and generally the proponents' of ontological multiversism) position that one may identify a set concept with the model of set theory to which it gives rise, in other words the refutation of the position that a set concept has no self-standing content but it should be rather referred to the description of the set-theoretic universe in which it is instantiated. On this account a key argumentation of ontological multiversism, namely the discarding of the absoluteness of the concept of set, and further of ordinals, of well-orderings and well-foundedness of sets, can be critically weakened by a counter-argumentation articulated on subjectively founded grounds. This is a position that seems partly in resonance and partly in conflict with the view expressed by Hamkins himself, namely: "The assertion that there are diverse concepts of set is a metamathematical as opposed to a mathematical claim, and one does not expect the properties of the multiverse to be available when undertaking an internal construction within a universe. That is, we do not expect to see the whole multiverse from within any particular universe. Nevertheless, set theory does have a remarkable ability to refer internally to many alternative set concepts, as when we consider definable inner models or various outer models to which we have access" (Hamkins 2012, p. 417). However the fact is that by Skolem's paradox the metamathematical meaning attributed to the notions of inside and outside a universe in a certain sense runs in conflict with Hamkin's position above, namely in terms of the ways an 'inhabitant' of a countable model M, in contrast with someone 'living' outside M, thinks of the countable ordinal ω_1^M (ω_1^M, the first uncountable ordinal relativized to the countable model M) as an uncountable one insofar as there is no function in M from ω onto ω_1^M. Obviously this would not be the case if there was no absolute notion of ω (ω, the first limit countable ordinal) to alter the assumption of the hypothetical 'inhabitant' of M about ω_1^M with regard to that of an 'external observer' to the model M (see Kunen 1982, p. 141).

Second, by refuting a central argument of Hamkins' in defending the ontological multiversism, namely his belief that forcing extensions, for instance the forcing extension $V[G]$ of the universe V (this latter taken as just one universe within a multiverse), can have a real existence even though the generic filter G may not be found in V.[12] As the argument goes even in the non-existence of the generic filter G within V, the extension $V[G]$ can be simulated in V either by the forcing principles or by the Boolean-valued structure V^B. Of course Hamkins concedes to the impossibility of establishing an isomorphic copy of the forcing extension $V[G]$ into the ground model V but he argues that the forcing methods come "maddeningly close" to this and in any case one can have a high accessibility to $V[G]$ from the ground model due to the character of forcing methods (Hamkins 2012, p. 420).

I argue against the position that forcing extensions can have an (ontologically meant) self-standing existence, particularly on the naturalist account of forcing,[13] by a metatheoretical interpretation of the same arguments by which Hamkins defends his brand of multiversism with regard to forcing extensions.

More concretely: Hamkins proved[14] that for any forcing notion P one can have an elementary embedding of the universe V into a class model $\overline{V}$ for which there is a $\overline{V}$-generic filter $G \subseteq \overline{P}$ so that the forcing extension $\overline{V}[G]$ is a definable class in V and $G \in V$. Consequently the universe V may have full access to the model $\overline{V}[G]$ including the generic set G and the way V is mapped to $\overline{V}$ for they are all definable classes in V. The models $\overline{V}$ and $\overline{V}[G]$ are not necessarily transitive or well-founded which makes that their membership relation $\overline{\in}$ is not the standard one associated with transitivity and well-foundedness. However the proof of this theorem relies on the Axiom of Foundation, in virtue of appealing to Scott's trick concerning reduced equivalence classes[15] to form $\overline{V}$ as the collection of equivalence classes of names τ inside V, with the generic set G being the equivalence class $[\dot{G}]_U$ in V where U is an ultrafilter in V 'imitating' the role of the generic set in standard forcing (for details: Hamkins (2012), pp. 423–424).

The payoff for the proponents of (ontological) multiversism is that even though the extended universe $\overline{V}[G]$ may not actually exist, since the proof of Hamkins' theorem does not provide a concrete V-generic filter G, it behaves like it exists in the sense that it is definable in all stages of its construction within V. Yet it is questionable whether the extended model $\overline{V}[G]$ would qualify for an ontologically

[12] The case of forcing extensions on the set-theoretic universe V as presumably legitimate universes in their own right can be addressed by universism, on the one hand, as simply model-theoretic representations within V with nothing non-trivial added to the semantics of V, and, on the other hand, by easily accommodating this situation by appealing to the reflection theorem and the downward Skolem-Löweheim Theorem to define a countable model on which to implement forcing (Barton 2016, p. 4).

[13] The naturalist account of forcing in Hamkins' approach is closely related to the Boolean valued model approach to forcing primarily in the sense that one can implement forcing entirely within **ZFC** without restriction to the kind of models, in particular the countable transitive models in the classical forcing techniques, or without the need to appeal to the metatheory in the proof-theoretic machinery.

[14] See (Hamkins 2012), p. 423.

[15] Scott's trick is a method for giving a definition of equivalence classes in a proper class by referring to the levels of the cumulative hierarchy $V_{\alpha \in ON}$. It is basically a way of assigning representatives to cardinal numbers in **ZF** theory without the Axiom of Choice using the fact that for every set A there is a least rank in the cumulative hierarchy when some set of the same cardinality as A appears. As such Scott's trick makes an essential use of the Axiom of Foundation (Axiom of Regularity).

existing universe given that firstly, it is entirely definable within V and secondly, in the naturalist account of forcing the definability in V is proved with the help of the Scott trick for defining equivalence classes which is conditioned in turn on the Axiom of Foundation. The Axiom of Foundation, however, is inherently linked with the absoluteness of well-ordering,[16] in the understanding of course that the notion of absoluteness of set-theoretical properties is radically denied by ontological multiversism.

At another point Hamkins argues that second-order categoricity arguments "requires one to operate in a context with a background concept of set" which as a matter of fact 'undermines' the absoluteness of the concept of finite natural numbers in terms of the natural procession '1,2,3,...' and *so on*. Hamkins goes as far as to wonder whether the scheme '1, 2, 3, . . .,' and *so on* is meaningful as an absolute characterization of natural numbers or whether there should be the other way round, namely a background concept of set would be *ante* to the concept of natural numbers, in the sense that two different concepts of sets need not agree even on the concept of the natural numbers. Following Hamkins' reasoning, Peano's categoricity proof and therefore the structure of the finite numbers as uniquely determined are conditioned on a background concept of set, and further on a vague understanding of which subsets of the natural numbers really exist, *a fortiori* on the vagueness of existence of the totality of the subsets of natural numbers. Hamkins asks therefore, "why are mathematicians so confident that there is an absolute concept of finite natural number, independent of any set-theoretic concerns, when all of our categoricity arguments are explicitly set-theoretic and require one to commit to a background concept of set?" (ibid., pp. 427–428). Consequently in Hamkins' view one might, just like in forcing methods, modify models of arithmetic by the invention of new technicalities and this would generate new models that would shake the confidence in a unique model of arithmetic, in analogy with the way forcing methods generate new forcing models in extension of ground-level ones.

However I find these arguments deficient for the following reasons: On the purely set-theoretical level one can have nonstandard models of arithmetic only as conservative extensions of the existing standard one. To the extent the axiomatical assumptions applied to generate the extended models are compatible with the existing ones over the standard domain, there can be in principle no reasonable claim to the existence of ontologically diverse models of arithmetic something that might shake our belief to a unique natural number structure.

On a metamathematical, in fact an extra-theoretical level, Hamkins' claim that set-theoretical concerns undermine the absoluteness of natural number statements by reducing them to set-theoretical ones, may fall short on subjectively founded grounds. By this I mean that the intuition of natural numbers and their procession in terms of '1, 2, 3, . . .' and *so on*, on the one hand, and the fixed concept of the set of all subsets of natural numbers, on the other, may be thought of as pointing to two diverse intuitions corresponding to distinct modes of object-constitution on the part of a subject. (**a**) The intuition of natural numbers as discrete enactments of a subject's consciousness (of an a priori intentional character in phenomenological

[16] See lemma 4.2, p. 124 in Kunen (1982).

terms), and (**b**) the intuition of a collection of objects, material or abstract ones as mental representations, in the form of a totality in the actual now. I point out that the intuition of a sequence of natural numbers is associated in Van Atten et al. (2002) with the phenomenologically motivated principle of two-ity applied in connection with the progression of choice sequences.[17]

On the other hand the intuition of a set as a totality may point, on phenomenological grounds, to Husserl's view of a set as an original objectivity pre-constituted by an act of colligation of disjunct objects-elements which is 'complemented' by what he called a retrospective apprehension, an act making possible the thematization of a collectivity of objects pre-constituted by the act of colligation into an identifiable and re-identifiable object possibly posited as a substrate of judgments (Husserl 1939, pp. 246–247). This latter act of thematization, in terms of the continuous unity of consciousness, points to the origin of the constituting temporal consciousness in the various denominations Husserl gave to this concept (absolute or pure ego among others). It is remarkable that well in advance of his properly meant transcendental phenomenology phase to which belong these views, Husserl had acknowledged that corresponding to the set-presentation is an objectivity proper to it (a position that probably influenced later Gödel's conceptual realism[18]), namely the set or the multiplicity, for which the "indefiniteness of the left-open continuation of concatenation still precedes the nominalization of the plural 'some' and then makes the transition to nominalization, where a multiplicity, a set, an aggregate results—all, properly understood, synonymous words" (Husserl 2019, p. 172).

In consequence it seems hardly convincing to argue in favor of an ontologically founded translatability between a concept of sets as potentially modifiable by the manipulation of corresponding universes and the concept of absoluteness (e.g., of natural numbers), in taking account that these concepts may be attributable to diverse intentional-constitutive capacities of a rational subject's consciousness. Naturally then Hamkins is led to the controversial assertion that "the multiverse view allows for many different set-theoretic backgrounds, with varying concepts of the well-founded, and there seems to be no reason to support an absolute notion of well-foundedness" (Hamkins 2012, p. 439).

In view of these arguments, Hamkins does not seem to me to have produced convincing grounds for an ontological need of multiversism neither in his metatheoretical arguments nor in his technical work on the modal logic of forcing and his (so-called) set-theoretic geology.

4 Why the Continuum Hypothesis Cannot be Resolved by Multiversism

Everybody in the community of set-theorists, not to say of mathematicians in general, knows that Cantor's *Continuum Hypothesis* is a virtually unresolved set-theoretical question since Cantor proposed his famous conjecture about it, $2^{\aleph_0} = \aleph_1$.

[17] See for details: (Van Atten et al. 2002, pp. 206–210).

[18] See for details (Livadas 2019).

Yet even though the question is still an undecidable one it has spawned, throughout decades of mathematical toil to assign it a truth value, a good deal of inspiring and innovative theories and results owing perhaps to the attractiveness and naturalness of the question given its claim to provide a link between the intuition of infinitely proceeding countability and that of the real-world or intuitive continuum. Of course one could go on and on with the philosophical discussions this question has raised, for instance, the view of Feferman that **CH** is an inherently vague question in Feferman (1999), or Martin's parallel view that "As long as no new axiom is found which decides *CH*, their case (*auth. note*: of those who argue that the concept of set is not sufficiently clear to fix the truth-value of **CH**) will continue to grow stronger, and our assertion that the meaning of *CH* is clear will sound more and more empty" (Martin 1976, pp. 90–91).

However as further discussion of this far-reaching matter is understandably not within the scope of this article, I will focus instead on some deficiencies of Hamkins' arguments for a non-bivalent position concerning the continuum question in the sense that it may have truth values acceptable within the context of the each time specific universe in which its proof is articulated. As with the main line of argumentation of this paper here also one may point to the fact that the existing proof-theoretic apparatus either for the consistency of the *Continuum Hypothesis* or of its negation with the existing **ZFC** theory is constrained to principles of a metatheoretical, in fact subjective foundation, that makes them determinately understood and absolute throughout any generic extension of a standard ground universe.

Hamkins argues that the mathematical knowledge of the universes in which **CH** or $\neg$**CH** statements hold would make hard to view these worlds as imaginary, in other words Hamkins expresses a common view among ontological multiversists that the **CH** question should not be characterized, as in standard terms, an undecidable question but as a question having a truth value corresponding to ontologically existing diverse universes of which we have abundant knowledge over decades of relevant mathematical experience especially after the introduction of the forcing method. For if by assuming Step 2 of the dream solution template[19] we have that $\Phi \Longrightarrow$ **CH** or that $\Phi \Longrightarrow \neg$**CH**, given a beyond doubt established sentence Φ, then for Hamkins either solution would mean that we nullify all those worlds in which the opposite case is true, and where we 'resided' and constructed the proof-theoretic machinery 'all those preceding years'. Hamkins' argument may be extended beyond the frame of the dream solution to include also Woodin's results though Ω-logic, in fact to any proof that decides the **CH** insofar as it would make illusory the experience gained so far on the matter.

As pointed out my arguments against the positions of multiversism in general, and on the question of **CH** in particular, are drawn primarily from the metatheoretical dimensions of the question. Having in mind that for Gödel the standard definition of a set through the impredicative expression of 'a collection or a

[19] Hamkins refers to the dream solution template in the following sense: Step 1: Produce a set-theoretic assertion Φ expressing an 'obviously true' set-theoretical principle and Step 2: Prove that Φ determines **CH**, i.e., $\Phi \Longrightarrow$ **CH** or $\Phi \Longrightarrow \neg$ **CH** (Hamkins 2012, p. 430).

multitude of elements x' already inserts a kind of vagueness to the semantics of **ZF** set theory, Feferman's concerns about the following problematic features of **ZFC** theory are even more telling:

"(i) abstract entities are assumed to exist independently of any means of human definition or construction; (ii) classical reasoning (leading to non-constructive existence results) is admitted, since the statements of set theory are supposed to be about such an independently existing reality and thus have a determinate truth value (true or false); (iii) completed infinite totalities and, in particular, the totality of all subsets of any infinite set are assumed to exist; (iv) in consequence of (iii) and the Axiom of Separation, impredicative definitions of sets are routinely admitted; (v) the Axiom of Choice is assumed in order to carry through the Cantorian theory of transfinite cardinals" (Feferman 1998, pp. 287–288).

As known the proof of the independence of **CH** is conditioned on at least the (ii), (iii), (iv) and (v) assumptions. The definition of a generic set in the forcing method is one obvious example of a non-constructive existence, while the totality of all subsets of the natural numbers must be assumed to exist to determine its cardinality. Even in proof steps where there is no explicit use of the assumption (v) of the *Axiom of Choice*, this latter is nonetheless implicitly applied through other statements or theorems conditioned in turn upon its acceptance.[20]

Besides, attempts to determine the **CH** question by other approaches are either dependent on set-theoretically ambivalent assumptions (Woodin's initial determination of the falsity of **CH** under Ω-conjecture) or on conjectures that need verification (Woodin's Ultimate-L hypothesis). Steel has the view that "None of our current large cardinal axioms decide CH, because they are preserved by small forcing, whilst CH can both be made true and made false by small forcing. Because CH is provably not generically absolute, it cannot be decided by large cardinal hypotheses that are themselves generically absolute" adding furthermore that there is no obvious way to state **CH** in the multiverse (MV) language he has laid down (Steel (2014), p. 163).

I have already provided some hints to the extra-theoretical, in fact subjectively founded (therefore not platonic) character of some guiding principles in the body of the established **ZFC** set theory such as those of set formation, absoluteness, the conception of infinities as complete totalities for which I enter into a more detailed discussion in the last section.[21] These principles are made part, in an explicit or implicit fashion, of the proof-theoretic machinery leading to the independence of **CH** in ways that would be 'immune' to the structural properties of another universe (within a multiverse) which by this reason alone weakens any claim to the ontological independence of universes. Consequently one cannot hope the **CH** to be placed in a radically new context by the multiversist approach to the extent that it is constrained by the same subjective-constituting principles as the universist one.

[20] See for instance the way by which the forcing conditions consisting of functions FN (I, J), with I arbitrary and J countable sets have, by the application of the Delta Lemma, the countable chain condition (*CCC*) so as to preserve cardinals among the ground and forcing model in the proof of the negation of **CH** (Kunen (1982), pp. 205–206).

[21] These concepts are discussed in more detail and in a phenomenological motivation in Livadas (2013); Tieszen (2005) and (Tieszen 2011).

5 Is There an Ontological Basis for a Multiverse Axiomatization?

A major motivation of the proponents of ontological multiversism is the possibility to resolve the question of absolutely undecidable statements in mathematics as it is the case with **CH** discussed in the preceding section. This will presumably provide a counterargument to Gödel's claim, namely that there should be a unique well-determined universe of mathematical objects where mathematical propositions would be true or false, by providing the alternative existence of 'parallel' universes working inside **ZFC** theory itself, hence not by a metamathematical approach nor as a consequence of Gödel's first incompleteness theorem. In the latter cases one would be led to an 'outside' view of **ZFC** theory, thus leading to ambivalences of reasoning to the extent that considering **ZFC** as referring to the standard universe V of our time everything should be in principle carried out within it.

In what follows I argue against the possibility of an 'ontological' axiomatization of multiversism, at least of the kind undertaken in Väätänen's work in Väänänen (2014). My primary arguments against this kind of axiomatization, which adds to the corpus of the known **ZFC** axioms some special axioms, termed multiverse dependent logic axioms, to ensure 'ontologically' diverse parallel multiverses, are again for the most part extra-theoretical and subjectively founded. These are the following:

(A) As it happened with Hamkins' approach, Väätänen assumes the domain of set theory as a multiverse of parallel universes in which variables of set theory simultaneously range over each parallel universe, thus pointing to the multiverse as a kind of Cartesian product of all its parallel universes (Väänänen 2014, p. 182). This assumption even as a mental image bears, in Väätänen's own admission, the deficiency of 'isolating' each universe from any other in the class and, what is more important from my point of view, is conditioned on the invariability of first-order logical propositions across the universes which would be unattainable if it weren't for the absoluteness of individuals and relations of individuals in terms of $\in$-inclusion in well-founded structures. Even if one discards any subjectively founded connection with the mathematical notion of absoluteness, still he might hardly interpret the intranslatability of the variables of set theory across the universes if the latter are to be thought of as ontologically distinct ones. All the more so, in case these variables are bound to quantifiers and by this token acquire, according to Quine, ontological claims insofar as they are construed as universals demanding attributes or classes as values (Quine 1947, pp. 74–77).

(B) Both the Axiom of Foundation and the Axiom of Choice, axioms of a strong metamathematical significance and susceptible of a subjectively founded interpretation, arise in multiverse theory for the same reasons as in a single universe theory. In the iterative concept of set-formation, sets are formed in stages and if these are going to be well-founded there must be an element x in each stage formed earlier than the actual set, so it is irrelevant whether this process will run, insofar as variables range simultaneously over universes, over one universe or over a multitude of universes. Further, as Väätänen concedes, the essence of the Axiom of Choice, that is, the act of choosing among an infinite class of sets is "problematic even in one universe when there are infinitely many sets to choose from [..]. The extra complication arising from choosing simultaneously in many universes is simply not part of the setup of multiverse set theory" (Väänänen 2014, pp. 190-191). Therefore, both at the level of **ZFC** theory itself and on metamathematical or further extra-theoretical, subjective grounds it makes no difference whether the act of choice in the specific sense is implemented over a unique universe or a multitude of universes. As known a standard practice of ontological multiversism is to apply first order logic in the structural conception of the multiverse which makes truth to be reflected in each structure of the multiverse.

(C) The way Väätänen chooses to establish a multiverse that would not be by metamathematical reasoning a trivial multiverse of all possible models of **ZFC** is the introduction of a team semantics described as a variation of ordinary Tarski semantics of first order logic.[22] This can be seen as essentially a means of formally expressing dependence and independence situations across the universes through a set of assignment functions defined on each universe M of a multiverse structure $\mathcal{M}$.

The critical step is to extend first order logic to multiverse dependent logic, i.e. the MD logic, by adding the dependence atomic formulas $=(\overrightarrow{y}, \overrightarrow{x})$ with appropriate axioms, which can be associated with the intuition: "the values of $\overrightarrow{y}$ functionally determine the values of $\overrightarrow{x}$ in the team." However the way it is applied by Väätänen, for instance, through the sentence

$$(\forall z)(\forall x)(\exists y)(=(y,x) \wedge \neg y = z)$$

which says that there is a one-one function from the universe to a proper subset, in other words the universe has infinite points, amounts in fact to an implicit application of a process of choice reducible, at least metamathematically, to the uniqueness of each act implied by the Axiom of Choice. This can be seen as yet another indication of the impossibility of 'transcending' the metatheoretical constraints that underlie and to a large extent determine a single universe theory. This is indirectly admitted by Väätänen in claiming that the semantics of dependent logic do not depend on either universe or multiverse set theory for the reason that "although dependence logic goes beyond first-order logic and dependence logic truth cannot be reduced to truth in all universes, we do not use dependence logic in metatheory. Our metatheory is just first-order logic" (ibid., fn 20, p. 198). This means that in effect Väätänen constructs the multiverse by adding new predicates through (single universe) first order logic in the metatheory.

(D) Although one of the main motivations of the multiverse theory is to provide a new context for resolving the question of **CH** once and for all, Väätänen's attempt to a metatheoretical approach through MD logic and the Boolean disjunction

$$\begin{aligned} &\mathcal{M} \models_{\mathcal{S}} \phi \vee_B \psi \text{ if and only if} \\ &\mathcal{M} \models_{\mathcal{S}} \phi \text{ or } \mathcal{M} \models_{\mathcal{S}} \psi \end{aligned}$$

[22] In team semantics the basic concept is not that of an assignment s satisfying a formula φ in a model M, but of a set $(\mathcal{S})$ of assignments, called a team, satisfying the formula φ. See Väänänen (2014, p. 197).

leads again to an impasse (ibid., pp. 200–201). More concretely, given that the multiverse formula $=(\phi)$ is made to represent $\phi \vee_B \neg\phi$, then $\neq(\phi)$ would mean the sentence ϕ is absolutely undecidable over the multiverse. It follows that adopting $\neq(\mathbf{CH})$ as an axiom would mean that as such is beyond controversy which goes against the mathematical experience and of course against Hamkins' appreciation of the accessibility due to existing experience. On the other hand adding $=(\mathbf{CH})$ to the other first order axioms of set theory does not resolve the question either, since **CH** would then have a definite truth value, yet in the current state of affairs it may well have, by forcing techniques on the ground model, any truth value we might wish to assign.

If Väätänen's approach has, in spite of his primary goal of a multiverse axiomatization, a certain sense of leaning toward a single universe by 'smoothing the edges' of multiple universes, we can find approaches favorable to the single universe by indirect means and in a totally different context, as Clarke-Doane has done in Clarke-Doane (2019). Clarke-Doane's view is one of vindication of the single universe approach on account of Benacerraf's challenge.[23]

Clarke-Doane has undertaken to refute the view that set-theoretical pluralism rightfully answers Benacerraf's epistemological challenge, among them Hamkins' position on the abundance of set-theoretical possibilities offered by our experience working with different models of set theory, by naively arguing that the real existence of various models in set theory does not help to causally explain set theorists' psychological states in the first place. For Clarke-Doane the special kind of perception, according to Gödel, we have of the objects of set theory - as it is seen from the fact that the axioms force themselves upon us as true - scarcely helps to explain the justification of our set-theoretic beliefs let alone their reliability on the grounds that the content of such mathematical intuitions could not necessarily be an essential feature of being true.

Even as Clarke-Doane proceeds on the basis of the reliability (juxtaposed to justification) criterion for our set-theoretic beliefs to defend the argument that the pluralist is no better positioned than the universist to answer Benacerraf's challenge and therefore it is not clear how someone could epistemologically argue in favor of universe pluralism[24], he falls short, in my view, of establishing a consistent, extra-theoretical level of discourse on which to dialectically link knowledge about mathematical objects

[23] The Benacerraf problem, known subsequently also as the Benacerraf-Field challenge, was initially presented in Benacerraf's article on mathematical truth, (Benacerraf 1973), in which Benacerraf claimed to be in favor of "a causal account of knowledge on which for (*auth. add.*: a subject) X to know that S is true requires some causal relation to obtain between X and the referents of the names, predicates, and quantifiers of S". By this measure and on the subjectively based principle of the knowing person, Benacerraf argued that in view of the 'asymmetry' between the truth conditions of a proposition p put in formal terms and the grounds on which p is said to be known, e.g. in terms of reliability in connection with general mathematical or other knowledge "[..] makes it difficult to see how mathematical knowledge is possible". See: (Benacerraf 1973, pp. 671–673).

[24] Clarke-Doane seeks to show through various subsumed interpretations of Benacerraf-Fields' challenge, e.g., indispensability, counterfactual persistence, etc., that the pluralists do not have the edge over universists in all such cases. In his own words "if there is a reason to be a set theoretic pluralist, then it is not related to the challenge to establish a causal, explanatory, logical or even counterfactual dependence between our set-theoretic beliefs and the truths" (Clarke-Doane 2019, p. 13).

to conditions of formal truth about them. I refer in the immediately next to some prompts on the matter which are, from my standpoint, better elucidated from a subjectively rather than an ontologically founded point of view.

6 A Philosophically Inclined Argumentation in Favor of Universism

If someone, on account of Clarke-Doane's dealing with the Benacerraf challenge above, has the intention to explain the real existence of various models in set theory in connection with the set theorists' psychological states or generally the postulation of truth of a proposition P inside a logical-mathematical system in connection with the causal relation obtained between a subject X and the logical structure of P, he must have a clear view of at least the following questions: (i) the way that can be founded the meaning and truth of mathematical objects, more generally of logical-mathematical states-of-affairs, (ii) the question of whether there is an extra-logical meaning of the mathematical notions of absoluteness, well-foundedness, well-ordering, etc., (iii) the question of whether mathematical models are self-standing entities or are committally determined by corresponding formal theories and their expressional means which could be again subjected to inquiry as to their origin. Given my phenomenological motivation I offer some clues to these questions without entering deeper into a philosophical discussion to avoid going too far afield from the intended scope and the rather technical content of this article.

(i) Husserl had oriented over the years the meaning and truth of logical propositions to his predilection for the reduction of all logical-mathematical concepts, in fact of all phenomena of the external world and of mental sphere, to representations in consciousness and consequently to the a priori modes prescribed by the nature of this latter. Therefore meaning in general is associated, on the soil of experience-within-the-world, in an essential way with the ways consciousness displays an a priori directedness toward objects, in the rough sense that meaning becomes each time the content of this a priori directedness called intentionality. Further, meaning associated with each actually given experience is subject to the "[..] unconditional norm that it must first comply with all the a priori 'conditions of possible experience' and the possible thinking of such experience: that is, with the conditions of its pure possibility, its representability and positability as the objectivity of a uniformly identical sense" (Husserl 1939, *Engl. transl.*, p. 353). In other words, to the extent meaning is fundamentally constrained by the a priori 'conditions of possible experience' becomes not only dependent on the constitutive mental faculties of each knowing person but it is also pre-determined and invariably the same for all humans according to eidetic attributes. In such view existence in terms of mathematical propositions can be considered an a priori possibility of existence something that may be taken as a vindication of Gödel's naively thought non-realist attitude on the question of mathematical existence in his article against Carnap's syntactical account of mathematical foundations .[25]

[25] In the footnote 20 of the article *Is Mathematics Syntax of Language?* Gödel offered as an example of a transfinite (i.e., non-constructive) concept the phrase 'there exists', if this phrase "means object existence irrespective of actual producibility" (Gödel 1995, p. 341).

In these terms one may define knowledge as the consciousness of the ‘agreement’ between an anticipatory, in eidetic sense, predicative belief and the corresponding first-hand experience of the object of the belief. Therefore truth is taken as what is experienced in the verification of the anticipatory turning toward an ‘empty’ predicative belief with the first-hand experience of the state-of-affairs of the object itself.

(ii) In contrast to Hamkins’ and other multiversists’ notion of absoluteness of mathematical entities as sensitive to a background set-theory, I propose a radical subjective reduction of the formal notion of absoluteness that links it to a subject’s intentional directedness (i.e., intentionality) toward formal individuals and their categorial properties. In *Experience and Judgment* Husserl proposed a new understanding of the concept of absolute substrate in which “a ‘finite’ substrate can be experienced simply for itself and thus has its being-for-itself. But necessarily, is at the same time a determination, that is, it is experienceable as a determination as soon as we consider a more comprehensive substrate in which it is found. Every finite substrate has determinability as being-in-something, and this is true *in infinitum*.” (ibid., p. 137).

In consequence absolute substrates may be seen as completely indeterminate from the point of view of analyticity, in virtue of being lowest-level individuals of intentionality devoid of any ‘inner’ analytical content themselves, thus being ultimate substrates of all first order logical activity. Consequently they exclude by their own essential being everything that may be their determination by a logical activity of a higher level. In this sense a reduction of absolute statements to atomic formulas of individuals-substrates bound by logical connectives and possibly accompanied by fundamental categorial properties (e.g., those of inclusion, order, permutability, etc.), may come to terms with a subjectively founded and invariant by any formal-mathematical transformation (including inter-model translatability) notion of absoluteness. In such radical attitude notions like well-foundedness and well-ordering associated to the one or the other degree with absoluteness may be interpreted in the same subjectively founded terms, while sticking at the same time to a notion of infinite sets as completed totalities in actuality no matter the level of infinity attained in formal terms.

(iii) In recent literature it is pretty much debated the question of a possible ontology of mathematical models and the interrelation with the mathematical theories they represent. Rather than addressing the controversy between proponents of the syntactic and the semantic account of theories, I point to the view that the status of mathematical models “ is largely defined by the way models subsume a set-theoretical structure whose constraints, reducible to an extra-linguistic level of discourse, may implicitly condition the epistemic status of models as representations of axiomatic theories” (Livadas 2020, p. 13).

Further and on account of the non-categoricity of first order **ZFC** theory due to the Skolem-Löwenheim Theorem, one might argue that the inner constraints posed on the logical-mathematical level should someway be reflected in the model-theoretic one. For instance, one could not have a sound notion of isomorphism or for that matter of partial isomorphism of models if there wasn’t a prior intuition of syntactical individuals as irreducible unities, or (for that matter) of the ‘ontic’ invariability of ordinals and ordinal definable formulas, or yet of the concept of actual infinity, e.g., as the indefinite scope of quantifiers in predicative formulas, etc. The following passage is perhaps indicative of the virtues of a subjectively based argumentation against the supposedly ontological merits of multiversism: " countability (*add of the auth.*: of models) is in fact imposed extra-theoretically in logico-linguistic and more concretely set-theoretical constructions by the way the predicative (you may prefer to call it noematic) activity, pertaining to well-meant objects of a system, is carried through by the mental activity of a subject: that is, in finitely many steps ideally extensible in infinitum, an extrapolation that can also accommodate a universal quantification over an indefinite domain." (ibid., p. 33).

There is no kind of multiversism as far as I know that, in view of my arguments in this article, can possibly accommodate the proposed subjective principles of set-theoretical construction in a way that would justify an ontological foundation of multiversism in mathematical theories.

7 Appendix

As noted in Sect. 2, par 8, this technical in character Appendix serves to show that the proof-theoretical machinery employed in the (indirectly) touching on the multiverse debate axiom $V = \text{HOD}$, and *a fortiori* to its stronger version $V \subseteq \text{HOD}^{V[G]}$, applies certain 'standard' notions without the need to appeal to radically different concepts which would in turn imply the need for alternative universes.

Woodin outlines in Woodin (2017) the proof of the following theorem which summarizes some key consequences of the axiom $V = \text{Ultimate} - L$, where the Generic-Multiverse is the generic multiverse generated by the set-theoretical universe V.

Theorem 7.1 (***V=Ultimate-L***)

- **CH** holds
- $V = \text{HOD}$
- V is the minimum universe of the Generic-Multiverse.

Crucial in the proof of the two latter cases of Theorem 7.1 is the following theorem which is a stronger case than $V = \text{HOD}$ and moreover establishes a set-generic extension of the set-theoretical universe V, a fact that might meet Hamkins' standards of ontological multiversism.

Theorem 7.2 (***V=Ultimate-L***) *Suppose* $V[G]$ *is a set-generic extension of* V. *It follows*:

$$V \subseteq (\text{HOD})^{V[G]}$$

(Woodin 2017, p. 102).

The key approach to the proof of Theorem 7.2 is to fix a partial order $P \subseteq V$ with G V-generic for P and cardinal $\lambda = \text{ card } (P)^V$. Then for all regular cardinals $\kappa > \lambda$ one may prove that $\mathcal{P}(\kappa)^V \subseteq \text{HOD}^{V[G]}$ which will eventually prove that $V \subseteq (\text{HOD})^{V[G]}$.

The proof hinges on the following notions which in spite of their technical nature are invariant and unconstrained by the structure of extended models forged by forcing or collapsing principles.

First: It is important, on fixing a regular cardinal $\kappa > \lambda$, to have a partition of stationary sets $\mathcal{F} = <S_\alpha; \alpha < \kappa>$ inside the universal set V that is also a partition in $V[G]$. This can be done by letting $\mathcal{F}$ be a partition of the set

$$S = \{\alpha < \kappa; (\text{cof } (\alpha))^V = \omega\}$$

into stationary sets such that by definition of stationarity there is a closed unbounded set $N_o \subset \kappa$ such that $N_o \in V$ and moreover for each $\sigma \in N_o \cap S$

$$\sigma \in \cup \{S_\xi;\ \xi < \sigma\}$$

Now if $N \subseteq \kappa$ is a closed cofinal set such that $N \in V[G]$ then there must be a closed cofinal subset $M \subseteq N$ such that $M \in V$, having as a consequence that each S_α is stationary in $V[G]$.

So what is the trick in getting the S_αs to be stationary sets in $V[G]$? The answer is that the possibility of finding a closed cofinal subset $M \subseteq \kappa$ of N (this latter being in $V[G]$) in V that shares the same intersection properties with N_o regarding the elements of $\mathcal{F}$ is fundamentally due to the absoluteness of ω between transitive models of **ZF**, in virtue of the cofinality of the elements α of S. This would not be the case if the first limit ordinal ω was not an absolute concept (like all other ordinals).

Second: 7.2 depends on a previous theorem of Woodin's[26] which is also a consequence of $V = \text{ Ultimate } - L$, namely for each cardinal κ there exists an elementary embedding

$$\pi : (H(\kappa^+))^V \longrightarrow K$$

such that $K \in (\text{HOD})^{V[G]}$ and $(\pi, K) \in V$. The stationarity of the subsets $<S_\alpha;\ \alpha < \kappa>$ of the (regular cardinal) κ in $V[G]$ is again the decisive trick of the proof, although in the context of the elementary embedding π above into the inner model $(\text{HOD})^{V[G]}$. Of course it would be absurd to talk about inner models via elementary embeddings without a fairly absolute notion of ordinals to the extent that by definition a class N is an inner model iff N is a transitive model of **ZF** under the $\in$ predicate and contains the class of all ordinals, i.e., $\text{ON } \subseteq N$.

This is an *a fortiori* condition in the present situation given the structure of $(\text{HOD})^{V[G]}$ in terms of hereditarily ordinal definable sets whose definition is straightforwardly associated with the notion of ordinals.

[26] See for details: Theorem 7.25, p. 101 in Woodin (2017).

Further Woodin proves that a consequence of $V =$ Ultimate $-L$ is that the universe V is the minimum universe of the Generic-Multiverse.[27] This could be of some importance for certain proponents of multiversism, yet I note that a key part of the proof depends on the use of the collapsing poset Coll (ω, δ), with δ a specially defined cardinal in V. It depends, in particular, in a substantial way on its homogeneity through a 'codification' by the countable ordinal ω such that in V

$$\mathrm{RO}\ (P \times \ \mathrm{Coll}\ (\omega, \delta)) \cong \ \mathrm{RO}\ (\ \mathrm{Coll}\ (\omega, \delta))$$

and in another universe V_0

$$\mathrm{RO}\ (P_0 \times \ \mathrm{Coll}\ (\omega, \delta)) \cong \ \mathrm{RO}\ (\ \mathrm{Coll}\ (\omega, \delta))$$

Again it would be totally impossible to reach this result without reliance to absoluteness properties, e.g. of the functions $f_\omega(\delta) : \omega \times \delta \longrightarrow \delta$ and of course the absoluteness of ω.

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[27] See for details: Theorem 7.28, p. 103 in Woodin (2017).

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