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# COMPOSITIONALITY, RELEVANCE, AND PEIRCE'S LOGIC OF EXISTENTIAL GRAPHS

ABSTRACT. Charles S. Peirce's pragmatist theory of logic teaches us to take the context of utterances as an indispensable logical notion without which there is no meaning. This is not a spat against compositionality *per se*, since it is possible to posit extra arguments to the meaning function that composes complex meaning. However, that method would be inappropriate for a realistic notion of the meaning of assertions. To accomplish a realistic notion of meaning (as opposed e.g. to algebraic meaning), Sperber and Wilson's Relevance Theory (RT) may be applied in the spirit of Peirce's Pragmatic Maxim (PM): the weighing of information depends on (i) the practical consequences of accommodating the chosen piece of information introduced in communication, and (ii) what will ensue in actually using that piece in further cycles of discourse. Peirce's unpublished papers suggest a relevance-like approach to meaning. Contextual features influenced his logic of Existential Graphs (EG). Arguments are presented *pro* and *con* the view in which EGs endorse noncompositionality of meaning.

KEY WORDS: compositionality, existential graphs, Peirce's pragmaticism, relevance

## 1. INTRODUCTION: RELEVANCE AND CONTEXTUAL EFFECTS

I will approach the topics of compositionality, relevance and context from both systematic and historical perspectives, but this is purposely so, since these viewpoints are intertwined to the extent that is singularly unforeseen in previous research. The general systematic topic that I wish to emphasise concerns the interplay between *semantic* – or, given the historical delineation in late 19th and early 20th-century discourse, *semeiotic* – and *relevance-theoretic* notions. This is to say that some recent formulations of compositional semantics (Hodges 1997) provide a realistic interpretation of logical expressions only if they spell out ways – something they have not done – in which relevance enters the wide selection of semantic entities such systems provide.

On the other hand, in the Relevance Theory (RT) introduced by Sperber and Wilson in early 1980s, no convincing account of how such contexts are constructed exists against which the relevant semantic entities (of a linguistic communication or discourse) are defined by the contextual effects that they have. A context-construction task of this type may, at least partially, be realised in semantic terms; however, these methods are likely to be not compositional in the sense in which a tension exists between semantic context-dependence and *strong* compositionality, namely the standard formulation of compositionality of meaning plus its inverse, the compositional expression of complex content (Pagin 2003).<sup>1</sup>

My historical perspective involves tying these notions into the outlook on logic and semantics shared by early pragmatist philosophers, most notably that expressed by the American philosopher and scientist Charles S. Peirce (1839–1914). In particular, his iconic, diagrammatic logic of Existential Graphs (EG), together with the semantics of these graphs which earned the ahead-of-time but little-known title of the "Endoporeutic Method" (EM) during the first decade of the 1900s (Pietarinen 2004a), highlights the relevance of context construction and its updating during interpretation. In conjunction with EGs, Peirce also emphasised strategic communication, mutual knowledge, and the presence of the common ground (Pietarinen 2005a).

Accordingly, my purpose is to shed light on some of the ways in which EGs and the EM instantiate aspects of compositionality as well as non-compositionality, and to put these aspects into a new historical perspective.

Peirce's one-time student Dewey (1859–1952) stated that the neglect of context is "the greatest single disaster which philosophic thinking can incur". It is the notion that is "inescapably present" without which we cannot "grasp the meaning of what is said in our language" (Dewey 1931). I return to Dewey briefly in Section 5.

On the logical side, Peirce's pragmatist theory of logic addresses precisely the Dewey contention. There is no sentence, expression or depiction that has a meaning independent of the environment, context or circumstance within which it is uttered and within which it gets interpreted (Peirce 1931–58, 1980–). This is not, of course, an argument against compositionality of meaning *per se* (apart perhaps from some very strong notions of compositionality in which the meaning of expression would be *exhaustively* defined by its parts), since context may itself be provided as a set of utterances, or perhaps

some representational schema, script, frame, or what have you, to which a linguistic or logical description can be attached.

More precisely, as results by Janssen (1997) and many others have shown, it is possible to encode any contextual information into the model-theoretic consequence relation, perhaps directly into the subexpressions of language, as has been the case in context-carrying in logic programming or in dynamic theories of meaning, or else into extensions of assignment as in compositional semantics for 'Independence-Friendly' (IF) first-order logic (Hintikka 1996) intended to provide proper semantic attributes via non-empty sets of sequences of assignments instead of merely using sequences of them (Hodges 1997; Janssen 1997, 2002; Sandu and Hintikka 2001).

The context-dependent nature of the formulations of IF logics has not been applauded without dissent. Attempts exist to downplay proposals of this type because they appear to hamper what many regard as the essential ingredient of not only any feasible logical system, but also of the "guilty secret" logicians and linguists may have possessed concerning natural language (Sandu and Hintikka 2001), namely compositionality as a prerequisite for issues such as learnability, systematicity, and perhaps something like the communicativity and comprehension of language. For, if proper subformulas are context dependent, it is said, they do not necessarily have a self-supporting meaning. It is for this reason, advocates of the compositional approach claim, that they cannot be considered to be natural constituents of a larger unit, typically a formula or a sentence, the meaning of which ought to be morphically imaged on those constituents.

To put across my main argument, I need not and will not delve into the details of these semantics and logics. What these results show is that the meaning of a sentence which varies between contexts is not in conflict with compositionality as such, for contexts may be provided as extra arguments in the meaning function that contributes to the complex meaning. But this runs the risk of making the principle methodologically empty (Westerståhl 1998).

Above all, the strategy of enforcing compositionality at all costs is entirely inappropriate for a *realistic* notion of the meaning of assertions, since it tells nothing about the question of which descriptions of context or environment, or which systems of mutual background knowledge and common ground between the interlocutors, are actually *relevant* to the meaning of the assertion, sentence, or a fragment of discourse.

By assertions, I mean *symbolic, iconic* and *indexical* assertions. Current discussion concerning compositionality has been confined to symbolic systems supposed to underlie natural language and algebraic frameworks alike, occasionally perhaps taking a recourse to indexical aspects of meaning (especially in terms of different ways of drawing a division between semantics and pragmatics). Hardly ever do we find systematic treatments of the compositionality of iconic representations of assertions. This imbalance defines my broader agenda.

Towards meeting the concern about the relevance of the diversity of semantic attributes that compositional semantics are forced to introduce, I wish to highlight Sperber and Wilson's (1995) RT, which claims to have provided a logical and cognitive account of relevance. It attempts to capture the notion of relevance in communicative situations through *contextual effects*. Since it is impossible to know in advance which descriptions of circumstances or parts of the common ground will actually be relevant to the dynamic, on-going processes of linguistic communication, they define the notion in terms of a context-change potential. In other words, relevant factors or properties of an expression are those which intrude into the context of discourse. This is an argument from *cognitive economy*: the goal of communication is to maximise the relevance of the phenomena available to language users while minimising the amount of mental or cognitive processing effort that is required. The grounds for believing in cognitive economy are, in turn, evolutionary. This opens up a number of hard questions that I will not dwell on in this paper to any great extent (see Section 5).

# 2. Relevance and the pragamtic maxim

According to RT, the inferential model of communication involves attempts to share, distribute and recognise acts of intention, emotion and other modalities that are delivered in the communication of information. These attempts are what contribute to the relevance of an utterance that communicates some intended piece of information. What agents recognise as relevant is largely related to common traces in their experience. The notion of context is therefore central to this theory, since what is relevant is that which produces a tangible contextual effect, or which penetrates the context of discourse.<sup>2</sup> The goal of RT is to provide a theory of communication for a range of pragmatic phenomena.

The basic idea of RT is thus neither entirely psychological nor epistemic, though admitting a modicum of both. It aims at providing a theory which is psychologically and psycholinguistically realistic, but not overly so. It is an attempt to make sense of linguistic pragmatics at the cognitive level which is, according to (Carston 1988, p. 713), "the first account of pragmatics which is grounded in psychology". This may not be a major compliment after all. Most have regarded H. Paul Grice's program of analysing literal meaning in public language through conversational maxims as psychological, since it involves notions of the speaker's and the hearer's intentions and beliefs (Grice 1989). I consider this assimilation to be a gross oversimplification even on Grice's own account (Pietarinen 2004b).

That the aim of relevance would lie in psychological explanation has some additional drawbacks. Even though announced as one of the main aims of RT, the idea of relevance has not been tame enough to suit the needs of a rigorous logical modelling of discourse, since such an enterprise would hinge on effective ways of representing contextual information and its change. As will be seen in later sections, the diagrammatic system of EGs, together with its semantic interpretation, tackles contextual phenomena by virtue of non-compositional interpretation, thus extracting contextual propositional input from the early stages of interpretation. The later stages are then open to the processing of further utterances using this readily-extracted content.

My key concern, given Peirce's pragmatic outlook on the meaning of concepts, is the placing of Sperber and Wilson's proposal in a wider historical perspective. I wish to suggest that it may be thought of as an instance of Peirce's Pragmatic Maxim (PM), which says that the meaning of a concept is the sum total of its implications for possible observations and possible actions. In precise terms:

The rule for attaining the third grade of clearness of apprehension is as follows: Consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object' (5.402, 1878, *How to Make our Ideas Clear*).<sup>3</sup>

This formulation of the pragmatic maxim first appeared in the January 1878 issue of *Popular Science Monthly*. Several versions of it exist in Peirce's large corpus. A very succinct and unambiguous one says that "the maxim of logic [is] that the meaning of a word lies in the use that is to be made of it" (CN 2.184, 2 February 1899, *Matter, Energy, Force and Work*).<sup>4</sup>

For my initial purposes, PM may be read such that the weight of the information depends to a large extent on (i) the practical consequences of accommodating the chosen piece of information introduced in communication, and (ii) what will ensue as a consequence of actually using that piece in further cycles of discourse.<sup>5</sup> According to PM, the most relevant information is that which provides the best toehold for agents to continue the dialogue or action.

We may thus think of practical bearings as the contextual effect which an item of information, or a belief, has on the context within which they are located. The implications that the infiltration of information into the context have may thus be read as Sperber and Wilson's preferred properties of relevance in linguistic utterances, which balance the *inferences* made in choosing between possible rival interpretations against the notion of the *cost* of making such inferences. In the light of PM, and *mutatis mutandis*, RT turns out to be a markedly pragmatic theory of pragmatics.<sup>6</sup>

PM also states that practical consequences need not be actualised, even though they need to be actualisable. They may illustrate Peirce's "would-be", a modality presented to the consciousness of future deliberation. According to Peirce, possibilities are just as real as actual objects and events. Meaning as a list of the practical consequences of a concept is characterised in terms of subjunctive rather than indicative conditionals.

This subjunctive nature of meaning is all the better for the gaining of further philosophical support for RT, as it is exceedingly difficult to say which among the wide variety of relevant semantic attributes will finally materialise. Such hypotheticals manifest themselves in the relevance of the proposition "Diamonds are hard but not very firm" as expressed by a conditional such as "If diamonds are rubbed then they are unlikely to be scratched" as well as in "If diamonds are struck against something rock hard they are likely to break into pieces". This later subjunctive formulation has caused some to take Peirce's pragmaticism to be both too liberal and too broad in its characterisation of concept meaning in terms of possible, potential or expected practical effects that may never be manifested.

Such a criticism is nevertheless not effective in those formulations of PM that take relevance into account. What is expected to be relevant is often not only sufficient for a contextual change to happen. A mere potentiality may also intrude into the context of utterances and change their constitution in a hypothetical way, as a form of *possible* 

*contextual chance*, rather than actually realising any particular observable effect. One example of a potential effect is the asymmetric case of the hearer's determination of what would count as relevant in contradistinction to what the speaker intended to be relevant, in other words relevance to the hearer remaining as a potential change in the shared environment of the utterance.

One should also note that, according to Sperber and Wilson's original formulation of RT, relevance is something that is not *determined* by context but *constrained* by context. On the contrary, thus, a particular context is determined by the search for relevance, the interpreter acting according to the version of the Principle of Charity that concerns relevance maximisation.

This has the unfortunate effect that the power of RT ceases precisely at the point in which an utterance makes the earliest contribution to context, since the theory does not presuppose computing the effects in all contexts – doing so would be cognitively too costly. The evolutionarily hardwired principle of *least effort* will cut in and select the first and most accessible contextually-effective interpretation.

I believe that acquiring a comprehensive account of the strategic nature of communication compels us to also include *suboptimal relevancies* into the scope of the theory, and to make it bi-directional in terms of also accommodating what the hearer takes to be relevant into the formal framework of computing relevance in terms of possible contextual change instead of actual change.

Sandu and Hintikka (2001) have asked whether the semantic attributes provided by compositional systems of interpretation are also *natural*. Here we may reinstate the question in terms of the *relevance* of attributes. This is because naturalness is a vaguer notion than relevance for formal regimentation, and hence we would be better off to start with the latter, especially in view of its increasingly-mature status in theories concerning cognitive aspects of communication.

How are these points concerning compositionality, relevance and context-dependence in semantics related to Peirce's PM and his logical system presented over the years? Peirce offers a rich theory of how different contexts are formally set up which involves not only the presence of previous utterances, but also the collateral observation, experience and memory of the utterer and interpreter. It includes both semantic and pragmatic elements. The prevalence of such contextual elements suggests, in its turn, that Peirce's remarks have a bearing on the compositionality issues.

## 3. PEIRCE'S LOGIC OF EXISTENTIAL GRAPHS

Did Peirce himself believe in anything like compositionality? Surely the term is much more recent? To answer this question, I will focus on his system of EG, which are strongly influenced by contextual (congraphical) features.

# 3.1. The system of existential graphs

Let us review Peirce's logic of EGs. He divided EGs into ALPHA, BETA and GAMMA parts, which correspond, approximately, to propositional, first-order and modal logic. GAMMA also involves elements of higherorder logic and abstraction. Peirce wanted it to also tackle nondeclarative assertions such as interrogatives and imperatives.

The first concept is a *spot*, which is a predicate  $S_i$ , i = 1..., k surrounded by a finite number n of hooks at its periphery:<sup>7</sup>  $S_1$  The number of hooks corresponds to the arity of a predicate.

Spots may be linguistic assertions. For instance, the spot "\_\_ is good" has one hook (the blank line of expression), and the spot "\_\_ loves\_" has two hooks (two blank lines of expression).

The system of BETA EGs consist of the following components:

1. Sheets of Assertion (sA) on which graph-instances  $G_i$  are scribed:

We may think of an sA as an interpreted structure. If nothing is scribed on an sA, it represents tautology (T, *verum*).

2. Juxtapositions, which are placements of  $G_i$  on the same sa:

If  $G_1$  is scribed as a graph-instance of BETA EGs, then

 $G_2$  is scribed as a graph-instance of BETA EGs.

Juxtaposition is an iconic analogue to Boolean conjunction.

3. Cuts, which are thin, simple, non-overlapping closed curves enclosing  $G_i$ :

If

 $G_1$  is scribed as a graph-instance of BETA EGs, then



) is scribed as a graph-instance of BETA EGs.

A cut is an iconic analogue to Boolean negation.

4. Lines of Identities (LI), which are either *dots* or thick continuous *lines* composed of a set of contiguous dots, attached to the hooks on the periphery of a spot:

If 
$$(S_1)$$
 is scribed as a graph-instance of BETA EGs, then  
 $(S_1)$  and  $(S_1)$  are scribed as graph-instances of

BETA EGS.

An LI is an iconic analogue to existential quantification, identity and predication of first-order logic.

Dropping the fourth formation rule amounts to the ALPHA part of EGs.

Taking juxtaposition to define isotopy-equivalence classes, it is seen that the orientation of juxtaposed graphs on an sA does not matter for truth or falsity, and so the operation of conjunction it defines is commutative and associative.

In what follows, we explicitly omit drawing out the sAs. The ALPHA graph on the left corresponds to the formula of propositional logic on the right:

An LI connects spots such that any hook at the periphery of a spot may be connected by an LI. Any unconnected extremity of an LI is a *loose end*. An LI that connects to only one hook and does not have a loose end reduces to a zero-dimensional dot. A dot on an SA singles out an individual subsisting in the universe of discourse.

Like juxtaposition, LIS with a loose end that is enclosed within the same area of a cut, give rise to an isotopy-equivalence class. Therefore, their order of their interpretation is irrelevant to the truth-value of a graph.

The number of hooks at the periphery of a spot  $S_i^n$  corresponds to the arity of a predicate. The number of occupations at the periphery of  $S_i^n$  corresponds to the number of bound variables of a predicate. No more than one LI or dot may occupy any one hook. LIS may be connected to each other. The totality of connected LIS gives rise to a

*ligature*. Ligatures are not graph-instances but "composites" of several graph-instances (MS 669, 1911, *Assurance Through Reasoning*).

Any line that crosses a cut is actually a ligature composed of two lines. Like graph-instances on an SA, LIS in ligatures are compositions read as juxtaposed signs ('there exists *b* and this *b* is not S').

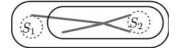
For example, the BETA graph on the left is a diagrammatic analogue of the first-order formula on the right



This EG has two cuts, two spots  $S_1$  and  $S_2$ , and one ligature abutting a cut which is composed of two LIS that meet at the boundary of the inner cut. The LI asserts at its outermost extremity that an individual exists in the universe of discourse of the sA. The EG asserts that, given any individual object of the universe of discourse of sA about which  $S_1$  is true,  $S_2$  is true. The two nested cuts denote conditional.<sup>8</sup>

The outermost free extremity of an LI or ligature determines whether the quantification it represents is *universal* (the free extremity lies within an *odd* number of cuts) or *existential* (the free extremity lies within an *even* number of cuts).

The following EG pertains to the BETA part of the system, containing two LIS corresponding to the existential and universal quantifiers ( $\forall x \text{ and } \exists y$ ) of first-order logic, as well as to the identity of two variables expressed by the two place spot  $S_2$  and the two lines abutting each other within the inner cut.



Prima facie, this graph may be correlated with the symbolic formula  $\forall x \exists y (S_1(x) \rightarrow S_2(x, y))$  of first-order logic, in which  $S_2(x, y)$  is taken as the identity relation. However, doing things in this iconic and diagrammatic fashion has the consequence that many of the characteristics routinely attributed to symbolic expressions no longer carry over to iconic representations (Pietarinen 2005b).

# 3.2. Semantics

The semantics of EGs may be given in the Tarski-type manner. However, I believe that the iconicity of diagrams recommends that a

*game-theoretic semantics* be given. This would also be in accordance with Peirce's own intentions.

A semantic game is played between the GRAPHIST and the GRAPHEUS on a given graph-instance  $G_i$  and a model  $\mathcal{M}$ . We may think of these as players in the sense of the game-theoretic semantics (Hintikka 1973):

1. *Juxtaposition* of positively (negatively) enclosed graphs: the GRA-PHEUS (resp., the GRAPHIST) chooses between two graphs. The evaluation proceeds with respect to that choice.

2. *Cut:* roles switch between the GRAPHIST and the GRAPHEUS. Also the winning conventions change.

3. Polarity of the area of the outermost extremity of a *ligature* determines whether the GRAPHIST (if positive polarity) or the GRAPHEUS (if negative polarity) is to make a choice from the domain of  $\mathcal{M}$  to be the interpretation of the LI. Evaluation proceeds with respect to that choice and with a graph-instance that has the interpreted part of the ligature removed up to its next connexion with another line or with a spot.

4. When a *spot* is reached, its truth-value determines the winner: if a spot is true, then the GRAPHIST wins a particular play of the game; if false, then the GRAPHEUS wins.

5. The existence of a winning strategy for the GRAPHIST agrees with  $G_i$  being true in  $\mathcal{M}$ . Likewise, the existence of a winning strategy for the GRAPHEUS agrees with  $G_i$  being false in  $\mathcal{M}$ .

Graph-instances are interpreted according to the Endoporeutic Method: the outermost cuts or contexts are evaluated on the model  $\mathcal{M}$  before proceeding to the inner, contextually constrained cuts until spots are reached. The evaluation of lines involves an instantiation of a value to the outermost end of an LI, and this value then propagates

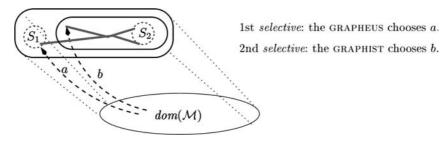


Figure 1. EG BETA graph, first two steps of the evaluation.

continuously along the LI towards the interiors of the inner cuts and to the spots to which the lines are attached. Peirce termed the process of instantiation together with the type of the LI the *selective* (Figure 1). This is the basic way in which contexts are created and updated.

As regards the EG in Figure 1, the evaluation proceeds, after the first two steps of choosing individuals, by the GRAPHEUS choosing between the spots  $S_1$  and  $S_2$  that is being cut. If he chooses  $S_1$  and  $S_1$  is satisfied in  $\mathcal{M}$ , the GRAPHIST wins. If the GRAPHEUS chooses  $S_1$  and  $S_1$  is not satisfied in  $\mathcal{M}$ , he wins. The EG is true precisely in the case where a winning strategy exists for the GRAPHIST, and false precisely in the case where a winning strategy exists for the GRAPHEUS.

As noted, the evaluation takes place between two parties "in our make believe" (MS 280, *The Basis of Pragmaticism*)<sup>9</sup> the GRAPHIST who scribes the graphs and proposes modifications to them, and the GRAPHEUS who authorises the modifications.

The graphs scribed by the GRAPHIST are true, because "the truth of the true consists in his being satisfied with it" (MS 280: 29).<sup>10</sup> Hence, the GRAPHIST is the verifier of the graphs. To end with a true atomic graph amounts to a win for the GRAPHIST, and to end with a false one amounts to a win for the GRAPHEUS. The truth of the whole graph agrees with the notion of the existence of a winning strategy for the GRAPHIST, in Peirce's terms a *habit* that is of "a tolerable stable nature" (MS 280: 30). Likewise, falsity is the existence of such habits for the GRAPHEUS.

Peirce further assumes that these players have common knowledge concerning the universe of discourse and thus a reasonable common ground well understood between the two of them, without which the discoursing that proceeds according to the presumption of collaborative enterprise would not be possible. We must skip over further details concerning EGs here and refer to (Zeman 1964; Roberts 1973; Sowa 2001; Pietarinen 2004a, 2005a, 2005b).

# 4. COMPOSITIONALITY AND EXISTENTIAL GRAPHS

## 4.1. The pragmatic principle of compositionality

What are Peirce's views on the matters related to the compositionality of such graphs? Quite remarkably, when speaking about his EGs circa 1905, he made the following note in unpublished papers: "The *meaning* of any graph-instance is the meaning of the composite of all the

propositions which that graph-instance would under all circumstances [= in all contexts, *A.-V.P.*] empower the interpreter to scribe" (MS 280). A similar statement exists among the assorted draft pages of the same manuscript: "The *meaning* of any graph-instance is the meaning of the sum total or aggregate of all the propositions which that graph-instance enables the interpreter to scribe, over and above what he would have been able to scribe" (MS 280: assorted pages 35).

What the interpreter is empowered to scribe are thus, on the one hand, *experimental and evidenced facts* derived from experiments upon these diagrams, and on the other, *inferential propositions* that follow from graphs by the rules of transformation. Meaning therefore involves both inductive and deductive elements of reasoning.

These passages suggest that, given PM, according to which the meaning of an assertion is the sum totality of all its actual and possible practical consequences under a given interpretation, Peirce had in mind an approach to compositionality quite close to PM. A pragmatic principle of compositionality would be thus:

**The Pragmatic Principle of Compositionality (PPComp):** The meaning of a sentence is the meaning of all sentences that follow from that sentence either by inductive or deductive principles and permissions under all authorised circumstances (i.e., those arising out of mutual consent by the GRAPHIST and the GRAPHEUS).

Here we have an outward-looking, indefinitely-progressing principle for meaning. Noteworthy is the employment of both inductive and deductive forms of reasoning. What exactly is meant by them falls outside the scope of the present treatise. In general, they are intended to account for everything those practical, conceivable, observable or sensible effects referred to in PM would ideally amount to in logical terms. Experimental verification is one aspect of the logic of induction, and in his EGs Peirce indeed claimed that further clarification of the meaning of the assertion put forward by iconic graphs is attained by rationally experimenting upon them to form beliefs about the relations involved in such representations.

Note that such diagrams should not be equated with the logical empiricists "protocol sentences", which were meant to provide the robust, independent logical bedrocks of scientific theories. For, protocol sentences that can be "confirmed" or "disconfirmed" relate to the practical consequences of meaningful diagrams that are being experimented upon for the purpose of hypothesis generation. In this wide sense, the consequences, not the diagrams, may be directly and

inductively tested and be prone to the kind of logical atomism that at one time was claimed to be the proper way of constructing protocol sentences from simple sense data.

Hence, some protocol representations may well be subject to compositionality. But no logical atomism applies to the parental iconic representations generating these sentences, since they admit of multiple interpretations depending on the purpose of the scientific theory at hand.

As regards the formulation of meaning in terms of its consequences, one may claim that it no longer deals with compositionality of an expression E, since the term is intended to refer to the inner constituents of E. If so, we may then think of PPComp as a pragmatised version of the *Context Principle* (CP) of meaning. According to CP, a word meaning cannot exist unless there is sentence in which words are embedded.

Remarks on CP are found in Frege's (1884) *Grundlagen der Arithmetik:* "The meaning [*Bedeutung*] of a word must be asked for in the context of a proposition [*Satzzusammenhang*], not in separation" (Beaney 1997, p. 90). Wundt made similar but slightly earlier remarks on contextual meaning in his first volume of *Logik* (1880–1883). Unlike Frege's overtly anti-psychologistic stance, Wundt's approach was more prone to psychological undertones, and characteristically so in relation to context-dependent expressions that deal with the 'psychic associations of lexical meaning. Peirce was well versed in Wundt's writings, whose psychologism he disapproved.

Translated into pragmatist nomenclature, CP may be stated thus:

**The Pragmatic Principle of Context (PPCont):** A proposition has no meaning in isolation from its consequences.

If a proposition has no consequences, it is meaningless. I believe that PPCont is how Peirce would have restated CP, had he been informed of its existence in Frege's writings.

# 4.2. Aspects of compositionality in existential graphs

Peirce's *prima facie* belief in compositionality of his logical systems was, I submit, primarily a result of three facts.

First, in separating (as he did) the notions of *meaning* and *truth*, the game or dialogue-theoretic interpretation of EGs – in terms of strategic interaction between the GRAPHIST and the GRAPHEUS – is the sundry classical one. In other words, the semantic game is one of

*perfect information* (Hilpinen 1982; Pietarinen 2003a, 2005a). Because of the assumptions of perfect information and the recursively nested system of cuts, and the presumption that no graph-instance may cross a cut, Peirce may well have thought that the truth of EGs was indeed compositional in the sense of being determined by the truth of the component graph-instances and their spatial relations. This does not imply that the *meaning* would be compositional (let alone *inwardly* compositional, as we may call the received principles in contradistinction to the kind of pragmatic, *outward* compositionality that Peirce suggested), since such an implication would refer to the pragmatic consequences of graphs.

Second, a related point, is that Peirce restricted his ALPHA, BETA and GAMMA parts of EGs to two-dimensional assertions. In the IF extension that I have presented in (Pietarinen 2004c), cuts and identity lines may be scribed on sheets layered in a three-dimensional *space of assertions*. Figure 2 gives an example of an IF EG in which the graph from the previous examples is duplicated and separated into two layers in the assertion space.

The IF EG in Figure 2 has the symbolic counterpart in the IF formula

1

$$\forall x \forall z (S_1(x,z) \to ((\exists y/z)(\exists u/x)(S_2(x,y) \land S_2(z,u)))).$$

The expressions  $(\exists y/z)$  and  $(\exists u/x)$  mean that the selections for u and u are made in ignorance of the selections for z and u respectively (Hintikka 1996; Sandu and Hintikka 2001; Pietarinen 2004c).<sup>11</sup>

Such IF extensions of EGs are much more context-dependent than Peirce's original graphs, because the interpretation of sub-expressions, including the saturation of predicate terms (spots), may depend not only on the *local* context within which parts of the threedimensional peripheries of the spots reside, but also on the *global* values of the environment in all those assertion sheets that appear

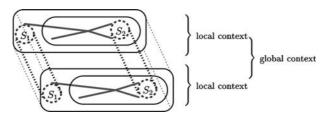


Figure 2. IF EG BETA graph in three dimensions.

elsewhere in the space but through which the peripheries span (Figure 2). No compositional semantics exists for these graphs.

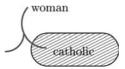
Third, since in EGs co-referential phenomena such as anaphora are handled uniformly by the use of identity lines, unlike the situation of having to move between non-compositional and compositional semantics – such as from discourse-representation theories to dynamic theories of meaning – the semantics, as prescribed by the EM, do not need to be re-defined for EGs that make diagrams out of cross-sentential anaphora.

The versatility of representing natural-language expressions by diagrams was thus another of the central reasons why even the question of a preference for compositional or non-compositional semantics did not arise in Peirce's writings.

## 4.3. Aspects of non-compositionality in existential graphs

This said, there is a dazzling juncture at which Peirce comes close to announcing a version of non-compositional graphs. In Manuscript 490, he discovered a graph that crosses a cut.<sup>12</sup> Three years earlier, in *Syllabus of Logic* from 1903 (MS 480), he had stated the fact that no graph could be partly in one area and partly in another as a rule of the system, since there would be no interpretation for a graph that crosses a cut. Such a case would be in violation of the standard principle of compositionality. But now, as a result of his unfinished attempts to capture modalities by one of the versions of the GAMMA apparati, Peirce had found that comparisons between actual and possible individuals could not be avoided.

What he found was that the *verso* of the assertion sheet on the area of the cut represents a kind of possibility. The reversal of the sheet's *verso*, that is, the *recto*, represents actuality. But in such cases, a ligature (which is a composite of several graph-instances) may connect actually extant beings scribed on the *recto* of the sheet and mere possibilities scribed on the *verso*, because this connection presupposes



*Figure 3.* Modal EG of "There is a woman who is not and could not be identical with any possible catholic".

composition of "something is a woman" and "something is other than any possible catholic" (Figure 3).

He thus saw, rightly, that there is a need to represent assertions concerning *identification* (or perhaps non-identification) between actually extant things scribed on the blank sheet and possible things scribed on the enclosure of a *shaded area*, in other words on a graph that refers to the universes of possible beings. In such cases, it is ligatures (and not merely LIS) that signify these identities across 'possible worlds', to apply the later idea of modal semantics. These identity relations Peirce called *references*. This term, like so many others, did not catch on with 20th-century logicians.

We may think of the discovery of references as constituting the core of the explanation of what Peirce is searching for in his argument for the existence of *non-propositional* concepts.<sup>13</sup> According to Peirce, "non-propositional signs can only exist as constituents of propositions", and it is not true that "a proposition can be built up of non-propositional *signs*" (MS 490). This can be deciphered by saying that a non-propositional sign is assigned a semantic attribute or a meaning by neither the GRAPHIST nor the GRAPHEUS, and may exist only in the context of a larger proposition.

The concept of non-propositionality is, in reality, much wider than that generated solely by notions in modal predicate logic such as possible individuals or references suggest. Schildknecht (2002) studies incarnations of non-propositionality as that undulating, truth-valueless, non-structured stuff distinct from the propositionalities with more tangible ontological status, the latter being routinely characterised by descriptions such as sentences, mental images, intentional acts of assent or dissent, attitudes, expressions of judgements, bearers of inferential relations, facts, and so on. By contrast, non-propositionality looks farther, encompassing *knowing how*, ideas, perceptions, qualia, appearances, identification and re-identification, intuition, collaterality, introspection, awareness and self-awareness, plus countless other cognitive or private brain states and nervous operations as well as private mental capacities.

The general view of propositionality resonates in a significant way with Frege's CP. What is pivotal to Peirce's conception is the way in which a version of CP is stated in terms of the meaning of nonpropositional signs that only exist within a wider conception of propositional signs. Non-propositionality itself is an aspect of Peirce's concept of the *Phaneron*, and hence, properly speaking, subservient to his phenomenological division of philosophical

inquiry, namely *phaneroscopy*, the "collective total of all that is in any way or in any sense present to the mind, quite regardless of whether it corresponds to any real thing or not" (1.284).<sup>14</sup>

Accordingly, non-propositionality points towards that overlapping area of inquiry that is strictly and exclusively contained neither in normative science nor in phenomenology (Pietarinen 2004d).

Given the aforementioned perspective on the interpretation of the modal part of EGs with individual objects, we have indeed found a case in Peirce's logic that violates one of the weak versions of the compositionality of truth – namely that the truth of a proposition is a function of the truth of its constituents and their proper grammatical combinations. This case is similar to the so-called *identification semantics* for first-order modal logic argued for in (Hintikka 1972). In identification semantics, the "world lines" (partial functions from several possible worlds to individuals as the referents of singular terms) identifying different aspects of individuals across possible worlds via the actual world, have a special status in the semantics, drawn as they are according to different cognitive "modes of identification" rather than by any predetermined semantic rule.

More precisely, Peirce argued that the identities between actual and possible individuals must be characterised as three-place lines of *teridentities*, not lines of identities. Teridentities are primitive signs not irreducible to one or two-place identities. Moreover, teridentities have, according to Peirce, a peculiar feature in that they possess one loose end that is not connected to a spot. In modal contexts, he termed these lines "multiple indefinite identities" (4.583). Figure 3 reproduces an EG with the line of teridentity that was used in Manuscript 490, but not printed in the partial transcription of Manuscript 490 in 4.583.<sup>15</sup>

We may think of the discovery of such multiple identities as a remarkable prophecy of the modal-logical idea of individuals in different possible worlds being diluted into manifold 'manifestations' or 'aspects' of individuals.<sup>16</sup>

In identification semantics, the question of which manifestations or aspects of individuals will appear in which alternative worlds has to be drawn according to an agent's cognitively-defined modes of identification. Speaking of cognitive modes therefore calls for yet another contextual parameter to be taken into account in compositional semantics for quantified modal logic or, for that matter, in the logic of EGs representing modal assertions concerning individuals.

This complication is primarily due to the varying domains of different *rectos* or possible worlds, and the lines of multiple indefinite identities, in other words Peirce's references, that span several *rectos*.

Furthermore, it is a time-honoured assumption in quantified modal logics that world lines, which we may here associate with Peirce's concept of references, cannot always be extended from one world to another (Hintikka 1970). The individuals subsisting in any actual world may come into existence and disappear in alternative worlds, which is the case if the world lines branch or merge when moving from one possible world with its particular domain of individuals to another world and another domain.

#### 5. ON PRAGMATISM AND RELEVANCE

To return to our blanket topic, several other congenialities obtain between Peirce's pragmatism and RT than those that meet the eye solely within the context of EGs. I list here five of perhaps the most vital such connections, and follow these by discussing some wider philosophical issues pointing out also some differences between the two.

First, the RT principle of *least effort* used in choosing the first and the most accessible interpretation or interpretive hypothesis as the relevant one turns on an argument from cognitive economy. This argument is closely related to Peirce's evolutionary principles, including his quality of *incomplexity* in the economics of research, which suggests that the hypotheses requiring the least effort should be tested before any others. Incomplete hypotheses, which in rational inquiry are bound to be so, should in Peirce's own words "give a good leave" (Peirce 1998, p. 110), because they are in any case likely to be eventually overridden by new hypotheses. They should point towards future investigation rather than past. Hypotheses *per se* are closer to the good and fruitful conducts to be followed than any static set of scientifically tested propositions.

In communicative situations, this method may be understood to refer to things such as the acceptance of both micro-and macro-level data in recreating contexts, in other words, the taking into account of evidence from both the cognitive and biological sides of a given set of theoretical assumptions. It is also related to the Principle of Charity, of taking others' utterances, in large respect and under normal circumstances, as intended to communicate optimal relevance.

Second, supplementing the effect of belief-strengthening in relevance-theoretic comprehension tasks has its correlate in Peirce's notion of a *habit-change*, which means an updating of the belief set held by the communicators, and which gives rise to logical interpretants. Whenever a habit-change occurs, what has been communicated must be taken as relevant. What is also notable in RT is the appeal to the concept of interaction between relevant information and the already-existing assumptions of speaker and hearer concerning the world. This innocent-looking notion that is implicit in the theory may be unravelled by reflecting it against Peirce's interactional interpretation of the relationship between the producer of the information (the utterer) and the receiver who has assumptions about the world which are contested by the information produced by the utterer.

To make the close connections between Peirce's philosophy and later notions in pragmatics absolutely clear, the notion of utterance meaning, or in relevance-theoretical terms the "assumption ostended by an utterance" can be assimilated with Peirce's notion of the *intentional interpretant* that he introduced in his correspondence with Lady Victoria Welby (Peirce 1977). In the same letter, the meaning that the interpreter has to work out is termed by Peirce the *effectual interpretant*. Their mutual merger produces the *communicational interpretant*, or *cominterpretant*, which ultimately accounts for how any form of communication is possible in the first place. The relevance-theoretic outlook on communication has now come strikingly close to this venerable Peircean perspective.

Third, since the degree of effort required when changing the background assumptions measures the degree of relevance, the induced minimax reasoning may be fitted into the strategic framework of game theory, making it explicit that context update in discourse is a rational matter of (optimal) strategy selection. However, as soon as we do this, we are close to what the EM is intended to achieve, since it may be re-instantiated as a form of strategic evaluation method in the sense of game-theoretic semantics.

More pertinent to the topic of interplay between strategic communication and relevance is that, whether the effort of bringing forth relevant information is recommended depends largely on the outcomes (payoffs) of the relevant strategies in the associated game of discourse interpretation. Likewise, it is necessary to deduct the costs incurred by inferences to the best (in the sense of the most-relevant) interpretation from the payoff values assigned to such strategies. While the strategies are chosen according to the general principle of rationality of action,

since they encode information about the context in which discourse is performed and are not confined to isolated utterances, they make the relevance-theoretic notion of the context-change potential of information introduced in communication amenable to rationalistic, although not hyper-rationalistic, game-theoretic analysis.

Fourth, as far as the history and the emergence of the idea of relevance is concerned, it would be make-believe to claim that the core component of relevance (or maxim of relation) would have been something novel with Grice, let alone Sperber and Wilson's RT. From a purely textual viewpoint, Peirce offered the following passage:

The answer to what relevance theorists have been searching for is implicit in this example: it is the *collaterality* of what is given in observation for both the utterer and the interpreter of the utterance that determines relevance. In the light of Peirce's phenomenology, the notion of "what is given" naturally refers not only to real, dynamic, or physical objects, but also to the ideas that signs produce in consciousness. They thus consist of both factual and conceptual elements. There is no analytic/synthetic division in such collaterality. It must however be borne in mind that ideas evoked by conscious minds depend on the situations or environments in which collateral observations can be made, even though the assertions that the signs make in such situations are independent of them in the sense that they can be made just as well in other situations, in which case the ideas produced are, of course, likely to be different.

Since Peirce's logic and his theory of communication (Pietarinen 2003b) is purpose-driven and full of accounts of meaningful intention, and especially since every utterance is made with some goal in sight that an agent tries to reach, the notion of what is relevant must also be assessed with that purpose in mind. What is relevant is relative to the circumstances prevailing in the communicative situation, but what is truly relevant is also, and most likely first and foremost, calculated to be so. Put in the form of a slogan, the Peircean category of Thirdness ought to be ever-present in relevance.<sup>17</sup>

If the utterer says "Fine day!" he does not dream of any possibility of the interpreter's thinking of any mere desire for a fine day that a Finn at the North Cape might have entertained on April 19, 1776. He means, of course, to refer to the actual weather, then and there, where he and the interpreter have it near the surface of their common consciousness. (MS 318: 32–33, c.1907, *Pragmatism.*)

Fifth, it was not only Peirce's pragmaticism (and pragmatics) that foreshadowed the ideas of relevance theorists. Dewey argued forcefully for relevance as a context-effective, context-changing potential: "The existence of the problematic situation to be resolved exercises control over the selective discrimination of relevant and effective evidential qualities as means" (Dewey 1925–53/1986, my emphasis). I believe it will be difficult to find direct textual evidence from the early pragmatists' writings that would come closer than Dewey's remarks on logic to the essentials of the later relevance-theoretic idea of relevance as an effective, inferential, context-sensitive and contextaffecting notion. From the pragmatist perspective, such effects are natural consequnces of the open-systemic nature of language and the organisms that use it, embedded as they are in the background from which mutual collaterality is gained, and which is both affected by and constituted via the selective bias of actions taken by these organisms.

Considerable differences also obtain between Peirce and the kind of pragmatics set up by Grice and his followers and pseudo-followers. The communicative dimensions of Peirce's sign theory are by no means exhausted in what RT attempts to achieve. What is avoided in Peirce's theory of communication is the untoward tendency in current theories to reduce variability in linguistic meaning into the one-sided problem of the speaker's meaning and recognition of his or her intentions. In RT, for instance, the hearer's role has not been incorporated in full.

No one-shot interpretation would have been approved of by Peirce, however, for whom the reciprocal, open-ended and triadic nature of sign meaning is irreducible. I suppose that the reason for the mild reductionism advocated by Sperber and Wilson lies in the unpremeditated domination of Grice's original proposal, in which he laid considerable emphasis on the role of speaker-meaning in linguistic comprehension. Followers of Grice took his suggestions too literally: he never claimed that by focusing on what is different and what is similar in speaker-meaning versus literal meaning, one would reach an exhaustive account of what linguistic and logical meaning amounts to in general.

Moreover, the *soi-disant* followers of Grice were misled by what they took to be his key suggestion: that the proper exposition of speaker-meaning ought to be conducted, first and foremost, by psychological means. In sober reality, this suggestion was, at best, an afterthought for Grice. He de-emphasised the use of psychological

notions in explaining speaker-meaning, a fact which comes out very clearly and forcefully in his writings once it is realised that (i) Grice's main occupation was the meaning of logical particles (most notably of conditionals and other logical connectives) rather than linguistic utterances, and that (ii) his remark that psychological concepts, required for the formulation of an adequate theory of language, refers to intensional concepts of believing and intending which can be tackled in a logical manner. After all, in Grice's writings, references to psychological terminology are few and far between. His theory of meaning is no more psychological than, say, game theory or epistemic logic are matters which concern psychic activity.

RT may have emerged in the wake of Grice, but it subsequently redefined its goals to the extent of now being somewhat of a rival. The emphasis on the search for principles of cognitive processing from which it is hoped that implicatures and other pragmatic notions ensue has had the effect of diminishing the force and depth of the all-powerful rationality postulate upon which Grice's programme was built. In so doing, relevance theorists have rubbed shoulders with the computational sciences, sciences for the efficient accounting of information transmission and manipulation, while turning a blind eye to the conceptual analysis of information. Accordingly, RT has gained in status compared to theories of lessthan-hyper-rational reasoning and action. They all share the methodological concern that effort spent on any act of uttering and interpreting, or believing and decision-making, ought to be weighed against the practical consequences of such acts, and thus continue the venerable economy of research methodology and pragmaticism originated by Peirce. This methodological attitude was also Grice's main preoccupation.

What will happen to RT in the light of Peirce's pragmaticism? Perhaps relevance is being drawn in a somewhat unhealthy direction by attempts to build the notion upon the psychological and cognitive theory of the competence of intelligent agents, while simultaneously claiming to be able to give some support to its inferential and logical dimensions. In sharp contrast, Peirce's goal was not to spell out the theory of cognition of intelligent agents, let alone their psychology, but to dispense with these as much as possible. This may not have been an undertaking that was entirely realistic, but at least Peirce's theory shows the priorities he thought were required in the brands of rational inquiry that concern language and thought.

## 6. CONCLUSIONS

Relevant items of information are those that are context-effective. This means that they are context-dependent. In communicative situations, context-dependence is inferred by the interpreter who is given evidence concerning the intended meaning by the utterer. This view of relevance is nothing like a coding-encoding method of expressing thought and then comprehending the meaning or content of the expressed thought, let alone an endorsement of the view that the actual meaning or content resides in such expressions. The hearer has to infer the intended meaning, and the speaker has to effectuate the intended meaning. Varieties of meaning cannot be lumped together under any single method.

Moreover, since inference takes in collateral information and experience, relevant items of information are delineated by the negative contexts, in other words information residing within cuts in terms of Peirce's EGs, and are in that sense not only merely rejected or accepted according to context but also determined by it. This is the juncture at which the mutual interplay between context-creation and the idea of relevance as context-change potential manifests itself in a logical way.

That Peirce would have been keen to repudiate the utility of any alternative compositional version of semantics over his noncompositional one is supported not only by his overall pragmatic and exterior-to-interior method of interpretation, but also by his everpresent *holistic* attitude to recognition, perception, and other cognitive tasks. Such an Aristotelian holon was supposed to take graph-instances, definite but incomplete snapshots of the contents of the mind, as a unity regulated by the true continuity of *synechism*, mathematically rendered as topological, structure-preserving mappings subject to continuous deformations from one graphinstance to another. If the relative positions of graph-instances on a given graph change as a result of violating transformation rules, this must affect the composition of the given graph as a whole. Likewise, if the consequences that follow from the graph and which constitute the aggregate of meaning as a collection of graph-instances change, this must affect the nature of the meaning of the given graph.

In doing this, Peirce came to anticipate the later Gestalt theories of representation and Gestalt-like organisation and the "mental composite photograph" (2.438, c.1893, *The Grammatical Theory of Judgment and Inference*) type of composition of percepts (Pietarinen

2004d), as indeed the later non-monotonic systems as regimented versions of his *tychims* (Maróstica 1997), a doctrine of absolute chance which, when synthesised with pragmaticism, gave rise to his synechist metaphysics.

But these were already the beginnings of other stories.

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#### NOTES

<sup>1</sup> The standard formulation states that the meaning of a compound expression is the morphic image of the meaning of its constituents.

 $^{2}$  Apart from cognitive economy, the idea hints at an element of utilitarianism in the definition of communicative goals in terms of the maximisation of something (in this context, relevance).

<sup>3</sup> The reference is to Peirce (1931–58) by volume and paragraph number.

<sup>4</sup> The reference is to Peirce (1975–1987) by volume and paragraph number. This is a striking foreshadowing of Wittgenstein's mantra that "the word has meaning by the particular use we make of it" (Wittgenstein, 2000–, item 147: 39v, *Grosses Notizbuch*, cf. Pietarinen, 2003a).

<sup>5</sup> Witness the infamous Frame Problem in AI: it is only the most idiotic robots who would not distinguish between relevant and non-relevant input from their environment, and those are unlikely to survive for long.

<sup>6</sup> Not all theories of pragmatics are pragmatic in the same sense. For instance, Richard Montague's theory of grammar claims to explain the resolution of contextual matters using a hybrid of higher-order and possible-worlds concepts.

<sup>7</sup> Spot is alternatively termed a *rhema* or a *rheme* by Peirce.

<sup>8</sup> Peirce used the term *scroll* to refer to the graph of two nested cuts scribed by one continuous closed line.

<sup>9</sup> The reference is to Peirce Manuscripts (Peirce 1967) by manuscript number, and, if applicable, followed by page number. I also give the title of the manuscript when first referred to.

<sup>10</sup> We may even associate the "being satisfied with" used here with Tarski's less concrete idea of satisfaction. Löwenheim used similar term of being "satisfied" ["erfüllt"] in his early model-theoretic explorations well before Tarski (Badesa, 2004, p. 139).

<sup>11</sup> An alternative symbolic notation is in terms of branching (alias Henkin) quantifiers:  $\frac{\forall x \exists y}{\forall z \exists u} (S_1(x, z) \to (S_2(x, y) \land S_2(z, u))).$ 

<sup>12</sup> Introduction to Existential Graphs and an Improvement on the Gamma Graphs, written in 1906, a couple of months after Peirce's pragmatic account of the principles of 'compositionality' and 'context' in terms of practical effects.

<sup>13</sup> The argument is found later in the Manuscript 490 which was transcribed only partially in 4.583, and some crucial intermediate passages were not reproduced.

<sup>14</sup> For Peirce, phaneroscopy is the third part of philosophical inquiry alongside with the normative sciences and metaphysics.

<sup>15</sup> This omission by the editors of the *Collected Papers* was, I believe, one of the main reasons why Peirce's idea of "multiple indefinite identities" has, to the best of my knowledge, received scarcely any attention in earlier literature.

<sup>16</sup> If cross-identification of individuals carries too heavy metaphysical baggage, multiple identities may be thought of in Lewis' (1968) sense as 'counterparts' of actual individuals in contradistinction to actual individuals with a self-supporting identity.

<sup>17</sup> Refraining from going into the details of Peirce's categories, Thirdness was, according to him, "that mode or element of being whereby a subject is such as it is to a second and for a third, prominent in the ideas of instrument, organon, method, means, mediation, betweenness, representation, communication, community, composition, generality, regularity, continuity, totality, system, understanding, cognition, abstraction, etc." (MS L 107: 21–23, 1904, *Auto-Biography for Matthew Mattoon Curtis, Draft C*, marked 'final' by Peirce).

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