DOI 10.1007/s10511-024-09829-y
Astrophysics, Vol. 67, No. 2, August, 2024

ISOSPIN-ASYMMETRIC COLD NUCLEAR MATTER IN THE RELATIVISTIC MEAN-FIELD THEORY WITH A SCALAR-ISOVECTOR INTERACTION CHANNEL

G. B. Alaverdyan and A. G. Alaverdyan

The properties of isospin-asymmetric cold nuclear matter are studied in terms of the relativistic mean-field theory in which, besides the fields of σ, ω, and ρ mesons, and the isovector, Lorentz-scalar field of the δ-meson is also taken into account. The properties of purely nucleonic np matter are studied as a function of the baryon density n_B and the asymmetry parameter α, as well as the properties of electrically neutral β-equilibrium npeμ matter as a function of the baryon density n_B. For different values of n_B and a, such characteristics of np matter as the energy per baryon, the specific energy owing to isospin asymmetry, the effective proton and neutron masses, and the specific binding energy, are determined. It is shown that the energy owing to the asymmetry for a fixed value of α is a monotonically increasing function of the baryon density n_B. For npem matter, the effective proton and neutron masses $M_p^{(eff)}$, $M_n^{(eff)}$, the specific binding energy E_{bind}, the symmetry energy E_{sym}, the quantitative fraction of protons $Y_p = n_p/n_B$ are studied, as well as the average meson fields $\tilde{\sigma}$, $\tilde{\omega}$, $\tilde{\delta}$, and $\tilde{\rho}$ as functions of the baryon density n_B.

Keywords: *Mean field, nucleon matter: neutron star matter: symmetry energy: effective mass: binding energy*

Erevan State University, Armenia; e-mail: galaverdyan@ysu.am

Original article submitted February 10, 2024; accepted for publication June 10, 2024. Translated from Astrofizika, Vol. 67, No. 2, pp. 229-243 (May 2024)

1. Introduction

The thermodynamic description of nuclear matter is an important problem on the way to understanding the properties and structure of nuclei, the dynamics of heavy ion collisions, the structure of compact stars, the dynamics of supernova explosions, and the process of merging of neutron stars. For densities higher than the density of nuclear saturation n_0, β-equilibrium nucleon matter is isospin-asymmetric. This circumstance explains precisely the great interest devoted to the study of the properties of isospin-asymmetric nucleon matter [1-5]. The isospin dependence of the equation of state of nuclear matter for a specified value of the baryon density n_B is determined by the energy of nuclear symmetry $E_{sym}(n_B)$.

The modern feasibility of earthbound nuclear experiments and astrophysical observations have ensured substantial progress in the study of the dependence of the symmetry energy on density [6-8]. In recent years new possibilities have arisen for obtaining limits on the physical characteristics of asymmetric nuclear matter using data from the NICER and XMM-Newton scientific programs for simultaneous determination of the mass and radius of a neutron star (NS) [9], along with data on tidal deformation obtained from an analysis of gravitational radiation during merger of the binary neutron star GW170817 [10,11].

Theoretical studies of the properties of nuclear matter and finite nuclei as systems of strongly-interacting relativistic baryons and mesons are based on a quantum-field approach in the framework of quantum hadrodynamics (QHD). One of the successfully applied models of this kind is the relativistic mean-field theory (RMF). In the original model, the interaction between nucleons took place through exchange of an isoscalar, Lorentz-scalar σ-meson and an isoscalar, Lorentz-vector ω-meson [12-14]. For satisfactory reproduction of nuclear incompressibility and the properties of the unstable nuclei, self-interaction terms for the σ- and ω-mesons were included in the model leading to the appearance of nonlinear terms in the equations for the meson fields [15-17]. To describe the thickness of the neutron shell of the heavy nuclei and the characteristics of isospin-asymmetric nuclei, the composition of the exchange mesons was expanded, and the isovector, Lorentz-vector ρ-meson was also added to the scheme [18].

For completeness of the transformation properties of the meson fields it was also necessary to have an isovector, Lorentz-scalar δ-meson in the composition of the exchange mesons. This was done in Refs. 19-21. This sort of expansion of the model was used to study scattering processes in excess-neutron heavy ions at medium energies and to clarify the prospects for formation of a mixed hadron-quark phase during a collision event [22,23]. Studies of the influence of a δ-meson field on the characteristics of a hadron-quark phase transition and on the observed parameters of hybrid stars were made in Refs. 24 and 25.

The purpose of this paper is to study single-particle properties of isospin-asymmetric nuclear matter in terms of the relativistic mean-field theory, in which the isovector, Lorentz-scalar field of the δ-meson is also taken into account. The article is organized as follows. A brief description of the model in which a system of equations is introduced for the average meson fields and formulas for such single-particle characteristics of nuclear matter as the energy of symmetry, the energy per baryon, the specific binding energy, the effective masses of the proton and neutron, and the chemical potentials of the baryons. Both purely nucleonic np matter and electrically neutral with

β-equilibrium nuclear matter consisting of nucleons and charged e^- and μ^- leptons (*npe*μ matter) are examined. In the last section the main results of this paper are summarized.

2. Description of the model

This section gives a brief discussion of the model that we used for a thermodynamic description of dense nucleonic matter. In terms of the relativistic mean-field model [12-14] based on quantum hadron dynamics, we examine a system of particles consisting of nucleons (*n*, *p*), with the strong interaction between them realized by exchange of σ, ω, δ, and ρ mesons.

The Lagrangian density for this sort of system has the form

$$\begin{aligned}\mathcal{L}_{RMF} = \sum_{i=p,\,n} \bar{\psi}_i \left[\gamma_\mu\left(i\partial^\mu - g_\omega \omega^\mu - \frac{1}{2} g_\rho \boldsymbol{\rho}^\mu \boldsymbol{\tau}_i\right) - \left(m_N - g_\sigma \sigma - g_\delta \boldsymbol{\delta}\boldsymbol{\tau}_i\right)\right]\psi_i \\ -\frac{1}{3} g_{\sigma 3}\left(g_\sigma \sigma\right)^3 - \frac{1}{4} g_{\sigma 4}\left(g_\sigma \sigma\right)^4 + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right) + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\ -\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}\left(\partial_\mu \boldsymbol{\delta}\partial^\mu \boldsymbol{\delta} - m_\sigma^2 \boldsymbol{\delta}^2\right) + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \boldsymbol{\rho}^\mu - \frac{1}{4}\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}\ ,\end{aligned} \tag{1}$$

where ψ_i is the spin field of the nucleon, $\boldsymbol{\tau}_i$ are the isospin Pauli matrices, σ, ω_μ, $\boldsymbol{\delta}$, and $\boldsymbol{\rho}_\mu$ are the fields of the exchange mesons, which depend on the space-time coordinates $x_\mu = (t,\ x,\ y,\ z)$, m_N is the mass of a bare nucleon, m_σ, m_ω, m_δ, and m_ρ are the masses of the exchange mesons, $\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $\mathcal{R}_{\mu\nu} = \partial_\mu \boldsymbol{\rho}_\nu - \partial_\nu \boldsymbol{\rho}_\mu$ are the antisymmetric tensors of the vector fields ω_μ and $\boldsymbol{\rho}_\mu$. The terms $g_{\sigma 3}\left(g_\sigma \sigma\right)^3/3$ and $g_{\sigma 4}\left(g_\sigma \sigma\right)^4/4$ in Eq. (1) leading in the equations of motion to a nonlinear dependence on the isoscalar Lorentz-scalar field σ were introduced in Ref. 15 to reach an acceptable quantitaative reprodcction of the properties of the ground state of symmetric nuclear matter. The coupling constants of a nucleon with the corresponding meson are denoted by g_σ, g_ω, g_δ, and g_ρ.

In the mean-field approximation the meson fields, which in general depend on the space-time coordinates, are replaced by the nonuniform time-invariant mean fields $\bar{\sigma}$, $\bar{\omega}_\mu$, $\bar{\boldsymbol{\delta}}$, and $\bar{\boldsymbol{\rho}}_\mu$. The Euler-Lagrange equations for the nucleon and meson fields lead to a closed system of equations for a specified value of the density of the baryon number $n_B = n_n + n_p$ and the asymmetry parameter $\alpha = \left(n_n - n_p\right)/n_B$. Of course, these equations will also include the masses of the mesons, but with a convenient change of the notation for the binding constant and average meson fields it is possible to avoid these parameters (see, e g., Refs. 20 and 24):

$$g_\sigma \overline{\sigma} \to \tilde{\sigma}, \quad g_\omega \overline{\omega} \to \tilde{\omega}, \quad g_\delta \overline{\delta^{(3)}} \to \tilde{\delta}, \quad g_\rho \overline{\rho^{(3)}} \to \tilde{\rho},$$
$$\left(\frac{g_\sigma}{m_\sigma}\right)^2 \to \alpha_\sigma, \left(\frac{g_\omega}{m_\omega}\right)^2 \to \alpha_\omega, \left(\frac{g_\delta}{m_\delta}\right)^2 \to \alpha_\delta, \left(\frac{g_\rho}{m_\rho}\right)^2 \to \alpha_\rho . \tag{2}$$

The equations for the redesignated average meson fields $\tilde{\sigma}$, $\tilde{\omega}_\mu$, $\tilde{\delta}$, and $\tilde{\rho}_\mu$ have the form

$$\tilde{\sigma} = \alpha_\sigma \left(n_p^{(s)}(n_B, \alpha) + n_n^{(s)}(n_B, \alpha) - g_{\sigma 3}\tilde{\sigma}^2 - g_{\sigma 4}\tilde{\sigma}^3\right), \quad \tilde{\omega} = \alpha_\omega n_B ,$$
$$\tilde{\delta} = \alpha_\delta \left(n_p^{(s)}(n_B, \alpha) - n_n^{(s)}(n_B, \alpha)\right), \quad \tilde{\rho} = -\frac{1}{2}\alpha_\rho n_B \alpha . \tag{3}$$

Here $n_p^{(s)}(n_B, \alpha)$ and $n_n^{(s)}(n_B, \alpha)$ are the scalar densities of the protons and neutrons, which are given by [24,25]

$$n_p^{(s)}(n_B, \alpha) = \frac{M_p^{(eff)}(\tilde{\sigma}, \tilde{\delta})}{\pi^2} \int_0^{k_F(n_B)(1-\alpha)^{1/3}} \frac{1}{\sqrt{k^2 + M_p^{(eff)}(\tilde{\sigma}, \tilde{\delta})^2}} k^2 dk ,$$
$$n_n^{(s)}(n_B, \alpha) = \frac{M_n^{(eff)}(\tilde{\sigma}, \tilde{\delta})}{\pi^2} \int_0^{k_F(n_B)(1+\alpha)^{1/3}} \frac{1}{\sqrt{k^2 + M_n^{(eff)}(\tilde{\sigma}, \tilde{\delta})^2}} k^2 dk , \tag{4}$$

where $k_F(n_B) = \left(3\pi^2 n_B/2\right)^{1/3}$, while $M_p^{(eff)}(\tilde{\sigma}, \tilde{\delta})$ and $M_n^{(eff)}(\tilde{\sigma}, \tilde{\delta})$ are the effective masses of the nucleons, which are given by formulas of the form

$$M_p^{(eff)}(\tilde{\sigma}, \tilde{\delta}) = m_N - \tilde{\sigma} - \tilde{\delta}, \quad M_n^{(eff)}(\tilde{\sigma}, \tilde{\delta}) = m_N - \tilde{\sigma} + \tilde{\delta} . \tag{5}$$

In terms of the relativistic mean-field theory the energy density $\varepsilon_{NM}(n_B, \alpha)$ of nucleon (neutron-proton) matter as a function of the baryon density n_B and asymmetry parameter α is given by

$$\varepsilon_{NM}\left(n_B,\ \alpha\right)=\frac{1}{\pi^2}\int\limits_0^{k_F(n_B)(1-\alpha)^{1/3}}\sqrt{k^2+M_p^{(eff)^2}}\,k^2dk$$

$$+\frac{1}{\pi^2}\int\limits_0^{k_F(n_B)(1+\alpha)^{1/3}}\sqrt{k^2+M_n^{(eff)^2}}\,k^2dk \tag{6}$$

$$+\frac{1}{2}\left(\frac{\tilde{\sigma}^2}{\alpha_\sigma}+\frac{\tilde{\omega}^2}{\alpha_\omega}+\frac{\tilde{\delta}^2}{\alpha_\delta}+\frac{\tilde{\rho}^2}{\alpha_\rho}\right)+\frac{1}{3}g_{\sigma 3}\tilde{\sigma}^3+\frac{1}{4}g_{\sigma 4}\tilde{\sigma}^4 .$$

In the case of an electrically neutral and β-equilibrium cold hadron material, consisting of neutrons and protons, as well as charged leptons — electrons e and muons μ, the energy density $\varepsilon_{HM}\left(n_B\right)$ is given by the expression

$$\varepsilon_{HM}\left(n_B\right)=\varepsilon_{NM}\left(n_B,\ \alpha\right)+\sum_{l=e,\ \mu}\frac{1}{\pi^2}\int\limits_0^{\sqrt{\mu_l^2-m_l^2}}\sqrt{k^2+m_l^2}\,k^2dk\ , \tag{7}$$

where m_l are the masses, while μ_l are the chemical potentials of the corresponding charged leptons ($l=e,\mu$). For matter of this type, the conditions of β-equilibrium and electrical neutrality must hold:

a) for a baryon density below the threshold for creation of muons:

$$\mu_n\left(n_B,\ \alpha\right)-\mu_p\left(n_B,\ \alpha\right)=\mu_e\left(n_e\right)\ ,\ n_p=n_e\ . \tag{8}$$

b) for a baryon density greater than the threshold for creation of muons:

$$\mu_n\left(n_B,\ \alpha\right)-\mu_p\left(n_B,\ \alpha\right)=\mu_e\left(n_e\right)=\mu_\mu\left(n_\mu\right)\ ,\ n_p=n_e+n_\mu\ . \tag{9}$$

Here $\mu_p\left(n_B,\ \alpha\right)$ and $\mu_n\left(n_B,\ \alpha\right)$ are the chemical potentials, respectively, of protons and neutrons, which are defined by the formulas

$$\mu_p\left(n_B,\ \alpha\right)=\sqrt{k_F\left(n_B\right)^2\left(1-\alpha\right)^{2/3}+M_p^{(eff)^2}}+\tilde{\omega}+\frac{1}{2}\tilde{\rho}\ ,$$

$$\mu_n\left(n_B,\ \alpha\right)=\sqrt{k_F\left(n_B\right)^2\left(1+\alpha\right)^{2/3}+M_n^{(eff)^2}}+\tilde{\omega}-\frac{1}{2}\tilde{\rho}\ . \tag{10}$$

Equations (8) and (9) make it possible to express the asymmetry parameter of an electrically neutral and β-equilibrium material in terms of the baryon density. In this case, the energy density depends only on the baryon density n_B.

The energy per baryon E_B and the specific binding energy E_{bind}, as functions of the baryon density n_B and the asymmetry parameter α are given by the formulas

$$E_B(n_B,\alpha)=\frac{\varepsilon_{NM}(n_B,\alpha)}{n_B}\ ;\ E_{bind}(n_B,\alpha)=E_B(n_B,\alpha)-m_N\ . \tag{11}$$

We denote the energy per baryon owing to the isospin asymmetry of the system by $\Delta E_B(n_B,\alpha)=E_B(n_B,\alpha)-E_B(n_B,0)$.

The symmetry energy $E_{sym}(n_B)$ is determined from an expansion of the function $E_B(n_B,\alpha)$ in a series with respect to the asymmetry parameter: $E_B(n_B,\alpha)=E_B(n_B,0)+E_{sym}(n_B)\alpha^2+o(\alpha^4)$, so that

$$E_{sym}(n_B)=\frac{1}{2}\frac{\partial^2 E_B(n_B,\alpha)}{\partial\alpha^2}\bigg|_{\alpha=0}\ . \tag{12}$$

Using Eq. (6) for the energy density $\varepsilon_{NM}(n_B,\alpha)$ and Eqs. (3) for the meson fields, it is possible to obtain an expression for the symmetry energy $E_{sym}(n_B)$. The terms that do not contain coupling constants for nucleon-meson interactions will represent the kinetic part of the symmetry energy $E_{sym}^{(kin)}$, and the terms that contain the coupling constants, the potential part of the symmetry energy $E_{sym}^{(pot)}$:

$$E_{sym}^{(kin)}(n_B)=\frac{k_F^2}{6\sqrt{k_F^2+M_N^{(eff)^2}}}\ , \tag{13}$$

$$E_{sym}^{(pot)}(n_B)=\frac{n_B}{2}\left\{\frac{\alpha_\rho}{4}-\frac{\alpha_\delta M_N^{(eff)^2}}{k_F^2+M_N^{(eff)^2}}J_{sym}\right\}\ . \tag{14}$$

Here J_{sym} denotes the expression

$$J_{sym}=\left[1+3\alpha_{\delta}\left(\frac{n_B^{(s)}}{M_N^{(eff)}}-\frac{n_B}{\sqrt{k_F^2+M_N^{(eff)^2}}}\right)\right]^{-1},$$

$$k_F=\left(3\pi^2 n_B/2\right)^{1/3},\quad M_N^{(eff)}=M_p^{(eff)}\left(\tilde{\sigma},\tilde{\delta}=0\right)=M_n^{(eff)}\left(\tilde{\sigma},\tilde{\delta}=0\right)=m_N-\tilde{\sigma}.$$

The scalar baryon density $n_B^{(s)}$ in Eq. (14) is the sum of the scalar densities of the proton and neutron in symmetric nuclear matter, $n_B^{(s)}=n_p^{(s)}\left(n_B,\ 0\right)+n_n^{(s)}\left(n_B,\ 0\right)$:

$$n_B^{(s)}=\frac{2}{\pi^2}\int_0^{k_F(n_B)}\frac{M_N^{(eff)}}{\sqrt{k^2+M_N^{(eff)^2}}}k^2dk\ . \tag{15}$$

A similar expression for the symmetry energy of nucleonic matter can be obtained [26,27], based on the statement of the Huyghenholts-van Hove theorem [28,29] that the chemical potential of a nucleon in asymmetric nuclear matter should equal its Fermi energy.

A determination of the phenomenological constants of the theory also requires knowledge of the compressibility modulus of nuclear matter at the saturation density n_0:

$$K_0=9n_0^2\left.\frac{d^2E_B\left(n_B,\ \alpha\right)}{dn_B^2}\right|_{\substack{n_B=n_0\\ \alpha=0}}\ .$$

The compressibility modulus also consists of kinetic and potential parts:

$$K^{(kin)}\left(n_B\right)=\frac{3k_F^2}{\sqrt{k_F^2+M_N^{(eff)^2}}}\ , \tag{16}$$

$$K^{(pot)}\left(n_B\right)=9n_B\left(\alpha_{\omega}-\frac{\alpha_{\sigma}M_N^{(eff)^2}}{k_F^2+M_N^{(eff)^2}}J_K\right)\ . \tag{17}$$

J_K denotes the expression

$$J_K = \left[1 + 3\alpha_\sigma \left(\frac{n_B^{(s)}}{M_N^{(eff)}} - \frac{n_B}{\sqrt{k_F^2 + M_N^{(eff)2}}} + \frac{2}{3} g_{\sigma 3}\, \tilde{\sigma} + g_{\sigma 4}\, \tilde{\sigma}^2 \right)\right]^{-1}.$$

3. Numerical results

Making numerical calculations of the thermodynamic quantities requires a determination of the coupling constants characterizing the interaction of nucleons with the meson fields. The constants α_σ, α_ω, $g_{\sigma 3}$, and $g_{\sigma 4}$ corresponding to the isoscalar σ and ω - mesons can be determined using the values of the known parameters of symmetric nuclear matter at saturation. We used the following values: bare nucleon mass m_N=939.9 MeV, nuclear saturation density n_0=0.16 fm^{-3}, effective mass of a nucleon at saturation $M_0^{(eff)} = 0.78 m_N$, specific binding energy f_0=–16.3 MeV, and compressibility modulus at saturation K_0=300 MeV. For determining the constants corresponding to the isovector δ and ρ-mesons, it is necessary to use the values for the characteristics of asymmetric nuclear matter. For the constant α_δ [20], a value of $\alpha_\delta = 2.5$ fm^{-3} was used. The constant α_ρ was determined using the value of the symmetry energy at saturation, $E_{sym}^{(0)} = 32.5$ MeV.

The above characteristics of nuclear matter are reproduced with the following values of the parameters of the model of Ref. 30: $\alpha_\sigma = 9.15$ fm^2, $\alpha_\omega = 4.83$ fm^2, $\alpha_\delta = 2.5$ fm^2, $\alpha_\rho = 13.62$ fm^2, $g_{\sigma 3} = 0.0165$ fm^{-1}, and $g_{\sigma 4} = 0.0132$.

Once the nucleon-meson coupling constants were known, along with the self-interaction constants of the σ-field, $g_{\sigma 3}$ and $g_{\sigma 4}$, for different values of the baryon density n_B and the asymmetry parameter α, the physical characteristics of the isotopically-asymmetric nuclear matter consisting of protons and neutrons were calculated.

Figure 1a shows the energies $E_B(n_B, \alpha)$ and $E_B(n_B, 0)$, respectively, per baryon for isospin-asymmetric and symmetric nucleon matter as functions of the baryon density n_B and the asymmetry parameter α. Figure 1b shows the energy $\Delta E_B(n_B, \alpha) = E_B(n_B, \alpha) - E_B(n_B, 0)$ per baryon owing to the isospin asymmetry of nucleonic matter as a function of n_B and α. This figure shows that the energy $\Delta E_B(n_B, \alpha)$ owing to the asymmetry for a fixed value of the asymmetry parameter α is a monotonically increasing function of the baryon density n_B.

Figure 2 shows the effective masses of the proton and neutron for values of the asymmetry parameter $\alpha = \{0;\ 0.5;\ 1\}$ as functions of the baryon number density n_B. The effective masses of the proton and neutron, as is to be expected, are the same for symmetric nucleonic matter. A separation in the effective masses of the nucleons is caused by the presence of the field of the isovector Lorentz scalar δ-meson. In the model we are examining, the

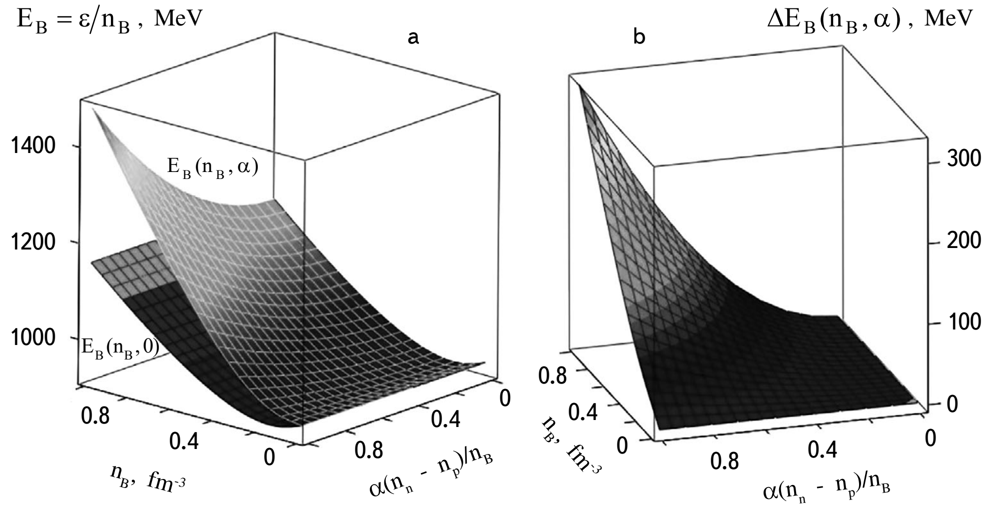

Fig. 1. a. The energy per baryon $E_B(n_B, \alpha)$ as a function of the baryon number density n_B and the asymmetry parameter α. The lower surface corresponds to the function $E_B(n_B, 0)$. b. The energy per baryon owing to the isospin asymmetry of nucleonic matter $\Delta E_B(n_B, \alpha) = E_B(n_B, \alpha) - E_B(n_B, 0)$ as a function of the baryon number density n_B and the asymmetry parameter α.

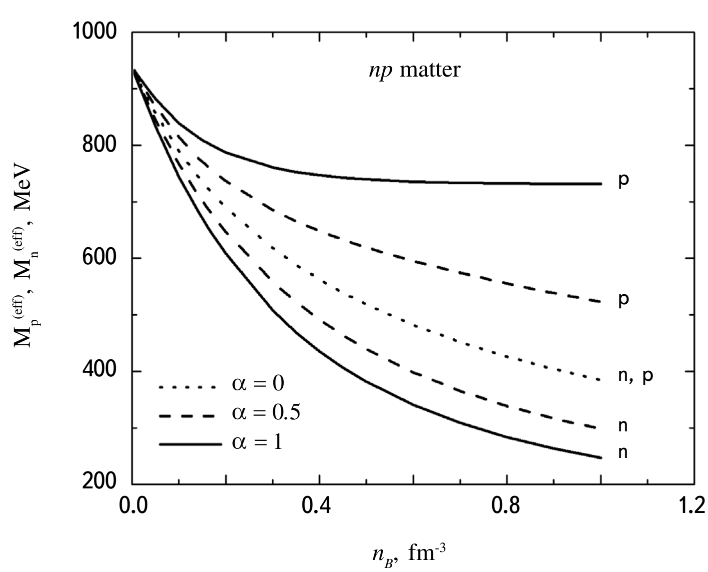

Fig. 2. The effective masses of a proton and a neutron as functions of the baryon number density n_B for different values of the asymmetry parameter α.

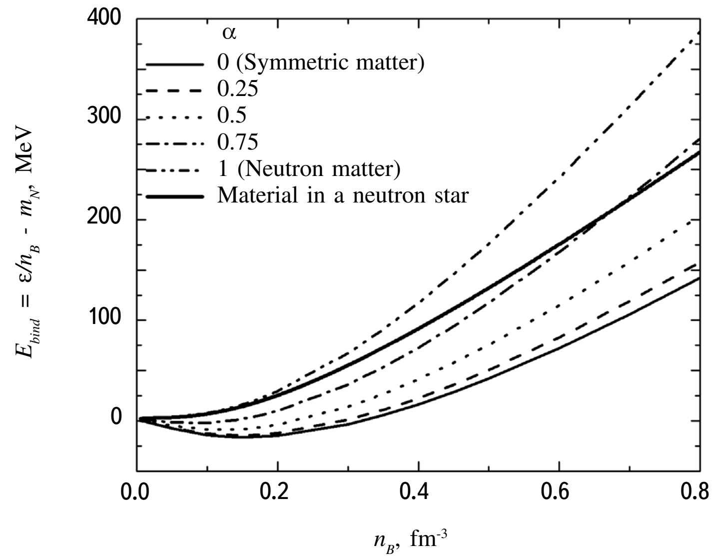

Fig. 3. The binding energy per unit baryon as a function of the baryon number density n_B for different values of the asymmetry parameter α. The thick smooth curve corresponds to electrically neutral β-equilibrium *npe*μ matter (N3 matter).

effective mass of the proton is greater than the effective mass of the neutron. For a specified value of the baryon density n_B the effective mass of the neutron decreases as the asymmetry parameter increases, while the effective mass of the proton increases. The difference in the effective masses of the proton and neutron for a given value of the baryon density n_B increases as the asymmetry parameter increases, reaching its maximum value in the case of pure neutronic matter ($\alpha = 1$).

The dependence of the specific binding energy $E_{bind}(n_B, \alpha)$ of nucleonic matter on the baryon density n_B for asymmetry parameters equal to $\alpha = \{0;\ 0.25;\ 0.5;\ 0.75;\ 1\}$ is shown in Fig. 3. For comparison, this figure shows the analogous dependence in the case of electrically neutral and β-equilibrium *npe*μ (N3 matter) in the form of a thick continuous curve. It is clear that in the range of values of the baryonic density $n_B \in [0;\ 0.8]$, typical of the matter with a hadronic structure in neutron stars, the asymmetry parameter a ranges from values $\alpha \approx 1$ (for $n_B \approx 0$) to $\alpha \approx 0.7$ (for $n_B \approx 0.8$ fm^{-3}).

Figure 4 shows the effective masses of the proton and neutron as functions of the baryon number density n_B for electrical neutrality and β-equilibrium of *npe*μ matter. Like the increase in the baryon density, the asymmetry parameter in N3 matter decreases (cf. Fig. 3), and for a given value of the baryon density n_B the difference in the effective masses of the proton and neutron is smaller in the case of electrical neutrality for β-equilibrium of *npe*μ matter than in the case of purely neutronic matter.

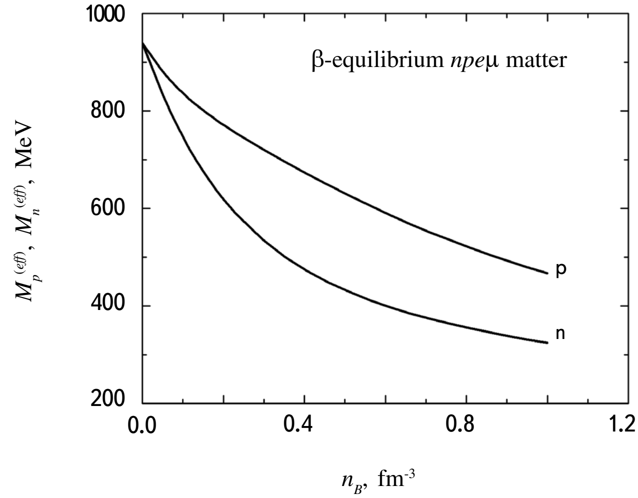

Fig. 4. Effective masses of a proton and neutron as functions of the baryon number density n_B for electrically neutral and β-equilibrium $npe\mu$ matter (N3 matter).

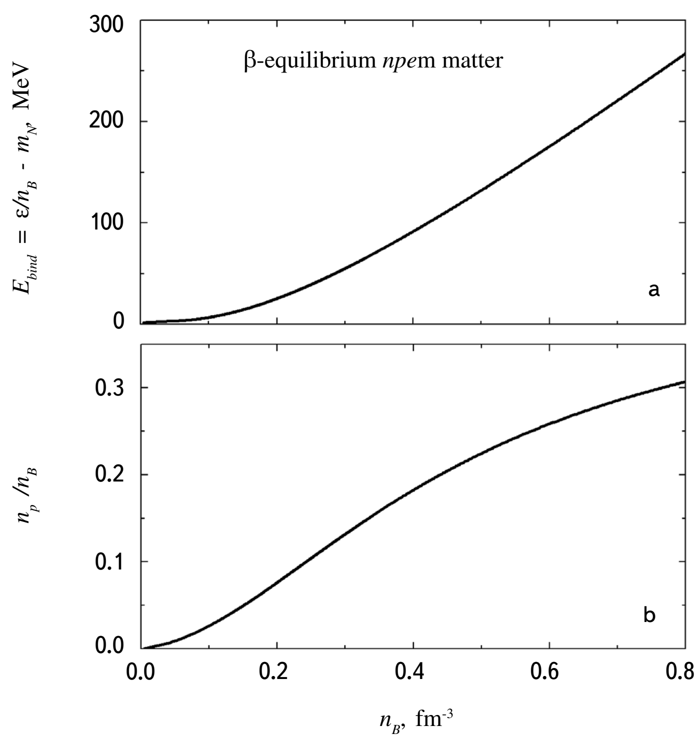

Fig. 5. a. The binding energy per baryon as a function of the baryon number density n_B for electrically neutral and β-equilibrium $npe\mu$ matter. b. The fraction of the number of protons $Y_p = n_p/n_B$ as a function of the baryon number density n_B for electrically neutral and β-equilibrium $npe\mu$ matter.

Figure 5a shows the dependence of the specific binding energy E_{bind} on the baryon density n_B for neutral and β-equilibrium *npe*μ matter (N3 matter). The fraction of the number of protons $Y_p = n_p / n_B = (1-\alpha)/2$ as a function of the baryon density n_B for N3 matter is shown in Fig. 5b. Given that for high densities the conditions are created for quark deconfinement and, because of a phase transition, quark matter will be formed, we conclude that in the hadron component of a neutron star the maximum value of the specific energy will be on the order of 250-300 MeV. The number of protons in the hadron component of a neutron star will not exceed ~30% of the number of nucleons.

Figure 6a shows the dependences of the symmetry energy E_{sym} and its components $E_{sym}^{(kin)}$ and $E_{sym}^{(pot)}$ on the baryon density n_B for electrically neutral and β-equilibrium *npe*μ matter. Figure 6b illustrates the contributions of the isovector δ and ρ-mesons, $E_{sym}^{(\delta)}$ and $E_{sym}^{(\rho)}$, to the potential part $E_{sym}^{(pot)}$ of the symmetry energy. It is clear that in the region of densities below the nuclear saturation density n_0, the kinetic energy of symmetry $E_{sym}^{(kin)}$ and the potential energy of symmetry $E_{sym}^{(pot)}$ are of the same order of magnitude. For high densities the potential part $E_{sym}^{(pot)}$ of the energy of symmetry makes a dominant contribution to the energy of symmetry E_{sym}.

It is clear from Fig. 6b that the isovector, Lorentz-vector r-meson makes a positive contribution to $E_{sym}^{(\rho)}$ in the potential part of the symmetry energy $E_{sym}^{(pot)}$, while the analogous contribution from the isovector Lorentz-scalar δ-meson, $E_{sym}^{(\delta)}$, is negative. For higher densities the contribution of the ρ-meson to the potential energy of symmetry $E_{sym}^{(\rho)}$ becomes far greater than the absolute value of the contribution of the δ-meson, $E_{sym}^{(\delta)}$.

As the equations for the average fields (3) imply, the average field $\tilde{\omega}$ for a specified value of the baryon density n_B does not depend on the asymmetry parameter α. This means that the average field of the isoscalar Lorentz-

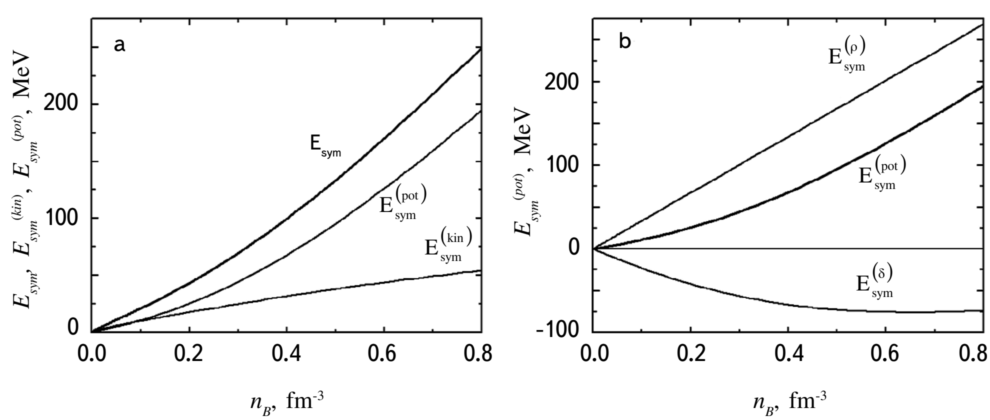

Fig. 6. a. The symmetry energy E_{sym} and its components $E_{sym}^{(kin)}$ and $E_{sym}^{(pot)}$ as functions of the baryon number density n_B for electrically neutral and β-equilibrium *npe*μ matter. b. The potential part $E_{sym}^{(pot)}$ of the symmetry energy and its components $E_{sym}^{(\delta)}$ and $E_{sym}^{(\rho)}$ as functions of the baryon number density n_B for electrically neutral and β-equilibrium *npe*μ matter.

vector ω-meson for a specified value of n_B has the same value for an arbitrary ratio of the amounts of neutrons and protons, n_n/n_p. The average field $\tilde{\omega}$ for *np* matter and electrically neutral and β-equilibrium *npe*μ matter has the same value for a specified value of n_B. Because of the dependence on the asymmetry parameter α, for a given value of n_B, the average fields $\tilde{\sigma}$, $\tilde{\delta}$, and $\tilde{\rho}$ will not be the same for *np* matter and for electrically neutral and β-equilibrium *npe*μ matter (N3 matter).

Figure 7 shows the average fields $\tilde{\sigma} = g_\sigma \overline{\sigma}$, $\tilde{\omega} = g_\omega \overline{\omega}$, $\tilde{\delta} = g_\delta \overline{\delta^{(3)}}$, and $\tilde{\rho} = g_\rho \overline{\rho^{(3)}}$ of the exchange mesons as functions of the baryon number density n_B for electrically neutral and β-equilibrium *npe*μ matter. The average fields of the isoscalar σ and ω-mesons have positive values and are monotonically increasing functions of the baryon density n_B. The average fields of the isovector δ and ρ-mesons have negative values. Here the average field of an isovector Lorentz-scalar δ-meson is a decreasing function of n_B for $n_B < 0.44$ fm^{-3} and a slowly increasing function of n_B for $n_B > 0.44$ fm^{-3}. The average field of the isovector Lorentz-vector ρ-meson is a monontonically decreasing function of n_B.

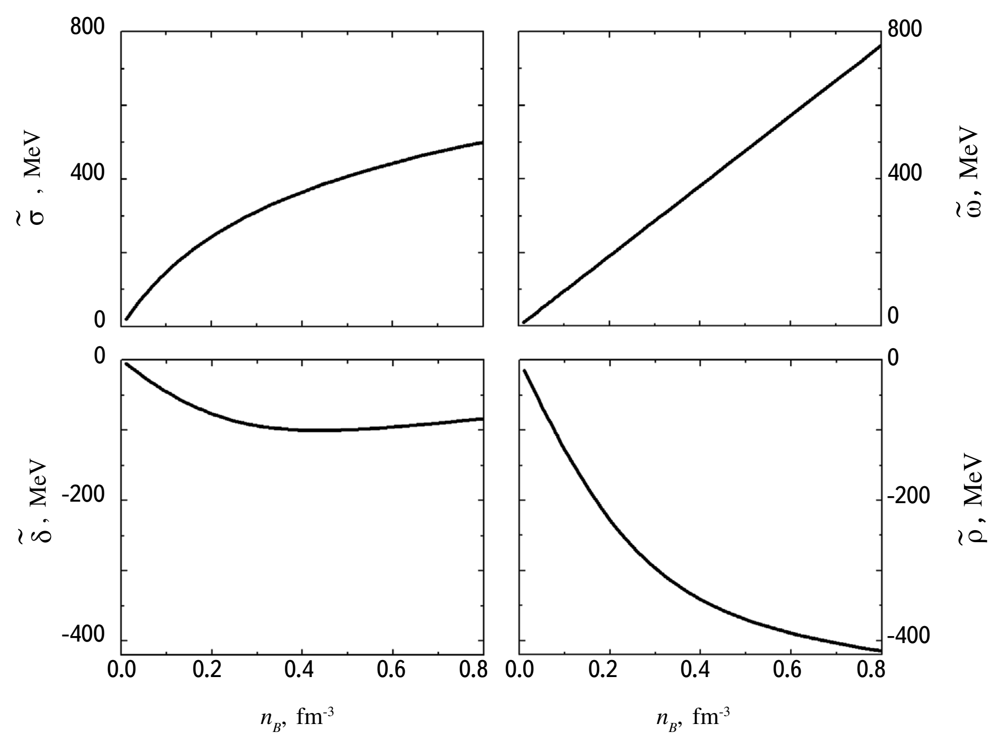

Fig. 7. The average fields of the exchange mesons as functions of the baryon number density n_B for electrically neutral and β-equilibrium *npe*μ matter.

4. Conclusion

This paper is a study of the properties of isospin-asymmetric cold nucleon matter. The relativistic mean-field theory has been used; besides the fields σ, ω, and ρ of the exchange mesons, the isovector, Lorentz-scalar field of the δ-meson has also been taken into account. The interaction constants of a nucleon with the mesons and the coupling constants $g_{\sigma 3}$ and $g_{\sigma 4}$ characterizing the self-interaction of the σ-meson and leading in the equations of motion to nonlinearities of second and third order, respectively, are chosen so as to reproduce the known values of the characteristics of symmetric nucleonic matter at the nuclear saturation density n_0. The model was used to study the properties of purely nucleonic *np* matter, as well as electrically neutral and β-equilibrium nuclear matter, consisting of nucleons and charged e^- and μ^- leptons (*npe*μ matter). The dependences on the baryon density n_B and the asymmetry parameter α of these sorts of *np* matter such as the energy $E_B(n_B, \alpha)$ ascribed to a single baryon, the specific energy owing to the isospin asymmetry $\Delta E_B(n_B, \alpha)$, the effective masses of the proton and neutron $M_p^{(eff)}(n_B, \alpha)$ and $M_n^{(eff)}(n_B, \alpha)$, and the specific binding energy $E_{bind}(n_B, \alpha)$ have been analyzed numerically. It has been shown that the resulting energy asymmetry $\Delta E_B(n_B, \alpha)$ for a fixed value of a is a monotonically increasing function of the baryon density n_B.

The splitting of the effective masses of the proton and neutron $\Delta M^{(eff)} = M_p^{(eff)} - M_n^{(eff)}$ in terms of our model is positive and for a given value of the baryon density n_B rises with increases in the asymmetry parameter. For a given value of n_B the splitting in the effective masses is maximal for purely neutronic matter. We note that at present, there is no single opinion regarding whether $\Delta M^{(eff)}$ is negative, zero, or positive [31].

The effective masses of the proton and neutron $M_p^{(eff)}$ and $M_n^{(eff)}$, the specific binding energy E_{bind}, the symmetry energy E_{sym}, the quantitative fraction of protons $Y_p = n_p / n_B$, and the average meson fields $\tilde{\sigma}$, $\tilde{\omega}$, $\tilde{\delta}$, $\tilde{\rho}$ have been studied as functions of the baryon density n_B for electrically neutral and β-equilibrium *npe*μ matter. It has been shown that for a specific value of n_B the splitting $\Delta M^{(eff)}$ in the effective masses of the proton and neutron in electrically neutral and β-equilibrium *npe*μ matter is smaller than in pure neutronic matter. Given that if at high densities a phase transition will take place from hadronic matter to quark matter, the maximum value of the specific binding energy for the hadronic component of a neutron star will be of order 250-300 MeV, and the resulting number of protons will not exceed ~30% of the number of nucleons.

Our results for the symmetry energy E_{sym} show that below the nuclear saturation density the kinetic and potential parts of the symmetry energy are quantities of the same order of magnitude. At high densities the contribution of the potential energy of symmetry $E_{sym}^{(pot)}$ is considerably greater than the contribution of the kinetic component $E_{sym}^{(kin)}$. The potential part of the energy of symmetry arises from exchange of isovector δ- and ρ-mesons. The contribution of the δ-mesons to the potential energy of symmetry $E_{sym}^{(\delta)}$ is negative, while the contribution of the ρ-mesons to $E_{sym}^{(\rho)}$ is positive. For high densities, the inequality $E_{sym}^{(\rho)} > \left|E_{sym}^{(\delta)}\right|$ holds.

This work was done in the scientific-research laboratory for the physics of superdense stars in the Department of Applied Electrodynamics and Modeling at Erevan State University, financed by the Committee on Science of the Ministry of Education, Science, Culture, and Sport of the Republic of Armenia.

REFERENCES

1. A. W. Steiner, M. Prakash, J. M. Lattimer, et al., Phys. Rep. **411**, 325 (2005).
2. J. M. Lattimer and M. Prakash, Phys. Rep. **442**, 109 (2007).
3. B.-A. Li, P. G. Krastev, D.-H. Wen, et al., Eur. Phys. J. A **55**, 117 (2019).
4. S. Wang, H. Tong, Q. Zhao, et al., Phys. Rev. C, **106**, 021305 (2022).
5. S. Choi, T. Miyatsu, M.-K. Cheoun, et al., Astrophys. J. **909**, 156 (2021).
6. M. Baldo and G. F. Burgio, Prog. Part. Nucl. Phys. **91**, 203 (2016).
7. X. Roca-Maza and N. Paar, Prog. Part. Nucl. Phys. **101**, 96 (2018).
8. B.-A. Li, B.-J. Cai, W.-J. Xie, et al., Universe **7**, 182 (2021).
9. M. C. Miller, F. K .Lamb, A. J. Dittmann, et al., Astrophys. J. Lett. **918**, L28 (2021).
10. B. P. Abbott, R. Abbott, T. D. Abbott, et al., Phys. Rev. Lett. **121**, 161101 (2018).
11. B. P. Abbott, R. Abbott, T. D. Abbott, et al., Phys. Rev. X **9**, 011001 (2019).
12. J. D. Walecka, Ann. Phys. **83**, 491 (1974).
13. B. D. Serot and J. D. Walecka, Adv. in Nucl. Phys. **16**, 1 (1986).
14. B. D. Serot and J.D.Walecka, Int. J. Mod. Phys. E **6**, 515 (1997).
15. J. Boguta and A. R. Bodmer, Nucl. Phys. A **292**, 413 (1977).
16. Y. Sugahara and H.Toki, Nucl. Phys. A **579**, 557 (1994).
17. G. A. Lalazissis, J. Konig, and P. Ring, Phys. Rev. C **55**, 540 (1997).
18. H. Mueller and B. D. Serot, Nucl. Phys. A **606**, 508 (1996).
19. S. Kubis and M. Kutschera, Phys. Lett. B **399**, 191 (1997).
20. B. Liu, V. Greco, V. Baran, et al., Phys. Rev. C **65**, 045201 (2002).
21. V. Greco, M. Colonna, M. Di Toro, et al., Phys. Rev. C **67**, 015203 (2003).
22. T. Gaitanos, M. Colonna, M. Di Toro, et al., Phys. Lett. B **595**, 209 (2004).
23. M. Di Toro, A. Drago, T. Gaitanos, et al., Nucl. Phys. A **775**, 102 (2006).
24. G. B. Alaverdyan, Astrophysics **52**, 132 (2009).
25. G. B. Alaverdyan, Research in Astron. Astrophys. **10**, 1255 (2010).
26. T. Miyatsu, M.-K. Cheoun, C. Ishizuka, et al., Phys. Lett. B **803**, 135282 (2020).
27. T. Miyatsu, M-K. Cheoun, and K.Saito, Astrophys. J. **929**, 82 (2022).

28. N. M. Hugenholtz and L. van Hove, Physica **24**, 363 (1958).
29. P. Czerski, A. De Pace, and A .Molinari, Phys. Rev. C **65**, 044317 (2002).
30. G. B. Alaverdyan, Symmetry **13**, 124 (2021).
31. H.-Y. Kong, Y. X. Jun Xu, L.-W. Chen, et al., Phys. Rev. C **91**, 047601 (2015).