DOI 10.1007/s10511-021-09694-z
Astrophysics, Vol. 64, No. 3, September, 2021

EFFECT OF CONTINUUM SCATTERING ON THE STATISTICAL AND TEMPORAL CHARACTERISTICS OF SPECTRAL LINE FORMATION

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Some new results from our study of the effect of continuum scattering on different observed characteristics of spectral lines are presented. The main purpose is to study temporal variations in line profiles and some statistical averages describing the diffusion of radiation in a medium. For simplicity the one dimensional problem of the formation of a spectral line of unit intensity in the continuum during reflection from a semi-infinite atmosphere is examined. The possibility of solving the steady-state problem analytically in the form of a Neumann series makes it possible to construct the corresponding solution of the nonstationary problem quite easily. Numerical calculations illustrate the differences in the evolution of the profiles and other characteristics between emission and absorption lines.

Keywords: *continuum scattering: Schuster mechanism: Erlang-n distribution: emission and absorption spectral lines*

1. Introduction

The need to study the role of scattering in the continuum spectrum arises during interpretation of the spectra of various astrophysical objects. Even for relatively low values of the scattering coefficient the effect may become observable in the continuum. as well as at the frequencies of a spectrum line [1-4]. As in our previous work, this

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Original article submitted April 7, 2021. Translated from Astrofizika, Vol. 64, No. 3, pp. 389-399, August 2021.

sort of study was motivated by questions involved in the interpretation of the spectra of class A supergiants and related classes with extended atmospheres. As they exist near the Eddington instability limit, from time to time these exotic objects eject their surface layers to form a fairly dense stellar wind and, thereby, a rather extended opaque shell. Besides a high luminosity, the nonstationary manifestations of these stars offer a good prospect for studying the changes in their spectra which, in turn, make it possible to obtain further information on the physics of the phenomena taking place in them.

The main purpose of this paper is a theoretical study of the effect of continuum scattering on temporal changes during the formation of spectrum lines. The scattering mechanism customarily assumed in this area is Thomson scattering on free electrons. The high degree of ionization in the atmospheres of these stars ensures the existence of sufficient free electrons for the effect to be measurable (e.g., see Refs. 5 and 6). It should also be noted that the theory developed in Ref. 1 and this paper is general and can be applied for other scattering mechanisms.

2. The problem of diffuse reflection from a semi-infinite atmosphere

In this part of the paper we limit ourselves to examining the simplest one-dimensional problem and assume that the scattering is monochromatic, both in a spectrum line and in the continuum. The problems in this approximation have at least two advantages. First, the various characteristics of the line spectrum of interest to us are found analytically, which substantially simplifies the construction of a solution of the nonstationary problem. The other advantage is that by neglecting radiative redistribution effects with respect to frequencies and directions, it is possible to obtain a relatively more precise idea of the variations undergone by spectrum lines over time.

We begin our study with the steady-state problem of diffuse reflection from an atmosphere which scatters and reflects at the frequencies of a spectrum line and in the continuum. It is assumed that the atmosphere is illuminated by radiation of unit intensity in the continuum. It is evident that the reflected radiation is produced by scattering in the line, as well as the continuum. If no differences are assumed between these two processes in the observed reflected radiation, then the reflection coefficient is given by [1,3]

$$\rho(x)=\frac{1}{\tilde{\lambda}}\left[2-\tilde{\lambda}(x)-2\sqrt{1-\tilde{\lambda}(x)}\right], \tag{1}$$

where

$$\tilde{\lambda}(x)=\frac{\lambda\alpha(x)+\gamma}{\alpha(x)+\beta+\gamma}. \tag{2}$$

The customary notation is used in these formulas: λ is the reradiation coefficient for a photon in an elementary scattering event at the frequencies of the spectrum line, $\alpha(x)$ is the profile of the absorption coefficient in the line as a function of the dimensionless frequency measured by the shift from the line center in Doppler widths, β and γ are, respectively, the ratios of the absorption and scattering coefficients in the continuum spectrum (χ_ν, σ) to the absorption coefficient at the center of the spectrum line κ_0. Assuming that continuum scattering takes place on free electrons, we can write $\sigma = n_e \sigma_0$, where n_e is the electron density and

$$\sigma_0 = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2, \tag{3}$$

where *e* and *m* are the electronic charge and mass, and *c* is the speed of light. It has been shown [1] that whether the spectrum line is observed in emission or in absorption will depend on the parameters l, b, and g in Eq. (2). In the case of a semi-infinite *isothermal* atmosphere with two-level atoms and the continuum taken into account, the condition for observation of an emission line can be written as the inequality

$$(1-\lambda)\gamma > \lambda\beta. \tag{4}$$

The physical significance of this inequality is easily understood on considering two events that take place in mutually reverse directions: a photon is scattered in the continuum on electrons and then absorbed at the frequencies of a spectrum line, on one hand, and a photon is emitted in the line and absorbed in the continuum, on the other. Depending on which of these two processes predominates in the medium, the line will be observed either in emission or in absorption. In other words, the effect of the Schuster mechanism depends on the balance between the emitting and absorbing capabilities at the frequencies of the spectrum line and in the continuum. It follows from the above inequality that the most probable candidates for observation in emission are relatively faint lines. It should be noted that there is a simple relationship between the profile of a line formed in an isothermal atmosphere, R_0, and a profile produced by reflection in a semi-infinite atmosphere, R_*: $R_0(x) + R_*(x) = 1$ [7], so that an absorption line in one of these problems corresponds to an emission line in the other. Everywhere in the following we shall be interested in lines formed by reflection of radiation from a semi-infinite atmosphere.

The influence of scattering in the continuum spectrum is easily detected in the behavior of various statistical averages at the frequencies of lines. We introduce the following three quantities into the discussion representing the average number of scattering events (ANS), depending on the type of scattering. We denote the ANS by $\langle N_L(x)\rangle$ and $\langle N_\gamma(x)\rangle$, respectively, if scattering is taken into account, respectively, only in a line or in the continuum. Finally, let $\langle N(x)\rangle$ denote the total ANS when the difference between the types of scattering is not set. These quantities are defined by the standard formulas ([8,9]; see Ref. 10, as well)

$$\langle N_L(x)\rangle = \lambda\frac{\partial \ln\rho(x)}{\partial\lambda}, \quad \langle N_c(x)\rangle = \gamma\frac{\partial \ln\rho(x)}{\partial\gamma}, \quad \langle N(x)\rangle = \tilde{\lambda}\frac{\partial \ln\rho(x)}{\partial\tilde{\lambda}}. \tag{5}$$

Besides these quantities, there is also interest in the average time spent by a photon during the process of diffusion in the medium. For this quantity, we have [11]

$$\langle\Omega(x)\rangle = -\frac{\partial \ln\rho(x)}{\partial\beta}. \tag{6}$$

All of these quantities are given explicitly by

$$\langle N(x)\rangle = \sqrt{[v(x)+\gamma]/u(x)}\ , \quad \langle\Omega(x)\rangle = 1\Big/\sqrt{u(x)v(x)}\ , \tag{7}$$

$$\langle N_L(x)\rangle = \sqrt{v(x)/u(x)}\ , \quad \langle N_c(x)\rangle = \sqrt{[v(x)+\gamma]/v(x)}\ , \tag{8}$$

where for brevity we have introduced the notation $u(x) = (1-\lambda)\alpha(x)+\beta$ and $v(x) = \alpha(x)+\beta$.

The curves in Figs. 1 and 2 illustrate the variations in these quantities on going from the center of a line to its wings. In particular, it may be concluded from Fig. 1 that the presence of continuum scattering, as expected, is not reflected in the frequency dependence of the ANS and average time in the line core: because of the transparency of the medium, photons in the wings of a spectrum line are scattered less, but spend more time in it because they cover large distances. The increased role of scattering in the continuum spectrum facilitates the emergence of photons from the medium, thereby shortening the time photons spend in the medium. The left of Fig. 2 shows separately the frequency dependence of the ANS in the absence of continuum scattering. In the figure on the right it is assumed that there is no scattering in the line (the dashed curve denotes the limiting value of the ANS, when all types of scattering are absent), These figures indicate that in the case of a strong line (here λ = 0.9) the effect of continuum

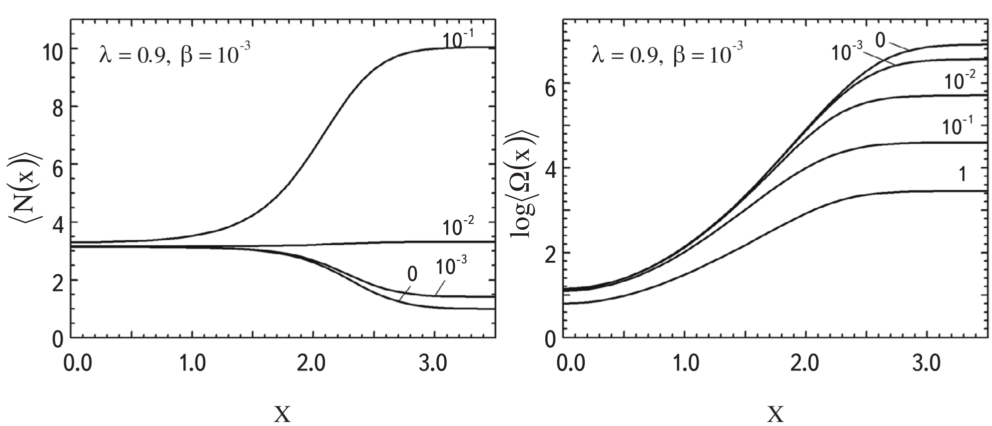

Fig. 1. The frequency dependences of the average number of scattering events and the average time spent by a photon during diffusion in a medium for the indicated values of the parameter γ.

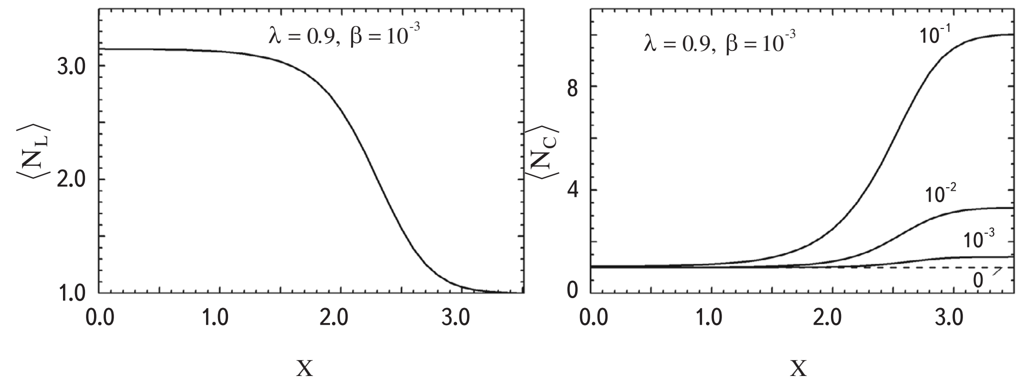

Fig. 2. The average number of scattering events accounting only for scattering in a line (left) and only in the continuum (right) for the indicated values of γ.

scattering begins to show up in the wings of the line. However, for relatively high free electron densities, it is also important in the line core.

3. Effect of continuum scattering on the evolution of the profile of a line formed by reflection from a semi-infinite atmosphere

We now proceed to determining the temporal characteristics of the formation of a spectrum line formed during reflection. For this purpose we examine the nonstationary problem, including in the calculation the time spent by photons at different frequencies as they diffuse in the medium. Problems in the nonstationary theory of radiative transfer have been discussed by various authors. Here we limit ourselves to mentioning the work of Sobolev [12], Minin [13,14]. Matsumoto [15], and Ganapol [16]. Various methods of solving problems with one or another simplifying assumption have been proposed. In particular, in the above papers of Minin, a method based on determining the Laplace transforms of the corresponding known radiative transfer characteristics in the stationary problem was proposed. In this way the problem of diffuse reflection from a semi-infinite atmosphere was solved in the case of isotropic scattering. Another method that proposes the determination of the unknown quantities in the form of a Neumann series expansion was developed in Refs. 15 and 16. The central point in these papers is occupied by an analogy between the forms of Neumann series for the unknowns in the nonstationary and corresponding stationary problems. This circumstance makes it possible to obtain the necessary multipliers in the unknown that depend on a time series directly through corresponding factors in the stationary problem. As for the latter, they are determined by recurrence relations (e.g., Refs. 10 and 16).

Turning to the problem that we are examining, instead of the variable time t, we introduce the dimensionless time $\omega = t/(t_1 + t_2)$, where t_1 is the average time necessary for an excited atom to radiate and t_2 is the average time spent by a photon on the path between two successive scattering events. Assuming that the medium consists of two-level atoms, then with continuum scattering included, we can write $t_2 = 1/\left[\left(1 + n^+/n_1\right) n_1 k_0 c\right]$, where n_1 is the number of atoms in the ground state and c is the speed of light, and it is also assumed that $n_e \approx n^+$. It should be noted that the time spent by a photon in moving between scattering events and the time required by an atom for radiating are independent random quantities distributed in the same way, following an exponential law.

Solving the nonstationary problem, as pointed out above, becomes substantially simpler if an analytic solution of the steady-state problem is known. In the problem we are examining this solution is given by Eq. (1). Proceeding to the nonstationary analog of the problem, we consider the function $\overline{\rho}(x,\omega)$ such that $\overline{\rho}\, d\omega$ is the probability of the reflection of a photon of frequency x from the medium within the time interval $\omega, \omega + d\omega$.

Recall that the Neumann series for the reflection function in the steady-state problem can be written in the form

$$\rho(x) = \sum_{n=1}^{\infty} \rho_n \tilde{\lambda}^n(x), \tag{9}$$

where $\rho_1 = 0.25$ and $\rho_2 = 0.125$, while the remaining coefficients of higher order are determined by the recurrence relation

$$\rho_n = \frac{1}{2}\left(\rho_{n-1} + \frac{1}{2}\sum_{k=1}^{n-2} \rho_k \rho_{n-k-1}\right). \tag{10}$$

It is easy to see that each term in the sum (9) is nothing other than the probability of reflection of a photon from the medium after a certain number n of scattering events.

For a temporal description of the reflection process it is important to note that the times spent by a photon between two successive scattering evens are mutually independent random quantities. Then the total time spent by a photon in the medium during diffusion is a random quantity equal to the sum of a certain number of identically distributed random quantities. The distribution function for the time spent in n-fold scattering is given by an Erlang-n distribution,

$$e^{-\omega} \frac{\omega^{n-1}}{(n-1)!}, \tag{11}$$

which is a special case of the gamma distribution. Then the distribution function for the random quantity $\overline{\rho}(x,\omega)$ is written in the form

$$\overline{\rho}(x,\omega) = e^{-\omega} \sum_{n=1}^{\infty} \rho_n \tilde{\lambda}^n(x) \frac{\omega^n}{(n-1)!}, \tag{12}$$

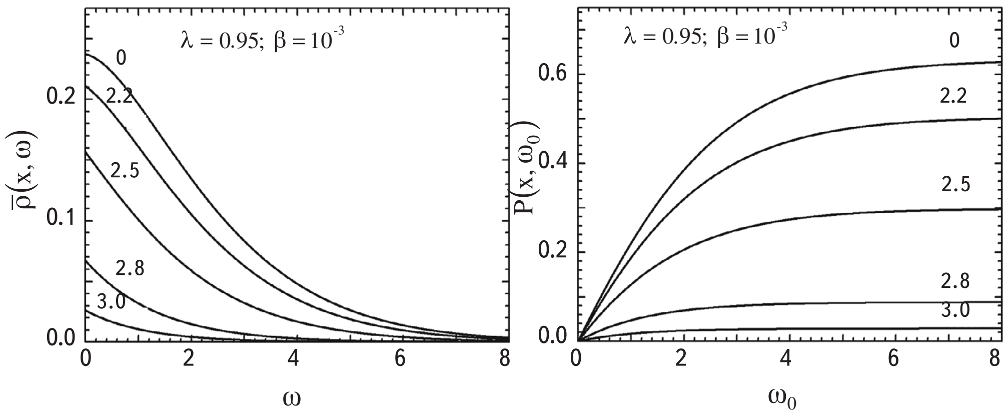

Fig. 3. The probability density function for reflection of photons at different frequencies inside a line as a function of time (left frame). The right hand frame shows the cumulative distribution function describing the evolution of the profile of a spectrum line at different frequencies (indicated above the curves) when illumination of the medium continues up to a fixed time ω_0.

It describes the time distribution of the probability of reflection of photons with different frequencies from the medium. At the same time, there is considerable interest in the so-called cumulative distribution function describing the evolution of the profile of a spectrum line up to a certain time ω_0. Denoting it by $P(x, \omega_0)$ and given Eq. (12) we find

$$P(x, \omega_0) = e^{-\omega_0} \sum_{n=1}^{\infty} \overline{\rho}_n \tilde{\lambda}^n(x) \sum_{k=0}^{\infty} \frac{\omega_0^{n+k}}{(n+k)!}. \tag{13}$$

Figure 3 illustrates the dependence of these two distributions on frequency within a spectral line. The location of the curves in the figures shows that in this case ($\lambda = 0.95$) an emission line is formed (for an isothermal atmosphere, an absorption line). At the same time, the curves in the right frame show that different parts of a spectrum line take their final form after different time intervals. It is easy to see that in this case the wings of the line are established earlier, preceding the line center by $\Delta\omega_0 > 10$, which, depending on the density of the medium and the degree of ionization, may correspond to a rather large time interval. The example shown n Fig. 3 is for a strong line, as resonance lines usually are. At the same time, as we have shown [1,2] the effect of continuum scattering is large for faint lines and whether an emission or absorption line is formed depends on it.

This effect is illustrated in Figs. 4 and 5 which show the results of calculations for a faint line with $\lambda = 0.5$, but with two different relationships between the values of the parameters β and γ. The latter were chosen so as to see in simplest fashion whether the spectrum line is in emission or absorption. It follows from Eq. (4) that in this case the line shape depends only on the relative effectiveness of the scattering and absorption processes in the continuum spectrum. On comparing location of the curvatures, depending on x in Figs. 4 and 5 it is easy to see that

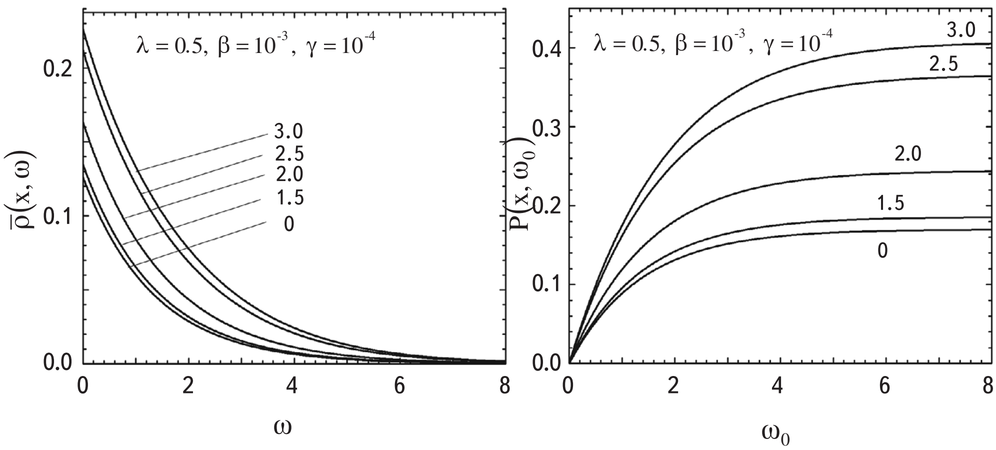

Fig. 4. Same as in Fig. 3 for a faint line when it is observed in absorption; the scattering coefficient in the line is chosen equal to 0.5, so that the shape of the line and its time variation depend on which of the emission and absorption processes predominate in the continuum spectrum (in this case $\beta > \gamma$).

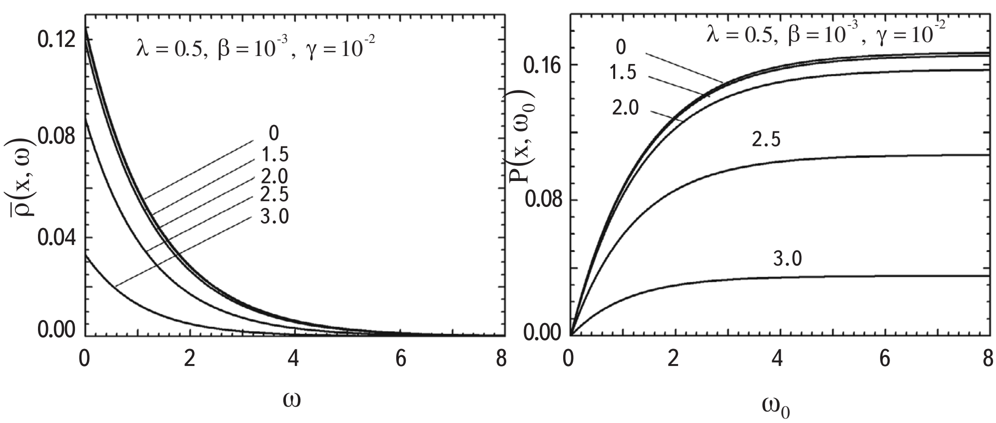

Fig. 5. Same as in Fig. 4 for an emission line ($\gamma > \beta$).

it is mutually reciprocal in them. This is evidently because the curves in Fig. 4 correspond to an absorption line, and in Fig. 5, to an emission line. The figures on the right show that these lines evolve in different ways: while the central part of the spectrum line is established more rapidly in the first case, the curves saturate more rapidly in the wings of the lines in the second case. The essentially identical time behavior of an emission line at its central frequencies (Fig. 5) is noteworthy; it is related to the bell-shaped line profile [1].

In studies of the temporal characteristics of the variation in observed spectra, besides the spectrum line profiles, there is interest in the evolution of the average characteristics describing the diffusion of radiation in the medium. Figures 6 and 7 illustrate the variation in the average number of scattering events in a line at its center ($x = 0$) and in its wings (e.g., for $x = 2.5$), depending on the role of scattering in the continuum. These figures show how and to what extent the average number of scattering events decreased in a spectrum line owing to continuum scattering. It is easy to see that the effect of this scattering is especially large in the wings of a line, which are established earlier in this case. An analogous picture is observed for the average time spent by photons in the center and wings of a line as they wander in the medium.

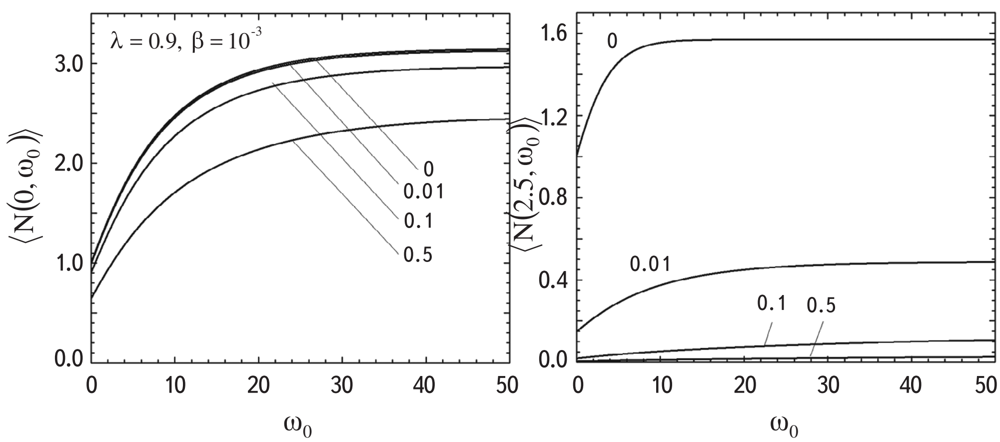

Fig. 6. Evolution of the average number of scattering events as a function of the magnitude of γ (values of which are indicated on the curves) with illumination of the medium maintained up to a fixed time ω_0.

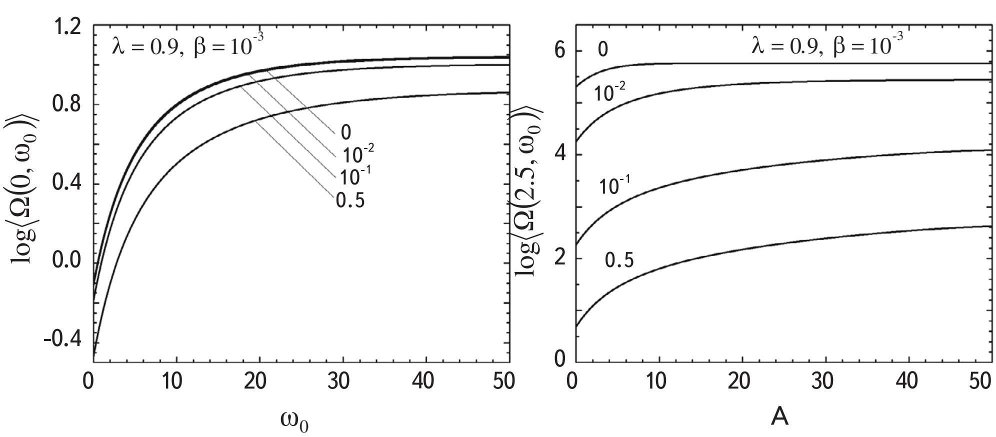

Fig. 7. Evolution of the average time spent by a photon during the process of diffusion in a medium as a function of the value of γ (values of which are indicated on the curves) with illumination of the medium maintained up to a fixed time ω_0.

4. Conclusion

A detailed study of two questions that often arise during interpretation of the spectra of different astrophysical objects has been made for the specific example of one of the simplest radiative transfer problems. The first concerns the effect of scattering in the continuum spectrum on the formation of a line spectrum, in particular on line profiles and quantities associated with them that statistically describe the diffusion of radiation in them. The other task has been to investigate variations in the line profiles with the passage of time and study the process by which an emission spectrum is converted into an absorption spectrum and *vice versa*. These results yield an idea of how these variations take place and make it possible to estimate the characteristic time intervals for their occurrence. Their magnitudes as functions of the density of the medium and the degree of ionization can range over quite wide limits. The approach used here can easily be generalized and applied to the case of media with finite optical thicknesses, as well as for quite general assumptions regarding elementary scattering events.

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