DOI 10.1007/s10511-018-9524-y
Astrophysics, Vol. 61, No. 2, June, 2018

TORUS DYNAMO MODEL FOR STUDY OF MAGNETIC FIELDS IN THE OUTER RINGS OF GALAXIES

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At present, there is essentially no doubt that magnetic fields of a few μG exist in some spiral galaxies. The fields are produced by a dynamo mechanism. Because the equations of the dynamo theory are quite complicated, a two-dimensional approximation is often used since the galactic disk is sufficiently thin, so it is possible to replace some partial derivatives with algebraic expressions. Some galaxies have outer rings in which magnetic fields may also exist. The generation of these fields can also be studied using a two-dimensional approximation, but because that approximation was not developed for rings, but for thin disks, in this case it only yields qualitative results. Therefore, a torus dynamo model is used to study this process. This model is used to analyze possible scenarios for the evolution of magnetic fields in outer rings. It is found that for motions that are not too intense, a field with a quadrupole symmetry is generated. For faster motions a dipole component of the field may develop, which is fundamentally impossible in the two-dimensional approximation.

Keywords: *outer rings of galaxies: dynamo theory*

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Original article submitted September 16, 2017; accepted for publication March 7, 2018. Translated from Astrofizika, Vol. 61, No. 2, pp. 165-177 (May 2018).

1. Introduction

It has been reliably established that a number of spiral galaxies have magnetic fields with strengths of a few μG. Observational evidence of these fields is provided by the Faraday rotation of the plane of polarization of the electromagnetic emission measured by modern radio telescopes [1]. Significant results obtained over recent decades with instruments such as LOFAR [2], VLA [3], and others [4] should be noted. In the future there are plans to study galactic magnetism with the SKA radio telescope which is under construction [5,6]. The formation of these fields is described by a dynamo mechanism. It is based on a combination of differential rotation and the alpha-effect, which is associated with twisting of turbulent motions of the ionized component of the interstellar gas. As the same time, the magnetic field is dissipated by turbulent diffusion. Because of this, the generation of the magnetic field is a threshold effect. For certain values of the corresponding controlling parameters of the dynamo, the magnetic field increases, while in the opposite case, it can only be damped [6].

The evolution of large-scale galactic fields is described by the so-called Steenbeck-Krause-Rädler equation, which is a consequence of averaging the equations of magnetohydrodynamics over characteristic scales of 50-100 pc [7,8]. This equation is complicated, so field generation is usually described with so-called two-dimensional approximation which uses the fact that the magnetic field lies in the equatorial plane and the components of the field perpendicular to it can be neglected. In addition, the partial derivatives of the magnetic field along the vertical direction can be replaced by algebraic expressions [9,13]. In this case the equation for field generation becomes much simpler and does not contain dependences on the distance to the equatorial plane. The results of the two-dimensional approximation are in good agreement with astronomical observations. The possibility of generating a magnetic field is characterized by the so-called dynamo number, which has a certain critical value, above which the damping of the field is replaced by its growth [11,12].

A number of galaxies have so-called outer rings [14]. They lie at some distance from the main part of the galaxy and have relatively narrow widths. They also contain ionized gas which is characterized by turbulent motion. In addition, differential rotation can also be observed in outer rings. All of these things suggest the existence of magnetic fields in these objects and these fields should also grow as a result of a dynamo mechanism [15].

At present there is shortage of observational data on magnetic fields in outer galactic rings. Thus, it is important to examine possible scenarios for the evolution of these fields and identify the parameters of the objects such that the field should grow and the parameters for which the dynamo mechanism is improbable.

The feasibility of using the two-dimensional approximation in the case of outer rings is extremely controversial. The radial dimensions of the main part of a galaxy greatly exceed its half thickness, while the width of an external ring is fully comparable to the former. Thus, other representations of the magnetic field which account for its dependence on the distance to the equatorial plane must be used.

Since the shape of an external ring is fairly close to toroidal, it is convenient to use a model of a magnetic field in a torus. In this case the magnetic field can be treated as a combination of a toroidal component and a part of the vector potential characterizing the poloidal component of the magnetic field. With axial symmetry, the model reduces to a system of two equations which can be solved numerically [16-19].

This paper examines the possibility of generating a magnetic field using two different approaches. The first is based on a two-dimensional approximation and the second, on a torus dynamo model. In order to compare the results of the two models, the equations of the two-dimensional model are rewritten in the same dimensionless variables as the torus dynamo model. Threshold values of the controlling parameters are found for which a magnetic field can grow in the two cases. Although they differ for fully understandable reasons, the results of the two cases are qualitatively similar. The influence of nonlinear effects (associated with saturation of the growing magnetic field as it approaches the value corresponding to energy equipartition) on the evolution of the magnetic field is examined.

It can be concluded that a magnetic field can be generated in an entire series of outer rings. Of course, this requires rather strict conditions that are not satisfied in all objects. In particular, the outer rings must be sufficiently wide; otherwise damping of the field will predominate over its generation owing to the magnetic dynamo. Typical time dependences of the magnetic field are shown for the various cases.

We note that magnetic field growth by these phenomena is not the only possible way that magnetic fields can develop in outer galactic rings. In particular, a field can be transferred from inner parts of a galaxy owing to propagation of nonlinear waves from the main part of the galaxy; this is known in mathematical physics as the Kolmogorov-Petrovskii-Piskunov effect [15,20-22]. This mechanism should play an especially important role in the case of so-called polar rings, which lie in a plane that is not coincident with the equatorial plane of the galaxy.

2. Model for magnetic fields in galaxies

The magnetic field $\mathbf{H}$ of the galaxy, as well as of the outer rings, consists of two main components, large-scale $\mathbf{B}$ and small-scale $\mathbf{b}$ [6,13], with

$$\mathbf{H} = \mathbf{B} + \mathbf{b}.$$

The evolution of the small-scale component is described by the Steenbeck-Krause-Rädler equation, which is obtained by averaging the basic equations of magnetohydrodynamics [8]:

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{rot}[\mathbf{V}, \mathbf{B}] + \mathrm{rot}(\alpha \mathbf{B}) + \eta \Delta \mathbf{B}, \tag{1}$$

where $\mathbf{V} = r\Omega \mathbf{e}_\varphi$ is the velocity of the large-scale motions (r is the distance to the galactic center and Ω is its rotation rate), α is a coefficient characterizing the alpha-effect (associated with twisting of the turbulent motions), and $\eta = lv/3$ is the characteristic turbulent diffusion coefficient (l is the characteristic turbulence scale length, on the order of 50-100 pc and equal to the typical sizes of the region in which the small-scale component of the galactic magnetic field is concentrated and ν is the turbulence velocity).

Equation (1) is usually not solved explicitly (because it is quite complicated and requires substantial computer resources) but by using an approximation that reduces the dimensions of the problem. In the case of outer rings, axial symmetry can be assumed and this makes the problem easier to solve. In the case of the main part of the galaxy, the two-dimensional approximation is well recommended [10].

2.1. The two-dimensional approximation. Let us assume that the magnetic field lies in the plane of the disk, so in the cylindrical coordinate system $r-\varphi-z$ only the components B_r and B_φ will be important. In addition, we assume that the alpha-effect is an odd function of distance to the equatorial plane [9,23], i.e.,

$$\alpha = \alpha_0 \frac{z}{h},$$

where h is the half-thickness of the ring. The characteristic magnitude of the alpha-effect is related to the operation of the Coriolis force and can be described by

$$\alpha_0 = \frac{\Omega l^2}{h}.$$

The magnetic field can be taken to depend on the distance to the equatorial plane as follows [11]:

$$B_r(r,z) = B_r(r,0)\cos\left(\frac{\pi z}{2h}\right), \quad B_\varphi(r,z) = B_\varphi(r,0)\cos\left(\frac{\pi z}{2h}\right),$$

so that the derivatives of the magnetic field along the vertical direction can be replaced by the fairly simple expressions [13]

$$\frac{\partial^2 B_r}{\partial z^2} = -\frac{\pi^2}{4h^2} B_r, \quad \frac{\partial^2 B_\varphi}{\partial z^2} = -\frac{\pi^2}{4h^2} B_\varphi.$$

Then the vector equation (1) reduces to a system of two scalar equations (it is assumed that a rotation curve for which $d\Omega/dr = -\Omega/r$ is present):

$$\frac{\partial B_r}{\partial t} = -\frac{\Omega l^2}{h^2} B_\varphi - \eta \frac{\pi^2}{4h^2} B_r + \eta \Delta_r B_r, \tag{2}$$

$$\frac{\partial B_\varphi}{\partial t} = -\Omega B_r - \eta \frac{\pi^2}{4h^2} B_\varphi + \eta \Delta_r B_\varphi, \tag{3}$$

where Δ_r is the part of the Laplacian associated with derivatives with respect to the distance to the galactic center.

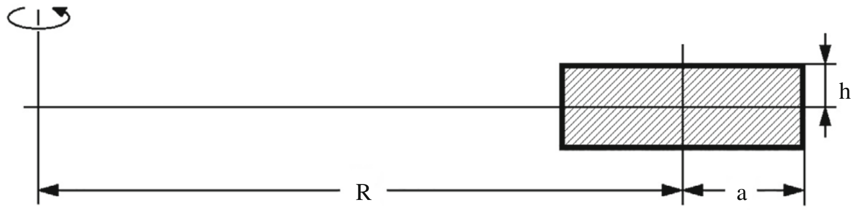

Fig. 1. A sketch of an outer galactic ring.

It is convenient to use dimensionless variables. Time can be measured in units of a^2/η, where a is the half-width of the outer ring (see Fig. 1). Distances are measured in terms of the distance R to the galactic center. Thus, the variables $\tilde{r} = r/R$, $\tilde{z} = z/R$, and $\tilde{t} = t\eta/a^2$ are used. For brevity we omit the "tildes" in the following. Then the system of Eqs. (2)-(3) for the magnetic field will have the following form:

$$\frac{\partial B_r}{\partial t} = -S_\alpha B_\varphi - \frac{\pi^2 k^2}{4} B_r + \lambda^2 \Delta_r B_r \,, \tag{4}$$

$$\frac{\partial B_\varphi}{\partial t} = -S_\omega B_r - \frac{\pi^2 k^2}{4} B_\varphi + \lambda^2 \Delta_r B_\varphi \,, \tag{5}$$

where $k = a/h$ is the ratio of the half-width and the half-thickness of the outer ring and $\lambda = a/R$ is the ratio of the half width to the distance from the center. In these variables, the distance to the galactic center will be unity and the half width of the ring will equal λ. The coefficient $S_\alpha = 3\Omega l a^2/v h^2$ characterizes the alpha-effect and $S_\omega = 3\Omega a^2/l v$, the differential rotation.

We now examine the possibility of magnetic field generation. The radial part of the Laplacian can be replaced by the expression

$$\Delta_r \approx -\frac{\pi^2}{4\lambda^2} \,.$$

Then the system of Eqs. (4)-(5) (here the partial derivatives can be replaced by total derivatives) reduces to

$$\frac{dB_r}{dt} = -S_\alpha B_\varphi - \frac{\pi^2 k^2}{4} B_r - \frac{\pi^2}{4} B_r \,, \tag{6}$$

$$\frac{dB_\varphi}{dt} = -S_\omega B_r - \frac{\pi^2 k^2}{4} B_\varphi - \frac{\pi^2}{4} B_\varphi . \tag{7}$$

Assuming that the magnetic field increases exponentially, i.e.,

$$B_r \propto \exp(\gamma t), \quad B_\varphi \propto \exp(\gamma t),$$

yields the following growth rates:

$$\gamma = -\frac{\pi^2}{4}\left(1 + k^2\right) \pm \sqrt{S_\alpha S_\omega} .$$

We introduce a number characterizing the combined action of the alpha-effect and differential rotation:

$$Q = S_\alpha S_\omega = \frac{9\Omega^2 a^4}{v^2 h^2} .$$

One of the growth rates for the magnetic field will always be negative and the sign of the second depends on the dimensionless parameter Q characterizing the relationship between differential rotation, the alpha-effect, and dissipation associated with turbulence. When the intensities of the first two mechanisms are high compared to the dissipation, Q is larger. Magnetic field growth is possible if even one of the growth rates is positive. This occurs when $Q > Q_{cr}$, where

$$Q_{cr} = \frac{\pi^4}{16}\left(1 + k^2\right)^2 .$$

The significance of Q is roughly the same as that of the dynamo number D in the galactic dynamo theory [11]. The possible growth of a magnetic field in the outer ring is associated with the relationship between its radial and vertical dimensions, which is characterized by k. In particular, if the half-width is equal to the half-thickness, $k = 1$. Then,

$$Q_{cr} = 24.4 .$$

This estimate was obtained using fairly crude arguments. Thus, it makes sense to verify it numerically. The problems was solved with the boundary conditions

$$B_r\big|_{r=1-\lambda} = B_r\big|_{r=1+\lambda} = B_\varphi\big|_{r=1-\lambda} = B_\varphi\big|_{r=1+\lambda} = 0 .$$

The toroidal component of the magnetic field in the case where the field generation threshold is exceeded is shown

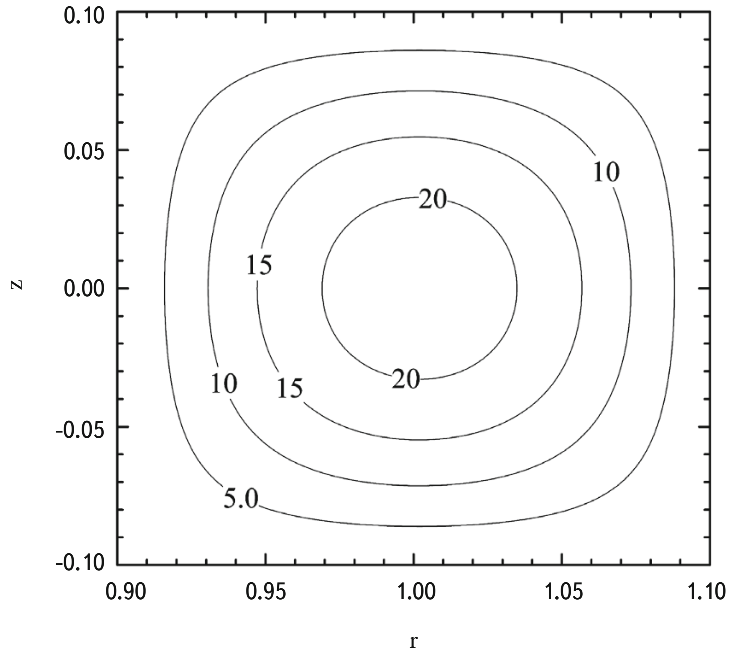

Fig. 2. A toroidal magnetic field obtained using the two-dimensional model for $t = 3$, $S_\alpha = 4$, $S_\omega = 15$.

in Fig. 2. Note that in this case only a quadrupole magnetic field can develop: this is a fundamental limit that follows from the construction of the two-dimensional approximation. In addition, the two-dimensional approximation arises from the fact that the size of the object in the vertical direction is considerably smaller than in the equatorial plane. Thus, our case of $k = 1$ (where they are equal) is not entirely correct. Because of this, it makes sense to examine another model that accounts for the dependence on distance from the equatorial plane.

2.2. Dynamo in a torus. The shape of an external ring is rather close to a torus. Thus, the torus dynamo model [16-19] is of great relevance for studying magnetic fields in external rings. The large-scale component of the magnetic field can be divided into two parts:

$$\mathbf{B} = B\,\mathbf{e}_\varphi + \mathrm{rot}\left(A\,\mathbf{e}_\varphi\right), \tag{8}$$

where B is the toroidal component and A is the part of the vector potential characterizing the poloidal component.

Then, assuming that the toroidal component is considerably greater than the poloidal component and that the field configuration is symmetric, we obtain the following equations for the evolution of the field [19]:

$$\frac{\partial A}{\partial t} = \frac{\Omega l^2}{h} \frac{z}{a} B + \eta \Delta A, \tag{9}$$

$$\frac{\partial B}{\partial t} = \Omega \frac{\partial A}{\partial z} + \eta \Delta B. \tag{10}$$

Note that the Laplacian indicates differentiation with respect to the variables r and z. The system of Eqs. (9)-(10) is conveniently rewritten using the same dimensionless variables as in the two-dimensional approximation (see above):

$$\frac{\partial A}{\partial t} = S_\alpha z B + \lambda^2 \Delta A, \tag{11}$$

$$\frac{\partial B}{\partial t} = S_\omega \frac{\partial A}{\partial z} + \lambda^2 \Delta B. \tag{12}$$

It is logical to assume the possibility of magnetic field generation will also be determined by the number Q. This model is more complicated for making estimates than the two-dimensional approximation. Thus, we make qualitative estimates that yield only an approximate result.

The product of the toroidal magnetic field and the distance to the equatorial plane can be estimated roughly as

$$zB \sim \frac{\lambda B}{2}.$$

We estimate the derivative of the component of the vector potential corresponding to the poloidal field as follows (assuming that the characteristic spatial scale for variation of the vector potential is equal to λ):

$$\frac{\partial A}{\partial z} \sim \frac{A}{\lambda}.$$

We note that in this case all the linear dimensions are measured in dimensionless variables (the unit of measurement corresponds to the radius of the outer ring). In terms of these units the variables range over $-\lambda < z < \lambda$ and $1-\lambda < r < 1+\lambda$.

In a way similar to that used for the two-dimensional approximation, the Laplacians can be replaced as follows:

$$\Delta A = -\frac{\pi^2}{2\lambda^2} A, \quad \Delta B = -\frac{\pi^2}{2\lambda^2} B.$$

Then the system of Eqs. (11)-(12) for the evolution of the magnetic field can be replaced by the qualitative analog

$$\frac{dA}{dt} = S_\alpha \frac{\lambda B}{2} - \frac{\pi^2}{2} A , \tag{13}$$

$$\frac{dB}{dt} = S_\omega \frac{A}{\lambda} - \frac{\pi^2}{2} B . \tag{14}$$

Exactly as in the previous case, it can be assumed that the field will increase exponentially, i.e.,

$$A \propto \exp(\gamma t), \quad B \propto \exp(\gamma t).$$

For the growth rate we obtain

$$\gamma = -\frac{\pi^2}{2} \pm \sqrt{\frac{S_\alpha S_\omega}{2}} .$$

It can be positive when the threshold for generation of a field is exceeded, i.e.,

$$Q = S_\alpha S_\omega > Q_{cr} .$$

For the critical value we obtain

$$Q_{cr} \approx 49 .$$

We verify this result numerically. We solve the problem in the region

$$\rho = \sqrt{(r-1)^2 + z^2} < \lambda .$$

For the boundary conditions we choose [18]

$$B\Big|_{\rho=\lambda} = \left.\frac{\partial A}{\partial \rho}\right|_{\rho=\lambda} = 0.$$

The numerical result, as opposed to the two-dimensional approximation, substantially corrects the qualitative estimates. Growth with positive values is found if $Q > 42$ [19]. For these values, differential rotation and the alpha-effect are strong enough to overcome dissipation.

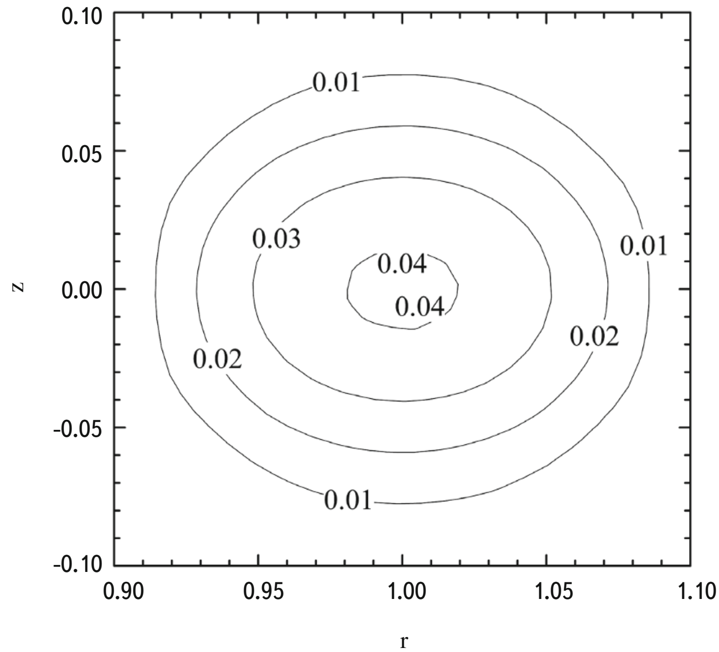

Fig. 3. A toroidal magnetic field obtained using the torus dynamo model in a torus with t = 3, $S_\alpha = 4$, and $S_\omega = 15$.

One characteristic of the results of the toroidal component of the magnetic field when the generation threshold is reached can be seen in Fig. 3. The results in this case are similar, on the whole, to the more two-dimensional approximation based on simpler arguments. The resulting magnetic field also has a quadrupole symmetry; that is, it is an even function of the distance to the equatorial plane, i.e.,

$$B(z) = B(-z).$$

The torus dynamo model also implies a possibility of generating structures with a dipole symmetry, where the field is an odd function of z, i.e.,

$$B(z) = -B(-z).$$

In this case, $B = 0$ in the equatorial plane ($z = 0$). Since the processes aiding the generation of a magnetic field are most intense right in the equatorial plane and fall off rapidly with increasing z, much more rigid conditions must be imposed on the velocities of the motions in the galaxy. Calculations show that dipole magnetic fields can be generated for $Q > 1190$ [19]. This can occur in situations where the angular rotation velocity of the outer ring is roughly an order of magnitude greater than the typical values for the central portion. In addition, generation of magnetic fields of this type requires initial conditions which are antisymmetric with respect to the equatorial plane.

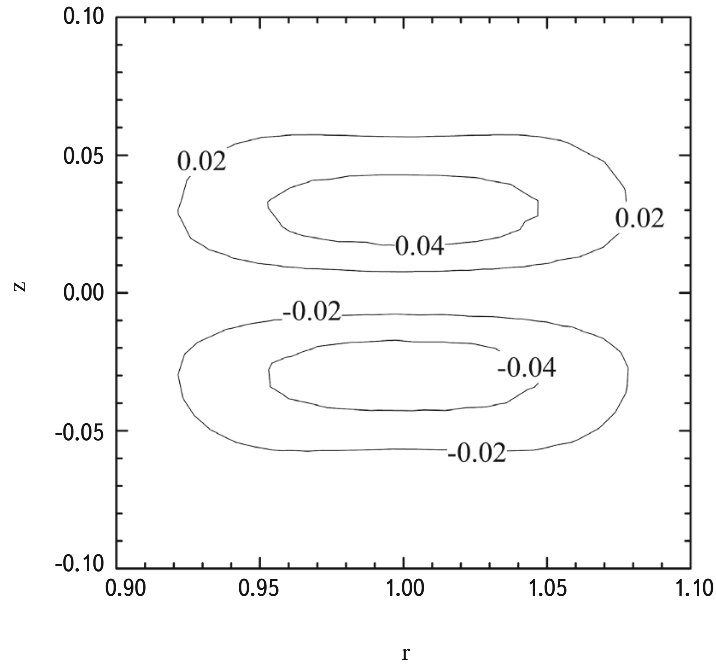

Fig. 4. A toroidal magnetic field obtained with the torus dynamo model in a torus with $t = 3$, $S_\alpha = 4$, and $S_\omega = 300$.

When even a small symmetric component is present in the initial conditions, the symmetry of the magnetic field may switch from dipole to quadrupole over time.

An example of the generation of a dipole magnetic field is shown in Fig. 4. We note that this result cannot, in principle, be obtained using the two-dimensional approximation.

A magnetic field can also be generated when Q is negative. As for positive values, with negative values a quadrupole or a dipole magnetic field can develop. Here the absolute value of this number must be higher than for positive values [19].

3. Conclusions

The generation of magnetic fields in outer rings of galaxies has been studied. Two different models have been used for this: a two-dimensional approximation developed for thin galactic disks and the torus dynamo model which is more realistic for outer rings. It has been shown that the magnetic field generation process is a threshold process and the critical values of the controlling parameters corresponding to increasing solutions have been determined. The magnetic fields for different magnetic field configurations have been plotted. It is demonstrated that, as opposed to the two-dimensional approximation, in the torus dynamo model quadrupole as well as dipole structures can be generated. Although a magnetic field with dipole symmetry can be generated subject to highly specific conditions on the kinematics of the motions in an outer ring, this result may be useful in the future for studies of magnetic fields in other toroidal objects, such as accretion tori.

We note that the operation of an in situ dynamo mechanism is not the only way of producing and maintaining magnetic fields in outer galactic rings. Thus, in a number of papers it has been shown that in peripheral regions, as well as outer rings of galaxies, one of the mechanisms for transport of field structures is the so-called Kolmogorov-Petrovskii-Piskunov effect. It is well known in mathematical physics and is related to the propagation of nonlinear waves. A magnetic field initial rises to a saturation level in the main part of a galaxy and then propagates with the aid of a wave to the outer regions [15,20-22]. The propagation velocity can be estimated using the asymptotic theory of contrasting structures. In addition, in the case of polar rings, which intersect the main part of a galaxy, "pumping" a magnetic field out of the main part of the galaxy into the ring is possible. This can be driven by linear transport phenomena.

The author thanks Prof. D. D. Sokolov for useful recommendations during preparation of this paper, as well as the organizing committee of the conference "Modern Stellar Astronomy-2017" for the opportunity of presenting the results of this work.

This work was supported by the Russian Foundation for Basic Research (project 16-32-00056 mol_a).

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