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DETERMINATION OF THE SUPERFLARE FREQUENCY DISTRIBUTION FUNCTION OF SOLAR-TYPE STARS

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This is a statistical study of a sample of 1547 superflares in 270 stars found during an analysis of data on more than 80000 stars of the sun's type (obtained by the Kepler orbital observatory during the first 500 days of observations). Estimates are given of the total number of stars capable of superflares and the superflare distribution frequency function is determined.

Keywords: *superflare: flare frequency*

1. Introduction

A superflare in a star of the sun's type is a sudden, sharp rise in brightness followed by a relatively slow decay, with an overall duration ranging from a few minutes to tens of hours. The energy release during superflares is 10 to 10^6 times the energy of the most powerful solar flares ever recorded [1] and is comparable to the energy of powerful flares in flaring stars. This naturally raises questions about the mechanisms for formation of superflares and about the possible relationships between superflares, solar flares, and flares in flaring stars. Interest in superflares is also related to the rapid development of astrobiology and research on exoplanets [1].

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Data on superflares published up to 2012 were obtained in unconnected, inspecific studies and were substantially random in character. Systematic discovery and studies of solar-type superflare stars became possible only after the launching of the Kepler orbital observatory, which was intended for discovering extrasolar planets and planetary systems by means of high precision, continuous photometric measurements of an enormous number of stars. At the same time, the Kepler observations provide valuable and uniform data on variable stars and variable phenomena of almost all types, including superflares. With regard to superflares, the Kepler data are of double value, since they can both record these stars and, to a certain extent, establish whether massive planets of stars are in the vicinity of flaring stars. They can also track the behavior of starspots which are invoked in different models (of a planet/star, spot) to explain the mechanism of superflares.

Systematic studies of superflares in solar-type stars began with the work of Maehara, et al. [2], who presented data on 365 superflares detected in 148 solar-type stars. A total of more than 83000 stars of this type were examined. Light curves obtained by the Kepler observatory during the period from April to December 2009 were studied.

These data were used in a statistical study [3] of these stars drawing on methods used to solve similar problems for flare stars. The total number of stars capable of superflares was estimated and the flare frequency distribution was determined for the complete set of stars and for separate subsamples constructed by breaking the complete sample into two parts in terms of the rotation periods and variability amplitudes of the stars.

Later, studies of superflares over an observation period (about 500 days) that was roughly four times longer were published [4,5]. A total of 1547 flares were detected in 279 stars. This paper is a statistical study of that sample for the purpose of refining and correcting the results of Ref. 3 on the superflare frequency distribution function for the entire sample of stars, as well as on the estimate of the total number of stars capable of superflares. As opposed to the earlier work [3], here the distribution function is found using Ambartsumyan's method, which became possible because of the large amount of data.

2. New estimates of the number of stars capable of superflares

The total number of stars capable of superflares can be estimated using Ambartsumyan's estimate [6] for ordinary flare stars:

$$n_0 = \frac{n_1^2}{2n_2}, \qquad (1)$$

where n_0 is the number of flare stars for which flares have not yet been detected, while n_1 and n_2 are the number of known flare stars for which one and two flares, respectively, have been observed. This estimate only gives a lower bound for n_0 [7].

Therefore, the total number of flare stars in the system is determined by the sum of the already known and still unknown flare stars:

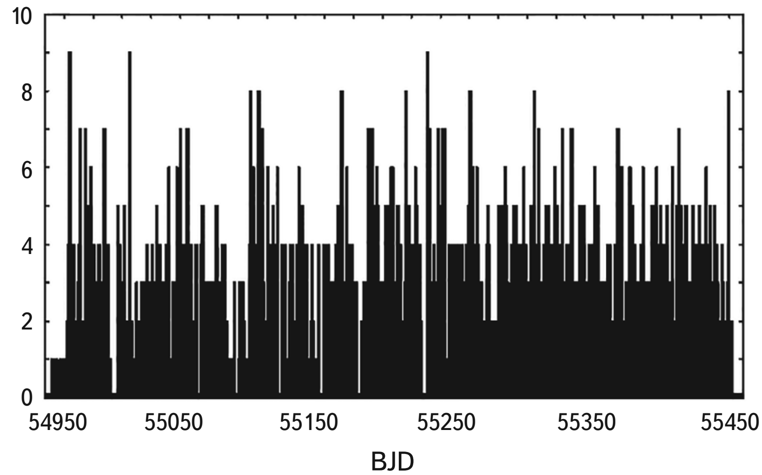

Fig. 1. The distribution of the flares over time (daily).

TABLE 1. Final Distribution of the Number of Flares for the Stars in the Complete Sample

k	n_k	k	n_k	k	n_k	k	n_k
1	102	6	10	11	5	16	4
2	42	7	9	12	6	17	1
3	28	8	6	13	1	18	2
4	19	9	6	14	3	19	2
5	11	10	2	15	6	≥ 20	14

$$N = n_0 + N_{obs}\ . \tag{2}$$

The dispersion and confidence intervals for the estimate of n_0 have been obtained in Ref. 8, where the dispersion is given by

$$\sigma_{n_0}^2 = n_2\left[\frac{1}{2}\left(\frac{n_1}{n_2}\right)^2 + \left(\frac{n_1}{n_2}\right)^3 + \frac{1}{4}\left(\frac{n_1}{n_2}\right)^4\right]. \tag{3}$$

The available data can be used to determine the temporal distribution of the flares (Fig .1) which, in turn, can be used to determine the distribution of the number of flares at an arbitrary time (the final distribution of the number of flares for stars in the complete sample is given in Table 1).

This, in turn, makes it possible to track the time behavior of the estimate (2) of the number of stars capable of superflares (Fig. 2). It can be seen that, beginning at the time BJD-55240, when about 800 superflares had been detected in 220 stars, the estimate "stabilizes" at a level of about 380-400 stars (at the latest time n_0 = 124 and

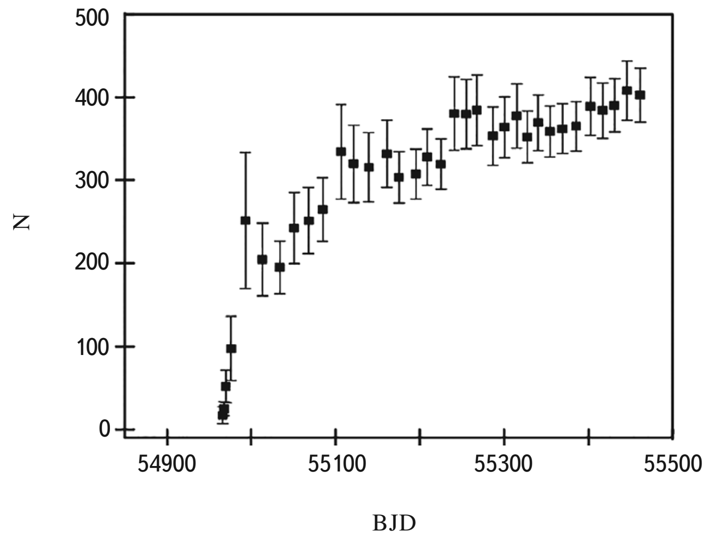

Fig. 2. The estimated number of stars capable of superflares. The X axis is the barycentric Julian date.

N = 403, with a mean square deviation of $\sigma_{n_0} = 33$), which represents about 0.5% of the total number of stars.

3. Determination of the superflare frequency distribution function

3.1. Ambartsumyan's method. It is essentially impossible to determine the frequency distribution of superflares, $\varphi(\nu)$, by direct counting because of the comparatively small number of detected flares for individual stars. In 1978 Ambartsumyan [9] proposed a statistical method for determining $\varphi(\nu)$ that avoided this difficulty. The solution used the inverse Laplace transform (L^{-1}) of the observed function $m_1(t)/m_1(0)$:

$$\varphi(\nu) = \frac{\nu_m}{\nu} L^{-1}\left[\frac{m_1(t)}{m_1(0)}\right], \tag{4}$$

where

$$m_1(t) = N \int \nu e^{-\nu t} \varphi(\nu) d\nu$$

is the number of new flare stars per unit time at time t. In particular, for the initial time t = 0,

$$m_1(0) = N \nu_m = \frac{n(t)}{t}, \tag{5}$$

where ν_m is the average frequency of stellar flares and $n(t)$ is the number of detected flares up to time t.

The observed function $m_1(t)/m_1(0)$ is subject to strong fluctuations, so it is best to smooth the function at the start. Ambartsumyan proposed smoothing with the aid of the statistics of second flare stars. Here, smoothing is carried out using the formula

$$\frac{m_1(t)}{m_1(0)}=\frac{n_1(t)}{n(t)}, \tag{6}$$

where $n_1(t)$ is the number of stars with one flare up to time t.

Figure 3 shows the function $m_1(t)/m_1(0)$ calculated by direct counting (squares) and by smoothing (circles) using Eq. (6), as well as an approximation of these data by an analytic function

$$\frac{m_1(t)}{m_1(0)}=\frac{1}{(1+at)^b},$$

where a = 0.03 and b = 1.

It follows from Eq. (4) that

$$\varphi(\nu)=\frac{\nu_m}{\nu}\frac{e^{-\nu/a}}{a}. \tag{7}$$

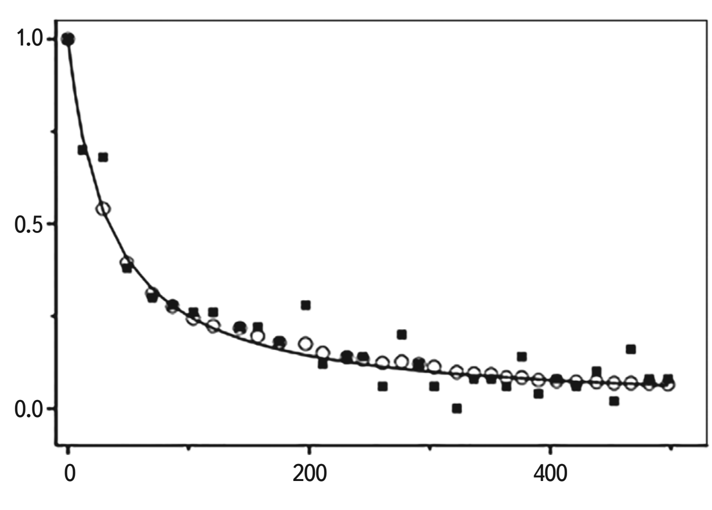

Fig. 3. The function $m_1(t)/m_1(0)$ calculated by direct counting (squares) and by smoothing (circles), along with an approximation of these by an analytic function (curve).

The expression (7) is not a probability density since it is nor normalized because of the singularity of the integrant at the point $\nu = 0$. A similar situation occurred in Ambartsumyan's paper [9] which proposed and justified the idea that the actual density function can have the form $\varphi(\nu)g(\nu)$, where $g(\nu)$ within some neighborhood of the point $\nu = 0$ takes values equal or close to zero, but is equal to unity outside this neighborhood. Here it becomes possible only to determine the number of stars with a flare frequency equal to or greater than a specified flare frequency. This approach is used in this paper. The number of stars corresponding to this approach with a flare frequency $\nu \geq \nu_0$ is equal to

$$N(\nu_0) = N \int_{\nu_0}^{\infty} g(\nu) \frac{\nu_m}{\nu} \frac{e^{-\nu/a}}{a} d\nu .$$

Outside some neighborhood of zero, where it can be assumed that $g(\nu) = 1$,

$$N(\nu_0) = N \nu_m \int_{\nu_0}^{\infty} \frac{e^{-\nu/a}}{\nu a} d\nu .$$

In this method, the number of nonflaring stars n_0 is assumed unknown, so that N and ν_m cannot be determined separately, but their product $N\nu_m$ is easily estimated with the aid of Eq. (5) using observational data:

$$N\nu_m \approx 3.1 \text{ day}^{-1} .$$

TABLE 2. Calculated Numbers of Stars with a Superflare Frequency $\nu \geq \nu_0$

ν_0	$N(\nu_0)$	ν_0	$N(\nu_0)$	ν_0	$N(\nu_0)$	ν_0	$N(\nu_0)$	ν_0	$N(\nu_0)$
0.002	227	0.022	36	0.042	12	0.062	5	0.082	2
0.004	162	0.024	32	0.044	11	0.064	4	0.084	2
0.006	126	0.026	28	0.046	10	0.066	4	0.086	2
0.008	103	0.028	25	0.048	9	0.068	4	0.088	1
0.01	86	0.03	23	0.05	8	0.07	3	0.09	1
0.012	73	0.032	20	0.052	7	0.072	3	0.092	1
0.014	62	0.034	18	0.054	7	0.074	3	0.094	1
0.016	54	0.036	16	0.056	6	0.076	2	0.096	1
0.018	47	0.038	15	0.058	6	0.078	2	0.098	1
0.02	41	0.04	13	0.06	5	0.08	2	0.1	1

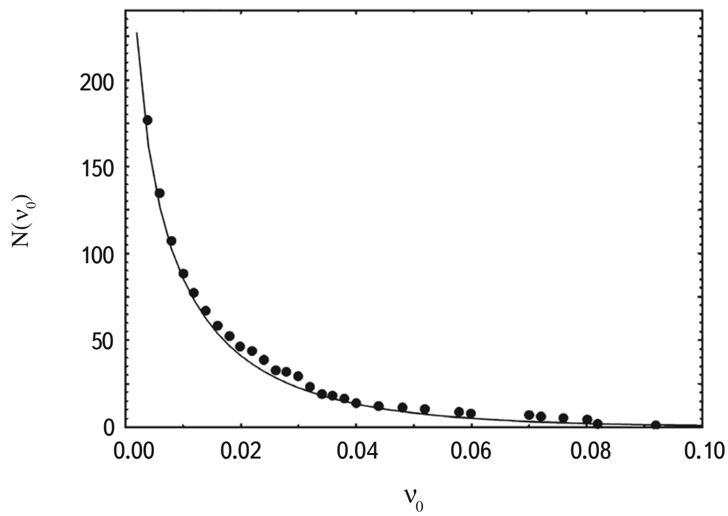

Fig. 4. A comparison of calculated values of $N(\nu_0)$ with analogous observations.

Calculated values of $N(\nu_0)$, beginning with $\nu_0 = 0.002$ which corresponds to one flare over the entire observation period *T*, are listed in Table 2.

Figure 4 shows a comparison of the calculated values with analogous measurements calculated using the formula

$$N(\nu_0) = \sum_k^{\infty} n_i \, ,$$

where *k* successively takes values *k* = 1, 2, 3, ..., and $\nu_0 = k/T$. Good agreement with the observed data can be seen beginning with a frequency of $\nu_0 = 0.002$.

3.2. The method of moments. Another method for determining the flare frequency distribution function has been proposed [10]. This method essentially involves determining the desired function in terms of the characteristic moments of the distribution. The initial data are the observed numbers of flares, which can be used to calculate the moments of the distribution of the number of flares. Using the equations

$$\mu k_1 = \int \sum_{k=0}^{\infty} k \frac{(\nu t)^k}{k!} e^{-\nu t} \varphi(\nu) d\nu \, ,$$

$$\mu k_j = \int \sum_{k=0}^{\infty} (k - \mu k_1)^j \frac{(\nu t)^k}{k!} e^{-\nu t} \varphi(\nu) d\nu, \quad j = 2, 3, 4,$$

$$\mu\nu_1 = \int \nu\varphi(\nu) d\nu, \quad \mu\nu_j = \int (\nu - \mu\nu_1)^j \varphi(\nu) d\nu, \quad j = 2, 3, 4,$$

where, in particular, μk_i are the statistical moments of the distribution of the number of flares and $\mu\nu_i$ are the statistical moments of the frequency distribution of the flares, it is possible to express the moments of the flare frequency distribution function in terms of the corresponding moments of the number of flares:

$$\mu\nu_1 = \frac{\mu k_1}{t}, \quad \mu\nu_2 = \frac{\mu k_2 - \mu k_1}{t^2}, \quad \mu\nu_3 = \frac{\mu k_3 - 3\mu k_2 + 2\mu k_2}{t^3},$$
$$\mu\nu_4 = \frac{\mu k_4 - 6\mu k_3 - 6\mu k_2 \cdot \mu k_1 + 11\mu k_2 - 6\mu k_1 + 3\mu k_1^2}{t^4}. \tag{8}$$

Substituting the empirical moments of the distribution of the number of flares in Eq. (8), we obtain the corresponding empirical moments of the flare frequency distribution function. Thus, the problem reduces to determining the distribution function using the known moments of the distribution. This was done [10] using a method for fitting curves from a family of Pearson distributions by the method of moments [11]. The type of distribution is determined by the quantities β_1, β_2, and k, where

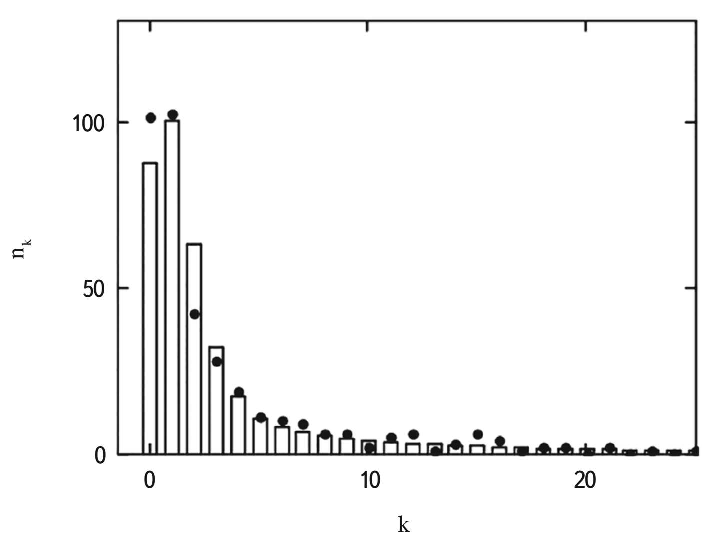

Fig. 5. A comparison of the observed distribution of the number of flares with the theoretically calculated distribution.

$$\beta_1 = \frac{\mu\nu_3^2}{\mu\nu_2^3}, \quad \beta_2 = \frac{\mu\nu_4}{\mu\nu_2^2}, \quad \kappa = \beta_1 \frac{(\beta_2+3)^2}{4(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)}.$$

By using the estimate of the overall number of stars found in the previous section and varying it within the limits of error, it is possible to calculate β_1, β_2, and k and to determine the type of distribution for which the agreement between the observed distribution of the number of flares and the theoretically calculated distribution (Fig. 5) will be best.

It turned out that the distribution function can be represented in a Pearson type I distribution (β distribution),

$$\Phi(\nu) = C \int_{\hat{\nu}-a}^{\nu} \left(1 + \frac{\eta - \hat{\nu}}{a}\right)^{m_1} \left(1 - \frac{\eta - \hat{\nu}}{b}\right)^{m_2} d\eta,$$

where $\hat{\nu}$ is the average distribution and ($\hat{\nu} - a$) is the lower limit of the distribution. The corresponding upper limit will be ($\hat{\nu} + b$).

The parameters of the distribution are: $\hat{\nu} = 0.008$, $a = 0.006$ $b = 0.140$, $C = 16.368$, $m_1 = -0.861$, and $m_2 = 2.176$. Figure 6 shows a comparison of the density functions for the distribution obtained by Ambartsumyan's method and by the method of moments. These functions essentially coincide after a superflare frequency of $\nu = 0.004\ \mathrm{d}^{-1}$.

A comparison of the distribution functions obtained in this paper and in Ref. 3 by the method of moments is shown in Fig. 7, where the points correspond to the distribution from Ref. 3.

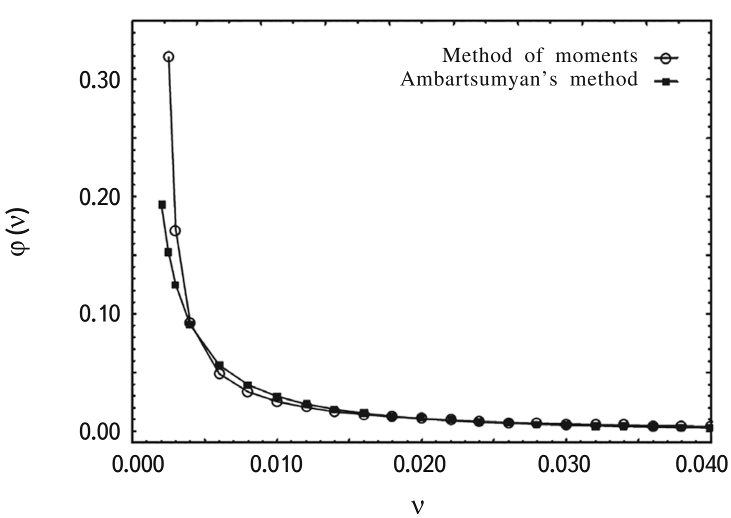

Fig. 6. A comparison of the density functions of the distribution obtained by Ambartsumyan's method and by the method of moments.

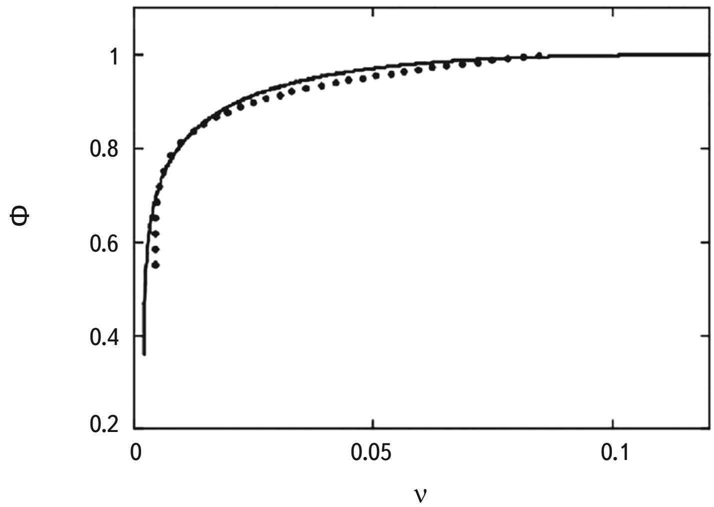

Fig. 7. A comparison of the distribution functions obtained in this paper and in Ref. 3 by the method of moments.

4. Conclusion

This paper discusses a statistical study of a sample of 1547 superflares in 279 stars discovered during an analysis of data on more than 80000 solar-type stars (obtained with the Kepler orbital observatory over its first 500 days of observations).

An estimate has been obtained for the total number of stars capable of superflares. The temporal behavior of this estimate indicates, as opposed to the previous estimate [3], that it is very close to the true value. We may conclude that the number of stars capable of superflares is roughly 0.5% of the total number of all the solar-type stars that were studied. This most likely indicates that "superflaring" stars of the solar-type either have some rarely encountered feature(s) or are in a short-duration phase of their evolution. Unavoidable observational errors, incorrect classification of the stars, etc., can hardly affect the main conclusion that the overwhelming majority of solar-type stars cannot produce superflares over a time comparable to the lifetime of these stars.

The superflare frequency distribution function for the complete sample of stars has been determined by two independent methods. The resulting functions are in good agreement with one another and with an analogous function obtained previously, as well as with the available observational data. A slight discrepancy can be seen within a narrow range of low superflare frequencies; this is not surprising, since observations with a duration of T = 500 days contain little information on frequencies on the order of $1/T$ or below. Applications of these functions will be discussed in subsequent papers.

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