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HYPERSURFACE HOMOGENEOUS COSMOLOGICAL MODEL IN MODIFIED THEORY OF GRAVITATION

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We study a hypersurface homogeneous space-time in the framework of the $f(R,T)$ theory of gravitation in the presence of a perfect fluid. Exact solutions of field equations are obtained for exponential and power law volumetric expansions. We also solve the field equations by assuming the proportionality relation between the shear scalar (σ) and the expansion scalar (θ). It is observed that in the exponential model, the universe approaches isotropy at large time (late universe). The investigated model is notably accelerating and expanding. The physical and geometrical properties of the investigated model are also discussed.

Keywords: *Hypersurface homogeneous: perfect fluid: $f(R,T)$ gravity*

1. Introduction

Recent observational data regarding high red shift from the Type Ia supernova and cosmic microwave background anisotropy indicate that the universe is accelerating [1-4]. The explanation of the late time accelerated expansion of the universe as well as the existence of dark energy (DE) and dark matter (DM) have received

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considerable attention. In the past few decades, the general theory of relativity has been modified in several ways to consider natural gravitation as a viable alternative to DE [5]. These modified theories are obtained by modifying the Einstein-Hilbert action in the general theory of relativity because it provides a means for understanding the problem of DE as well as the possibility for reconstructing the gravitational field theory potentially reproducing late-time acceleration. Among the various modifications the $f(R)$ theory of gravity is most suitable to explain the exact nature of accelerated expansion of the universe. The $f(R)$ theory provides a natural unification of early-time inflation and late -time acceleration [6]. Modified gravity can be categorized into several classes, including $f(G)$ gravity, $f(R,G)$ gravity, $f(T)$ gravity and $f(R,T)$ gravity.

Bertolami et al. [7] proposed a new class of modified theories of gravity by explicitly coupling the arbitrary function of the Ricci scalar (R) with matter Lagrangian density L_m. Herko et al. [8] extended this model by coupling geometry and matter. The $f(R,T)$ gravity is a modification of the $f(R)$ theory, where T dependence is induced by quantum effects or exotic nonideal matter configurations [9]. The $f(R,T)$ action [8] is given as follows:

$$S = \int \sqrt{-g}\left(\frac{1}{16\pi G} f(R,T) + L_m\right) d^4x\,, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the R and T the energy tensor of the matter is T_{ij}, and L_m represents matter Lagrangian density. Harko et al. [8] derived the field equations of $f(R,T)$ gravity by varying the action S of the gravitational field with respect to the metric tensor components g_{ij}.

In the $f(R,T)$ theory of gravity, the variation of the matter-energy tensor can be considered with respect to the metric. Therefore, reconstructing the Friedman-Robertson-Walker (FRW) cosmologies as an appropriate choice of the function $f(T)$ is possible. Moreover, Azizi [10] studied the wormhole solutions in the framework of $f(R,T)$ gravity. Naidu et al.[11] explored the FRW space time in relation to the $f(R,T)$. Reddy, Kumar [12] considered the LRS Bianchi type II space-time with the perfect fluid in the framework of the $f(R,T)$ gravity. A spherically symmetric fluid cosmological model with an anisotropic stress tensor in general relativity was studied by Pawar et al. [13]. Sharif et al. [14] investigated the energy condition in the $f(R,T)$ gravity for the FRW universe with the perfect fluid. Jamil et al. [15] reconstructed cosmological models in the context of $f(R,T)$ gravity and demonstrated that the dust fluid reproduces the ΛCDM, nonphantom era and phantom cosmology. Houndjo [16] investigated the cosmological models by using the function $f(R,T) = f_1(R) + f_2(T)$ in $f(R,T)$ gravity. Myrzakulov [17] studied metric-dependent torsion with the $f(R,T)$ theory of gravity and derived a model from the geometrical viewpoint. Motivated by the aforementioned studies, we studied the hypersurface homogeneous space-time in the $f(R,T)$ theory of gravity.

2. The $f(R,T)$ theory of gravity and field equations

The stress energy tensor of the matter is as follows:

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\partial \sqrt{-g}}{\partial g^{ij}} L_m . \tag{2}$$

Hence, we obtain the field equations of the $f(R,T)$ gravity model as follows:

$$\begin{aligned} f_R(R,T)R_{ij} - \frac{1}{2} f(R,T) g_{ij} + \left(g_{ij} \nabla^i \nabla_j - \nabla_i \nabla_j\right) f_R(R,T) = \\ = 8\pi T_{ij} - f_T(R,T) T_{ij} - f_T(R,T) \Theta_{ij} , \end{aligned} \tag{3}$$

where $\theta_{ij} = -2T_{ij} - pg_{ij}$, $f_R = \frac{\partial f(R,T)}{\partial R}$, $f_T = \frac{\partial f(R,T)}{\partial T}$, and ∇ is the covariant derivative.

We assume that the function $f(R,T)$ is given by

$$f(R,T) = R + 2 f_1(T), \tag{4}$$

where the $f_1(T)$ is an arbitrary function of the trace *T*. Recently, Chaubey and Shukla [18] discussed the Bianchi type I space time in the context of the $f(R,T)$ theory of gravitation using the special form of the average scale factor and obtained a new class of cosmological models. Ram et al. [19] obtained a new class of exact solutions of the Bianchi type cosmological models in the presence of a perfect fluid for a particular choice of the function $f(R,T) = R + 2\lambda T$, where λ is a constant.

We consider the hypersurface homogeneous space-time as follows

$$ds^2 = -dt^2 + A^2(t) dx^2 + B^2(t) \left[dy^2 + S^2(y,k) dz^2 \right], \tag{5}$$

where $S(y,k) = \sin y, y, \sinh y$ for k = 1, 0, –1 respectively. $A(t)$ and $B(t)$ are the cosmic scale functions. The hypersurface homogeneous space-time is cosmologically crucial. Ram and Verma [20] studied the hypersurface homogeneous space-time with a bulk viscous term and found some exact solutions. Reddy et al. [21] studied the Kantowski-Sachs space-time in the presence of a massless scalar field with a flat potential. Katore [22] investigated the magnetized Kantowski-Sachs inflationary cosmological model in the presence of a mass less scalar field with a flat potential.

The energy momentum tensor for a perfect fluid is given as follows:

$$T_{ij} = (\rho + p) u_i u_j + p g_{ij} , \tag{6}$$

where ρ is the energy density of the fluid, P is the pressure, and u^i represents the four velocity vector of the fluid, with components (0, 0, 0.1) satisfying $u_i u^i = -1$. The equation of state of a perfect fluid is $p = \gamma\rho$ with $\gamma \in [0,1]$. The condition $0 \le \gamma \le 1$ is necessary for the existence of local mechanical stability. Here, the matter Lagrangian can be taken as $L_m = -p$. Further, we choose

$$f_1(T) = \mu T. \tag{7}$$

The corresponding field equations for the metric (7) can be written as follows:

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{k}{B^2} = (8\pi + 2\mu)p + (5\mu p - \mu\rho), \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = (8\pi + 2\mu)p + (5\mu p - \mu\rho), \tag{9}$$

$$\frac{2A_4 B_4}{AB} + \frac{B_4^2}{B^2} + \frac{k}{B^2} = -(8\pi + 2\mu)\rho + (5\mu p - \mu\rho), \tag{10}$$

where the subscript 4 is used to denote differentiation with respect to time t.

The volume of the universe is given as follows:

$$V = AB^2. \tag{11}$$

From Eqs, (8), (9), and (10), we obtain

$$\frac{A_4}{A} - \frac{B_4}{B} = \frac{\lambda}{V} + \frac{1}{V}\int \left[\frac{k}{B^2}\right] V dt, \tag{12}$$

where λ represents the constant of integration.

The directional Hubble parameter in the direction of the x, y, and z axes are H_x, and H_y, H_z respectively, defined as follows:

$$H_x = \frac{A_4}{A}, \quad H_y = H_z = \frac{B_4}{B}. \tag{13}$$

The mean Hubble parameter is given by the following:

$$H = \frac{1}{3}\frac{V_4}{V} = \frac{1}{3}\left(\frac{A_4}{A} + \frac{2B_4}{B}\right). \tag{14}$$

The anisotropy parameter is defined as follows:

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 , \tag{15}$$

where H_i (i = 1, 2, 3) are the directional Hubble parameters in the direction of the x, y and z axes respectively. The expansion scalar (θ) and shear scalar (σ) are defined as follows:

$$\theta = 3H = \left(\frac{A_4}{A} + \frac{2B_4}{B}\right), \tag{16}$$

$$\sigma^2 = \frac{3}{2}\Delta H^2 = \frac{1}{3}\left(\frac{A_4}{A} - \frac{B_4}{B}\right)^2 . \tag{17}$$

Taking k = 0, we see that Eq. (12) leads to

$$\frac{A_4}{A} - \frac{B_4}{B} = \frac{\lambda}{V} . \tag{18}$$

Further, integration of Eq. (18) gives

$$B^3 = \frac{V}{c_1}\exp\left(-\int \frac{\lambda}{V} dt\right). \tag{19}$$

Now, we have three independent equations (8)-(10) in four unknowns P, ρ, A, and B. For complete determinacy of the system, we consider two volumetric expansions namely, exponential and power [25], Because the law of variation for the Hubble parameter proposed by Berman yields a constant value of the deceleration parameter. This law is not consistent with our observations. Thus, a new variation of the Hubble parameter is proposed, which led to two volumetric expansions [25]. The volumetric expansions are as follows [23,24]:

$$V = c_2 e^{3lt} , \tag{20}$$

$$V = c_3 t^{3m} , \tag{21}$$

where c_2, c_3, l and m are arbitrary positive constants. When $0 < m < 1$, the power law model yields the constant deceleration parameter q, whereas when $m > 1$ it yields accelerated expansion. Notably we get an inflationary universe for $q = 0$ and $m = 1$. The exponential expansion model reveals accelerating volumetric expansion.

3. Exponential expansion model

Considering Eq. (19) for the volumetric exponential expansion in Eq. (20), we obtain

$$A = \left(c_2 c_1^2\right)^{1/3} \exp\left(lt - \frac{2\lambda}{9mc_2} e^{-3lt} \right), \tag{22}$$

$$B = \left(\frac{c_2}{c_1} \right)^{1/3} \exp\left(lt + \frac{\lambda}{9mc_2} e^{-3lt} \right). \tag{23}$$

Clearly, from Eqs. (22) and (23), in the early stage of the universe, the values of scale factors of the universe are approximately constant and increase very slowly for $l > 0$. At a specific time, the universe suddenly exploded and expanded to a large extent, which is consistent with the Big Bang scenario. A similar result was obtained by Singh and Beesham [24] as well as by Katore and Hatkar [26].

Using Eq. (8) and with the help of Eqs. (22) and (23), we write the energy density and pressure as follows:

$$\rho = \frac{15l^2}{16\pi} + \frac{5\lambda^2}{48\pi c_2^2} e^{-6lt} , \tag{24}$$

$$p = \frac{3l^2}{16\pi} + \frac{\lambda^2}{48\pi c_2^2} e^{-6lt} . \tag{25}$$

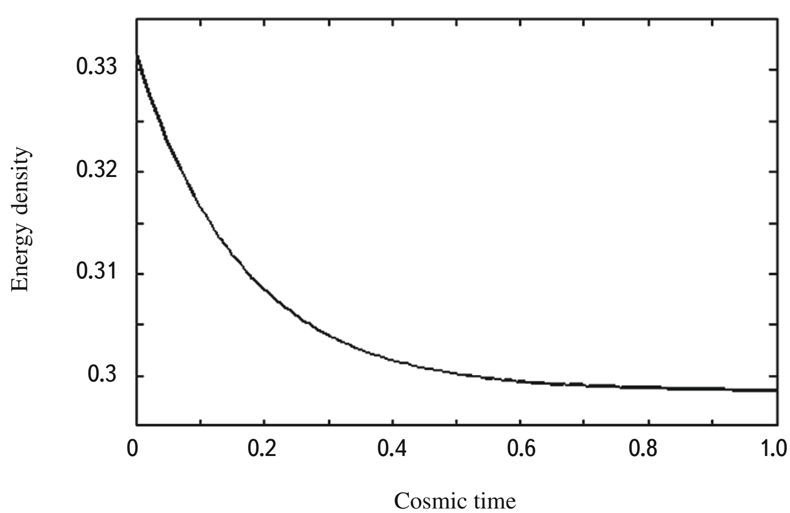

Fig.1. Plot of energy density for $l = c_2 = \lambda = 1$.

From Eqs. (24) and (25), we observe that $p > 0$ and $\rho > 0$ for a specific constant. Notably, the energy density is a decreasing function of time (see Fig.1). The model behaves like a steady-state model of the universe at large time. This is analogous to the findings of Das and Sarma [27]. In the literature, the stability of the model was investigated using the sign of the ratio $dP/d\rho$. Stability occurs when the ratio $dP/d\rho$ is positive. Here, the ratio is $dP/d\rho = 1/5 > 0$; therefore, the model is stable.

The expansion scalar θ, shear scalar σ, and deceleration parameter q are obtained as follows:

$$\theta = 3l, \tag{26}$$

$$\sigma^2 = \frac{\lambda}{\sqrt{3}\, c_2} e^{-3lt}, \tag{27}$$

$$q = -1. \tag{28}$$

The deceleration parameter from Eq. (28) indicates that the universe is accelerating. The value of the expansion scalar is constant; that is, the rate of expansion of the universe is constant. At the early stages of the evolution of the universe the ratio of the shear scalar to the expansion scalar was nonzero, and as the time increases, it tended to be zero, which means that the universe was initially anisotropic and at a late time it approached isotropy. The condition of homogeneity and isotropy, that is, $\lim_{t \to \infty} \frac{\sigma}{\theta} = 0$, formulated by Collins and Hawkins [28], is satisfied by the present model. The results are similar to those of Singh and Beesham [24], Katore and Hatkar [26], and Adhav [29].

4. Power Law model

Considering Eq. (19) for the power law volumetric expansion in Eq. (21), we obtain the following:

$$A = \left(c_3 c_1^2\right)^{1/3} t^m e^{\frac{2\lambda}{3(1-3m)c_3} t^{1-3m}}, \tag{29}$$

$$B = \left(\frac{c_3}{c_1}\right)^{1/3} t^m e^{-\frac{\lambda}{3(1-3m)c_3} t^{1-3m}}. \tag{30}$$

The expressions (29) and (30) show that A and B vanish at $t = 0$. Hence, the model has initial singularity. Afterwards, A and B increase indefinitely with the passage of time, which is in complete agreement with the Big-Bang

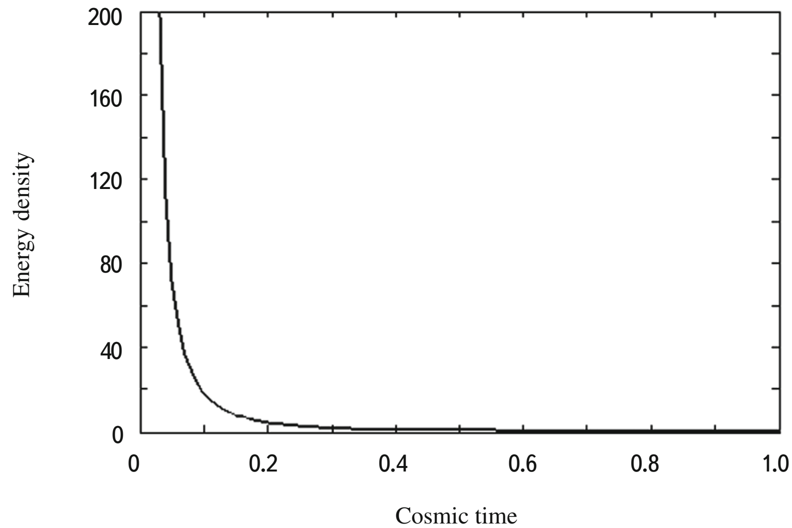

Fig.2. Plot of energy density for $\lambda = 2$, $\mu = 1$.

model of the universe. The model is similar to those of Akarsu and Kilinc [23] and Adhav [19]. Moreover, the solution of the field equations is obtained for $\gamma = 0.2$. Depending on its numerical values, γ describes the dust universe ($\gamma = 0$), radiation universe ($\gamma = 1/3$), hard universe ($\gamma \in (1/3, 1)$), and Zeldovich universe or stiff fluid ($\gamma = 1$) [30]. Therefore, in this model, $\gamma = 0.2$ represents the inflationary universe.

The energy density and pressure are obtained as follows:

$$\rho = \frac{5(\lambda^2 - 1)}{3(8\pi + 2\mu)t^2}, \tag{31}$$

$$p = \frac{(\lambda^2 - 1)}{3(8\pi + 2\mu)t^2}. \tag{32}$$

Clearly, from Eqs. (31) and (32), $p > 0$, $\rho > 0$, the energy density is a decreasing function of time. The energy density was very large at the early stages of evolution of the universe, and as the time increases, it tends to zero. Thus, the universe may be empty in the far future. A similar result was obtained by Singh [25]. In the present model, the ratio is $dP/d\rho = 1/5 > 0$; therefore the model is stable. The behavior of energy density is depicted for the appropriate choice of physical parameters and integration constants in Fig.2.

The expansion scalar θ , shear scalar σ , and deceleration parameter q are expressed as follows:

$$\theta = \frac{3m}{t}, \tag{33}$$

$$\sigma = \sqrt{\frac{\lambda}{3c_3 t}}, \tag{34}$$

$$q = 2. \tag{35}$$

The ratio of the shear scalar to the expansion scalar indicates that at the early epoch, the universe was anisotropic and, as time passes, it approaches isotropy. The universe has singularity at $t = 0$. It starts with an infinite rate of expansion and an infinite measure of anisotropy. For large time, that is, as $t \to \infty$, the shear becomes insignificant. The condition of homogeneity and isotropy, that is, $\lim_{t\to\infty}(\sigma/\theta) = 0$, formulated by Collins and Hawkins, [28] is satisfied in the present model. The observations by the differential radiometers on the NASA's Cosmic Background Explorer registered anisotropy in various angle scales. These anisotropies are believed to contain the entire history of cosmic evolution, including the recombination, and are considered indicative of the geometry and the material composing the universe. The theoretical arguments [31] and modern experimental data support the existence of an anisotropic phase, which is transformed into an isotropic one [32]. Our investigations indicate that the deceleration parameter is positive; that is, the universe was decelerating at the time of inflation; this is in accordance with modern cosmological observations [1-2].

5. Model III

In this model, we have assumed the proportionality relation of the shear scalar and the expansion scalar for solving the field equations. The work of Thorne [33] explains the reasons for the assumption. The observations of the velocity redshift relation for extragalactic sources suggest that the Hubble expansion of the universe is isotropic today within approximately 30%. More precisely, the redshift studies limit the ratio of the shear scalar to the Hubble constant to $\sigma/H \leq 0.3$ in the neighborhood of our galaxy [34-36]. In this connection, Bali etal. [37] pointed out that for LRS type spatially homogeneous space-time, the normal congruence to the homogeneous hypersurface satisfies the condition σ/θ as constant. Many authors have used this relation to obtain solutions of the field equations [27,38]. This leads to

$$A = B^n. \tag{36}$$

Using Eqs. (8),(9) and (36), we get

$$2B_{44} + 2(n+1)\frac{B_4^2}{B} = \frac{2k}{(n-1)B}. \tag{37}$$

Equation (37) further reduces to

$$B_4^2 = \frac{k}{n^2-1} + CB^{-2n-2}, \tag{38}$$

where C is the constant of integration.

Subcase I. $C = 0$.

From Eq. (38) for $C = 0$, we have

$$A = \left(\frac{k}{n^2-1}\right)^{n/2} t^n, \tag{39}$$

$$B = \left(\frac{k}{n^2-1}\right)^{1/2} t. \tag{40}$$

From equations (39) and (40), for $n \neq \pm 1$, A and B vanish at $t = 0$; thereafter, they start evolving as time increases, and, finally, they diverge at large time. The results are similar to those of Akarsu and Kilinc [23] and Adhav [29]. Moreover, the values of the scalar factors also vanish for $k = 0$. Thus, this model does not admit a solution for $k = 0$.

The energy density of the model is calculated as follows:

$$\rho = \left(\frac{n^2}{8\pi\gamma + 7\mu\gamma - \mu}\right)\frac{1}{t^2}, \tag{41}$$

where $n = \dfrac{\mu - 8\pi\gamma + 7\mu\gamma}{2\gamma(4\pi + \mu)}$.

The expression of energy density obtained in Eq. (41) shows that it is a decreasing function of time. The energy condition, that is, $\rho > 0$, is satisfied. In the case of dust fluid $\gamma = 0$, the value of density is positive infinite for $\mu < 0$. Furthermore, it is large at $t = 0$ and tends to zero at large time. Therefore, the universe may be empty in the far future [25]. The behavior for a suitable physical parameter and other constant is depicted in Fig.3.

The volume V, expansion scalar θ, shear scalar σ, and deceleration parameter q become

$$V = \left(\frac{k}{n^2-1}\right)^{(n+2)/2} t^{n+2}, \tag{42}$$

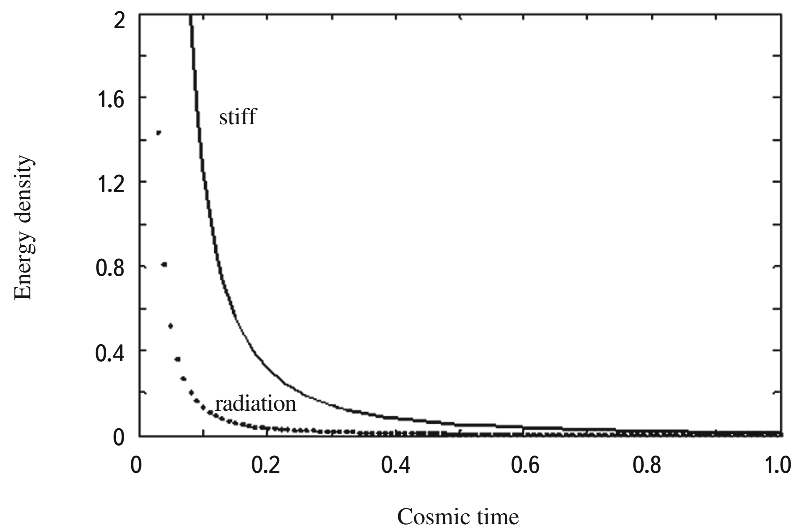

Fig.3. Plot of energy density for $\gamma = 1$, 1/3, $\mu = 1$.

$$\theta = \frac{3(n+2)}{t}, \tag{43}$$

$$\sigma = \frac{(n-1)}{\sqrt{3}\,t}, \tag{44}$$

$$q = \frac{1}{n+2} - 1. \tag{45}$$

The volume of the universe is clearly an increasing function of time *t*. The universe evolves with an infinite rate of expansion and anisotropy. Thus, the model represents the early era of the evolution of the universe. This is consistent with the Big Bang model of the universe. The shear scalar becomes insignificant as $t \to \infty$. Furthermore, the anisotropy is maintained throughout evolution of the universe. From Fig.4, the sign of the deceleration parameter is negative and positive for $n < -2$, $-1 < n$ and $-2 < n < -1$ respectively.

Subcase II. $C \neq 0$.

From Eq. (38) for $C \neq 0$, we obtain

$$A = \left((n+2)C\right)^{n/(n+2)} t^{n/(n+2)}, \tag{46}$$

$$B = \left((n+2)C\right)^{1/(n+2)} t^{1/(n+2)}. \tag{47}$$

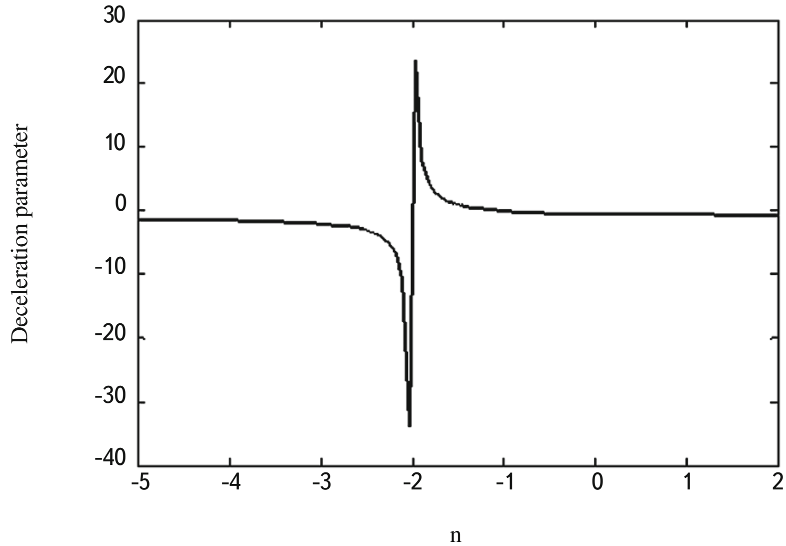

Fig.4. Plot of deceleration parameter.

The values of A and B vanish at $t = 0$; they start evolving with the passage of time, and as $t \to \infty$, they diverge, which is consistent with the Big Bang model. This solution of the field equations is subjected to the condition $k = 0$, $\gamma = 0.2$. Thus, the model represents the inflationary universe.

The energy density and pressure are obtained as follows:

$$\rho = \left(\frac{10n+5}{50-8\pi}\right)\frac{1}{(n+2)^2}\frac{1}{t^2}, \tag{48}$$

$$p = \left(\frac{2n+1}{50-8\pi}\right)\frac{1}{(n+2)^2}\frac{1}{t^2}. \tag{49}$$

For an appropriate choice of physical parameters and other integration constant, the energy density should be a decreasing function of time (see Fig.5). The energy conditions $p > 0$, $\rho > 0$ are satisfied. The energy density at the early epoch was large, and as time increases, it gradually decreases, approaching a constant value. Thus, the universe may be in a steady state in the far future. In that case, we obtain the same result as that obtained in the study by Das and Sarma [27]. In this model, the ratio is $dP/d\rho = 1/5 > 0$; that is, the model is stable.

The volume V, expansion scalar θ, shear scalar σ, and deceleration parameter q for this model are obtained as follows;

$$V = (n+2)Ct, \tag{50}$$

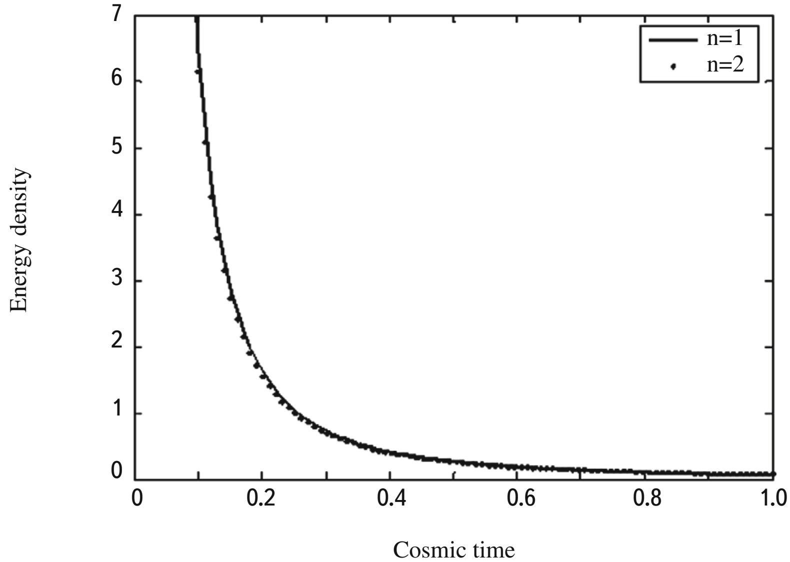

Fig.5. Plot of energy density.

$$\theta = \frac{1}{t}, \tag{51}$$

$$\sigma = \frac{(n-2)}{\sqrt{3}(n+2)t}, \tag{52}$$

$$q = 2. \tag{53}$$

From Eq. (50), the volume of the universe is clearly an increasing function of time. The expansion scalar in Eq. (51) shows that the rate of expansion is a decreasing function of time. The ratio of the shear scalar to the expansion scalar is nonzero; therefore, the universe is anisotropic. The value of the deceleration parameter is positive; that is, the universe is decelerating. The present model is consistent with recent observational data [1,33,39].

6. Conclusion

In the present paper, we investigated the perfect fluid cosmological model in the $f(R,T)$ theory of gravitation framework for the hypersurface homogeneous space-time. Under some specific choices of the parameters, in the exponential expansion model, the rate of expansion of the universe is constant. The universe is accelerating. The universe approaches isotropy at large time. In the power law model, we obtained an inflationary decelerating universe. The present models are consistent with the Big Bang model. The condition of homogeneity and isotropization formulated by Collins and Hawkins [28] is satisfied by the aforementioned models. The models are valid only for

$k = 0$. Both models are stable.

In subcase I of the model III, the anisotropy of the universe is maintained throughout the evolution. The universe is accelerating for $n < -2$, $-1 < n$ and decelerating for $-2 < n < -1$.The model is not valid for $k = 0$, while in the subcase II of model III the universe is inflationary. The model has initial singularity at $t = 0$, and the universe evolves with an infinite rate of expansion and anisotropy. The physical parameters such as energy density, pressure, and shear scalar become insignificant at large time. The model is stable.

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