

RELATIVISTIC MEAN-FIELD THEORY EQUATION OF STATE OF NEUTRON STAR MATTER AND A MAXWELLIAN PHASE TRANSITION TO STRANGE QUARK MATTER

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The equation of state of neutron star matter is examined in terms of the relativistic mean-field theory, including a scalar-isovector δ -meson effective field. The constants of the theory are determined numerically so that the empirically known characteristics of symmetric nuclear matter are reproduced at the saturation density. The thermodynamic characteristics of both asymmetric nucleonic matter and β -equilibrium hadron-electron npe-plasmas are studied. Assuming that the transition to strange quark matter is an ordinary first-order phase transition described by Maxwell's rule, a detailed study is made of the variations in the parameters of the phase transition owing to the presence of a δ -meson field. The quark phase is described using an improved version of the bag model, in which interactions between quarks are accounted for in a one-gluon exchange approximation. The characteristics of the phase transition are determined for various values of the bag parameter within the range $B \in [60, 120] \text{ MeV/fm}^3$ and it is shown that including a δ -meson field leads to a reduction in the phase transition pressure P_0 and in the concentrations n_N and n_Q at the phase transition point.

Keywords: (stars:) neutron: superdense matter: equation of state: quarks

1. Introduction

In addition to their independent, fundamental significance, studies of the structural characteristics and composition of the constituents of matter at extremely high densities and temperatures play an extremely important role in clarifying the physical nature of the internal structure and integral parameters of neutron stars. A quantum field

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approach in the framework of quantum hadrodynamics (QHD) provides a fairly adequate description of the properties of nuclear matter and of finite nuclei, treating them as a system of strongly interacting baryons and mesons. One theory of this type that has effective applications, is the relativistic mean-field theory [1-3]. This theory yields satisfactory descriptions of the structure of finite nuclei [4], the equation of state of nuclear matter [5], and the features of heavy ion scattering [6]. The parameters of the mean-field model characterizing the interaction of a nucleon with σ , ω , and ρ mesons can be self consistently determined starting with empirical data on symmetric nuclear matter near the saturation density. This, in turn, leads to the possibility of obtaining equations of state for superdense, isospin-asymmetric nuclear matter. In these studies it has been assumed that the masses of the scalar-isoscalar (σ), vector-isoscalar (ω), and vector-isovector (ρ) mesons and their coupling constants are independent of the density and of the values of the fields. In addition, the δ -scalar meson ($a_0(980)$) is not included among the exchange mesons.

Relativistic mean-field theory models have been constructed [7,8] assuming that the nucleon and exchange meson masses in nuclear media obey the Brown-Rho scaling law [9]. The results showed that including the density dependence of the mass leads to a more rigid equation of state for the matter. A scalar-isovector δ -meson has been added to the scheme and a study of its role for asymmetric nuclear matter at low densities has been made in Refs. 10-12. This approach has been used [13-15] to study scattering processes for neutron-excess heavy ions at medium energies and the feasibility of forming a hadron-quark mixed phase during the collision process.

This paper is a study of the equation of state for neutron star matter in terms of the relativistic mean-field theory and an examination of the variations in the parameters of the first order phase transition caused by including δ -meson exchange. These results show how these variations will affect the integral characteristics and structure of hybrid neutron stars with quark matter cores.

2. Lagrangian and thermodynamic characteristics of nucleonic systems

The nonlinear lagrangian density of an interacting many-particle system consisting of nucleons and isoscalar-scalar σ -mesons, isoscalar-vector ω -mesons, isovector-scalar δ -mesons, and isovector-vector ρ -mesons has the following form in QHD:*

$$\begin{aligned} \mathcal{L} = & \bar{\Psi}_N \left[\gamma^\mu \left(i \partial_\mu - g_\omega \omega_\mu(x) - \frac{1}{2} g_\rho \bar{\tau}_N \cdot \bar{\rho}_\mu(x) \right) - (m_N - g_\sigma \sigma(x) - g_\delta \bar{\tau}_N \cdot \bar{\delta}(x)) \right] \Psi_N + \\ & + \frac{1}{2} \left(\partial_\mu \sigma(x) \partial^\mu \sigma(x) - m_\sigma^2 \sigma(x)^2 \right) - U(\sigma(x)) + \frac{1}{2} m_\omega^2 \omega^\mu(x) \omega_\mu(x) - \frac{1}{4} \Omega_{\mu\nu}(x) \Omega^{\mu\nu}(x) + \\ & + \frac{1}{2} \left(\partial_\mu \bar{\delta}(x) \partial^\mu \bar{\delta}(x) - m_\delta^2 \bar{\delta}(x)^2 \right) + \frac{1}{2} m_\rho^2 \bar{\rho}^\mu(x) \bar{\rho}_\mu(x) - \frac{1}{4} \mathcal{R}_{\mu\nu}(x) \mathcal{R}^{\mu\nu}(x), \end{aligned} \quad (1)$$

where $x = x_\mu = (t, x, y, z)$, $\sigma(x)$, $\omega_\mu(x)$, $\bar{\delta}(x)$, and $\bar{\rho}_\mu(x)$ are the fields of the σ , ω , δ , and ρ exchange mesons, respectively, $U(\sigma)$ is the nonlinear part of the potential of the σ -field, given by [16]

* We shall use the natural system of units with $\hbar = c = 1$.

$$U(\sigma) = \frac{b}{3} m_N (g_\sigma \sigma)^3 + \frac{c}{4} (g_\sigma \sigma)^4, \quad (2)$$

$m_N, m_\sigma, m_\omega, m_\delta, m_\rho$ are the masses of the free particles, $\Psi_N = \begin{pmatrix} \Psi_p \\ \Psi_n \end{pmatrix}$ is the isospin doublet for nucleonic bispinors, and $\vec{\tau}$ are the isospin 2x2 Pauli matrices. The symbol " \rightarrow " denotes vectors in isotopic spin space. This lagrangian, as in quantum electrodynamics, also includes antisymmetric tensors of the vector fields $\omega_\mu(x)$ and $\rho_\mu(x)$ given by

$$\Omega_{\mu\nu}(x) = \partial_\mu \omega_\nu(x) - \partial_\nu \omega_\mu(x), \quad \mathcal{R}_{\mu\nu}(x) = \partial_\mu \rho_\nu(x) - \partial_\nu \rho_\mu(x). \quad (3)$$

Here $g_\sigma, g_\omega, g_\delta$, and g_ρ in Eq. (1) denote the coupling constants for a nucleon with the corresponding mesons. In the relativistic mean-field theory the meson fields $\sigma(x)$, $\omega_\mu(x)$, $\vec{\delta}(x)$, and $\vec{\rho}_\mu(x)$, which depend in general on the space-time coordinates, are replaced by the average (effective) uniform and time independent fields $\bar{\sigma}, \bar{\omega}_\mu, \bar{\vec{\delta}},$ and $\bar{\vec{\rho}}_\mu$. The Euler-Lagrange equations for the nucleon and meson fields can be used to solve the problem self consistently and find, as a final result, the equation of state of the matter in the nucleon phase. Changing the notation for the meson fields and coupling constants, with

$$g_\sigma \bar{\sigma} \equiv \sigma, \quad g_\omega \bar{\omega}_0 \equiv \omega, \quad g_\delta \bar{\vec{\delta}}^{(3)} \equiv \vec{\delta}, \quad g_\rho \bar{\vec{\rho}}_0^{(3)} \equiv \rho, \quad (4)$$

$$(g_\sigma/m_\sigma)^2 \equiv a_\sigma, \quad (g_\omega/m_\omega)^2 \equiv a_\omega, \quad (g_\delta/m_\delta)^2 \equiv a_\delta, \quad (g_\rho/m_\rho)^2 \equiv a_\rho \quad (5)$$

and introducing the asymmetry parameter

$$\alpha = (n_n - n_p)/n, \quad (6)$$

the equations for the fields can be rewritten in the form

$$\sigma = a_\sigma (n_{sp}(n, \alpha) + n_{sn}(n, \alpha) - b m_N \sigma^2 - c \sigma^3), \quad (7)$$

$$\omega = a_\omega n, \quad (8)$$

$$\vec{\delta} = a_\delta (n_{sp}(n, \alpha) - n_{sn}(n, \alpha)), \quad (9)$$

$$\rho = -\frac{1}{2} a_\rho n \alpha, \quad (10)$$

where

$$n_{sp}(n, \alpha) = \frac{1}{\pi^2} \int_0^{k_F(n)(1-\alpha)^{1/3}} \frac{m_p^*(\sigma, \delta)}{\sqrt{k^2 + m_p^*(\sigma, \delta)^2}} k^2 dk, \quad (11)$$

$$n_{sn}(n, \alpha) = \frac{1}{\pi^2} \int_0^{k_F(n)(1+\alpha)^{1/3}} \frac{m_n^*(\sigma, \delta)}{\sqrt{k^2 + m_n^*(\sigma, \delta)^2}} k^2 dk, \quad (12)$$

$$k_F(n) = \left(\frac{3\pi^2 n}{2} \right)^{1/3}. \quad (13)$$

The effective masses of the proton and neutron are given by

$$m_p^*(\sigma, \delta) = m_N - \sigma - \delta, \quad m_n^*(\sigma, \delta) = m_N - \sigma + \delta. \quad (14)$$

If the constants a_ω and a_ρ are known, then Eqs. (8) and (10) determine the functions $\omega(n)$ and $\rho(n, \alpha)$. Knowledge of the other constants a_σ , a_δ , b , and c makes it possible to solve the system of Eqs. (7), (9), (11), and (12) self consistently and to determine the two remaining functions for the meson fields, $\sigma(n, \alpha)$ and $\delta(n, \alpha)$.

The energy density of the nuclear np matter as a function of the concentration n and the asymmetry parameter np has the form

$$\begin{aligned} \varepsilon(n, \alpha) = & \frac{1}{\pi^2} \int_0^{k_F(n)(1-\alpha)^{1/3}} \sqrt{k^2 + (m_N - \sigma - \delta)^2} k^2 dk + \\ & + \frac{1}{\pi^2} \int_0^{k_F(n)(1+\alpha)^{1/3}} \sqrt{k^2 + (m_N - \sigma + \delta)^2} k^2 dk + \tilde{U}(\sigma) + \frac{1}{2} \left(\frac{\sigma^2}{a_\sigma} + \frac{\omega^2}{a_\omega} + \frac{\delta^2}{a_\delta} + \frac{\rho^2}{a_\rho} \right), \end{aligned} \quad (15)$$

where

$$\tilde{U}(\sigma) = \frac{b}{3} m_N \sigma^3 + \frac{c}{4} \sigma^4. \quad (16)$$

For the pressure of the nuclear matter we obtain

$$\begin{aligned}
P(n, \alpha) = & \frac{1}{\pi^2} \int_0^{k_F(n)(1-\alpha)^{1/3}} \left(\sqrt{k_F(n)^2(1-\alpha)^{2/3} + (m_N - \sigma - \delta)^2} - \sqrt{k^2 + (m_N - \sigma - \delta)^2} \right) k^2 dk + \\
& + \frac{1}{\pi^2} \int_0^{k_F(n)(1+\alpha)^{1/3}} \left(\sqrt{k_F(n)^2(1+\alpha)^{2/3} + (m_N - \sigma + \delta)^2} - \sqrt{k^2 + (m_N - \sigma + \delta)^2} \right) k^2 dk - \\
& - \tilde{U}(\sigma) + \frac{1}{2} \left(-\frac{\sigma^2}{a_\sigma} + \frac{\omega^2}{a_\omega} - \frac{\delta^2}{a_\delta} + \frac{\rho^2}{a_\rho} \right).
\end{aligned} \tag{17}$$

The chemical potentials of the proton and neutron are given by

$$\begin{aligned}
\mu_p(n, \alpha) &= \sqrt{k_F(n)^2(1-\alpha)^{2/3} + (m_N - \sigma - \delta)^2} + \omega + \frac{1}{2}\rho, \\
\mu_n(n, \alpha) &= \sqrt{k_F(n)^2(1+\alpha)^{2/3} + (m_N - \sigma + \delta)^2} + \omega - \frac{1}{2}\rho.
\end{aligned} \tag{18}$$

3. Determination of the constants for the model

Empirical characteristics of saturated nuclear matter and the constants of the theory. In order to determine the constants for the theory, a_σ , a_ω , a_δ , a_ρ , b , and c , we can derive a system of equations relating these parameters to known empirical characteristics of symmetric nuclear matter at the saturation concentration n_0 [17]. Given that the effective mass of a nucleon in symmetric nuclear matter ($\alpha=0$) at the saturation concentration n_0 is related to the bare nucleon mass by

$$m_N^* = \gamma m_N, \tag{19}$$

where γ is a constant between 0.7 and 0.8, for the σ field at the saturation concentration n_0 we have

$$\sigma_0 = (1-\gamma)m_N. \tag{20}$$

Equations (9) and (10) imply that $\delta_0=0$ and $\rho_0=0$ at the saturation concentration in saturated nuclear matter. Given the requirement that the energy $\varepsilon(n, \alpha)/n$ per nucleon should have a minimum at $n=n_0$ and $\alpha=0$, we obtain

$$\left. \frac{d\varepsilon(n, \alpha)}{dn} \right|_{\substack{n=n_0 \\ \alpha=0}} = \frac{\varepsilon(n_0, 0)}{n_0} = m_N + f_0, \tag{21}$$

where $f_0 = B/A$ is the specific binding energy of the nucleus, neglecting the Coulomb interaction and finite size of the nucleus.

Using Eq. (15), Eq. (21) yields

$$a_\omega = \frac{1}{n_0} \left(m_N + f_0 - \sqrt{k_F(n_0)^2 + (m_N - \sigma_0)^2} \right). \quad (22)$$

The ω_0 field for symmetric matter at n_0 , on the other hand, is given by

$$\omega_0 = a_\omega n_0 = m_N + f_0 - \sqrt{k_F(n_0)^2 + (m_N - \sigma_0)^2}. \quad (23)$$

From the equation for the σ field (7), we have

$$\frac{\sigma_0}{a_\sigma} = \frac{2}{\pi^2} \int_0^{k_F(n_0)} \frac{(m_N - \sigma_0)}{\sqrt{k^2 + (m_N - \sigma_0)^2}} k^2 dk - b m_N \sigma_0^2 - c \sigma_0^3. \quad (24)$$

The energy density $\varepsilon_0 = n_0(m_N + f_0)$ for saturated nuclear matter at the saturation concentration n_0 can be written in the form

$$\varepsilon_0 = n_0(m_N + f_0) = \frac{2}{\pi^2} \int_0^{k_F(n_0)} \sqrt{k^2 + (m_N - \sigma_0)^2} k^2 dk + \frac{b}{3} m_N \sigma_0^3 + \frac{c}{4} \sigma_0^4 + \frac{1}{2} \left(\frac{\sigma_0^2}{a_\sigma} + n_0^2 a_\omega \right). \quad (25)$$

An important empirical characteristic which, in a certain way, couples the phenomenological constants of the theory, is the modulus of compression of the nuclear matter, which is defined as

$$K = 9n_0^2 \frac{d^2}{dn^2} \left(\frac{\varepsilon(n, \alpha)}{n} \right) \Big|_{\alpha=0}^{n=n_0}. \quad (26)$$

Substituting Eq. (15) in Eq. (26) gives

$$K = 9a_\omega n_0 + 3 \frac{k_F(n_0)^2}{\sqrt{k_F(n_0)^2 + (m_N - \sigma_0)^2}} - 9 \frac{n_0(m_N - \sigma_0)^2}{k_F(n_0)^2 + (m_N - \sigma_0)^2} \frac{1}{\frac{1}{a_\sigma} + \frac{2}{\pi^2} \int_0^{k_F(n_0)} \frac{k^4 dk}{[k^2 + (m_N - \sigma_0)^2]^{3/2}} + 2b m_N \sigma_0 + 3c \sigma_0^2}. \quad (27)$$

In the semi-empirical Weizsäcker formula the term for the specific energy of the asymmetry of the nucleonic system is given by

$$\frac{\epsilon_{sym}}{n} = E_{sym}(n)\alpha^2. \quad (28)$$

The energy asymmetry coefficient $E_{sym}(n)$ is defined as

$$(29)$$

Using Eq. (15), we obtain the following expression for the symmetry energy at the saturation concentration of the nuclear matter, $E_{sym}^{(0)} = E_{sym}(n_0)$,

$$E_{sym}^{(0)} = \frac{n_0}{8} a_\rho + \frac{k_F(n_0)^2}{6\sqrt{k_F(n_0)^2 + (m_N - \sigma_0)^2}} - \frac{1}{2} \frac{n_0(m_N - \sigma_0)^2}{k_F(n_0)^2 + (m_N - \sigma_0)^2} \frac{1}{\frac{1}{a_\delta} + \frac{2}{\pi^2} \int_0^{k_F(n_0)} \frac{k^4 dk}{[k^2 + (m_N - \sigma_0)^2]^{3/2}}}. \quad (30)$$

3.1. Determination of the constants for the model.

Numerical determination of the constants for the theory. In order to determine the constants for the theory we have used the following values of the known nuclear parameters at saturation: $m_N = 938.93 \text{ MeV}$, $\gamma = m_N^*/m_N = 0.78$, saturation concentration of nuclear matter $n_0 = 0.153 \text{ fm}^{-3}$, specific binding energy $f_0 = -16.3 \text{ MeV}$, modulus of compression $K = 300 \text{ MeV}$, and $E_{sym}^{(0)} = 32.5 \text{ MeV}$. Equations (20) and (23) can be used to determine the σ_0 and ω_0 fields. Then Eqs. (22), (24), (25), (27), and (30) form a system of five equations for the six unknown constants, a_σ , a_ω , a_δ , a_ρ , b , and c . It can be seen from Eq. (30) that including the interaction channel involving the isovector-scalar δ -meson leads to a certain correlation between the values of a_δ and a_ρ .

Table 1 lists the values of a_ρ for various values of the constant a_δ . In order to clarify the role of the δ -meson, in the following we shall take $a_\delta = 2.5 \text{ fm}^2$ [11]. The absence of the δ interaction channel will correspond to an interaction constant $a_\delta = 0$. Note that the value used here, $a_\delta = 2.5 \text{ fm}^2$, is in good agreement with Ref. 18,

TABLE 1. Values of the Constant a_ρ for Different Values of a_δ

$a_\delta = (g_\delta / m_\delta)^2, \text{ fm}^2$	0	0.5	1	1.5	2	2.5	3
$a_\rho = (g_\rho / m_\rho)^2, \text{ fm}^2$	4.794	6.569	8.340	10.104	11.865	13.621	15.372

TABLE 2. Constants for the Theory without ($\sigma\omega\rho$) and with ($\sigma\omega\rho\delta$) a δ -Meson Field

Parameters	$\sigma\omega\rho$	$\sigma\omega\rho\delta$
$a_\sigma, \text{ fm}^2$	9.154	9.154
$a_\omega, \text{ fm}^2$	4.828	4.828
$a_\delta, \text{ fm}^2$	0	2.5
$a_\rho, \text{ fm}^2$	4.794	13.621
$b, \text{ fm}^{-1}$	$1.654 \cdot 10^{-2}$	$1.654 \cdot 10^{-2}$
c	$1.319 \cdot 10^{-2}$	$1.319 \cdot 10^{-2}$

where a microscopic Dirac-Bruckner-Hartree-Fock theory is applied to asymmetric nuclear matter and exotic nuclei in a study of the density dependence of the meson-nucleon coupling constants. According a plot of a_δ as a function of concentration n in Fig. 2 of Ref. 18, the average value of a_δ in the range $n \approx 0.1 \div 0.3 \text{ fm}^{-3}$ is on the order of $65 \text{ GeV}^{-2} \approx 2.5 \text{ fm}^2$.

Table 2 lists the values of the parameters obtained by numerical solution of the system of five Eqs. (22), (24), (25), (27), and (30) without ($\sigma\omega\rho$) and with ($\sigma\omega\rho\delta$) the isovector-scalar δ -meson interaction channel.

4. Characteristics of β -equilibrium npe plasmas and the equation of state of neutron star matter in the nucleonic phase

The values of the constants a_σ , a_ω , a_δ , a_ρ , b , and c for the relativistic mean-field theory obtained in the previous section (See Table 2.) can be used to calculate various characteristics of matter with an asymmetric proton-neutron composition (np -matter) and of β -equilibrium npe -matter. In terms of the relativistic mean-field theory, the lagrangian density for the npe -plasma is

$$\mathcal{L}_{NM} = \mathcal{L} + \bar{\psi}_e (i \gamma^\mu \partial_\mu - m_e) \psi_e, \quad (31)$$

where \mathcal{L} is the lagrangian of the system consisting of nucleons and $\sigma\omega\rho\delta$ mesons (See Eq. (1)), ψ_e is the electron wave function, and m_e is the electron mass. In this case, we find the energy density of the npe -plasma to be

$$\varepsilon_{NM}(n, \alpha, \mu_e) = \varepsilon(n, \alpha) + \varepsilon_e(\mu_e), \quad (32)$$

where $\varepsilon(n, \alpha)$ is the energy density of the $np\sigma\omega\rho\delta$ system defined by Eq. (26),

$$\varepsilon_e(\mu_e) = \frac{1}{\pi^2} \int_0^{\sqrt{\mu_e^2 - m_e^2}} \sqrt{k^2 + m_e^2} k^2 dk \quad (33)$$

is the contribution of the electrons to the energy density, and μ_e is the chemical potential of the electrons. For the pressure of the npe -plasma, we have

$$P_{NM}(n, \alpha, \mu_e) = P(n, \alpha) + \frac{1}{3\pi^2} \mu_e (\mu_e^2 - m_e^2)^{3/2} - \varepsilon_e(\mu_e). \quad (34)$$

Depending on the coefficient of surface tension, σ_s , it is known that a phase transition of nuclear matter into quark matter can occur in two ways [19]. It can either have the character of an ordinary first order phase transition with a discontinuous density change (Maxwell's rule) or mixed nucleon-quark matter can be formed with a continuous variation in the pressure and density [20]. In the second case, the condition of global electrical neutrality implies that, in order to determine the parameters of the phase transition and the equation of state of the mixed phase, it is necessary to know the equation of state of the β -equilibrium charged npe -plasma. To find the characteristics of the β -equilibrium, but not necessarily the neutral, npe -plasma, one has to solve the system of four equations (7)-(10) for specified values of the concentration n and asymmetry parameters α , and find the unknown mean meson fields

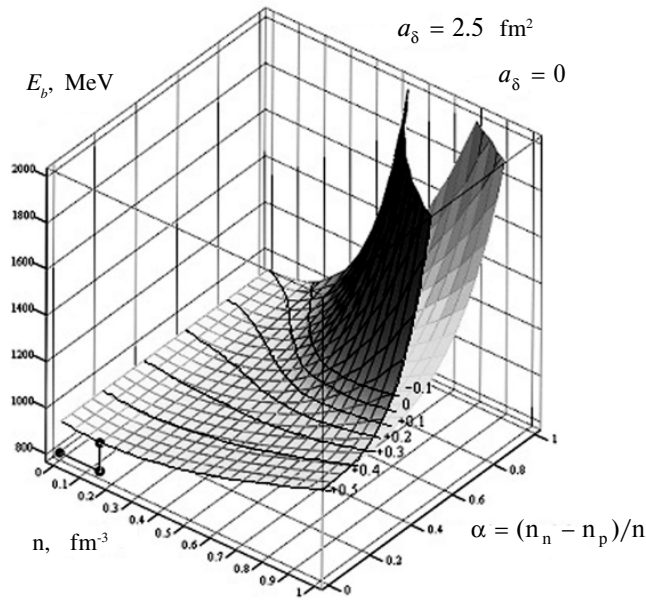


Fig. 1. Three dimensional representation of the energy E_b per baryon as a function of the concentration n and asymmetry parameter α in the case of a β -equilibrium, charged npe -plasma. The upper surface corresponds to the " $\sigma\omega\rho\delta$ " model and the lower, to " $\sigma\omega\rho$ ". The lines correspond to different values of the charge per baryon.

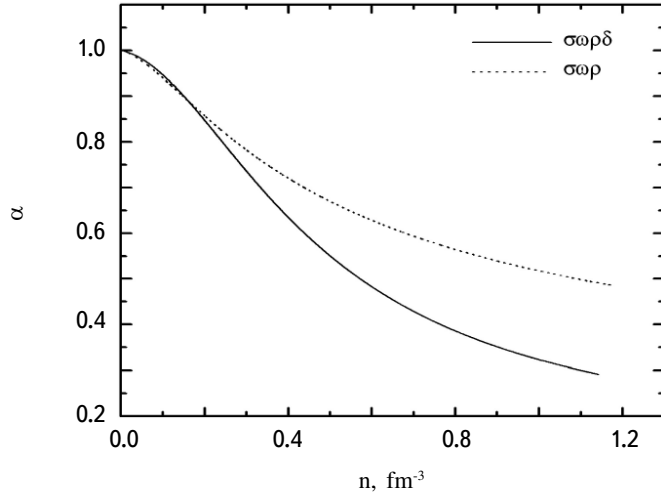


Fig. 2. The asymmetry parameter α as a function of the concentration n for a β -equilibrium, uncharged npe -plasma. The smooth curve corresponds to the " $\sigma\omega\rho\delta$ " model and the dashed curve, to " $\sigma\omega\rho$ ".

$\sigma(n, \alpha)$, $\omega(n)$, $\delta(n, \alpha)$ and $\rho(n, \alpha)$. Equations (18) can be used to determine the chemical potentials of the nucleons, $\mu_n(n, \alpha)$ and $\mu_p(n, \alpha)$, so that, using the β -equilibrium condition, it is possible to find the electron chemical potential,

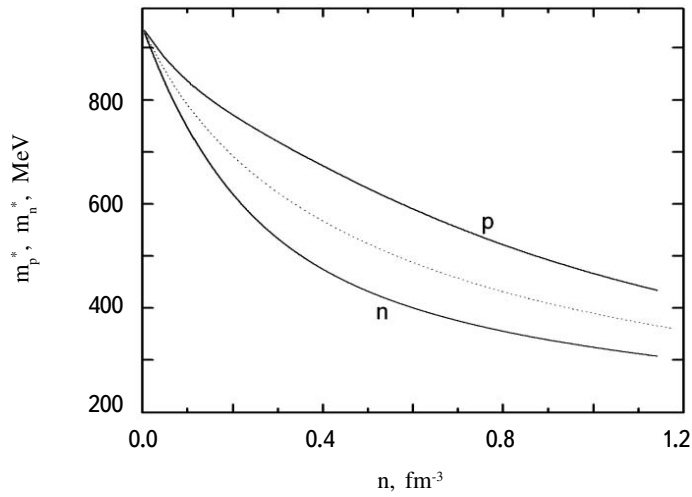


Fig. 3. The effective nucleon masses as functions of the baryon concentration n for a β -equilibrium, uncharged npe -plasma for the " $\sigma\omega\rho\delta$ " model.. The dashed curve corresponds to the " $\sigma\omega\rho$ " model.

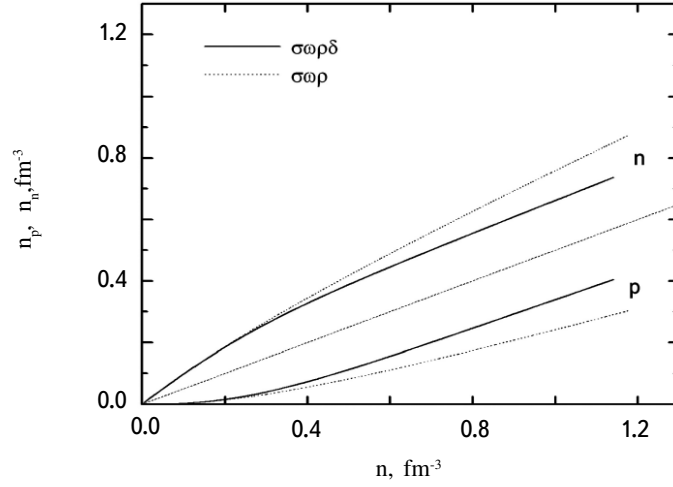


Fig. 4. The concentrations of protons and neutrons as functions of the baryon concentration n for a β -equilibrium, uncharged npe -plasma. The smooth curves correspond to the " $\sigma\omega\rho\delta$ " model and the dotted curves, to the " $\sigma\omega\rho$ " model. The dashed (straight) line corresponds to isospin symmetric matter.

$$\mu_e(n, \alpha) = \mu_n(n, \alpha) - \mu_p(n, \alpha) \quad (35)$$

and, ultimately, the energy density ε_{NM} and pressure P_{NM} of the β -equilibrium npe -plasma. Figure 1 is a three

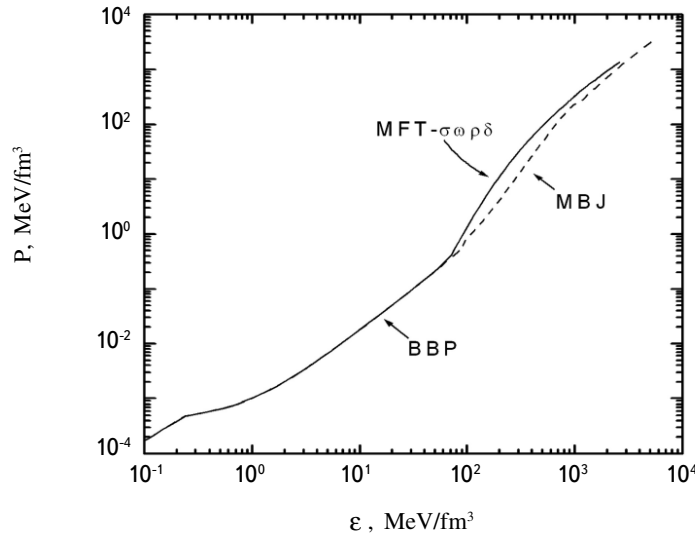


Fig. 5. The equation of state for neutron star matter in the nucleonic phase. The segment "MFG- $\sigma\omega\rho\delta$ " represents the results of this paper and "MBJ," those of Ref. 22. The region corresponding to nuclear-neutron (Aen) matter is described by the BBP equation of state [21].

dimensional plot of the energy per baryon, $E_b(n, \alpha) = \epsilon_{NM}/n$, as a function of the concentration n and asymmetry parameter α for the case of a β -equilibrium charged npe -plasma. The lines correspond to different fixed values of the charge per baryon, $q = (n_p - n_e)/n = (1 - \alpha)/2 - n_e/n$.

The thick line corresponds to β -equilibrium electrically neutral npe -matter. The lower surface corresponds to the " $\sigma\omega\rho$ " model and the upper, to the " $\sigma\omega\rho\delta$ " model. Clearly, including a δ -meson field increases the energy per nucleon, and the change is greater for larger values of the asymmetry parameter of the nuclear matter. For a fixed value of the specific charge, the asymmetry parameter falls off monotonically as the concentration is increased. Figure 2 is a plot of the asymmetry parameter as a function of the concentration n for the case of an electrically neutral npe -plasma for the " $\sigma\omega\rho$ " and " $\sigma\omega\rho\delta$ " models. It can be seen that including a δ -meson field for a fixed concentration n will reduce the asymmetry parameter α .

Figure 3 shows the effective masses of the protons and neutrons in a β -equilibrium uncharged npe -plasma as functions of the baryon concentration n for the " $\sigma\omega\rho\delta$ " model. Note that the effective masses of the protons and neutrons are the same in the " $\sigma\omega\rho$ " model. Including the δ -meson mean field breaks the symmetry, in this sense, between the protons and the neutrons; the effective mass of the protons in this kind of medium is greater than that of the neutrons, i.e., there is a split in the values of the effective masses for the protons and neutrons.

Figure 4 contains plots of the concentrations of the protons and neutrons as functions of the baryon concentration n for a β -equilibrium uncharged npe -plasma. The dashed (straight) line corresponds to the case of isospin-symmetric matter. It is clear from this figure that the presence of a δ -meson field reduces the neutron concentration and increases that of the protons.

Our calculated equation of state for electrically neutral β -equilibrium npe -matter (the material of a neutron star in its nucleonic phase) using the " $\sigma\omega\rho\delta$ " model is shown in Fig. 5. Our equation of state (the segment of the curve labelled "MFT- $\sigma\omega\rho\delta$ ") has been matched to the Baym-Bethe-Pethick (BBP) equation of state [21] in the region of normal nuclear densities. The Malone-Bethe-Johnson (MBJ) equation of state [22] is also shown for comparison.

5. The equation of state for a quark-electron (" $udse$ ") plasma

In order to describe the quark phase we have used an improved version of the MIT bag model [23], in which the interactions between the u , d , and s quarks inside the bag are accounted for in a one-gluon exchange approximation [24]. The quark phase consists of three quark flavors, u , d , and s , and electrons that are in equilibrium with respect to weak interactions via the reactions

$$d \rightarrow u + e^- + \tilde{\nu}_e, \quad u + e^- \rightarrow d + \nu_e, \quad s \rightarrow u + e^- + \tilde{\nu}_e, \quad u + e^- \rightarrow s + \nu_e.$$

Since the ν_e and $\tilde{\nu}_e$ particles leave the system, the energy of the system decreases and the reactions with neutrino emission continue until the condition $\mu_\nu = 0$ holds for the chemical potential of the neutrinos. Then, the following conditions hold for the chemical potentials of the u , d , s , and e particles:

$$\mu_d = \mu_s \equiv \mu, \quad \mu_u + \mu_e = \mu. \quad (36)$$

The following expression for the density of the thermodynamic potential Ω_f of the quark flavor f ($f=u, d, s$) has been obtained in a framework of quantum hadrodynamics (QHD) [24]:

$$\begin{aligned} \Omega_f(\mu_f) = & -\frac{1}{4\pi^2} \left\{ \mu_f \sqrt{\mu_f^2 - m_f^2} \left(\mu_f^2 - \frac{5}{2} m_f^2 \right) + \frac{3}{2} m_f^4 \ln \left(\frac{\mu_f + \sqrt{\mu_f^2 - m_f^2}}{m_f} \right) - \right. \\ & - 2 \frac{\alpha_s}{\pi} \left[3 \left(\mu_f \sqrt{\mu_f^2 - m_f^2} - m_f^2 \ln \frac{\mu_f + \sqrt{\mu_f^2 - m_f^2}}{\mu_f} \right)^2 - 2(\mu_f^2 - m_f^2)^2 - \right. \\ & \left. \left. - 3m_f^4 \ln^2 \left(\frac{m_f}{\mu_f} \right) + 6m_f^2 \ln \left(\frac{\tilde{\rho}}{\mu_f} \right) \left(\mu_f \sqrt{\mu_f^2 - m_f^2} - m_f^2 \ln \frac{\mu_f + \sqrt{\mu_f^2 - m_f^2}}{m_f} \right) \right] \right\}, \end{aligned} \quad (37)$$

where $\alpha_s = g^2/4\pi$, g is the QHD coupling constant, and $\tilde{\rho} \approx m/3 \approx 313$ MeV is the renormalization parameter. The quark concentrations are given by

$$n_f(\mu_f) = -\frac{\partial \Omega_f}{\partial \mu_f} = \frac{\mu_f^2 - m_f^2}{\pi^2} \left\{ \sqrt{\mu_f^2 - m_f^2} - 2 \frac{\alpha_s}{\pi} \left[\mu_f - \frac{3m_f^2}{\sqrt{\mu_f^2 - m_f^2}} \ln \frac{\mu_f + \sqrt{\mu_f^2 - m_f^2}}{\tilde{\rho}} \right] \right\}. \quad (38)$$

The thermodynamic potential Ω_e and concentration of the electrons are given by

$$\Omega_e(\mu_e) = -\frac{1}{\pi^2} \int_0^{\sqrt{\mu_e^2 - m_e^2}} (\mu_e - \sqrt{k^2 + m_e^2}) k^2 dk, \quad n_e(\mu_e) = \frac{(\mu_e^2 - m_e^2)^{3/2}}{3\pi^2}. \quad (39)$$

The condition of electrical neutrality for a "udse" plasma is

$$\frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0. \quad (40)$$

Using the functions $n_u(\mu, \mu_e)$, $n_d(\mu)$, $n_s(\mu)$, and $n_e(\mu_e)$ from Eqs. (38) and (40), this equation makes it possible to determine the function $\mu_e(\mu)$ and, ultimately, the functions $\Omega_u(\mu)$, $\Omega_d(\mu)$, $\Omega_s(\mu)$, and $\Omega_e(\mu)$.

The pressure of a "udse" plasma for a given chemical potential μ is given by

$$P_{QM}(\mu) = - \sum_{i=u,d,s,e} \Omega_i(\mu) - B, \quad (41)$$

where B is the "bag" constant, which characterizes the vacuum pressure and ensures confinement.

The energy density ε_{QM} and baryon concentration n_{QM} of a "udse" plasma are given by

$$\varepsilon_{QM}(\mu) = \sum_{i=u,d,s,e} (\Omega_i + \mu_i n_i) + B, \quad (42)$$

$$n_{QM}(\mu) = (n_u + n_d + n_s)/3. \quad (43)$$

Equations (41)-(43) give the equation of state of a quark-electron ("udse") plasma in the parametric form $\varepsilon_{QM}(P)$ and $n_{QM}(P)$. As in the case of an *npe*-plasma, the baryon chemical potential for quark-gluon matter is given by

$$\mu_{QM}(P) = (P_{QM} + \varepsilon_{QM}(P))/n_{QM}(P). \quad (44)$$

6. Phase transition to quark matter at constant pressure

The modern concept of the phase transition between nuclear matter and quark matter is based on a feature of this transition, first noted by Glendenning [20,17], to the effect that there are two conserved quantities in this transition: baryon number and electrical charge. The requirement of global electrical neutrality then leads to the possible formation of a mixed phase, where the nuclear and quark matter are, separately, electrically charged, while overall electrical neutrality is ensured by electrons (leptons). In the case of a phase transition of this sort, the energy density ε , baryon concentration n , and chemical potential μ_e of the electrons, as well as the pressure P , vary continuously.

The question of whether the formation of a mixed phase is energetically favorable, given the finite dimensions of the quark structures inside nuclear matter, the Coulomb interaction, and the surface energy, has been examined elsewhere [19,25-27]. It was shown there that the mixed phase is energetically favorable for small values of the surface tension between the quark matter and the nuclear matter.

In this paper we assume that the transformation of nuclear matter into quark matter is an ordinary first order phase transition described by Maxwell's rule. A separate paper will deal with the changes in the characteristics of the phase transition with formation of a mixed phase [20] when the contribution of a π -meson field is included, as well as the influence of these changes on the integral and structural parameters of hybrid stars. In the case of an ordinary first order phase transition, it is assumed that both nuclear and quark matter are separately electrically neutral and that at some pressure P_0 corresponding to the coexistence of the two phases, the baryon chemical potentials of the two phases are equal, i.e.,

$$\mu_{NM}(P_0) = \mu_{QM}(P_0). \quad (45)$$

Note that the chemical potential per baryon in nuclear matter is given by

$$\mu_{NM} = (\mu_p n_p + \mu_n n_n + \mu_e^{(NM)} n_e^{(NM)}) / n, \quad (46)$$

and in the case of neutral, β -equilibrium nuclear matter (because of the conditions $n_p - n_e^{(NM)} = 0$ and $\mu_p = \mu_n - \mu_e^{(NM)}$), coincides with the chemical potential μ_n for a neutron given by Eq. (29). In the case of a neutral, β -equilibrium quark-gluon plasma, the relationship between the baryon chemical potential and the chemical potentials of a d quark ($\mu_d = \mu$) and an electron ($\mu_e^{(QM)}$) has the form

$$\mu_{QM} = 3\mu - \mu_e^{(QM)}. \quad (47)$$

7. Numerical computations

Table 3 lists the calculated phase transition parameters for the " $\sigma\omega\rho\delta$ + MIT" model examined in this paper at constant pressure (Maxwell rule) for 12 different values of the "bag" parameter B . The quark masses are taken to be $m_u = 5$ MeV, $m_d = 7$ MeV, and $m_s = 150$ MeV, and the strong interaction constant, to be $\alpha_s = 0.5$. In this table μ_b is the baryon chemical potential at the phase transition point, n_N and n_Q are the baryon concentrations of the nuclear and quark matter, respectively, at the transition point, ε_N and ε_Q are the energy densities, $\mu_e^{(NM)}$ and $\mu_e^{(QM)}$ are

TABLE 3. Parameters of a Maxwellian Phase Transition for Different Values of the "Bag" Constant B

B	μ_b	n_N	n_Q	P_0	ε_N	ε_Q	$\mu_e^{(NM)}$	$\mu_e^{(QM)}$	λ
MeV/fm ³	MeV	fm ⁻³	fm ⁻³	MeV/fm ³	MeV/fm ³	MeV/fm ³	MeV	MeV	
60	965.9	0.1207	0.2831	2.11	114.5	271.4	99.14	9.205	2.327
65	999.7	0.1787	0.3161	7.218	171.4	308.8	138.0	8.350	1.728
69.3	1032	0.2241	0.3504	13.84	217.5	347.9	166.0	7.588	1.504
70	1038	0.2312	0.3564	15.10	224.9	354.9	170.2	7.464	1.479
75	1079	0.2810	0.4027	25.55	277.6	408.8	198.1	6.613	1.349
80	1119	0.3276	0.4525	37.95	328.8	468.6	221.9	5.842	1.278
85	1158	0.3704	0.5036	51.51	377.5	531.8	242.1	5.173	1.240
90	1194	0.4089	0.5541	65.54	422.8	596.2	259.0	4.605	1.221
95	1227	0.4435	0.6029	79.56	464.7	660.4	273.1	4.125	1.213
100	1257	0.4746	0.6497	93.30	503.3	723.5	285.2	3.717	1.213
110	1309	0.5281	0.7369	119.5	572.0	845.4	304.5	3.066	1.223
120	1354	0.5729	0.8165	143.9	631.7	961.4	319.5	2.568	1.240

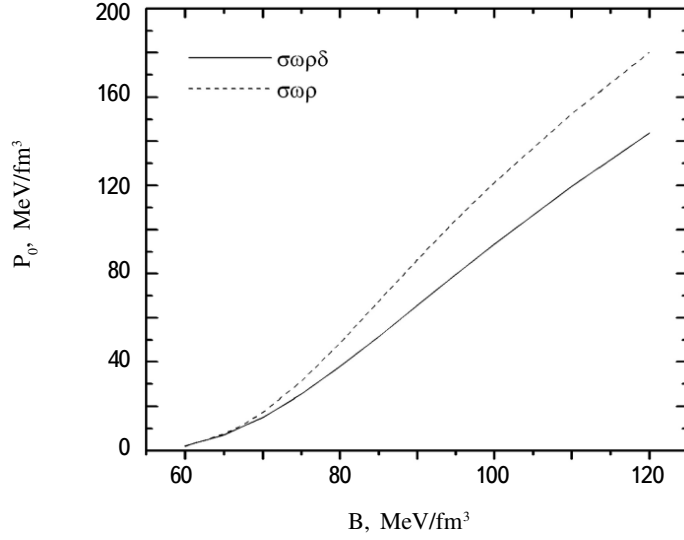


Fig. 6. The phase transition temperature P_0 as a function of the "bag" constant B . The smooth curve corresponds to the " $\sigma\omega\rho\delta$ " model and the dashed curve, to " $\sigma\omega\rho$ ".

the chemical potentials of an electron in nuclear and quark matter, respectively, and P_0 is the phase transition pressure.

It has been shown [28] that for a first order phase transition the density discontinuity parameter

$$\lambda = \varepsilon_Q / (\varepsilon_N + P_0) \quad (48)$$

plays a decisive in the stability of neutron stars with arbitrarily small cores made of matter from the second (denser)

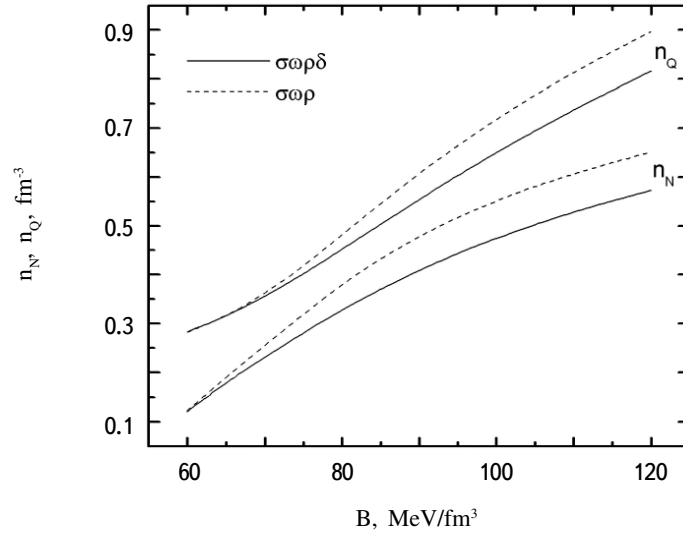


Fig. 7. Baryon concentrations of nuclear matter (n_N) and strange quark matter (n_Q) at the point of a maxwellian phase transition as functions of the "bag" constant B . Notation as in Fig. 6.

phase. Paraphrasing the conclusions of that paper [28], in the case of a hadron-quark first order phase transition, we have the following conditions. If $\lambda \leq 3/2$, then a neutron star with an arbitrarily small core of strange quark matter is stable. On the other hand, for $\lambda > 3/2$, neutron stars with small quark cores are unstable. In the latter case, there is a nonzero minimum value of the radius of the quark core for a stable star. Accretion of matter to a neutron star when $\lambda > 3/2$ will lead to a catastrophic (discontinuous) readjustment of the star, with formation of a star that has a quark core of finite size. This sort of catastrophic transition can also occur in the case of a rotating neutron star that is slowing down, when the pressure in the center rises and exceeds the threshold value, P_0 . The process of catastrophic readjustment with formation of a quark core of finite radius at the star's center will be accompanied by the release of a colossal amount of energy, comparable to the energy release during a supernova explosion. The last column of the table lists the discontinuity parameter λ for the various values of the "bag" constant B . The above mentioned catastrophic readjustment of a neutron star (during accretion of matter to its surface or as its rotation slows down) corresponds to the first three versions of the equation of state listed in Table 3, for which $B \leq 69.3 \text{ MeV/fm}^3$.

Figure 6 illustrates the dependence of the phase transition pressure P_0 on the value of the "bag" parameter B . It is clear that including a δ -interaction channel leads to a reduction in P_0 . Similar plots of the baryon concentrations of the nuclear (n_N) and quark (n_Q) phases at the phase transition point are shown in Fig. 7. Evidently, including a scalar-isovector effective δ -meson field reduces the baryon concentrations for both phases at the phase transition point. Then the density discontinuity parameter increases.

Figure 8 shows the equation of state for superdense matter with a maxwellian phase transition calculated in our " $\sigma\omega\rho\delta$ + MIT" model for five different values of the parameter B .

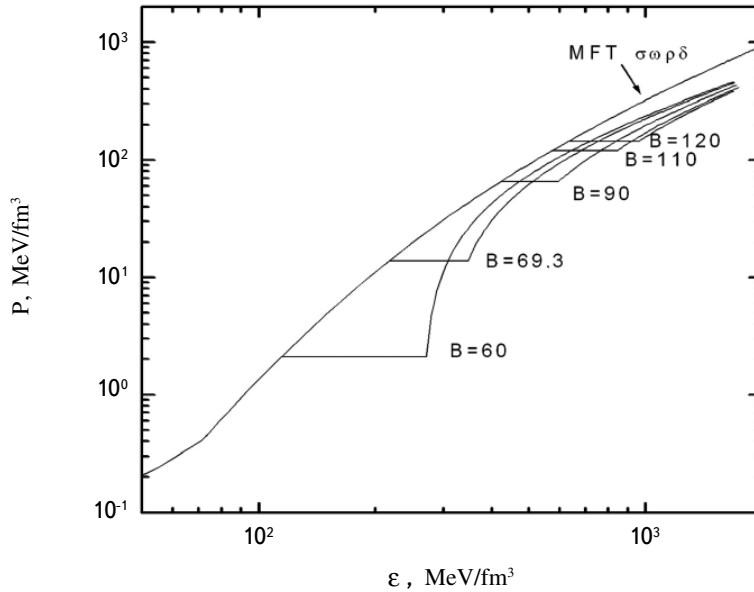


Fig. 8. The equations of state for superdense matter with a maxwellian phase transition calculated in the " $\sigma\omega\rho\delta$ " model for five different values of the parameter B .

8. Conclusion

In this paper we have studied the equation of state of superdense nuclear matter in terms of the relativistic mean-field theory, including a scalar-isovector δ -meson effective field. The values of the constants for the relativistic mean-field theory that we have found have enabled us to calculate the characteristics of asymmetric nuclear matter, as well as of β -equilibrium npe -plasmas. The dependences of the effective masses of protons and neutrons on the baryon concentration n for given values of the asymmetry parameter α have been studied and it has been shown that in an asymmetric nucleonic medium the effective mass of a proton exceeds that of a neutron.

The dependence of the asymmetry parameter α on the baryon concentration for β -equilibrium npe -plasmas has been studied for different values of the electrical charge per baryon and it was shown that including a δ -field reduces α .

Assuming that the phase transition between nuclear matter and strange quark matter is an ordinary first order phase transition obeying Maxwell's rule, we have made a detailed study of the effect of including a δ -meson field on the parameters of the phase transition. We have determined the phase transition parameters for 12 different values of the bag parameter within the range $B \in [60; 120] \text{ MeV/fm}^3$ and shown that including a δ -meson field leads to reduction in the phase transition pressure P_0 and in the concentrations n_N and n_Q for coexistence of the two phases. Here the density discontinuity parameter λ increases. A bag parameter $B \approx 69.3 \text{ MeV/fm}^3$ corresponds to the critical value $\lambda_{cr} = 3/2$. When $B < 69.3 \text{ MeV/fm}^3$ the density discontinuity parameter obeys $\lambda > \lambda_{cr}$ and neutron star configurations with infinitely small quark cores will be unstable.

This analysis shows that a scalar-isovector δ -meson field leads to more rigid equations of state for the nuclear matter owing to detachment of the effective proton and neutron masses, as well as to an increased asymmetry energy. It is known that a good source of information on the rigidity of an equation of state for dense matter is measurements of the mass of compact stars. The mass of the compact star in a binary system associated with the PSR pulsar B1516+02B was recently measured and found to be $M = 2.08 \pm 0.19 M_\odot$ [29]. The existence of neutron stars with such high masses argues for a more rigid equation of state than the equation which yields the standard value of $M = 1.44 M_\odot$.

Evidently, the above mentioned changes in the equation of state of superdense matter and in the phase transition parameters will lead to corresponding changes in both the structure and the integral characteristics of hybrid stars with strange quark cores. A separate article will be devoted to a study of the configuration of neutron stars of this type, calculated by integrating the system of Tolman-Oppenheimer-Volkoff equations based on the equations of state obtained in this paper, with and without the inclusion of a scalar-isovector effective δ -meson field.

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