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ORIGINAL ARTICLE

Effects of dust temperature and radiative heat-loss functions on the magnetogravitational instability of viscoelastic dusty plasma

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Abstract The effects of dust temperature and radiative heat-loss functions on the instability of selfgravitating homogeneous viscoelastic dusty plasma have been studied using the modified GH model. A general dispersion relation representing the dust temperature, radiative heat-loss functions, magnetic field, thermal conductivity and relaxation time parameters are obtained using the normal mode analysis method in the linearized perturbation equations system. Jeans criteria that represent the onset of instability of selfgravitating medium are obtained under the limits; when the medium behaves like a viscous liquid (strongly coupled limit) and a Newtonian liquid (weakly coupled limit) for some limiting cases. The effects of various parameters on the Jeans instability criteria and on the growth rate of selfgravitating viscoelastic dusty plasma have been discussed. It is found that the dust temperature and radiative heat-loss functions modify the instability criterion by reducing the critical wave number.

Keywords Gravitational instability · Radiative heat-loss functions · Dust temperature · Viscoelastic dusty plasma · Magnetic field · Strongly/weakly coupling limit

1 Introduction

A gravitational instability (GI) is a situation in which an object's self-gravity exceeds opposing forces such as; internal gas pressure or material rigidity and the object collapses. The problem of gravitational instability was first considered by Sir James Jeans (1902) so it is also called Jeans instability.

Dusty plasma is three components plasma with electrons, ions and a dispersed (low number density) phase of very massive charged grains of solid matter. The size of the heavy grains is typically in the range of 1 μm to 1 cm. Prajapati and Bhakta (2015a) reported that due to the presence of massive charge dust grains of the range $m_d \sim 10^{-5} g$ with $q_d \sim 10 - 100$ electronic charges, $a > 10$ μm in dusty plasma there is a possibility arises that either the gravitational and electromagnetic forces becomes equal or comparable and at this condition the ion densities distribution is given by the Boltzmann equation. This condition is considered to understand the consequences of the gravitational condensation of the dust grains in planetary system and interstellar medium and also to understand the features of electrostatic distributions in space plasma (cf. Mamun (1998)).

Dusty plasma is usually found in the interstellar clouds, interplanetary medium, cometary tails, planetary rings, in the vicinity of artificial satellites and space stations and laboratory (technological plasma applications, fusion devices) has increased making this area of research interesting and useful. The presence of strong coupling in dusty plasma was first predicted by Ikezi (1986). Sato et al. (1998, 2000, and 2001) and Konopka et al. (2000) carried out some experiments in strongly coupled dusty plasma, with the help of a simple theoretical model. Kaw et al. (2002) interpreted the experimental results of the above authors and observed that the theoretical model developed by them was similar to the models considered by Sato et al. (1998, 2000, 2001) and Konopka et al. (2000) to explain their experimental observations. They also reported that these studies are of importance in understanding the behavior of magnetized dusty plasma in

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various space, astrophysical and laboratory situations. Kaw and Sen (1998) reported that due to a large amount of charge on a single dust particle, the dust fluid can also exhibit strong coupling behavior, which shows the strong viscous properties of the medium and leading to viscoelastic behavior.

Rosenberg and Shukla (2011) reported that in the field of cosmic physics strongly coupled plasma (SCP) is of considerable interest because of its possible applications to various objects, such as white dwarf matter, the interior of heavy planets, the atmosphere of a neutron star and ultra-cold neutral plasma. These are assumed to be composed of viscoelastic fluid, which is strongly coupled and shows both viscous and elastic behavior.

In dusty plasma, strong coupling between particles provides a long-range correlation that causes the origin of elastic property. This strong coupling enables the system to support the transverse shear wave (cf. Banerjee (2013)). This shear wave in dusty plasma was first introduced by Kaw and Sen (1998) by using the generalized hydrodynamic (GH) model to study the dynamics of strongly coupled plasma (SCP) and later experimentally verified. The strength of the Coulomb coupling is characterized by the coupling parameter $\Gamma = q_d^2/k_B T_d a$, where q_d is the charge on the dust grains, ($\simeq n_d^{-1/3}$), a is the average distance between them for density n_d, T_d is the temperature of the dust component and k_B is the Boltzmann constant (cf. Ikezi (1986)). In the regime $1 \le \Gamma \le \Gamma_c$ (a critical value beyond which system becomes crystalline) both viscosity and elasticity are equally important and therefore such plasma exhibits visco-elastic behavior. When $\Gamma > \Gamma_c$, viscosity disappears and only elasticity reigns over the system. The experiments of Chu and Lin (1994), Thomas et al. (1994) and Hayashi and Tachibana (1994) have shown that in such dusty plasmas, the dust particles organize themselves into crystalline patterns.

As the temperature of the dust is increased and Γ decreased the dust crystals melts and then vaporizes, the media exhibits purely viscous effect so that one recovers the usual weakly coupled ideal Coulomb plasma, but as the dust temperature decreased, the coupling parameter increases, Coulomb interaction between neighboring particles becomes comparable to kinetic energy and hence the fluid also shows elastic property and the charged dust grain strongly interacts and known as strongly coupled dusty plasma (SCDP). Thus, strongly coupled plasmas cannot be classified as purely elastic or purely viscous. Therefore the experiments with such dusty plasmas give an excellent opportunity for the study of transitions from the strongly coupled to weakly coupled regimes. The values of typical parameters in the complex dusty plasma is taken as considered by Shukla and Sandberg (2003) and Mamun and Shukla (2001) and therein are $T_{e0} = 30$ K, $T_{i0} = 10$ K, $n_e = 103$ cm^{-3}, $n_{i0} = 2 \times 10^{-3}$ cm^{-3}, $T_d = 1$ K, $n_d = 5 \times 10^{-7}$ cm^{-3}, $z_d = 1000$ and $m_d = 10^{-11}$ g.

In GH model, the generalized hydrodynamical equations of momentum for viscoelastic medium incorporates the Frenkel's term or viscoelastic operator $\left(1 + \tau \frac{\partial}{\partial t}\right)$ (cf. Janaki et al. (2011)) which accounts for the relaxation effects arising out of growing correlations among the particles with an increase in the values of Γ_j. Kaw and Sen (1998) further suggested that the viscoelastic properties of the medium are characterized by the relaxation time τ which provides a characteristic timescale to distinguish two classes of low frequency modes; one when the frequency $\sigma \ll 1/\tau$ (known as hydrodynamic limit) and the other for frequencies $\sigma \gg 1/\tau$ (known as kinetic limit). Where, σ is the wave frequency and τ is the viscoelastic relaxation time.

Pandey et al. (1994) have discussed the Jeans instability of dusty plasma. Kaw and Sen (1998) have discussed the low-frequency wave modes in strongly coupled dusty plasma (SCDP) and found that for the well known dust acoustic mode strong correlations provide new corrections in dispersion property. Mamun (1998) studied the effects of dust temperature and fast ions on gravitational instability in selfgravitating magnetized dusty plasma and found that the growth rate decreases with dust temperature, fast nonthermal ions, and external magnetic field. Also, Avinash and Shukla (1994), Shukla and Verheest (1999), Mahanta et al. (1996) and Mace et al. (1998) studied the influence of gravitational force on massive dust particles and investigated the dust induced wave modes and Jeans instability in gravitating dusty plasma.

Shukla and Stenflo (2006) have studied the Jeans instability of selfgravitating dusty plasma with attraction force between charged dust grains. Janaki and Chakrabarti (2010) investigated the shear wave vortex solution in strongly coupled dusty plasma and found that, under the kinetic limit ($\sigma\tau \gg 1$), SCP support localized dipolar vortex-like solutions with amplitude-modulated periodically. Sharma (2014) studied the modified Jeans instability of strongly coupled inhomogeneous magneto dusty plasma in the presence of polarization force and it is observed that the decay in the growth rate is faster in the strong coupling limit in comparison to the weakly coupling limit. Prajapati and Chhajlani (2014) studied the effect of quantum corrections on the Jeans instability of selfgravitating viscoelastic dusty fluid and observed that the dispersion properties are affected due to the presence of viscoelastic effects and quantum statistical corrections. Prajapati and Bhakta (2015a) reported that many authors have considered the effect of gravitational force on the massive dust grains to investigate the modified dynamics, low-frequency waves and instabilities in dusty plasma.

In the field of astrophysics and plasma physics radiative effect plays a vital role. It is reported from the study of interstellar medium (ISM) structure that in the large astrophysical compact objects such as star formation and molecular cloud condensation process with thermal instability is strongly influenced by the radiative heat-loss mechanism (cf. Prajapati and Chhajlani (2010)). In the radiative heat-loss mechanism of a system, the decay of heat takes place with respect to local temperature and density. Thus many authors have considered radiative heat-loss function arises due to the heat-loss mechanism in various kinds of systems. Prajapati and Chhajlani (2010) supposed that in the astrophysical compact objects such as star formation, solar prominences, nebulae, and molecular dusty clouds the excitation of radiative condensation instability takes place. Dwivedi et al. (1996) have studied the effect of radiative condensation on the Jeans instability of dusty plasma and found that it enhances the dust acoustic stabilization of the Jeans instability. Pandey and Krishan (2001) have investigated the effect of radiative cooling of electrons on the gravitational collapse of the molecular cloud consisting of neutral particles and charged dust grains with dust charge fluctuations. Menou et al. (2004) have shown the importance of radiative effects in the Sun's upper radiative zone. Tsintsadze et al. (2008) studied the Jeans instability of magnetized dusty plasma considering the effect of radiation pressure. Prajapati and Chhajlani (2010) studied the effect of dust temperature on radiative condensation instability of selfgravitating magnetized dusty plasma and determined the *Jeans criterion* of instability, which depends on dust temperature and radiative cooling effects, whereas the presence of magnetic field has no effect on the Jeans instability criterion.

Prajapati and Bhakta (2015a) studied the radiative-condensation instability in gravitating strongly coupled dusty plasma with polarization force and concluded the dust thermal velocity and viscoelastic effects have to stabilize whereas polarization force and radiative cooling have a destabilizing influence on the growth rate of the Jeans instability. Prajapati and Bhakta (2015b) studied the influence of dust charge fluctuation and polarization force on radiative condensation instability of magnetized gravitating dusty plasma and observed that the dust charge fluctuation, radiative cooling and polarization force have destabilizing while dust thermal speed and dust cyclotron frequency have stabilizing influence on the growth rate of Jeans instability. Bhakta et al. (2019) studied the effects of radiation pressure and polarization force on Jeans instability in magnetized strongly coupled dusty plasma and the different velocities, Jeans wavenumber and Jeans mass have been calculated and the astrophysical consequences of the results are discussed. Dolai and Prajapati (2020) studied the effects of dust charge gradient and polarization forces on DAW and Jeans instability in the strongly coupled dusty plasma.

Thus, motivated by the above mentioned studies and the importance of dusty plasma in understanding the gravitational collapse of the protostar, in this analysis, we have considered the problem of magnetogravitational instability of strongly coupled dusty plasma to investigate the effects of dust temperature and radiative heat-loss functions on the gravitational instability criterion and on the growth rate of instability under both the strongly and weakly coupling limits. The considered fluid model is useful to describe the propagation of dust acoustic wave (DAW) mode and dust ion acoustic wave (DIAW) mode at the low frequency waves (Prajapati and Bhakta (2015b)). Further, the strong coupling plasma and radiative condensation effects plays a significant role in the gravitational collapsing process in the outer layer of the neutron star. The present work may be useful in understanding the core collapse of magnetized neutron star in presence of gravitational and radiative effects.

2 The mathematical formulation of the problem

In the present analysis, we have considered the selfgravitating strongly coupled dusty plasma consisting of electrons, ions and charged dust particles with a uniform magnetic field $H_0(z)$. Following the analysis of Prajapati and Chhajlani (2010), we have considered the negatively charged dust grains having mass m_d, radius a and charge q_d. The self gravitational effect of electrons and ions is not taken into account and are considered as inertialess. Further, it is considered that the ions are in the thermal equilibrium, whereas the electrons have finite thermal conductivity because moving fast ions have infinitely large thermal conductivity. Here, we have used the modified Generalized Hydrodynamic (GH) model which is modified due to the presence of dust temperature (cf. Prajapati and Bhakta (2015a)).

The dynamics of the dusty plasma consisting of electrons, ions and charged dust particles are described by the following set of equations (cf. Prajapati and Chhajlani (2010));

For ions Boltzmann distributions describe the dust dynamics and ion densities on the extremely slow time scale;

$$n_i = n_{i0} exp\left(\frac{-e\phi}{T_i}\right) \tag{1}$$

For electrons The electron dynamics are given by the following set of equations:

$$0 = en_e\nabla\phi - \nabla p_e \tag{2}$$

$$\frac{3}{2}n_e\frac{\partial T_e}{\partial t} + p_e\nabla.\vec{u_e} = \lambda\nabla^2 T_e - L(n_e, T_e) \tag{3}$$

$$\frac{\partial n_e}{\partial t} + \nabla.\left(n_e\vec{u_e}\right) = 0 \tag{4}$$

where, u_{e}, $\mathrm{T_e}$, $p_{\mathrm{e}}\,(=\mathrm{T_e}n_e)$, n_i and n_e are the electron velocity, electron temperature, electron pressure, total number density of ions and electrons, respectively. L and λ denote radiative heat-loss functions in terms of heating H and cooling C such that $L = H - C$ and thermal conductivity, respectively.

For the dust grains The dust grains dynamics are governed by the following equations;

The dust equation of motion gets modified due to dust temperature and is given by;

$$\left(1+\tau\frac{\partial}{\partial t}\right) \times \left[\frac{\partial \vec{u_d}}{\partial t} + \frac{q_d}{m_d}\nabla\phi - \frac{q_d}{m_d c}\left(\vec{u_d}\times\vec{H_0}\right) + \nabla\varphi + \frac{5\sigma_d}{3n_d^{1/3}}\nabla n_d\right] = \frac{\mu}{m_d n_d}\nabla^2\vec{u_d} + \frac{1}{m_d n_d}\left(\xi+\frac{\mu}{3}\right)\nabla\left(\nabla.\vec{u_d}\right) \tag{5}$$

Equation of mass transfer

$$\frac{\partial n_d}{\partial t} + \nabla.\left(n_d\vec{u_d}\right) = 0 \tag{6}$$

Poisson's equations for electrostatic and gravitational forces

$$\nabla^2\varphi = -4\pi\left[e\left(n_i - n_e\right) + q_d n_d\right] \tag{7}$$

and

$$\nabla^2\phi = 4\pi G m_d n_d \tag{8}$$

In the above equations; μ, τ and G respectively denote the coefficient of viscosity, viscoelastic relaxation time and the universal gravitational constant; $\vec{H_0}$, n_d, $\vec{u_d}$ is the magnetic field, dust particle number density, dust particle velocity, $\sigma_d = \frac{T_d}{q_d T_i}$, T_d is the dust temperature, T_i the ion temperature and φ and ϕ are electrostatic and gravitational forces, respectively. Also $\xi(=\lambda+\frac{2}{3}\mu)$ is a coefficient of bulk viscosity (cf. Batchelor (2000)).

Since, initially, when there is no motion, the system is in rest and

$$\begin{gathered}\vec{u_d} = \vec{u_e} = \vec{u_i} = 0, \varphi_0 = 0, \phi_0 = 0\vec{H_0} = (0,\ 0,\ H_z),\\ \phi = \phi_0, T_i = T_{i0}, n_d = n_{d0},\ n_e = n_{e0},\ n_i = n_{i0},\\ T_e = T_{e0}, L = L_0\end{gathered} \tag{9}$$

The basic equations (1)–(8) are identically satisfied. Here, we have neglected the zeroth-order gravitational field in this analysis $\phi_0 = 0$ (cf. Prajapati and Chhajlani (2010)) and thus avoiding the Jeans swindle.

3 Gravitational instability

In order to investigate the instability of the self-gravitating system governed by equations (1)–(8), let the initial stationary state be slightly perturbed by adding infinitesimal small perturbations δn_d, δn_i, δn_e, V_d, V_e, δT_e, $\delta\varphi$, $\delta\phi$, δL and $\vec{h}$ in the dust particle number density n_d, the total number density of ions n_i, electrons n_e, the dust particle velocity u_d, electrons velocity u_e, electrons temperature T_e, electrostatic φ, gravitational forces ϕ, radiative heat-loss functions L and magnetic field $\vec{H_0}$, respectively.

Thus, the equilibrium state (9) under perturbations is represented as;

$$\left.\begin{aligned} n_d &= n_{d0} + \delta n_d\\ n_i &= n_{i0} + \delta n_i\\ n_e &= n_{e0} + \delta n_e\\ \vec{u_d} &= \vec{u_d} + \vec{V_d}\\ \vec{u_e} &= \vec{u_e} + \vec{V_e}\\ T_e &= T_{e0} + \delta T_e\\ \varphi &= \varphi_0 + \delta\varphi\\ \phi &= \phi_0 + \delta\phi\\ L &= L_0 + \delta L\\ \vec{H} &= \vec{H_0} + \vec{h}\end{aligned}\right\} \tag{10}$$

Using the perturbed quantities (described in (10)) in the equations (1)–(8) and initial stationary state solutions and by neglecting the product of the perturbed quantities and second and higher-order perturbed quantities terms. We get the following linearized perturbed equations;

$$\delta n_i = -n_{i0}\left(\frac{e\delta\phi}{T_i}\right) \tag{11}$$

$$\delta n_e = n_{e0}\left(\frac{e\delta\phi}{T_{e0}} - \frac{\delta T_e}{T_{e0}}\right) \tag{12}$$

$$\frac{-3}{2}n_{e0}\sigma\delta T_e = \lambda\nabla^2\delta T_e - \left(\frac{\partial L_0}{\partial n_{e0}}\right)\delta n_e - \left(\frac{\partial L_0}{\partial T_{e0}}\right)\delta T_e \tag{13}$$

$$(1+\tau\sigma)\left[\sigma\vec{V_d} + \nabla\frac{q_d}{m_d}\delta\phi - \vec{V_d}\times\Omega_d + \nabla\delta\varphi + \nabla\frac{5\sigma_d}{3}\delta n_d\right] = -\frac{\mu}{m_d n_{d0}}\nabla^2\vec{V_d} - \frac{1}{m_d n_{d0}}\nabla^2\vec{V_d}\left(\xi+\frac{\mu}{3}\right) \tag{14}$$

$$\frac{\delta n_d}{n_{d0}}\sigma = \nabla\vec{V_d} \tag{15}$$

$$\nabla^2\delta\phi = -4\pi\left[e\left(\delta n_i - \delta n_e\right) + q_d\delta n_d\right] \tag{16}$$

$$-\nabla^2\delta\varphi = \frac{\omega_{jd}^2}{n_{d0}}\delta n_d \tag{17}$$

where, $\Omega_d = \frac{q_d H_0}{m_d c}$ is the dust cyclotron frequency, $\omega_{jd}^2 = 4\pi G m_d n_{d0}$ is the square of the Jeans dust frequency.

In order to solve equations (11)–(17), which are linear and homogeneous, let us assume the solution of the perturbed quantities of the form;

$$e^{ik+\sigma t} \tag{18}$$

where σ is the wave frequency and k is the wavenumber.

For this dependence of perturbed quantities, we can write

$$\frac{\partial}{\partial t} \equiv \sigma, \nabla \equiv ik \tag{19}$$

Using (19) in equations (11)–(17), we obtain the following equations;

$$\delta n_i = -n_{i0}\left(\frac{e\delta\phi}{T_i}\right) \tag{20}$$

$$\delta n_e = n_{e0}\left(\frac{e\delta\phi}{\mathrm{T_{e0}}} - \frac{\delta T_e}{\mathrm{T_{e0}}}\right) \tag{21}$$

$$\frac{-3}{2}n_{e0}\sigma\delta T_e = \lambda k^2\delta T_e - \left(\frac{\partial L_0}{\partial n_{e0}}\right)\delta n_e - \left(\frac{\partial L_0}{\partial T_{e0}}\right)\delta T_e \tag{22}$$

$$(1+\tau\sigma)\left[\sigma\overrightarrow{V_d} + ik\frac{q_d}{m_d}\delta\phi - \overrightarrow{V_d}\times\Omega_d + ik\delta\varphi + ik\frac{5\sigma_d}{3}\delta n_d\right]$$
$$= -\frac{\mu}{m_d n_{d0}}k^2\overrightarrow{V_d} - \frac{1}{m_d n_{d0}}k^2\overrightarrow{V_d}\left(\xi + \frac{\mu}{3}\right) \tag{23}$$

$$\frac{\delta n_d}{n_{d0}}\sigma = -ik\overrightarrow{V_d} \tag{24}$$

$$k^2\delta\phi = -4\pi\left[e\left(\delta n_i - \delta n_e\right) + q_d\delta n_d\right] \tag{25}$$

$$-k^2\delta\varphi = \frac{\omega_{jd}^2}{n_{d0}}\delta n_d \tag{26}$$

From equations (21) and (22), we obtain the expression for perturbed electron density as

$$\delta n_e = n_{e0}\left(\frac{e\delta\phi}{T_{e0}}\right)\left(1 - \frac{iL_n}{\Lambda}\right)^{-1} \tag{27}$$

Taking the divergence of equation (23) and using equations (24) and (26) in the resulting equation, we obtain the following expression for perturbed dust density;

$$\delta n_d = \frac{(1+\tau\sigma)k^2 q_d n_{d0}\delta\phi}{m_d\left[(1+\tau\sigma)\left\{-\sigma^2 + \sigma\Omega_d + \omega_{jd}^2 - \frac{5\sigma_d}{3}k^2 n_{d0}\right\} + \frac{k^2\sigma}{m_d n_d}\left(\xi + \frac{4\mu}{3}\right)\right]} \tag{28}$$

Substituting the values of δn_i, δn_e and δn_d from equations (20), (27) and (28), respectively in Poisson equation (25), we obtain the following dispersion relation:

$$\left[(1+\tau\sigma)\left\{-\sigma^2 + \sigma\Omega_d + \omega_{jd}^2 - \frac{5\sigma_d}{3}k^2 n_{d0}\right\}\right.$$
$$\left. + \frac{k^2\sigma}{m_d n_{d0}}\left(\xi + \frac{4\mu}{3}\right)\right]$$
$$\times\left[1 + \frac{iL_{n_{e0}}}{\left(1 + k^2\lambda_{De}^2 + \frac{n_{i0}T_{e0}}{n_{e0}T_i}\right)}\left(\Lambda - iL_{n_{e0}}\right)^{-1}\right]$$
$$-(1+\tau\sigma)k^2 c_{sd}^2 = 0 \tag{29}$$

where,

$$L_{n_{e0}} = \left(\frac{1}{T_{e0}}\right)\frac{\partial L_0}{\partial n_{e0}}$$

dust density – dependent heat loss function

$$\mathrm{L_{T_{e0}}} = -\frac{1}{n_{e0}}\left(\frac{\partial L_0}{\partial T_{e0}}\right)$$

dust temperature – dependent heat loss function

$\lambda_{De}^2 = \dfrac{T_{e0}}{4\pi n_{e0}e^2}$ is the Debye length of the electron

$\omega_{pd}^2 = \dfrac{4\pi n_{d0}q_d^2}{m_d}$ is dust plasma frequency

$$c_{sd} = \left(\frac{\lambda_{De}^2\omega_{pd}^2}{1 + \lambda_{De}^2 k^2 + T_{e0}n_{i0}/n_{e0}T_i}\right)^{1/2}$$

is the dust acoustic speed

$$\Lambda = \left[\left(\frac{3}{2}\right)\omega + i\lambda k^2 - iL_T\right].$$

Equation (29) is the required dispersion relation including the effects of various parameters such as radiative heat-loss functions, dust temperature magnetic field and thermal conductivity. Further, it is found that if the effects of viscoelastic medium are ignored i.e. if the effects of shear and bulk viscosities and viscoelastic operator are ignored and medium is considered as dusty plasma, then the dispersion relation (29) above becomes identical to the relation (22) as given by Prajapati and Chhajlani (2010). Also if we neglect the effect of dust temperature, we get the similar dispersion relation of Prajapati and Bhakta (2015a) with zero polarization force.

4 Jeans criterion of instability

We have used the above dispersion relation (29) to derive the instability criterion for the onset of gravitational collapse for the different cases under both strongly and weakly coupling limits.

Now, the effects of dust temperature, magnetic field and radiative cooling function ($\Omega_d = T_d \neq 0, L_{T_{e0}}, L_{n_{e0}} \neq 0$) on the gravitational instability of a selfgravitating viscoelastic medium has been studied.

Strongly coupling limit ($\sigma\tau \gg 1$) Under the strongly coupling limit and using the value of limiting case the dispersion relation (29) reduces to the following form;

$$\sigma^2 - \sigma\Omega_d - \omega_{jd}^2 + \frac{5\sigma_d}{3}k^2 n_{d0} + k^2 v_c^2 + \frac{k^2 c_{sd}^2}{(1-\delta')} = 0 \quad (30)$$

Here, $\delta' = \frac{L_{n_{e0}}}{L_{T_{e0}} + L_{n_{e0}} - \lambda k^2}$ radiative cooling function.

The equation (30) becomes identical to (29) of Prajapati and Bhakta (2015a) if the effects of polarization are ignored. Also if the effects of radiative cooling function, dust temperature are ignored then we get an equation similar (20) Prajapati and Chhajlani (2013), (11) of Janaki et al. (2011).

The constant term of the above equation yields the following instability criterion

$$k^2 < \omega_{jd}^2 \Big/ \left(\frac{5\sigma_d}{3}n_{d0} + v_c^2 + \frac{c_{sd}^2}{(1-\delta')}\right) \quad (31)$$

The *Jeans criterion* of gravitational instability gets modified due to the square of the velocity of viscoelastic mode, radiative cooling function and dust temperature couples with the dust acoustic speed. The obtained criterion similar to Prajapati and Bhakta (2015a) with zero polarization force and dust thermal velocity. Further, if viscoelastic effects and radiative cooling function are ignored then criterion becomes identical to (29) of Prajapati and Chhajlani (2010).

Weakly coupling limit ($\sigma\tau \ll 1$) Under the weakly coupling limit equation (29) reduces to the following dispersion relation

$$\sigma^2 - \sigma\Omega_d - \omega_{jd}^2 + \frac{5\sigma_d}{3}k^2 n_{d0} + \frac{k^2\sigma}{m_d n_d}\left(\xi + \frac{4\mu}{3}\right) + \frac{k^2 c_{sd}^2}{(1-\delta')} = 0 \quad (32)$$

The constant term of the above equation yields the following instability criterion

$$k^2 < \omega_{jd}^2 \Big/ \left(\frac{5\sigma_d}{3}n_{d0} + \frac{c_{sd}^2}{(1-\delta')}\right) \quad (33)$$

Also under the weakly coupling limit, the gravitational instability criterion gets modified due to the presence of radiative cooling function and dust temperature.

Special cases

Case I: $\Omega_d = T_d = L_{Te0}, L_{n_{e0}} = 0$
Here, we have considered the non-radiating, unmagnetized plasma having extremely low dust temperature and deduced the instability criterion under both strongly and weakly coupling limits.

Using these limiting values in the inequality (31), we get the following instability criterion;

$$k^2 < \omega_{jd}^2 \Big/ \left(v_c^2 + c_{sd}^2\right) \quad (34)$$

This is the modified *Jeans criterion* due to the viscoelastic behavior of the medium. Here the square of the velocity of viscoelastic mode couples with the dust acoustic speed and hence modifies the criterion.

Also, using the limiting values in inequality (33), we have the following instability criterion;

$$k^2 < \omega_{jd}^2 / c_{sd}^2 \quad (35)$$

This is the *Jeans criterion* given by Chandrasekhar (1961) for the gaseous medium. Thus, we can conclude that when the system is non-radiating, selfgravitating unmagnetized strongly coupled dusty plasma having extremely low dust temperature, the instability criteria obtained in (34) and (35) are similar as obtained by Dhiman and Sharma (2014) in the transverse mode of wave propagation under both the strongly and weakly coupling limit of the viscoelastic medium.

Case II: $\Omega_d = T_d \neq 0, L_T, L_{n_{e0}} = 0$
Here, we have considered the non-radiating dusty magnetized plasma having finite dust temperature and deduced the instability criterion under both strongly and weakly coupling limits.

Using these limiting values in the inequality (31), we get the following instability criterion;

$$k^2 < \omega_{jd}^2 \Big/ \left(\frac{5\sigma_d}{3}n_{d0} + v_c^2 + c_{sd}^2\right) \quad (36)$$

This is the modified criterion of instability and similar to the criterion as obtained in inequality (31) if the effects of radiative cooling function are ignored. If viscoelastic effects are ignored then criterion is similar to the (29) of Prajapati and Chhajlani (2010) and (27) of Prajapati and Bhakta (2015b) with zero polarization effect. Further, if the effects of dusty medium and viscoelastic effects are ignored then the criterion is identical to the Chandrasekhar (1961).

Also, using the limiting values in inequality (33), we have the following instability criterion;

$$k^2 < \omega_{jd}^2 / \left(\frac{5\sigma_d}{3} n_{d0} + c_{sd}^2 \right) \tag{37}$$

This is also the modified criterion of instability and similar to the criterion as obtained in equation (33) if the effects of radiative cooling function are ignored.

5 Growth rate of instability

Now, we shall study the effect of various physical parameters such as dust particle number density, dust cyclotron frequency, dust temperature, radiative cooling function on the growth rate of gravitational instability of a viscoelastic medium under the strongly and weakly coupling limits.

The growth rate of instability has been calculated under the strongly and weakly coupling limits by writing equations (30) and (32) in the following dimensionless form;

$$\gamma^2 - \gamma \Omega_d^* + \xi^* + \frac{c_{sd}^{*2}}{(1-\delta')} + \frac{5}{3}\sigma_d^* n_{d0}^* - 1 = 0 \tag{38}$$

$$\gamma^2 - \gamma \left(\Omega_d^* + c_{sd}^{*2} \varsigma \right) + \frac{c_{sd}^{*2}}{(1-\delta')} + \frac{5}{3}\sigma_d^* n_{d0}^* - 1 = 0 \tag{39}$$

The dimensionless parameters used here are

$$\Omega_d^* = \frac{\Omega_d}{\omega_{jd}}, \sigma_d^* = \frac{\sigma_d k}{\omega_{jd}}, \gamma = \frac{\sigma}{\omega_{jd}}, \xi^* = \frac{\nu_c k}{\omega_{jd}}, n_{d0}^* = \frac{n_{d0} k}{\omega_{jd}},$$
$$c_{sd}^* = \frac{c_{sd} k}{\omega_{jd}}$$

The effect of dust temperature on the growth rate of gravitational instability of dusty magnetized plasma in the presence of radiative cooling function has been calculated from the non-dimensional equations (38) and (39) under both the strongly and weakly coupling limits. The normalized growth rate (real positive γ) has been calculated under both the strongly and weakly coupling limits for different values of the dust temperature $\left(\sigma_d^* = 0.25, 0.75\right)$ and obtained values depicted graphically in Figure 1. Figure 1 represents the variation in the normalized growth rate (real positive γ) of Jeans instability against normalized dust acoustic speed with different values of the dust temperature $\left(\sigma_d^* = 0.25, 0.75\right)$ and fixed values of Ω_d^*, $n_{d0}^* = 0.5$, ξ^*, $\delta' = 0.25$, $\varsigma = 0.30$ under the strongly and weakly coupling limits. The solid and dotted lines represent the growth rate under strongly and weakly coupling limits, respectively, in all the Figures.

The effect of radiative cooling function on the growth rate of gravitational instability of dusty magnetized plasma has been calculated from the non-dimensional equations (38) and (39). The growth rate has been calculated under both the strongly and weakly coupling limits with different values of radiative cooling function $\left(\delta' = 0.0,\ 0.50\right)$ and the obtained values depicted graphically in Figure 2.

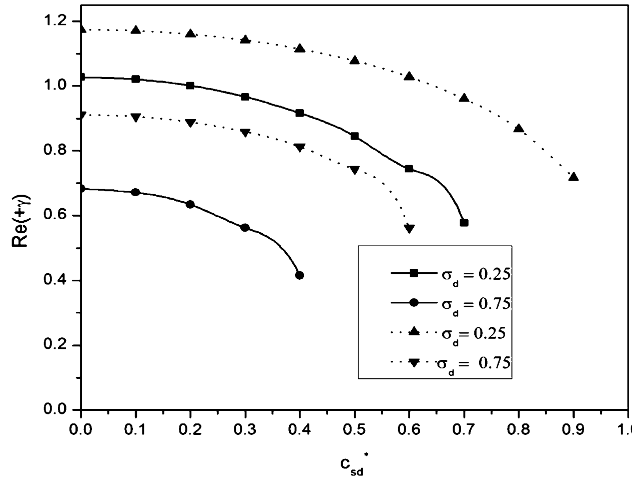

Fig. 1 Variation in the normalized growth rate of Jeans instability against normalized dust acoustic speed with different values of the dust temperature $\left(\sigma_d^* = 0.25,\ 0.75\right)$ and fixed values of Ω_d^*, $n_{d0}^* = 0.5$, ξ^*, $\delta' = 0.25$, $\varsigma = 0.30$ under the strongly and weakly coupling limits

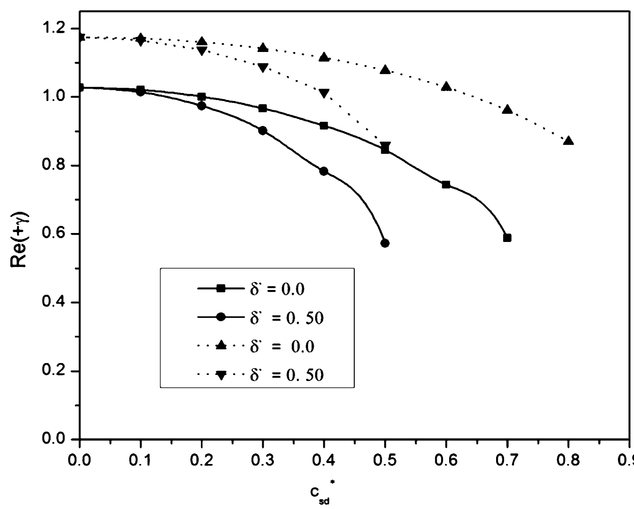

Fig. 2 Variation in the normalized growth rate of Jeans instability against normalized dust acoustic speed with different values of the radiative cooling function $\left(\delta' = 0.0,\ 0.50\right)$ and fixed values of Ω_d^*, $n_{d0}^* = 0.5$, ξ^*, $\sigma_d^* = 0.25\varsigma = 0.30$ under the strongly and weakly coupling limits

Figure 2 represents the variation in normalized growth rate (real positive γ) of Jeans instability against normalized dust acoustic speed with different values of the radiative cooling function $\left(\delta' = 0.0,\ 0.50\right)$ and fixed values of Ω_d^*, $n_{d0}^* = 0.5$, ξ^*, $\sigma_d^* = 0.25$, $\varsigma = 0.30$ under the strongly and weakly coupling limits.

Also, the effect of the magnetic field on the growth rate of gravitational instability of dusty magnetized plasma in the presence of radiative cooling function has been calcu-

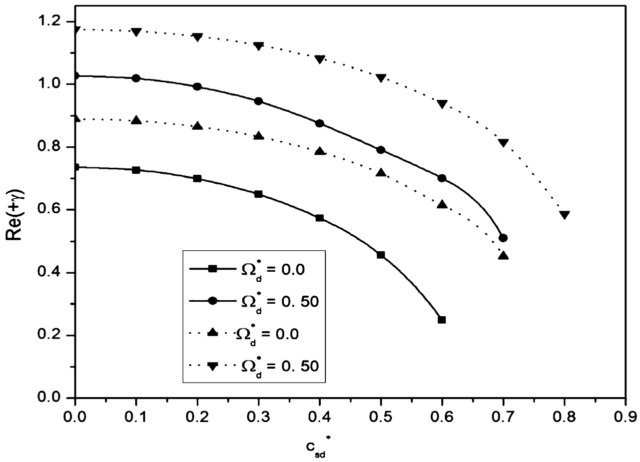

Fig. 3 Variation in the normalized growth rate of Jeans instability against normalized dust acoustic speed with different values of the magnetic field $\left(\Omega_d^* = 0.0,\ 0.50\right)$ and fixed values of $n_{d0}^* = 0.5$, ξ^*, δ', $\sigma_d^* = 0.25\varsigma = 0.30$ under the strongly and weakly coupling limits

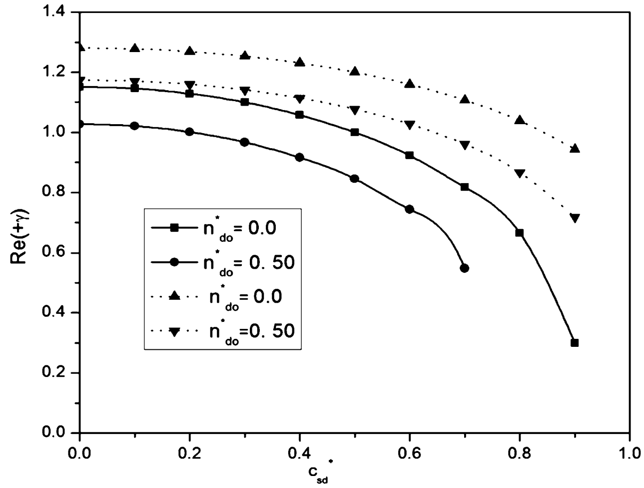

Fig. 4 Variation in the normalized growth rate of Jeans instability against normalized dust acoustic speed with different values of the dust particle number density $\left(n_{d0}^* = 0.0,\ 0.50\right)$ and fixed values of $\Omega_d^* = 0.5$, ξ^*, δ', $\sigma_d^* = 0.25\varsigma = 0.30$ under the strongly and weakly coupling limits

lated from the non-dimensional equations (38) and (39). The growth rate is calculated under both the strongly and weakly coupling limits with different values of a magnetic field $\left(\Omega_d^* = 0.0,\ 0.50\right)$ and the obtained values depicted graphically in Figure 3. Figure 3 represents the variation in normalized growth rate (real positive γ) of Jeans instability against normalized dust acoustic speed with different values of the magnetic field $\left(\Omega_d^* = 0.0,\ 0.50\right)$ and fixed values of $n_{d0}^* = 0.5$, ξ^*, δ', $\sigma_d^* = 0.25\varsigma = 0.30$ under the strongly and weakly coupling limits.

The effect of dust particle number density on the growth rate of gravitational instability of dusty magnetized plasma in the presence of radiative cooling function has been calculated from the non-dimensional equations (38) and (39). The growth rate is calculated under both the strongly and weakly coupling limits with different values of dust particle number density $\left(n_{d0}^* = 0.0,\ 0.50\right)$ and the obtained values are depicted graphically in Figure 4. Figure 4 represents the variation in normalized growth rate (real positive γ) of Jeans instability against normalized dust acoustic speed with different values of the dust particle number density $\left(n_{d0}^* = 0.0,\ 0.50\right)$ and fixed values of Ω_d^*, $n_{d0}^* = 0.5$, ξ^*, δ', $\sigma_d^* = 0.25\varsigma = 0.30$ under the strongly and weakly coupling limits.

6 Results and conclusions

In the present analysis, we have studied the effect of dust temperature and radiative condensation on the gravitational instability of strongly coupled magnetized dusty plasma. The physical problem is described by using the modified Generalized Hydrodynamic (GH) model. A general dispersion relation is obtained using the normal mode analysis of perturbed equations. The dispersion relation is discussed under the strongly and weakly coupling limit for the various limiting cases.

It is observed that when the effects of dust temperature, magnetic field, and radiative condensation considered the instability criterion gets modified due to dust temperature and radiative cooling functions under both the coupling limits, however, shear and bulk viscosities also modifies the criterion under the strongly coupling limit. Further, we have discussed the special cases in which we have considered the effect of non-radiating, selfgravitating unmagnetized plasma having extremely low dust temperature on the gravitational instability and it is found that under the strongly coupling limit the *Jeans criterion* is modified due to the presence of shear and bulk viscosities only whereas, in the weakly coupling limit the dust acoustic speed modifies the instability criterion. In the second case, the effects of non-radiating dusty magnetized plasma having finite dust temperature have been observed and it is found that the dust temperature term couples with the dust acoustic speed in order to modify the *Jeans criterion* of instability under both the coupling limits, whereas, under the strongly coupling limit, the viscoelastic properties of the medium also modifies the instability criterion. Thus, it is found that the dust temperature and radiative heat-loss functions modify the instability criterion by reducing the critical wave number.

The effects of various parameters on the growth rate of gravitational instability have been studied numerically and the obtained values are depicted graphically. Figure 1 represents the variation in the normalized growth rate (real positive γ) of Jeans instability against normalized dust acoustic speed for different values of the dust temperature $\left(\sigma_d^* = 0.25,\ 0.75\right)$ and fixed values of Ω_d^*, $n_{d0}^* =$

0.5, ξ^*, $\delta' = 0.25$, $\varsigma = 0.30$ under the strongly and weakly coupling limits. It is found that the unstable region is larger for $\left(\sigma_d^* = 0.25\right)$ but as the values of the dust temperature increases the growth of unstable Jeans mode and unstable region decreases. It is also observed that (real positive γ) decreases with increase in normalized dust acoustic speed, therefore it provides the stabilizing influence on the growth rate of Jeans instability of the radiating magnetized dusty plasma. On comparing the limits basis it is found that growth rate is larger in strongly coupling limit than the weakly coupling limit. On the other hand we can say that decay of the growth rate is faster in the strongly coupling limit than the weakly coupling limit.

Figure 2 represents the variation in normalized growth rate (real positive γ) of Jeans instability against normalized dust acoustic speed in the absence and presence of radiative cooling function $\left(\delta' = 0.0,\ 0.50\right)$ and fixed values of Ω_d^*, $n_{d0}^* = 0.5$, $\xi^*, \sigma_d^* = 0.25\varsigma = 0.30$ under the strongly and weakly coupling limits. It is observed that the presence of radiative cooling function decreases the growth of unstable Jeans mode and unstable region under both strongly and weakly coupling limits. Therefore, the unstable region is larger in the absence of radiative cooling function hence this parameter has a stabilizing effect on the growth rate of Jeans instability under both strongly and weakly coupling limits. Further, the decay of the growth rate is faster in the strongly coupling limit ($\omega\tau \gg 1$) than the weakly coupling limit ($\omega\tau \ll 1$).

Figure 3 represents the variation in normalized growth rate (real positive γ) of Jeans instability against normalized dust acoustic speed for different values of the magnetic field $\left(\Omega_d^* = 0.0,\ 0.50\right)$ and fixed values of $n_{d0}^* = 0.5$, ξ^*, $\delta', \sigma_d^* = 0.25\varsigma = 0.30$ under the strongly and weakly coupling limits. It is observed that the unstable region is smaller in the absence of the magnetic field, whereas the presence of magnetic field increases the growth of unstable Jeans mode and unstable region under both strongly and weakly coupling limits, hence have destabilizing influence on the growth rate of instability.

The effect of dust particle number density ($n_{d0}^* = 0.0$, 0.50) on the normalized growth rate (real positive γ) against the normalized dust acoustic speed on the radiating magnetized dusty plasma under the strongly and weakly coupling limits have been observed from Figure 4. It is found that the growth of unstable Jeans mode and unstable region is higher in the absence of dust particle number density, whereas for the increasing value of the dust particle number density the growth rate of Jeans instability decreases, hence have stabilizing effect under the strongly and weakly coupling limits. It is also observed that the growth rate is higher in weakly coupling limit than the strongly coupling limit. This means that in the strongly coupling limit ($\omega\tau \gg 1$) the decay of the growth rate is faster than the weakly coupling limit ($\omega\tau \ll 1$).

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