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ORIGINAL ARTICLE

A complete cosmological scenario with particle creation

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Abstract A flat Friedmann-Robertson-Walker (FRW) cosmological model with perfect fluid has been investigated within the framework of the particle creation mechanism. Perfect fluid isentropic particle creation rate as a function of Hubble parameter has been incorporated. By considering a simple parametrization of Hubble parameter, we discuss the cosmological scenarios through time evolution of the cosmological parameters. We examine the different regimes of the universe by studying the phase transition from decelerated to accelerated era. We also explore the classical stability of model and the state-finder diagnostic for highlighting the departure of the considered cosmological model from the ΛCold Dark Matter (ΛCDM) model.

Keywords FRW model · Particle creation · Inflation · Accelerating universe

1 Introduction

The existence of accelerated expansion of universe have been confirmed by recent observational surveys (Riess et al. 1998; Perlmutter et al. 1999; Spergal et al. 2003; Komatsu et al. 2011; Ade et al. 2014). The early-time expansion era can be described by a barotropic fluid model, a modified gravity like $f(R)$ gravity or by a combination of both (Capozziello et al. 2006; Nojiri and Odintsov 2007, 2011; Bamba et al. 2012, 2014; Nojiri et al. 2017). In literature, late-time accelerated expansion of universe has been explained by an exotic form of matter/energy called as dark energy (Li et al. 2014; Capozziello and Faraoni 2011; Clifton et al. 2012). Accelerated expansion era is a common property of the early and late-time stages of the universe.

Cosmological particle production has been intensively examined as part of the universe's early evolution. In literature, isentropic (adiabatic) production of perfect fluid particles has gathered special attention (Parker 1968; Zeldovich 1970; Prigogine et al. 1989; Calvao et al. 1992; Lima and Germano 1992); where entropy per particle (specific entropy) is conserved. Condition of conserved specific entropy during the production of fluid particles lead to a relation between creation pressure and particle creation rate. However, due to the enlargement of the phase space resulting from particle production, the total entropy is not conserved (Prigogine et al. 1989). A thermodynamically consistent cosmology with quantum creation of radiation due to vacuum decay may lead to a scenario in which Hubble parameter is determined by the particle number (Gunzig et al. 1998). The particle creation mechanism offers an alternative way to the explanation of early accelerated phase (Abramo and Lima 1996; Zimdahl 2000; Nunes 2016) and a period of late-time accelerated expansion have also been described in such a framework (Paul et al. 1998; Singh et al. 2000, 2002; Steigman et al. 2009; Jesus et al. 2011; Singh and Kale 2011; Lima et al. 2012; Chaubey 2012; Chaubey and Shukla 2014; Lima et al. 2014; Ramos et al. 2014; Fabris et al. 2014; Nunes and Pavon 2015; Harko 2015; Lima et al. 2016; Nunes and Pan 2016; de Haro and Pan 2016; Pan et al. 2016; Singh and Devi 2016; Singh et al. 2018; Sevinc and Aydiner 2019). It also allows two de Sitter phases with an intermediate radiation and matter dominated phases (Lima et al. 2012; Nunes and Pan 2016). In the context of particle creation, Chakraborty and Saha have investigated cosmological scenarios from inflation to late-time acceleration with non-equilibrium thermodynamical description (Chakraborty and Saha 2014). The dynamics of two fluids may entirely

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be controlled by a single non-linear differential equation involving the particle creation rate and may also be used to study stability of solutions (Pan et al. 2019). The thermodynamic constraints on the matter creation models have been derived for different particle creation rates in literature (Valentim and Jesus 2019). The non-minimal coupling of gravitational and matter sectors may result in a change of particle momentum on a cosmological time scale, irrespective of particle decay or creation (Azevedo and Avelino 2019). The dynamical analysis of particle creation models have also been investigated in literature (Biswas et al. 2017; Paliathanasis et al. 2017; Ivanov and Prodanov 2019). The possibility of transition from a decelerated to accelerated period of expansion based on effective equation of state and deceleration parameter have been investigated with particle creation mechanism in modified gravity (Singh et al. 2020; Hulke et al. 2020).

The particle creation process (Prigogine et al. 1989) can generate the same dynamic behaviour as a FRW model of universe with bulk viscosity but the models being quite different from thermodynamic point of view (Calvao et al. 1992; Lima and Germano 1992). In the bulk viscosity and particle creation concepts, universe can develop dynamically in the same manner but thermodynamic requirement for their identification is violated (Brevik and Stokkan 1996).

We aim to investigate the effects of particle production on the early stages and later evolution of the universe in the flat-FRW framework. We concentrate upon late-time dynamics of the universe with an accelerated phase. We write the cosmological equations of the model in Sect. 2 and investigate the dynamical properties of the model. In Sect. 3, we provide a summary of results.

2 Cosmological model

In this section, we introduce the basic cosmological equations of the cosmological model with particle creation mechanism.

We take the geometry of universe to be described by FRW metric:

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-\kappa r^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right) \quad (1)$$

where $a(t)$ is the scale factor and $\kappa = 0, +1, -1$ for flat, closed and open spatial sections respectively. We work in the units where $8\pi G = c = \hbar = k_b = 1$. The Einstein's field equations are given by

$$3H^2 + \frac{3k}{a^2} = \rho \quad (2)$$

$$2\dot{H} + 3H^2 + \frac{k}{a^2} = -(p + p_c) \quad (3)$$

where ρ and p are energy density and thermodynamic pressure of the matter content related by $p = \omega\rho$ with ω being equation of state parameter, p_c is the creation pressure and $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Overhead dot denotes the derivative with respect to cosmic time t. If the thermodynamical system is considered to be open, so that fluid particle are not conserved ($N^i_{;i} = n\Gamma \neq 0$ where $N^i = nu^i$ is the fluid flow vector and n stands for particle number density), then non-equilibrium thermodynamics comes into picture and p_c becomes an effective bulk viscous pressure due to particle creation. With the particle creation rate Γ being the rate of change of particle number in a co-moving volume V, the particle conservation equation can be written as:

$$\dot{n} + 3nH = n\Gamma \quad (4)$$

We take the present thermodynamic system to be adiabatic in nature. By adiabatic particle creation, we mean that the particle as well as entropy S with entropy per particle $\sigma = \frac{S}{N}$ being constant have been produced in the space-time. The entropy in this system is not conserved due to enlargement of phase-space resulting from particle production (Prigogine et al. 1989). If $\Gamma > 0$, then there is particle creation and $\Gamma < 0$ indicates particle annihilation, $\Gamma = 0$ shows no particle production. Due to second law of thermodynamics, for entropy to be never decreasing one must have $\Gamma > 0$. The creation pressure p_c is related to gravitationally induced 'adiabatic' particle creation rate Γ by the relation (Calvao et al. 1992; Lima and Germano 1992)

$$p_c = -\frac{\Gamma}{3H}(\rho + p) \quad (5)$$

Creation pressure p_c is zero or negative in the absence or presence of particle production. The energy conservation equation takes the form

$$\dot{\rho} + 3H(\rho + p) = -3Hp_c \quad (6)$$

We take the parametrization of particle creation rate Γ as (Zimdahl 2000)

$$\Gamma = 3\beta H^2 \quad (7)$$

where β is the constant of proportionality. In this paper we take $k = 0$. In flat-FRW model $H^2 \propto \rho$, one can observe that $\Gamma \propto \rho$ in the present model. From equation (7), the creation pressure (p_c) takes the form $p_c = -\frac{\beta}{\sqrt{3}}\sqrt{\rho}(\rho + p)$. The effective pressure (p_e) in the present cosmological model takes the form

$$p_e = p + p_c = \omega\rho - \frac{\beta(1+\omega)}{\sqrt{3}}\sqrt{\rho}\rho \quad (8)$$

Here, ρ and p are energy density and pressure of matter existing in the universe in form of barotropic fluid. $p = \omega\rho$ describe a radiation like fluid for $\omega = \frac{1}{3}$, a pressureless dark matter for $\omega = 0$. It is interesting to note that until now we do not have any exact explanation on the particles being created in the gravitational field. Here, we assume that the created particles are perfect fluid particles but with unknown nature (Paliathanasis et al. 2017). With the ansatz of particle creation rate, one can observe from equation (8) that the effective pressure is depending upon the parameters ω and β along-with energy density ρ. We continue by defining effective equation of state $\gamma = \frac{p_e}{\rho}$. In the effective scenario, the total fluid may behave like radiation for $\gamma = \frac{1}{3}$, or a dust fluid for $\gamma = 0$ or a pure cosmological constant for $\gamma = -1$. In terms of geometrical parameters, γ can be written as $\gamma = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}$ and $p_c = -2\dot{H} - 3(1+\omega)H^2$. The point-wise energy conditions that depends only on stress energy tensor at a given point in space-time are given as (Visser 1997)

- Null energy condition (NEC) $\Leftrightarrow \rho + p_e \geq 0$
- Weak energy condition (WEC) $\Leftrightarrow \rho \geq 0,\ \rho + p_e \geq 0$
- Dominant energy condition (DEC) $\Leftrightarrow \rho \geq 0, \rho \pm p_e \geq 0$
- Strong energy condition (NEC) $\Leftrightarrow \rho + 3p_e \geq 0,\ \rho + p_e \geq 0$

We now proceed with a specific, simple parametrization of the Hubble parameter $H(t)$ as:

$$H(t) = hn\tanh(m - nt) + c \tag{9}$$

where h, m, n, c are some constants. By implementing the first order formalism in $f(R, T)$ gravity with self interacting scalar field ϕ, this kind of Hubble parameter has been studied by Moraes and Santos (Moraes and Santos 2016). The expression of $H(t)$ can be used to get explicit form of scale factor $a(t)$ as follows:

$$a(t) = e^{ct}\cosh^{-h}(m - nt) \tag{10}$$

As $t \to 0$, $a(t) \to \cosh^{-h}(m)$ and for $t \to \infty$, $a(t) \to \infty$. The deceleration parameter $\left(q = -1 - \frac{\dot{H}}{H^2}\right)$ is given by

$$q = -1 + \frac{hn^2}{(hn\sinh(m - nt) + c\cosh(m - nt))^2} \tag{11}$$

The expression for effective equation of state γ takes the form

$$\gamma = -1 + \frac{2}{3}\frac{hn^2}{(hn\sinh(m - nt) + c\cosh(m - nt))^2} \tag{12}$$

In Figs. 1, 2, we plot Hubble parameter H, q and γ with t for $m = 3$, $n = 0.3$, $h = 3$ and $c = 0.9725$ respectively.

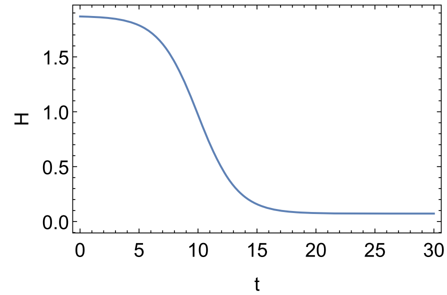

Fig. 1 Hubble parameter H with cosmic time t

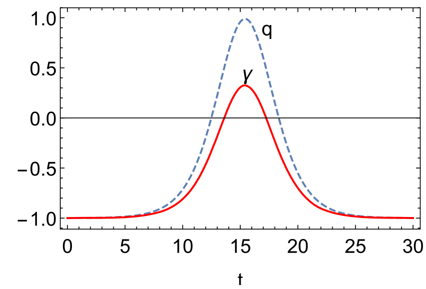

Fig. 2 Deceleration parameter q and effective equation of state γ with t

We keep these values of the parameters throughout in this paper, to discuss the properties of the universe. It can be observed that during initial stages of evolution of universe in our model, H is almost constant or very slowly varying, with $\gamma \sim -1$ and $q = -1$. The deceleration parameter indicates the rate at which the expansion of universe is slowing down. The de-Sitter expansion happens at $q = -1$, accelerating power-law expansion can be achieved when $-1 < q < 0$ and a super-exponential expansion happens for $q < -1$. Eternal acceleration is achieved when $q < 0$ (Bolotin et al. 2015). In our model, during the early evolution of universe, the expansion of universe is accelerated for a period of time where energy density ρ and H are almost constant or very slowly varying (see Fig. 3) with $\gamma < -\frac{1}{3}$. After the period of accelerated expansion (inflation), Hubble parameter is proportional to $\frac{1}{t}$, the same behaviour of H with t as in standard cosmology (Dodelson 2003). A sufficient negative pressure can drive the acceleration. A combination of p_c and p have been used to obtain the negative pressure in our model. The behaviour of creation pressure p_c with t has been shown in Fig. 4.

In our model, with the evolution of time $\dot{H} < 0$ and for large t, Hubble parameter H and ρ both are showing constant behaviour. There exist an era where $q > 0$ but with evolution of time we can observe that q changes its signature and at late times $q = -1$. In our model, there exist two eras of de-Sitter expansion, one at the early times and another at the late times. After inflationary era, effective equation of state γ evolves smoothly to $\frac{1}{3}$ and for high values

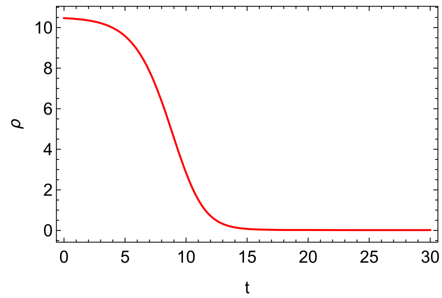

Fig. 3 Energy density ρ with t

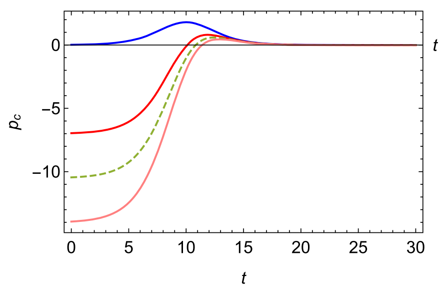

Fig. 4 Creation pressure p_c with t. Here blue, red, dashed, pink curve is for $w = -1$, -0.33, 0 and 0.33 respectively

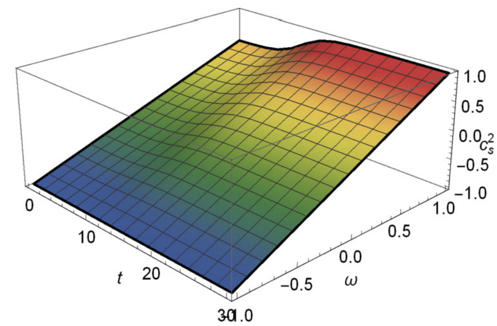

Fig. 5 ${c_s}^2$ with time t and ω

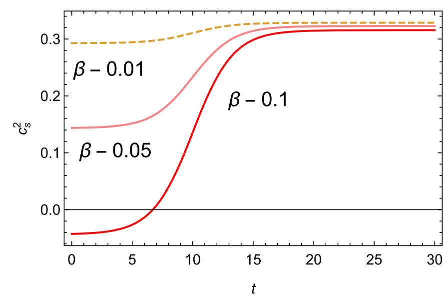

Fig. 6 ${c_s}^2$ with t for $\omega = 0.33$

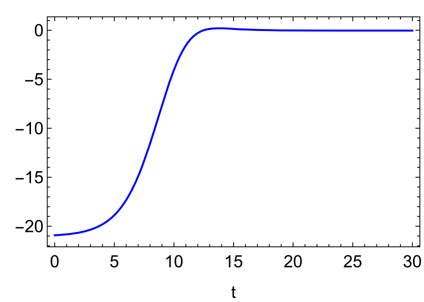

Fig. 7 $\rho + 3p_e$ with t

of t, $\gamma \sim -1$. As per recent observations of anisotropies in temperature of cosmic microwave background radiation (CMBR) (Hinshaw et al. 2013) $\gamma \sim -1$ and it is responsible for the phase of recent cosmic acceleration. The inflationary scenario in our model smoothly decays to radiation dominated era. This phenomena is also called as graceful exit (Perico et al. 2013; Basilakos et al. 2013). Our model also predicts the matter dominated phase $\gamma = 0$ and late-time accelerated phase in the universe (Riess et al. 1998; Perlmutter et al. 1999; Hinshaw et al. 2013).

The square of sound speed ${c_s}^2$ is defined by

$$ {c_s}^2 = \frac{\partial p_e}{\partial \rho} = \gamma - \frac{3}{2}\beta(1+\gamma)H \tag{13} $$

where $H = c + hn\tanh(m - nt)$. To ensure stability of the fluctuations in the fluid, the adiabatic speed of sound should be ${c_s}^2 \geq 0$ and for ensuring causality ${c_s}^2 \leq 1$. We plot ${c_s}^2$ with time t and fluid content having $\omega \in [-1, 1]$ in Fig. 5 for $\beta = 0.1$. We plot ${c_s}^2$ with t for $\omega = 0.33$ in Fig. 6. It is observed that parameter β affects the stability of model. The classical stability of the model also depends on the nature of particles present in the cosmological evolution of the universe.

The energy density remains positive (that is $\rho \geq 0$) in throughout cosmological history of the model. The expression $\rho + p_e$ takes the form $\rho + p_e = -2\dot{H}$ which is always greater than or equal to zero. Therefore phantom scenario ($\rho + p_e < 0$) does not arise in the considered model. We conclude that NEC, WEC are satisfied in our model but SEC is violated (see Fig. 7). Validity of $\rho + 3p_e \geq 0$ highlights the fact that gravity is attractive but in our model $\rho + 3p_e < 0$ implies that $\ddot{a} > 0$. WEC is always satisfied in our model implies the fact that the effective energy density is always be non-negative when measured by any observer. The expansion scalar is given by $\Theta = 3H$. The ratio $\frac{\Gamma}{\Theta} = A$ may be written as

$$ \frac{\Gamma}{\Theta} = \beta H = \beta(c + hn\tanh(m - nt)) \tag{14} $$

Due to the form of parametrization of Γ (see equation (7)), $A \propto H$. For inflationary regime, A is constant or very slowly varying while the particle production is strongly suppressed. We plot particle creation rate Γ with t and parameter β in Fig. 8 for $\omega = 0.33$. From Fig. 8, it can be observed that a very small range of values for β is available in order to have a continuous particle creation rate in our cosmologi-

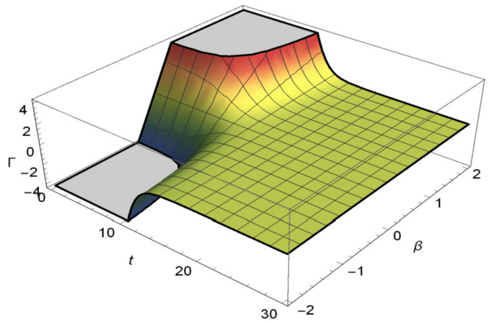

Fig. 8 Γ with time t and β

cal model. In the early universe, $\Gamma > H$ and therefore the created radiation behaves as a thermalized heat bath and subsequently at late times, the rate of particle creation becomes dynamically insignificant. The choice of $\Gamma \propto \rho$ is the simplest choice for modelling of the early universe scenario (Abramo and Lima 1996). One can have particle creation or particle annihilation in the model depending on the choice of value of parameter β, but at late times particle creation rate is constant. During the particle annihilation era, the second law of thermodynamics will be violated. The state-finder diagnostic pair $\{r, s\}$ are defined by (Alam et al. 2003)

$$r = \frac{\dddot{a}}{a}\frac{1}{H^3}, \quad s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} \tag{15}$$

State-finder pair probe the expansion dynamics of the universe through $\ddot{a}(t)$, $\dddot{a}(t)$. It is a geometric diagnostic in the sense that it is directly constructed from geometrical parameters. Departure of a model from $\{r, s\} = \{1, 0\}$ is a good way to highlight departure of the considered model from ΛCDM model. In $r - s$ plane, $\{r, s\} = \{1, 0\}$ is the fixed point of ΛCDM model. This value is independent of the red-shift z and the parameter of the ΛCDM model. In the present model, we have

$$r = 1 - \frac{3hn^2}{(hn\sinh(m-nt) + c\cosh(m-nt))^2} - \frac{2hn^3\sinh(m-nt)}{(hn\sinh(m-nt) + c\cosh(m-nt))^3} \tag{16}$$

$$s = \frac{-3hn^2 - \frac{2hn^3\tanh(m-nt)}{c+hn\tanh(m-nt)}}{3hn^2 - \frac{9}{2}[hn\sinh(m-nt) + c\cosh(m-nt)]^2} \tag{17}$$

We plot r with q and r with s in Figs. 9, 10. We can observe that $r = 1$ at $q = -1$ and curve passes through $\{1, 0\}$ in $r - s$ plane. The universe in our model starts with $\{r, q\} = \{1, -1\}$, goes to the decelerating phase with $q = 1$ and at late times returns to the $\{r, q\} = \{1, -1\}$. Upper and lower half of $r - q$ plane about the ΛCDM line ($r = 1$) is

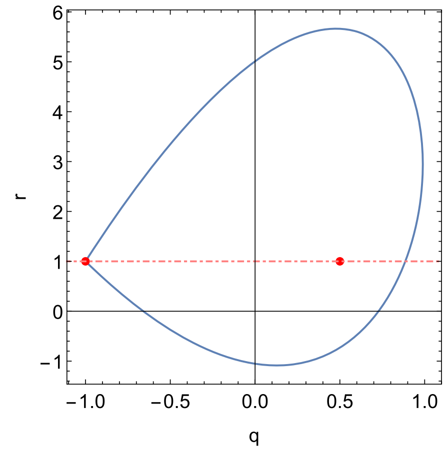

Fig. 9 r (vertical axis) with q (horizontal axis) with $r = 1$ being the ΛCDM line

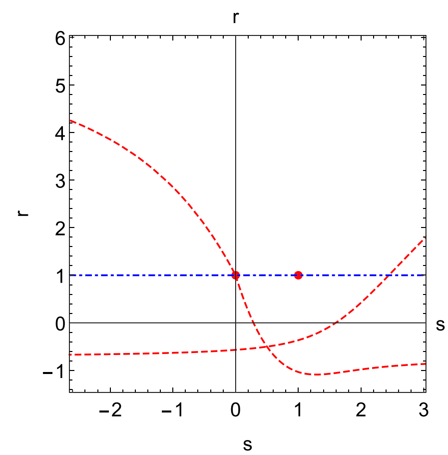

Fig. 10 r (vertical axis) with s (horizontal axis) with $r = 1$ being the ΛCDM line

occupied by the Chaplygin gas and quintessence models respectively (Alam et al. 2003). The state-finder r is < 1 (> 1) for quintessence (Chaplygin gas) models and state-finder s is positive for quintessence but negative for Chaplygin gas models (Alam et al. 2003). Figures 9, 10 exhibit the fact that the present toy model in particle creation framework resembles with quintessence as well as Chaplygin gas models during it's cosmological evolution.

The slow-roll parameters ϵ and η are defined as (Brevik et al. 2017, 2018; Singh et al. 2018a)

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{hn^2}{(hn\sinh(m-nt) + c\cosh(m-nt))^2} \tag{18}$$

$$\eta = -\frac{\ddot{H}}{2H\dot{H}} = -\frac{n\sinh(m-nt)}{hn\sinh(m-nt) + c\cosh(m-nt)} \tag{19}$$

Condition for inflation is $\ddot{a} > 0$. Slow-roll is then characterized by $|\epsilon| \ll 1$ and $|\eta| \ll 1$. The slow-roll parameters $|\epsilon|$,

$|\eta|$ are much smaller than unity during inflation. When ϵ, η increases, kinetic energy decreases and when ϵ becomes of order unity, the slow-roll approximation breaks down and inflation ends. With end of inflation, equation of state parameter smoothly evolves to $\frac{1}{3}$ (which is the maximum value γ should assume during the universe evolution). Therefore, this toy model traces the overall behaviour of the cosmological parameters, using ansatz of Hubble parameter with regard to the particles created with particle creation rate $\Gamma = 3\beta H^2$, and transfers energy and entropy to particles produced during the cosmological evolution.

3 Conclusion

In this paper, we investigate the flat FRW model with barotropic fluid satisfying $p = \omega\rho$ with particle creation mechanism. Hubble parameter in our model is very slowly varying or constant at early times as well as at late times. Particle creation mechanism in the considered toy model allows us to understand the particle production as well as annihilation during the evolution of the universe. In standard model, negative pressure is the exact mechanism responsible for accelerated universe.

The universe in our model starts with non-zero volume and during it's early stages of evolution $\gamma \sim -1$ and $q \sim -1$. Universe undergoes inflationary era and takes a graceful exit from inflationary stage to radiation dominated era. A transition from decelerated expansion to a period of accelerated expansion of universe may be possible due to the effective negative pressure available (which arises because of creation pressure and pressure of the barotropic fluid in our model). The energy density remains positive and NEC is also satisfied, and thus phantom scenario does not arises in our model. In the cosmological history of universe $\rho + 3p_e < 0$, which highlights the fact that $\ddot{a} > 0$.

Universe during early times evolves with de-Sitter expansion and behave as quintessence model for a period of time, advances to the decelerating era and then, re-enters to the current phase of accelerated expansion. The classical stability of model depends on the nature of matter created in the universe. At late times, particle creation rate is almost constant.

It is interesting to note that the ansatz based studies in literature, with or without particle creation mechanism in general relativity or modified gravity stresses upon the prediction of a transition from decelerated era to late-times accelerated era of the universe (Singh et al. 2000, 2002; Singh and Kale 2011; Chaubey 2012; Chaubey and Shukla 2014; Singh and Devi 2016; Singh et al. 2018, 2020; Hulke et al. 2020; Moraes and Sahoo 2017; Mishra et al. 2019; Nagpal et al. 2019). The complete evolution of universe has been realised (Chakraborty and Saha 2014) by choosing different particle creation rates for different phases of evolution. In the present model, we describe all the dynamical stages of universe (from inflation, to radiation, to matter and the current accelerated era) with the proper unique choice of particle creation rate as a function of Hubble parameter.

An analysis of power spectrum, distance measurements from type Ia supernovae and the position of the first peak in anisotropy of CMB indicates a cosmological late-time dark matter creation at 95% confidence level (Pigozzo et al. 2016). It would be interesting to investigate the observables of inflationary cosmology (Bamba and Odintsov 2016) by considering outcomes of our model in framework of GR as well as modified gravities (Capozziello and Faraoni 2011). The study of particle creation in context of non-equilibrium thermodynamics for bouncing models (Singh et al. 2016, 2018) may give interesting insight on the singularity problem of standard cosmology. We leave this for a future investigation.

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