



# Comparison among three types of relativistic charged anisotropic fluid spheres for *self*-bound stars

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**Abstract** The number of exact solutions for spherically symmetric anisotropic fluid spheres is derived. And the relativistic model of an electrically charged compact star whose energy density associated with the electric fields is on the same order of magnitude as the energy density of fluid matter itself, such as electrically charged *bare* strange stars is also investigated. Among the three models, Heintzmann and Durgapal IV, and V permit a simple method that provide bounds on the maximum possible mass of compact electrically charged self-bound stars, and numerically exhibit that the maximum compactness and mass increase in the presence of an electric field and anisotropic pressure. Based on analytic models developed in this work, it has been compared to investigate which one is better than the others; the values of some pertinent physical quantities have been considered by assuming the estimated masses and radii of some well-known potential strange star candidates like Vela X-1, Cen X-3, PSR J1903+327 and EXO 1785-248. This analysis depends on several mathematical assumptions.

**Keywords** General relativity · Einstein–Maxwell · Reissner–Nordström · Relativistic astrophysics · Compact star · Equation of state · Self-bound star · Anisotropic fluid sphere

## 1 Introduction

The Reissner–Nordström spacetime provides the generalized Schwarzschild exterior solution leading to the behav-

ior of the gravitational field outside a spherically symmetric perfect fluid including the effects of the electromagnetic field. The nonlinear Einstein–Maxwell field equations are required for the description of the behavior of relativistic gravitating matter with electromagnetic field distributions, and they are the key for modeling of the relativistic compact objects.

To integrate the field equations, various restrictions have been developed due to the geometry of space-time and content of the matter. The exact Einstein solutions can be obtained by specifying the geometry and form of the anisotropic factor when differential equations are resolved by computations using an equation of state.

The exterior solution of a compact object is unique, and also important for simple algebraic relations to the analytical solutions that permit the distribution of matter in the interior of the stellar object. Dev and Gleiser (2002, 2003), Herrera and Santos (1997), Herrera et al. (2008), and Lake (2003) (Lake’s algorithm), have all developed the analytical solution of locally anisotropic fluids. Bowers and Liang (1974) accomplished the original work in the field of anisotropic fluid models which shows the physical behavior of a star undergoing gravitational collapse, and promotes the researcher for further research in anisotropic effect. Herrera and Santos (1997) explored the properties of anisotropic self-gravitating spheres using the perturbation method. According to Herrera et al. (1998), local pressure anisotropy is one of the influencing factors for inhomogeneities in energy density.

Based on the behavior of matter, the analytic solutions to the equation of relativistic stellar structure of gravitational field equations are of two types; one is “normal” matter for neutron stars and another is “self-bound” strange quark stars. The Tolman VII solution is useful for realistic neutron star models which are bounded by gravity. Quark stars are self-bound by the strong interaction and are massive for their

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gravity. The self-bound strange quark star has finite density and is about 2–3 times of the normal nuclear matter saturation density (Postnikov et al. 2010). The Bodmar–Witten hypothesis states that strange quark matter is the ultimate ground state of matter and this hypothesis remains as a great opportunity in physics and astrophysics (Weber 1999, 2005; Glendenning 2000; Haensel 2003; Haensel et al. 2007).

The wide range of values of constant parameters permit the required maximum mass of charged fluid spheres. This aside, on the particular choice of stellar surface density  $\rho_s$ , various authors usually have chosen  $\rho_s = 2 \times 10^{14} \text{ g/cm}^3$  to calculate the mass and radius of the charged fluid spheres, which give rise to the stellar configuration as massive as 4–6  $M_\odot$  with much lower central density. Such a massive configuration may not serve as a realistic model for a self-bound star. This choice is, therefore, not a physical one.

Bare strange stars have own ultra-strong electric fields on their surface which is around  $10^{18} \text{ V/cm}$  (Alcock et al. 1986) and for color superconducting strange matter is  $10^{20} \text{ V/cm}$  (Usov 2004; Usov et al. 2005; Negreiros et al. 2010). Ray et al. (2003) and Malheiro et al. (2004) first inquired the influence of the energy density of ultra-high electric fields on the bulk properties of compact stars. Weber et al. (2007, 2009, 2010) and Negreiros et al. (2009) have also proposed that electric fields of this magnitude increase stellar mass by up to 30%, contingent on the strength of the electric field which is generated by charge distributions situated on the neighbor surfaces of strange quark stars. However, in the case of the neutron star, the surface electric field is absent. These characteristics may allow the observationally to distinguish quark stars from neutron stars.

The principal objective of this work is twofold. I seek a model that is physically acceptable in the relativistic sphere with an anisotropic fluid and compare the three types of models and try to investigate which one is better than the other by using physical analysis. The metric function is nonsingular, continuous and well behaved in the interior of the star. All the three models permit a simple method that bounds on the maximum possible mass of compact electrically charged self-bound stars, and numerically exhibit the maximum compactness and increase in mass in the presence of an electric field and anisotropic pressure. This analysis depends on several mathematical assumptions.

This paper is organized into six sections. In Sect. 2 presents Einstein’s field equations, electrically charge distribution and pressure anisotropy. Section 3 introduces physical quantities using boundary conditions. Some physical properties are discussed and comparison among the physical behavior of the strange stars candidates based on the three models is discussed in Sects. 4 and 5. And, lastly, Sect. 6 is a conclusion of this work.

## 2 Fundamental equations

### 2.1 Field equations

We intend to describe stellar structure with a statically symmetric matter distribution, in spacetime manifold (Sunzu et al. 2014), and whose stress tensor may be locally anisotropic. The interior metric in Schwarzschild coordinates  $x^\mu = (t, r, \theta, \phi)$  (Tolman 1939; Oppenheimer and Volkoff 1939) is given by the metric (Herrera et al. 2008; Bowers and Liang 1974; Cosenza et al. 1981; Herrera et al. 2001):<sup>1</sup>

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (2.1)$$

The function  $\nu(r)$  and  $\lambda(r)$  are arbitrary and satisfy the Einstein-Maxwell field equations,

$$G^\mu_\nu = R^\mu_\nu - \frac{1}{2}\delta^\mu_\nu R = \kappa(T^\mu_\nu + E^\mu_\nu), \quad (2.2)$$

where  $\kappa = 8\pi$  is Einstein’s constant. Consequently,  $T^\mu_\nu$  and  $E^\mu_\nu$  are energy-momentum tensor of fluid distribution and electromagnetic field, assumed in locally anisotropic fluid, defined by (Dionysiou 1982; Herrera and Ponce de León 1985)

$$T^\mu_\nu = (P_t + \rho c^2)v^\mu v_\nu - P_t \delta^\mu_\nu + (P_r - P_t)\chi^\mu \chi_\nu, \\ E^\mu_\nu = \frac{1}{4\pi} \left( -F^{\mu i} F_{\nu i} + \frac{1}{4}\delta^\mu_\nu F^{ij} F_{ij} \right),$$

where  $\rho$ ,  $P_r$ ,  $P_t$ ,  $v^\mu$ , denote the energy density, radial pressure, and tangential pressure of the fluid distribution respectively and velocity vector. Antisymmetric electromagnetic field strength tensor,  $F_{\mu\nu}$ , defined by

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}, \quad (2.3)$$

which satisfies the Maxwell equations,

$$F_{;\nu}^{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} (\sqrt{-g} F^{\mu\nu}) = -4\pi J^\mu, \quad (2.4a)$$

$$F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = 0, \quad (2.4b)$$

where  $g$  is the determinant of quantities  $g_{\mu\nu}$  in Eq. (2.1), defined by

$$g = \begin{vmatrix} e^\nu & 0 & 0 & 0 \\ 0 & -e^\nu & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{vmatrix} = -e^{\nu+\lambda} r^4 \sin^2\theta$$

<sup>1</sup>Throughout the work using  $c = G = 1$  except in the tables and figures.

and  $A_\nu = (\phi(r), 0, 0, 0)$  is the four-potential and  $J^\mu$  is the four-current vector, defined by

$$J^\mu = \frac{\rho_{ch}}{\sqrt{g_{00}}} \frac{dx^\mu}{dx^0},$$

where  $\rho_{ch}$  denotes the proper charge density. The nonvanishing components of electromagnetic field tensor are related to  $F^{01} = -F^{10}$ , which represents the radial component of the electric field. From Eq. (2.4a) the following expression for the electric field:

$$F^{01} = -e^{-\frac{\nu+\lambda}{2}} \frac{q(r)}{r^2},$$

where  $q(r)$  represents the total charge contained within the sphere of radius by

$$q(r) = 4\pi \int_0^r e^{\frac{\lambda}{2}} \rho_{ch} u^2 du. \tag{2.5}$$

Charge density

$$\rho_{ch} = \frac{e^{-\frac{\lambda}{2}} dq}{4\pi r^2 dr}. \tag{2.6}$$

The above equation treated as the relativistic version of Gauss’s law.

For the metric (2.1), the Einstein–Maxwell field equations with matter and charge are expressed as (Herrera and Ponce de León 1985):

$$\frac{\nu'}{r} e^{-\lambda} - \frac{1 - e^{-\lambda}}{r^2} = \kappa P_r - \frac{q^2}{r^4}, \tag{2.7}$$

$$\left( \frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r} \right) e^{-\lambda} = \kappa P_t + \frac{q^2}{r^4}, \tag{2.8}$$

$$\frac{\lambda'}{r} e^{-\lambda} + \frac{1 - e^{-\lambda}}{r^2} = \kappa \rho + \frac{q^2}{r^4} \tag{2.9}$$

where prime (') denotes the  $r$ -derivative.

For electrically uncharged case, introducing a quantity  $m(r)$  in the following expression:

$$e^{-\lambda} = 1 - \frac{2m(r)}{r} + \frac{q^2}{r^2}. \tag{2.10}$$

If  $M$  is the gravitational mass and  $R$  represents the radius of the fluid distribution then  $m$  is constant  $m(r = R) = M$  outside the fluid distribution. Murad and Fatema (2015) describe  $m(r)$  and finally get, using Eqs. (2.10) and also (2.7) to (2.9),

$$m(r) = \frac{\kappa}{2} \int \rho r^2 dr + \frac{q^2}{2r} + \frac{1}{2} \int \frac{q^2}{r^2} dr, \tag{2.11}$$

$$\nu' = \frac{\kappa r P_r + \frac{2m}{r^2} - \frac{2q^2}{r^3}}{1 - \frac{2m}{r} + \frac{q^2}{r^2}}, \tag{2.12}$$

$$\frac{dP_r}{dr} = -\frac{P_r + \rho}{2} \nu' + \frac{q}{4\pi r^4} \frac{dq}{dr} + \frac{2(P_t - P_r)}{r}. \tag{2.13}$$

Using (2.7), (2.8) and (2.9), following charged generalization of Tolman–Openheimer–Volkoff (TOV) equation of a hydrostatic equilibrium for the anisotropic stellar configuration (Ponce de León J. 1987)

$$\frac{dP_r}{dr} = -\frac{(P_r + \rho) (\kappa r P_r + \frac{2m}{r^2} - \frac{2q^2}{r^3})}{2 (1 - \frac{2m}{r} + \frac{q^2}{r^2})} + \frac{q}{4\pi r^4} \frac{dq}{dr} + \frac{2\Delta}{r}, \tag{2.14}$$

where  $\Delta \approx \kappa(P_t - P_r)$ .

To transform the system into relatively simpler form of Eqs. (2.7)–(2.9) with the help of following ansatz (Durgapal 1982; Korkina 1981),

$$e^\nu = B_N (1 + Cr^2)^N, \tag{2.15}$$

where  $N$  is a positive integer and  $B_N, C > 0$  are two constants to be determined by the appropriate physical boundary conditions. Equation (2.15) has chosen for giving us a very simple relation for the redshift from any region of the configuration. And also a simple expression for  $e^\nu$  can help calculate the trajectories of ultra-relativistic particles in the gravitational field. For each integral value of  $N$  the field equation can be solved exactly and also get a new exact solution. From (2.7) and (2.8) one can obtain the equation of “pressure anisotropy”,

$$\begin{aligned} & \left( \frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} - \frac{\nu' + \lambda'}{2r} \right) e^{-\lambda} + \frac{1 - e^{-\lambda}}{r^2} \\ & = \kappa(P_t - P_r) + \frac{2q^2}{r^4}. \end{aligned} \tag{2.16}$$

Equation (2.16), is a second order nonlinear differential equation in  $\nu$  and first order linear in  $\lambda$ , may have two generating functions,  $\kappa(P_t - P_r)$  and  $\nu$ , as Herrera et al. (2008) notified in all static spherically symmetric anisotropic solutions of Einstein’s field equations. Now we are going to introduce the following transformations:

$$e^{-\lambda} = Z, \quad x = Cr^2, \quad C > 0. \tag{2.17}$$

Equation (2.16) becomes the following equation by transforming Eq. (2.14):

$$\frac{dZ}{dx} + P(x)Z = Q(x), \tag{2.18}$$

where

$$P(x) = \frac{(N^2 - 2N - 1)x^2 - 2x - 1}{x(1+x)(1+(1+N)x)},$$

$$Q(x) = \frac{(1+x)}{x(1+(1+N)x)} \left( \frac{2Cq^2}{x} + \Delta x - 1 \right).$$

This equation is linear differential equation and the solution:

$$Z = e^{-\int P(x)dx} \left[ \int e^{-\int P(x)dx} Q(x)dx + A_N \right]. \tag{2.19}$$

Here  $A_N$  is the integral constant, which may be determined by imposing appropriate physical boundary conditions. When the metric potential  $Z$  is attained, other physical variables may be represented in terms of the generating functions and the equation of state may be extracted, and we may write the parametric equations:

$$\frac{\kappa}{C} P_r = \frac{1+(2N+1)x}{x(1+x)} Z - \frac{1}{x} + \frac{Cq^2}{x^2}, \tag{2.20}$$

$$\frac{\kappa}{C} P_t = \frac{2N+N^2x}{(1+x)^2} Z + \frac{1+(1+N)x}{(1+x)} \frac{dZ}{dx} - \frac{Cq^2}{x^2}, \tag{2.21}$$

$$\frac{\kappa}{C} \rho = -2 \frac{dZ}{dx} - \frac{Z}{x} + \frac{1}{x} - \frac{Cq^2}{x^2}. \tag{2.22}$$

Now we can generalize the metric potential  $Z$  in general form is

$$\begin{aligned} Z = & \frac{x}{(1+x)^{N-2} [1+(1+N)x]^{\frac{2}{1+N}}} \\ & \times \int \frac{(1+x)^{N-1} [1+(1+N)x]^{\frac{1-N}{1+N}}}{x^2} \\ & \times \left( \frac{2Cq^2}{x} - 1 + \Delta x \right) dx \\ & + A_N \frac{x}{(1+x)^{N-2} [1+(1+N)x]^{\frac{2}{1+N}}}. \end{aligned} \tag{2.23}$$

### 2.2 Electric charge distribution and pressure anisotropy

Anninos and Rothman intuitively remarked from their paper (Anninos and Rothman 2001) that, the “realistic” charge distribution inside the fluid sphere, is reasonable due to electrical repulsion the charge distribution should be weighted towards the surface. To obtain a closed form solution, one can imagine several plausible distributions to integrate the equation of pressure isotropy (2.16). Various authors presented a variety of solutions previously for different suitable choices of charge distributions. Some of the solutions will be found from Murad and Fatema (Murad and Fatema 2013). In this work we consider the following model distributions to avoid

the singularity at the center:

$$\frac{2Cq^2}{x^2} = kx(1+x)^{1-N} [1+(1+N)x]^{\frac{N-1}{N+1}}, \tag{2.24}$$

$$\Delta = \delta x(1+x)^{1-N} [1+(1+N)x]^{\frac{N-1}{N+1}}, \tag{2.25}$$

where  $k, \delta \geq 0$ .

These distributions are chosen, in term of  $x$ , in such a way that electric field intensity and anisotropy vanish at the center and remains continuous and bounded in the interior of the star for a wide range of values of the parameters  $k$  and  $\delta$ . Thus these choices are physically reasonable and useful in the study of the gravitational behavior of charged stellar objects. It has been shown by Maurya and Gupta (Maurya and Gupta 2011a, 2011b) for the uncharged and charged cases respectively that the ansatz is the metric function  $e^\nu = B_N(1+x)^N$  where  $N$  is a positive integer, produces an infinite family of analytic solutions of the self-bound type. Some of these were previously known ( $N = 1, 2, 3, 4$ , and  $5$ ). The most relevant case is for  $N = 2$ , for which the velocity of sound  $\approx 1/\sqrt{3}$  throughout most of the star, somewhat similar to the behavior of strange quark matter (Lattimer and Prakash 2005).

In our model calculation, the density at the stellar radius is a value taken from the work (Sharma et al. 2006) within the range  $4-10 \times 10^{14}$  g/cm<sup>3</sup> (Bombaci 2001) and rapidly to zero, as with all stellar models matching an interior metric to the external Reissner-Nordström form. This abrupt drop in density is a reasonable model approximation since the thickness of the “quark surface” is of order 1 fm, a negligibly small dimension compared to the stellar radius. The solutions obtained in this work are expected to provide simplified but easy to mathematically analyzed charged stellar models with nonzero super-high surface density which could reasonably model the stellar core of an electrically charged strange quark star by satisfying applicable physical boundary conditions.

In this work we keep our interest particularly to obtain the charged analogue of the types  $N = 3, 4, 5$  which correspond to Heintzmann (Heintzmann 1969) and Durgapal (Durgapal 1982; Delgaty and Lake 1998) models and derive corresponding equations of state.

For the cases  $N = 3, 4$  and  $5$ , the solution of the Einstein–Maxwell system (2.7)–(2.9), for the model charge distribution and pressure anisotropy considered in Eqs. (2.24)–(2.25), are then given by the following.

#### Type I:

$$e^\nu = B_3(1+x)^3, \tag{2.26a}$$

$$\frac{2Cq^2}{x^2} = kx(1+x)^{-2}(1+4x)^{\frac{1}{2}}, \tag{2.26b}$$

$$\Delta = \delta x(1+x)^{-2}(1+4x)^{\frac{1}{2}}, \tag{2.26c}$$

$$Z = \frac{x^2(k + \delta)}{(1+x)(1+4x)^{1/2}} + A_3 \frac{x}{(1+x)(1+4x)^{1/2}} + \frac{2-x}{2(1+x)},$$

$$\frac{\kappa}{C} P_r = \frac{1}{2(1+x)^2(1+4x)^{1/2}} \times [kx(3+18x) + 2\delta x(1+7x) + 9(1-x)(1+4x)^{1/2} + 2A_3(1+7x)], \quad (2.26d)$$

$$\frac{\kappa}{C} P_t = \frac{1}{2(1+x)^2(1+4x)^{1/2}} \times [kx(3+18x) + 2\delta x(2+11x) + 9(1-x)(1+4x)^{1/2} + 2A_3(1+7x)], \quad (2.26e)$$

$$\frac{\kappa}{C} \rho = -\frac{1}{2(1+x)^2(1+4x)^{3/2}} \times [kx(11+46x+32x^2) - 2\delta x(5+19x+8x^2) + 3(3+x)(1+4x)^{3/2} - 3A_3(1+3x)]. \quad (2.26f)$$

**Type II:**

$$e^\nu = B_4(1+x)^4, \quad (2.27a)$$

$$\frac{2Cq^2}{x^2} = kx(1+x)^{-3}(1+5x)^{3/5}, \quad (2.27b)$$

$$\Delta = \delta x(1+x)^{-3}(1+5x)^{3/5}, \quad (2.27c)$$

$$Z = \frac{x^2(k + \delta)}{(1+x)^2(1+5x)^{2/5}} + A_4 \frac{x}{(1+x)^2(1+5x)^{2/5}} + \frac{7-10x-x^2}{7(1+x)^2},$$

$$\frac{\kappa}{C} P_r = \frac{1}{14(1+x)^3(1+5x)^{2/5}} [7kx(3+23x) + 14\delta x(1+9x) + 32(2-7x-x^2)(1+5x)^{2/5} + 14A_4(1+9x)], \quad (2.27d)$$

$$\frac{\kappa}{C} P_t = \frac{1}{14(1+x)^3(1+5x)^{2/5}} [7kx(3+23x) + 28\delta x(1+7x) + 32(2-7x-x^2)(1+5x)^{2/5} + 14A_4(1+9x)], \quad (2.27e)$$

$$\frac{\kappa}{C} \rho = -\frac{1}{14(1+x)^3(1+5x)^{7/5}} [7kx(11+54x+27x^2) + 14\delta x(5+22x+x^2) - 16(9+2x+x^2)(1+5x)^{7/5} + 14A_4(3+10x-9x^2)]. \quad (2.27f)$$

**Type III:**

$$e^\nu = B_5(1+x)^5, \quad (2.28a)$$

$$\frac{2Cq^2}{x^2} = kx(1+x)^{-4}(1+6x)^{2/3}, \quad (2.28b)$$

$$\Delta = \delta x(1+x)^{-4}(1+6x)^{2/3}, \quad (2.28c)$$

$$Z = \frac{x^2(k + \delta)}{(1+x)^3(1+6x)^{1/3}} + A_5 \frac{x}{(1+x)^3(1+6x)^{1/3}} + \frac{112-309x-54x^2-8x^3}{112(1+x)^3},$$

$$\frac{\kappa}{C} P_r = \frac{1}{224(1+x)^4(1+6x)^{1/3}} [112kx(3+28x) + 224\delta x(1+11x) + 50(19-165x-42x^2-8x^3)(1+6x)^{1/3} + 224A_5(1+11x)], \quad (2.28d)$$

$$\frac{\kappa}{C} P_t = \frac{1}{224(1+x)^4(1+6x)^{1/3}} [112kx(3+28x) + 224\delta x(2+17x) + 50(19-165x-42x^2-8x^3)(1+6x)^{1/3} + 224A_5(1+11x)], \quad (2.28e)$$

$$\frac{\kappa}{C} \rho = -\frac{1}{224(1+x)^4(1+6x)^{4/3}} [112kx(11+62x+16x^2) + 1120\delta x(1+5x-2x^2) - 30(129+x+30x^2+8x^3)(1+6x)^{4/3} + 224A_5(3+11x-22x^2)]. \quad (2.28f)$$

**3 Determination of constants and physical quantities using boundary conditions**

**3.1 Conditions for physical acceptability**

For well-behaved nature of the solutions for anisotropic fluid sphere should be satisfied by the following conditions (Abreu et al. 2007):

- i. The solution should be free from physical and geometric singularities (Singh et al. 2015), i.e. it should yield finite and positive values of the central pressure, central density and nonzero positive value of  $e^{\nu(0)} = \text{constant}$ , and  $e^{-\lambda(0)} = 1$ .
- ii. The pressure ( $P_r$ ,  $P_t$ ) and density  $\rho$  should be positive inside the fluid configuration.

- iii. The interior solution for strong energy condition should be positive (Esculpi et al. 2007), i.e.  $(P_r + 2P_t)/\rho \geq 0$  and dominant energy condition  $\rho \geq P_r$  and  $\rho \geq P_t$ .
- iv. The relativistic adiabatic index is given by  $\Gamma = \frac{(P+\rho)}{P} \frac{dP}{d\rho}$ . The necessary condition for this exact solution to serve as a model of a relativistic star is that  $\Gamma > 4/3$ .
- v. The radial pressure must be vanished but the tangential pressure may not necessary to vanish at the boundary and the radial pressure is equal to the tangential pressure at the center of the fluid sphere. So,  $\Delta(0) = 0$  (Bowers and Liang 1974) and tangential pressure is greater than radial pressure,  $\Delta(r) > 0$  (Böhmer and Harko 2006; Maurya et al. 2017; Abbas et al. 2018).
- vi. Electric field intensity  $E$ , such that  $E(0) = 0$ , is taken to be monotonically increasing (Harrison et al. 1965) i.e.  $(dE/dr) > 0$  for  $0 < r < R$ .
- vii. Pressure and density, should maximum at the center and monotonically decreasing towards the pressure free interface i.e.  $(dP_r/dr)_{r=0} = 0$ ,  $(d\rho/dr)_{r=0} = 0$  and  $(d^2P_r/dr^2)_{r=0} < 0$ ,  $(d^2\rho/dr^2)_{r=0} < 0$  so that pressure gradient  $dP_r/dr \leq 0$  and density gradient  $d\rho/dr \leq 0$  for  $0 < r < R$ .
- viii. The redshift  $z$  should be positive, finite and monotonically decreasing in nature with the increase of  $r$ .
- ix. Buchdahl condition (Buchdahl 1959) must be satisfy for neutron star which has the maximally allowable mass- radius ratio  $2M/R \leq \frac{8}{9}$ . But Böhmer and Harko (2007) proved that for a charged compact object, there is a lower bound for mass-radius ratio,  $\frac{3}{2} \frac{Q^2}{R^2} \frac{1+Q^2/(18R^2)}{1+Q^2/(12R^2)} \leq \frac{2M}{R}$ .
- x. Since  $\Gamma > 4/3$  and  $P_r > 0$ , then for realistic star, the compression modulus  $\kappa_e$ ,  $\kappa_e = P_r \Gamma$ , must be decreasing outwards (Singh et al. 2016c).

### 3.2 Determination of the arbitrary constant A

To specify  $A$  the boundary condition  $P(r = R) = 0$  can be utilized

**Type I:**

$$A_3 = -\frac{1}{2(1+7X)} [kX(3+18X) + 2\delta X(1+7X) + 9(1-X)(1+4X)^{\frac{1}{2}}]. \tag{3.1a}$$

**Type II:**

$$A_4 = -\frac{1}{14(1+9X)} [7kX(3+23X) - 14\delta X(1+9X) - 32(2-7X-X^2)(1+5X)^{\frac{2}{5}}]. \tag{3.1b}$$

**Type III:**

$$A_5 = -\frac{1}{224(1+11X)} [112kX(3+28X)$$

$$- 224\delta X(1+11X) - 50(19-165X-42X-8X^3)(1+6X)^{\frac{1}{3}}], \tag{3.1c}$$

where  $X = CR^2$ .

### 3.3 Determination of the constant B

The constant  $B$  can be specified by the boundary condition  $e^{\nu(R)} = e^{-\lambda(R)}$ , which yields,

**Type I:**

$$B_3 = \frac{1}{2(1+X)^4(1+4X)^{1/2}} [2X^2(k+\delta) + 2A_3X + (2-X)(1+4X)^{\frac{1}{2}}]. \tag{3.2a}$$

**Type II:**

$$B_4 = \frac{1}{7(1+X)^6(1+5X)^{2/5}} [7X^2(k+\delta) + 7A_3X + (7-10X-X^2)(1+5X)^{2/5}]. \tag{3.2b}$$

**Type III:**

$$B_5 = \frac{1}{112(1+X)^8(1+6X)^{1/3}} [112X^2(k+\delta) + 112A_5X + (112-309X-54X^2-8X^3)(1+6X)^{\frac{1}{3}}]. \tag{3.2c}$$

### 3.4 Determination of the total charge to radius ratio

**Type I:**

$$\frac{Q^2}{R^2} = \frac{K}{2} \frac{X^2(1+4X)^{1/2}}{(1+X)^2}. \tag{3.3a}$$

**Type II:**

$$\frac{Q^2}{R^2} = \frac{K}{2} \frac{X^2(1+5X)^{3/5}}{(1+X)^3}. \tag{3.3b}$$

**Type III:**

$$\frac{Q^2}{R^2} = \frac{K}{2} \frac{X^2(1+6X)^{2/3}}{(1+X)^4}. \tag{3.3c}$$

### 3.5 Determination of the total mass to radius ratio

**Type I:**

$$\frac{2M}{R} = -\frac{1}{2(1+X)^2(1+4X)^{\frac{1}{2}}} [3kX^3 + 2\delta X^2 - 3X(1+4X)^{\frac{1}{2}} + 2A_3X]. \tag{3.4a}$$



**Type II:**

$$\frac{2M}{R} = -\frac{1}{14(1+X)^3(1+5X)^{2/5}} [7kX^2(1-3X) + 14\delta X^2 - 16X(3+X)(1+5X)^{2/5} + 14A_4X]. \quad (3.4b)$$

**Type III:**

$$\frac{2M}{R} = -\frac{1}{224(1+X)^4(1+6X)^{1/3}} [112kX^2(1-4X) + 224\delta X^2 - 30X(43+26X+8X^2)(1+6X)^{1/3} + 224A_5X]. \quad (3.4c)$$

**3.6 The central and surface redshift  $z_c, z_s$**

The central and surface redshifts of the charged fluid sphere are given by

$$z_c = \sqrt{e^{-\nu(0)}} - 1 = \frac{1}{\sqrt{B_N}} - 1, \quad (3.5a)$$

$$z_s = \sqrt{e^{-\nu(R)}} - 1 = \frac{(1+X)^{-\frac{N}{2}}}{\sqrt{B_N}} - 1. \quad (3.5b)$$

Type I:

$$z_s = \frac{(1+x)^{-3/2}}{\sqrt{B_3}} - 1. \quad (3.5c)$$

Type II:

$$z_s = \frac{(1+X)^{-2}}{\sqrt{B_4}} - 1. \quad (3.5d)$$

Type III:

$$z_s = \frac{(1+x)^{-5/2}}{\sqrt{B_5}} - 1. \quad (3.5e)$$

**4 Constructing physical realistic fluid spheres**

**4.1 Pressure and density gradients**

To analyze the analytical equation of state, a straightforward differentiation of the pressure and density equations (2.26a)–(2.28f) with respect to the auxiliary variable  $x$ . Due to the comparison of those types we obtain the pressure and density gradients as

**Type I:**

$$\frac{\kappa}{C} \frac{dP_r}{dx} = \frac{1}{2(1+x)^3(1+4x)^{3/2}} [k(3+39x+90x^2-36x^3) + 2\delta(1+15x+36x^2)$$

$$-9(3-x)(1+4x)^{\frac{3}{2}} + 6A_3(1-x+14x^2)],$$

$$\frac{\kappa}{C} \frac{dP_t}{dx} = \frac{1}{2(1+x)^3(1+4x)^{3/2}} [k(3+39x+90x^2-36x^3) + 2\delta(2+24x+54x^2-22x^3) - 9(3-x)(1+4x)^{\frac{3}{2}} + 6A_3(1-x+14x^2)],$$

$$\frac{\kappa}{C} \frac{d\rho}{dx} = -\frac{1}{2(1+x)^3(1+4x)^{\frac{5}{2}}} \times [k(11+59x+78x^2-52x^3-64x^4) + 2\delta(5+23x+12x^2-58x^3-16x^4) + 3(5+x)(1+4x)^{\frac{5}{2}} - 6A_3(5+23x+30x^2)].$$

**Type II:**

$$\frac{\kappa}{C} \frac{dP_r}{dx} = \frac{1}{14(1+x)^4(1+5x)^{7/5}} \times [7k(3+49x+125x^2-161x^3) + 14\delta(1+19x+51x^2-63x^3) - 32(13-12x-x^2)(1+5x)^{7/5} + 28A_4(1-2x-27x^2)],$$

$$\frac{\kappa}{C} \frac{dP_t}{dx} = \frac{1}{14(1+x)^4(1+5x)^{7/5}} \times [7k(3+49x+125x^2-161x^3) + 28\delta(1+15x+37x^2-49x^3) - 32(13-12x-x^2)(1+5x)^{\frac{7}{5}} + 336A_4(1-2x-27x^2)],$$

$$\frac{\kappa}{C} \frac{d\rho}{dx} = -\frac{1}{14(1+x)^4(1+5x)^{\frac{12}{5}}} \times [7k(11+64x+2x^2-432x^3-189x^4) + 14\delta(5+24x-38x^2-256x^3-7x^4) + 16(25+2x+x^2)(1+5x)^{\frac{12}{5}} - 56A_4(5+31x+47x^2-27x^3)].$$

**Type III:**

$$\frac{\kappa}{C} \frac{dP_r}{dx} = \frac{1}{224(1+x)^5(1+6x)^{4/3}} \times [112k(3+59x+164x^2-392x^3) + 224\delta(1+15x+36x^2)$$

$$\begin{aligned}
 &+ 50(8x^3 + 60x^2 + 411x - 241)(1 + 6x)^{4/3} \\
 &+ 1120A_5(1 - 3x - 44x^2)], \\
 \frac{\kappa}{C} \frac{dP_t}{dx} = &\frac{1}{224(1+x)^5(1+6x)^{4/3}} \\
 &\times [112k(3 + 59x + 164x^2 - 392x^3) \\
 &+ 224\delta(2 + 36x + 96x^2 - 238x^3) \\
 &+ 50(8x^3 + 60x^2 + 411x - 241)(1 + 6x)^{4/3} \\
 &+ 1120A_5(1 - 3x - 44x^2)], \\
 \frac{\kappa}{C} \frac{d\rho}{dx} = &-\frac{1}{224(1+x)^5(1+6x)^{7/3}} \\
 &\times [112k(11 + 69x - 114x^2 - 1096x^3 - 224x^4) \\
 &+ 1120\delta(1 + 5x - 22x^2 - 118x^3 + 28x^4) \\
 &+ 30(515 - 57x + 36x^2 + 8x^3)(1 + 6x)^{7/3} \\
 &- 1120A_5(5 + 39x + 66x^2 - 88x^3)].
 \end{aligned}$$

### 4.2 Stability and equilibrium conditions

In connection to stability of the stellar model, Stettner (1973) argued that a fluid sphere of uniform density with a net surface charge is more stable than without charge. Theoretically (Canuto and Chitre 1973; Canuto 1975; Ruderman 1972a, 1972b; Canuto and Lodenquai 1975), the pressure anisotropy is one of the most significant factor for compact stellar objects. The nuclear matter, relativistic interaction, may be very high density range ( $\rho > 10^{15}$  g/cm<sup>3</sup>) (Murad 2018; Ruderman 1972a, 1972b). In this point of view, radial pressure and tangential pressure may not be equal and considering  $P_t > P_r$  for  $\Delta > 0$ .

Herrera (1992) developed a criteria to check the stability of anisotropic gravitating source. This technique state that if radial speed of sound is greater than the transverse speed of sound in a region, then it is a potentially stable region, otherwise unstable. So, if the term  $|v_t^2 - v_r^2| \leq 1$  then local anisotropic matter distribution is stable. Figure 9 indicates that this model is stable within the specific configuration.

The static stability criterion (Zel'dovich and Novikov 1971; Harrison et al. 1965),  $\frac{dM}{d\rho_c} > 0$ , in the literature if the mass  $M$  increases consequently central density  $\rho_c$  is also growing. In this study the parameters may be set in such a way that the solution satisfies the necessary conditions of physical acceptability.

### 5 Discussions

For the particular set of values of  $(\delta, K, X)$  for which the fluid distribution satisfies the following inequalities;  $P(r) \geq$

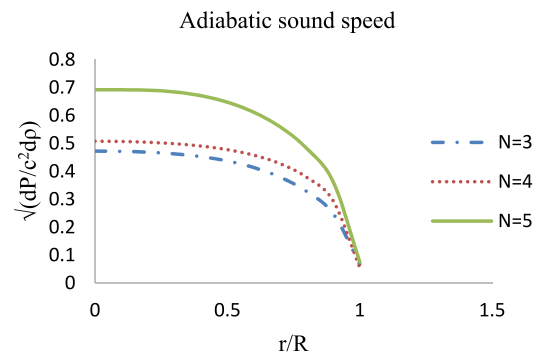


Fig. 1 Behavior of adiabatic sound speed

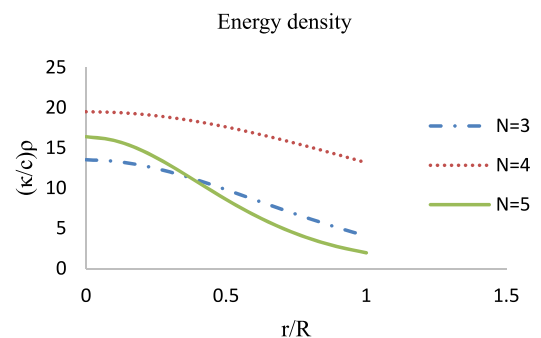


Fig. 2 Behaviors of the energy density  $c^2\rho$

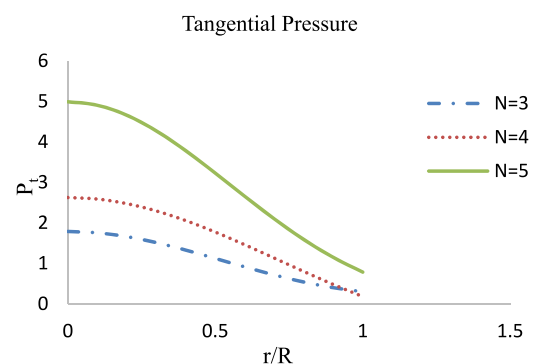
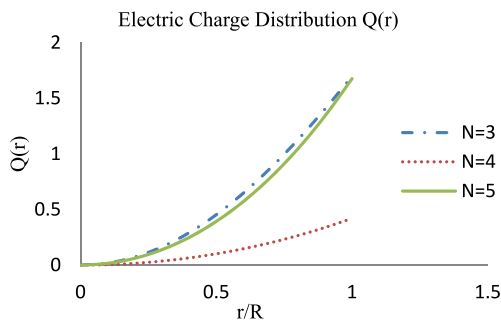


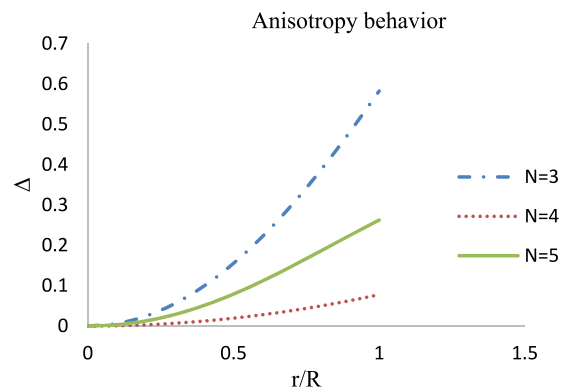
Fig. 3 Behavior of tangential pressure for same stellar configuration as in Fig. 1

$0, \rho(r) > 0, dP/dr < 0, d\rho/dr < 0$ , the speed of sound satisfies  $0 \leq \sqrt{dP/c^2 d\rho} \leq 1$ . A fluid sphere satisfying these inequalities will be termed as *well-behaved*. Though there is no explicit relation in between  $K$  and  $X$ , but for some particular choices of  $\delta$ , the parameters  $K$  and  $X$  found to follow the relation plotted in Figs. 1–13. Here considering the range of the parameters are  $0.3 < \delta < 5, 0.5 < k < 1.8$  and  $0 < X < 0.235$  use to illustrate the figures and also in Table 1. For particular input  $(\delta, K, X) = (5, 0.5, 0.063)$ , among three particular models, the graphs describe  $N = 3, 5$  are better than  $N = 4$ . The maximum value of  $X$  is fixed for one minimum value of  $K$  and every values depend on

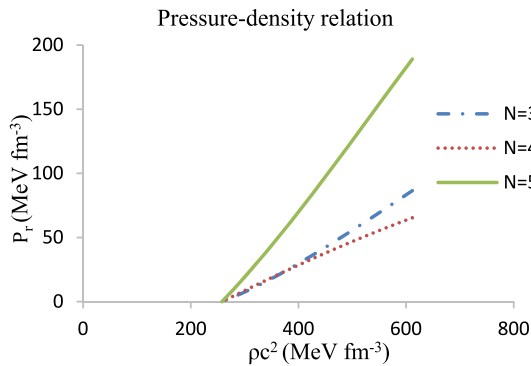




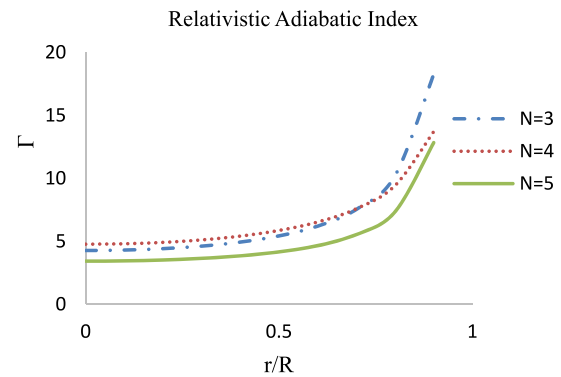
**Fig. 4** Behavior of electric charge distribution for same stellar configuration



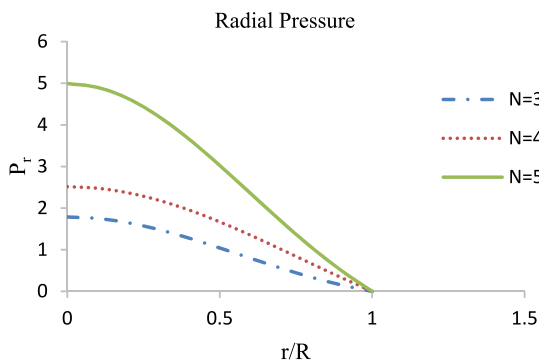
**Fig. 7** The same configuration for anisotropy behavior



**Fig. 5** Pressure-density outlines for same fluid spheres

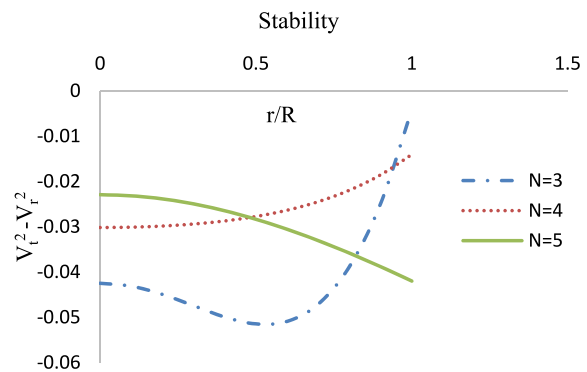


**Fig. 8** The relativistic adiabatic index  $\Gamma$  for the same stellar configuration



**Fig. 6** Behavior of radial pressure same configuration as Fig. 1

particular value of ‘ $\delta$ ’. To illustrate the behaviors of various physical variables in the interior of the star, among the three models, describe by various inputs we have plotted the pressure–density relation in Fig. 5. In Figs. 3, 6, and 2 have the behavior of anisotropic pressure, energy and density respectively. For instance, to generate an anisotropic fluid sphere just set  $\delta = 1.6$  which corresponding to the sound speed  $v_s = \sqrt{dP/c^2 d\rho} = 0.502546, 0.591875, 0.6990695$  for  $N = 3, 4$  and  $5$  respectively and which are less than the speed of light. If considering  $N = 3$ , for maximum value of compactness parameter is obtain  $(2M/R)_{max} = 0.541742$ , Solar mass  $M_{max} = 2.041M_\odot$ , radius = 11.197 km and also surface density  $\kappa\rho_s = 6.941 \times 10^{14} \text{ g/cm}^3$ , central den-



**Fig. 9** Plot of the  $v_t^2 - v_r^2$  for the parameter values

sity  $\kappa\rho_c = 1.0902 \times 10^{15} \text{ g/cm}^3$  for maximum value of  $(X, K, \delta) = (0.202, 1.4, 1.6)$  where as for  $N = 4$  and  $5$ , the results are not similar for sound and surface and center density i.e. for same value of  $X, K, \delta$  there are compactness parameter  $(2M/R)_{max} = 0.609125, 0.658835$  and surface density  $\kappa\rho_s = 5.7512 \times 10^{14} \text{ g/cm}^3, 6.6110 \times 10^{14} \text{ g/cm}^3$ , central density  $\kappa\rho_c = 7.63059 \times 10^{15} \text{ g/cm}^3, 1.15265 \times 10^{15} \text{ g/cm}^3$  respectively but sound speed are too high (see Fig. 1). From Fig. 11, solar mass  $M_{max} = 2.8227M_\odot$ , ra-

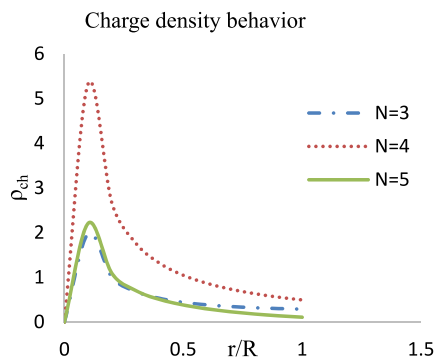


Fig. 10 Behavior of charge density

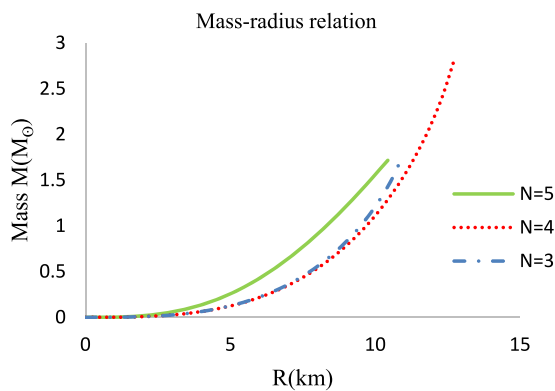


Fig. 11 Behavior of mass vs. radius relation

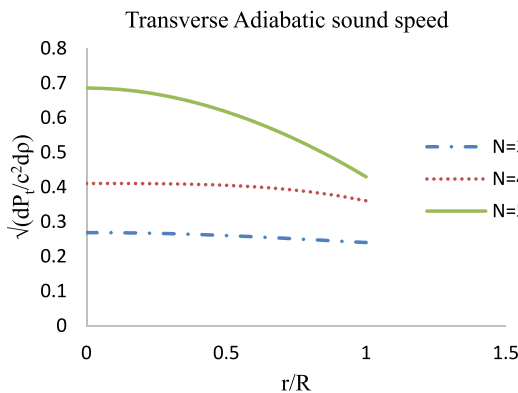


Fig. 12 Behavior of transverse adiabatic sound speed

radius = 12.733 km and solar mass  $M_{max} = 1.7164M_{\odot}$ , radius = 10.434 km for  $N = 4$  and  $N = 5$  respectively.

It is well known that anisotropy will be directed outward when  $p_t > p_r$  (i.e.  $\Delta > 0$ ), and inward when  $p_t < p_r$  (i.e.  $\Delta < 0$ ). It is evident from Fig. 7 that for the given model a repulsive force would exist as  $\Delta > 0$ , that permits the formation of a supermassive star, for large values of  $r/R$ ,  $\Delta = 0$ , where a star comes to the equilibrium position. This implies that the anisotropic force allows the construction of more massive stars. For  $N = 3$ , anisotropy behavior is better than the rest of the other types.

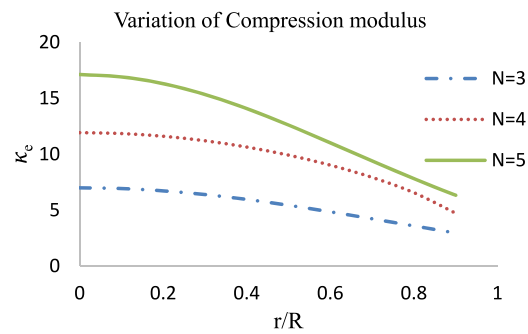


Fig. 13 Behavior of variation of compression modulus  $\kappa_c$

From Figs. 2 and 4, energy density is too low for  $N = 5$  but electric charge distribution is good enough. On the other hand for  $N = 4$ , electric charge distribution is low enough whereas energy density is very high. In Fig. 10, surface charge density is higher for  $N = 4$  then other two models. And it's not realistic for  $N = 4$ . It shows that in Fig. 4 the charge distribution is zero at the center and monotonically increasing towards the pressure free interface (boundary). Behavior of mass-radius relation from Fig. 11 all models i.e.  $N = 3, 4$  and also 5 are satisfied by Buchdahl condition (Buchdahl 1959) for which all are less than or equal to  $8/9$ . Figure 9 shows that all the models are stable, but  $N = 5$  is decreasing from center to surface, according to the stability condition.

## 6 Conclusions

This work has presented a new family of interior solutions of Einstein–Maxwell field equations for a static spherically symmetric distribution of perfect fluid with particular forms of charge distribution to construct electrically charged stellar models for better fit. And comparing which model is better than others. Based on ad hoc assumptions, metric potential, the electric charged distribution and the pressure anisotropy, the analytical equation of state has been computed from the resulting metric. It is shown that in the presence of charge the solutions satisfy all the physical requirements adopted in this work to construct physically acceptable electrically charged stellar models (Sect. 3.1). While, not only the case  $N = 3$ , Heintzmann model, is well satisfied the physical requirement but also other two models i.e. Durgapal models ( $N = 4, 5$ ) are considerable for static neutral solutions belongs to the class of 16 solutions (Delgaty and Lake 1998) at passes at least all the tests. The speed of sound is obtained  $\sim 1/\sqrt{3}$  at the center and remains almost the same throughout most of the fluid sphere for  $N = 3$  and 4 but  $N = 5$  is slightly greater than the speed of sound. Comparing behavior of the Strange stars candidate and graphical representation confirm that Heintzman  $N = 3$  and Durgapal Model  $N = 5$  are best fitted. Although there are some pitfall among

**Table 1** Physical values of energy density and pressure for different strange stars about  $N = 3, 4, 5$  respectively

Strange star candidate	$(\delta, k, X)$	$M (M_{\odot})$	$R$ (km)	$P_{c,35}$	$\rho_{c,15}$	$\rho_{s,14}$
PSR J1614-2230	(2.5, 0.05, 0.22)	1.978 [1,2,5]	9.612	1.82	1.618	7.90
	(3.25, 0.06, 0.086)	1.38	10.01	0.70	0.692	5.415
	(2.01, 0.02, 0.15)	1.978	9.74	1.54	1.012	6.50
PSR J1903+327	(4.5, 0.9, 0.16)	1.671 [2,3,6]	9.541	1.268	1.088	6.512
	(3.3, 1.2, 0.0815)	1.367	9544	0.710	0.705	5.125
	(10, 0.6, 0.08115)	1.667	9.776	1.528	1.020	6.559
4U 1608-52	(2.9, 0.92, 0.09)	0.896 [2]	7.866	0.529	0.712	4.443
	(2.2, 0.2, 0.068)	0.895	7.866	0.623	0.719	4.035
	(1.23, 1.2, 0.053)	0.893	7.866	0.612	0.689	3.965
Vela X-1	(4.5, 1, 0.18)	1.782 [2,3]	10.55	1.01	1.221	7.245
	(0.33, 1.17, 0.0817)	1.36	10.551	0.71	0.705	5.524
	(10, 6, 0.089)	1.77	10.88	0.76	0.954	6.569
4U 1820-30	(4.5, 1.71, 0.1408)	1.580 [2]	10.339	0.695	1.038	6.826
	(3.3, 1.25, 0.0746)	1.262	10.3388	0.641	0.683	5.450
	(10, 6, 0.076)	1.581	10.672	0.638	0.975	6.210
Cen X-3	(2.2., 1.1, 0.165)	1.489 [2,3,4]	9.17	0.630	0.972	6.603
	(1.1, 0.06, 0.13)	1.481	9.17	1.38	0.852	7.507
	(1.43, 1.4, 0.098)	1.485	9.17	1.24	0.831	4.396
EXO 1785-248	(2.2, 1.1, 0.165)	1.3 [2,3]	8.849	0.785	0.925	7.109
	(1.8, 0.8, 0.106)	1.312	8.849	1.02	0.839	4.387
	(2.2, 0.9, 0.083)	1.3	8.849	1.02	0.7889	5.838

[1]: Murad and Fatema 2015;  
 [2]: Rahman and Murad 2014;  
 [3]: Das et al. 2016; [4]: Singh et al. 2016b; [5]: Singh and Pant 2016; [6]: Singh et al. 2016a

the three models, but the pressure–density relation which almost linear for  $N = 3$  and  $N = 5$ . This behavior is like MIT bag model.

The charged stellar models obtained here crucially depend on the choice of metric potential and the forms of electric charge distribution. However, the numerical outcomes might be the indication of astrophysical significance of the equations of state obtained in parametric form (2.26d, e, f), (2.27d, e, f) and (2.28d, e, f) in the study of relativistic stellar structure of very cold compact self-bound charged star. If we juxtapose all the properties (comparing) of Heintzmann and Durgapal IV, and V models, our toy models may facilitate studies of anisotropic fluid sphere with an electromagnetic field distribution and provide possibility for further studies of other relativistic matter distributions.

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## References

- Abbas, G., Momeni, D., Qaisar, S., Zaz, Z., Myrzakulov, R.: Can. J. Phys. **96**, 1295–1303 (2018). <https://doi.org/10.1139/cjp-2017-0579>
- Abreu, H., Hernández, H., Núñez, L.A.: Class. Quantum Gravity **24**, 4631 (2007). <https://doi.org/10.1088/0264-9381/24/18/005>
- Alcock, C., et al.: Astrophys. J. **310**, 261 (1986). <https://doi.org/10.1086/164679>
- Anninos, P., Rothman, T.: Phys. Rev. D **65**, 024003 (2001). <https://doi.org/10.1103/PhysRevD.65.024003>
- Böhmer, C.G., Harko, T.: Class. Quantum Gravity **23**, 6479 (2006). <https://doi.org/10.1088/0264-9381/23/22/023>
- Böhmer, C.G., Harko, T.: Gen. Relativ. Gravit. **39**, 757 (2007). <https://doi.org/10.1007/s10714-007-0417-3>
- Bombaci, I.: In: Blaschke, D., Glendenning, N.K., Sedrakian, A. (eds.) Physics of Neutron Star Interiors. Lecture Notes in Physics, vol. 578, pp. 253–284. Springer, Berlin (2001)
- Bowers, R.L., Liang, E.P.T.: Astrophys. J. **188**, 657 (1974). <https://doi.org/10.1086/152760>
- Buchdahl, H.A.: Phys. Rev. **116**, 1027 (1959). <https://doi.org/10.1103/PhysRev.116.1027>
- Canuto, V.: Annu. Rev. Astron. Astrophys. **13**, 335 (1975). <https://doi.org/10.1146/annurev.aa.13.090175.002003>
- Canuto, V., Chitre, M.: Phys. Rev. Lett. **30**, 999 (1973). <https://doi.org/10.1103/PhysRevLett.30.999>
- Canuto, V., Lodenquai, J.: Phys. Rev. D **11**, 233 (1975). <https://doi.org/10.1103/PhysRevD.11.233>
- Cosenza, M., Herrera, L., Esculpi, M., Witten, L.: J. Math. Phys. **22**, 118 (1981). <https://doi.org/10.1063/1.524742>
- Das, A., et al.: Eur. Phys. J. C **76**, 654 (2016). <https://doi.org/10.1140/epjc/s10052-016-4503-0>
- Delgaty, M.S.R., Lake, K.: Comput. Phys. Commun. **115**, 395 (1998). [https://doi.org/10.1016/S0010-4655\(98\)00130-1](https://doi.org/10.1016/S0010-4655(98)00130-1)
- Dev, K., Gleiser, M.: Gen. Relativ. Gravit. **34**, 1793 (2002). <https://doi.org/10.1023/A:1020707906543>
- Dev, K., Gleiser, M.: Gen. Relativ. Gravit. **35**, 1435 (2003). <https://doi.org/10.1023/A:1024534702166>
- Dionysiou, D.D.: Astrophys. Space Sci. **85**, 331 (1982). <https://doi.org/10.1007/BF00653455>
- Durgapal, M.C.: J. Phys. A, Math. Gen. **15**, 2637 (1982). <https://doi.org/10.1088/0305-4470/15/8/039>

- Esculpi, M., Malaver, M., Aloma, E.: *Gen. Relativ. Gravit.* **39**, 633 (2007). <https://doi.org/10.1007/s10714-007-0409-3>
- Glendenning, N.K.: *Compact Stars. Nuclear Physics, Particle Physics, and General Relativity*, 2nd edn. Astrophysics and Space Science Library. Springer, New York (2000)
- Haensel, P.: *EAS Publ. Ser.* **7**, 249 (2003). <https://doi.org/10.1051/eas:2003043>
- Haensel, P., Potekhin, A.Y., Yakovlev, D.G.: *Neutron Stars I: Equation of State and Structure*. Astrophysics and Space Science Library, vol. 326. Springer, Berlin (2007)
- Harrison, B., Throne, K., Wakano, M., Wheeler, J.: *Gravitational Theory and Gravitational Collapse*. University of Chicago Press, Chicago (1965)
- Heintzmann, H.: *Z. Phys.* **228**, 489 (1969). <https://doi.org/10.1007/BF01558346>
- Herrera, L.: *Phys. Lett. A* **165**, 206 (1992). [https://doi.org/10.1016/0375-9601\(92\)90036-L](https://doi.org/10.1016/0375-9601(92)90036-L)
- Herrera, L., Ponce de León, J.: *J. Math. Phys.* **26**, 2302 (1985). <https://doi.org/10.1063/1.526813>
- Herrera, L., Santos, N.: *Phys. Rep.* **286**, 53 (1997). [https://doi.org/10.1016/S0370-157\(96\)00042-7](https://doi.org/10.1016/S0370-157(96)00042-7)
- Herrera, L., Prisco, A.D., Hernandez-Pastoraand, J.L., Santos, N.O.: *Phys. Lett. A* **237**, 113–118 (1998). [https://doi.org/10.1016/S0375-9601\(97\)00874-8](https://doi.org/10.1016/S0375-9601(97)00874-8)
- Herrera, L., Prisco, A.D., Ospino, J., Fuenmayor, E.: *J. Math. Phys.* **42**, 2129 (2001). <https://doi.org/10.1063/1.364503>
- Herrera, L., Ospino, J., Prisco, A.D.: *Phys. Rev. D* **77**, 027502 (2008). <https://doi.org/10.1103/PhysRevD.77.027502>
- Korkina, M.P.: *Sov. Phys. J.* **24**, 468 (1981). <https://doi.org/10.1007/BF00898413>
- Lake, K.: *Phys. Rev. D* **67**, 104015 (2003). <https://doi.org/10.1103/PhysRevD.67.104015>
- Lattimer, J.M., Prakash, M.: *Phys. Rev. Lett.* **94**, 111101 (2005). <https://doi.org/10.1103/PhysRevLett.94.111101>
- Malheiro, M., et al.: *Int. J. Mod. Phys. D* **13**, 1375–1379 (2004). <https://doi.org/10.1142/S0218271804005560>
- Maurya, S.K., Gupta, Y.K.: *Astrophys. Space Sci.* **334**, 145 (2011a). <https://doi.org/10.1007/s10509-011-0705-y>
- Maurya, S.K., Gupta, Y.K.: *Astrophys. Space Sci.* **334**, 301 (2011b). <https://doi.org/10.1007/s10509-011-0736-y>
- Maurya, S.K., Gupta, Y.K., Ray, S.: *Eur. Phys. J. C* **77**, 45 (2017). <https://doi.org/10.1140/epjc/s10052-017-4604-4>
- Murad, M.H.: *Eur. Phys. J. C* **78**, 285 (2018). <https://doi.org/10.1140/epjc/s10052-018-5712-5>
- Murad, M.H., Fatema, S.: *Int. J. Theor. Phys.* **52**, 2508 (2013). <https://doi.org/10.1007/s10773-013-1538-y>
- Murad, M.H., Fatema, S.: *Eur. Phys. J. C* **75**, 533 (2015). <https://doi.org/10.1140/epjc/s10052-015-3737-6>
- Negreiros, R.P., et al.: *Phys. Rev. D* **80**, 083006 (2009). <https://doi.org/10.1103/PhysRevD.80.083006>
- Negreiros, R.P., et al.: *Phys. Rev. D* **82**, 103010 (2010). <https://doi.org/10.1103/PhysRevD.82.103010>
- Oppenheimer, J.R., Volkoff, G.M.: *Phys. Rev.* **55**, 374 (1939). <https://doi.org/10.1103/PhysRev.55.374>
- Ponce de León, J.: *Gen. Relativ. Gravit.* **19**, 797 (1987). <https://doi.org/10.1007/BF00768215>
- Postnikov, S., Prakash, M., Lattimer, J.M.: *Phys. Rev. D* **82**, 024016 (2010). <https://doi.org/10.1103/PhysRevD.82.024016>
- Rahman, A.H.M.M., Murad, M.H.: *Astrophys. Space Sci.* **351**, 255 (2014). <https://doi.org/10.1007/s10509-014-1823-0>
- Ray, S., et al.: *Phys. Rev. D* **68**, 084004 (2003). <https://doi.org/10.1103/PhysRevD.68.084004>
- Ruderman, R.: *Annu. Rev. Astron. Astrophys.* **10**, 427 (1972a). <https://doi.org/10.1146/annurev.aa.10.090172.002235>
- Ruderman, M.: *Annu. Rev. Astron. Astrophys.* **10**, 427 (1972b). <https://doi.org/10.1146/annurev.aa.10.090172.002235>
- Sharma, R., et al.: *Int. J. Mod. Phys. D* **15**, 405 (2006). <https://doi.org/10.1142/S0218271806008012>
- Singh, K.N., Pant, N.: *Indian J. Phys.* **90**, 7 (2016). <https://doi.org/10.1007/s12648-015-0815-4>
- Singh, K.N., Pradhan, N., Pant, N.: *Int. J. Theor. Phys.* **54**, 3408 (2015). <https://doi.org/10.1007/s10773-015-2581-7>
- Singh, K.N., Bhar, P., Pant, N.: *Int. J. Mod. Phys. D* **25**, 11 (2016a). <https://doi.org/10.1142/S0218271816500991>
- Singh, K.N., Pant, N., Govender, M.: *Indian J. Phys.* **90**, 11 (2016b). <https://doi.org/10.1007/s12648-016-0870-5>
- Singh, K.N., Rahaman, F., Pant, N.: *Can. J. Phys.* **94**, 1017 (2016c). <https://doi.org/10.1139/cjp-2016-0307>
- Stettner, R.: *Ann. Phys.* **80**, 1 (1973). [https://doi.org/10.1016/0003-4916\(73\)90325-4](https://doi.org/10.1016/0003-4916(73)90325-4)
- Sunzu, J.M., Maharaj, S.D., Ray, S.: *Astrophys. Space Sci.* **354**, 517 (2014). <https://doi.org/10.1007/s10509-014-2131-4>
- Tolman, R.C.: *Phys. Rev.* **55**, 364 (1939). <https://doi.org/10.1103/PhysRev.55.364>
- Usov, V.V.: *Phys. Rev. D* **70**, 067301 (2004). <https://doi.org/10.1103/PhysRevD.70.067301>
- Usov, V.V., Harko, T., Cheng, K.S.: *Astrophys. J.* **620**, 915 (2005). <https://doi.org/10.1086/427074>
- Weber, F.: *Pulsars as Astrophysical Laboratories for Nuclear and particle Physics*. Studies in High Energy Physics, Cosmology and Gravitation. Institute of Physics Publishing, Bristol (1999)
- Weber, F.: *Prog. Part. Nucl. Phys.* **54**, 193 (2005). <https://doi.org/10.1016/j.pnpnp.2004.07.001>
- Weber, F., et al.: *Int. J. Mod. Phys. E* **16**, 1165 (2007). <https://doi.org/10.1142/S0218301307006599>
- Weber, F., et al.: In: *Astrophysics and Space Science Library*, vol. 357, pp. 213–245. Springer, Berlin (2009)
- Weber, F., et al.: *Int. J. Mod. Phys. D* **19**, 1427–1436 (2010). <https://doi.org/10.1142/S0218271810017329>
- Zel'dovich, Y., Novikov, I.: *Relativistic Astrophysics Stras and Relativity*. University of Chicago Press, Chicago (1971)