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ORIGINAL ARTICLE

Dynamics of perfect fluid cosmological model in the presence of massive scalar field in $f(R, T)$ gravity

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Abstract In this paper, we are interested to investigate spatially homogeneous and totally anisotropic perfect cosmological model in the presence of an attractive massive scalar field in $f(R, T)$ gravity in the background of Bianchi type-*III* space-time. Here R is the Ricci scalar and T is the trace of the energy momentum tensor. In order to solve the field equations, we have used (i) the expansion scalar of the space-time is proportional to the shear scalar which leads to a relationship between metric potentials and (ii) a power law between the scalar field and the average scale factor. We obtain a cosmological model of the universe with variable deceleration parameter. We have computed all the cosmological parameters of the model and discussed their physical importance.

1 Introduction

The recent concept of cosmic acceleration of our universe has been confirmed by the observational data and the reason for this is supposed to be the presence of mysterious force known as dark energy (DE) (Riess et al. 1998; Perlmutter et al. 1999; Hanany et al. 2000; Peebles and Ratra 2003;

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Padmanabhan 2003; Komatsu et al. 2009). Two approaches have been proposed to explain this late time acceleration of the universe. One of them is to study the dynamics of DE model like the Chaplygin gas (Kamenshchik et al. 2001; Bento et al. 2002; Zhang et al. 2006), holographic models (Hsu 2004; Li 2004), new age graphic model (Cai 2008), the polytropic gas model (Karami et al. 2009) and the pilgrim model (Wei 2012; Sharif and Jawad 2013; Santhi et al. 2016, 2017a). The above models have been studied because of the fact that the simple DE model namely cosmological constant is plagued with the coincidence and other serious problems in general relativity. Another approach to explain the cosmic acceleration is to modify Einstein's theory of gravitation. Among the several modifications the theories which are significant are $f(R)$, $f(R, T)$ (Capozziello et al. 2003; Nojiri and Odintsov 2003; Harko et al. 2011) theories of gravity. Here we are, mainly, concerned with $f(R, T)$ gravity. In this particular theory, the gravitational Lagrangian has been taken as an arbitrary function of the Ricci scalar R and the trace T of the matter energy tensor.

There has been a lot of interest in the investigation of cosmological models in $f(R)$ and $f(R, T)$ theories of gravitation in the presence of various source terms which represent stress energy tensor. This is mainly because $f(R, T)$ gravity models describe early inflation and late time acceleration of the universe. A review of modified theories of gravity to explain dark energy is made available by Capozziello and De Laurentis (2011), Nojiri and Odintsov (2011) and Nojiri et al. (2017). FRW models are obtained in $f(R)$ gravity by Paul et al. (2009) while Sharif and Shamir (2009) and Shamir (2010) have discussed Bianchi type-*I*, *III*, *V* and Kantowski-Sachs models in this theory. Several authors (Sheykhi 2012; Katore 2015; Santhi et al. 2018; Aditya and Reddy 2018) have studied various anisotropic Bianchi type cosmological models in this theory under different physical

situations. In particular Reddy et al. (2012, 2014) have discussed Bianchi type-*III* and Kantowshi-Sachs cosmological model in $f(R, T)$ gravity in the presence of perfect fluid and bulk viscous strings as source while Reddy and Santhi Kumar (2013) have obtained some anisotropic models in the background of Bianchi type-*II* space time in this theory of gravitation. Mishra et al. (2015) have discussed non-static cosmological model in $f(R, T)$ gravity. Sahoo et al. (2016) have investigated Bianchi type string cosmological models in $f(R, T)$ gravity. Aditya et al. (2016) have studied various Bianchi type models in this theory of gravitation with variable Λ. Santhi et al. (2017b) have obtained Kantowski-Sachs scalar field cosmological models in this modified theory of gravity.

The study of scalar fields in general relativity has attracted the attention of many researchers because of their physical importance in cosmology. Scalar fields are supposed to cause the accelerated expansion of the universe and help to solve the horizon problem. In cosmology, scalar fields represent matter fields with spin less quanta and describe gravitational field. Scalar fields are of two types. They are zero mass scalar fields and massive scalar fields. Long range interactions are described by zero mass scalar fields while massive scalar field describe short range interactions. Mass less and massive scalar meson fields in general relativity have been extensively studied. FRW models in the presence of mass less scalar fields have been studied by Ellis (1971) while Mohanty and Pradhan (1992) have discussed attractive massive scalar field models in this particular space time. Singh and Ram (1996) and Singh (2005) have investigated cosmological models with viscous fluid in the presence of attractive massive scalar field. Singh (2009) obtained Bianchi type-*V* model in Lyra's (1951) geometry in the presence of massive scalar field. Singh and Rani (2015) presented Bianchi type-*III* cosmological model in Lyra's geometry in the presence of massive scalar field. Recently, Reddy (2018) obtained Bianchi type-*V* dark energy model with scalar meson fields in general relativity while Naidu (2018) investigated Bianchi type-*II* modified holographic Ricci dark energy model in the presence of attractive massive scalar field. It is worth mentioning that Bianchi models in $f(R, T)$ gravity in the presence of anisotropic dark energy and attractive massive scalar field have not been investigated in literature.

The main aim of the present work is to study the dynamics of Bianchi type-*III* cosmological model in the presence of perfect fluid and an attractive massive scalar meson field. Investigation of Bianchi type-*III* models are important in cosmology because they are among the simplest models with an anisotropic background. Our work in this paper is organized as follows: Sect. 2 consists of the general framework of $f(R, T)$ gravity. In Sect. 3, we derive the field equations $f(R, T)$ gravity in Bianchi type-*III* space-time in the

presence of perfect fluid and attractive massive scalar field. In Sect. 4, we solve the field equations and present the corresponding cosmological model in this theory of gravitation. The cosmological parameters corresponding to our model are computed and their physical significance with reference to the modern cosmology is discussed in Sect. 5. The last section is devoted to summary and conclusions.

2 Basic $f(R, T)$ theory field equations

Here, the field equations of $f(R, T)$ theory of gravity are obtained from the following action incorporating gravity, matter and scalar field (Harko et al. 2011; Sharif and Nawazish 2017):

$$S = \frac{1}{16\pi} \int f(R, T)\sqrt{-g}\, d^4x + \int (L_m + L_\varphi)\sqrt{-g}\, d^4x, \tag{1}$$

where L_m and L_φ represent matter and scalar field Lagrangian densities. Here gravity Lagrangian $f(R, T)$ admitting minimal coupling only with L_m and L_φ (Harko et al. 2011). Here T is the trace of the energy-momentum tensor of matter and R is the Ricci scalar. The combined energy-momentum tensor of matter and scalar field is defined as

$$\mathfrak{T}_{ij} = -\frac{2\delta(\sqrt{-g}(L_m + L_\varphi))}{\sqrt{-g}\,\delta g^{ij}}, \quad i, j = 1, 2, 3, 4. \tag{2}$$

By assuming that matter Lagrangian L_m and scalar field Lagrangian L_φ depend only on the metric tensor components g_{ij}, we have obtained the field equations of $f(R, T)$ gravity as

$$f_R(R, T)R_{ij} - \frac{1}{2}f(R, T)g_{ij} - (\nabla_i\nabla_j - g_{ij}\Box)f_R(R, T)$$
$$= 8\pi\mathfrak{T}_{ij} - f_T(R, T)(\mathfrak{T}_{ij} + \Theta_{ij}) \tag{3}$$

and

$$\Theta_{ij} = -2\mathfrak{T}_{ij} + g_{ij}(L_m + L_\varphi) - 2g^{lk}\frac{\partial^2(L_m + L_\varphi)}{\partial g^{ij}\partial g^{lm}}. \tag{4}$$

Here

$$\Box = \nabla^i\nabla_i,$$

$$f_R(R, T) = \frac{\partial f(R, T)}{\partial R}, \qquad f_T(R, T) = \frac{\partial f(R, T)}{\partial T}$$

and ∇_i denotes the covariant derivative.

For a perfect fluid, the energy-momentum tensor is

$$T_{ij} = (\rho_m + p_m)u_i u_j - p_m g_{ij} \tag{5}$$

where ρ_m and p_m define energy density and pressure, respectively whereas u_i represents the four-velocity vector of the fluid. Also, we assume an attractive massive scalar field whose energy-momentum tensor is given by

$$T_{ij}^{\varphi} = \varphi_{;i}\varphi_{;j} - \frac{1}{2}g_{ij}\left(\varphi_{;k}\varphi^{,k} - M^2\varphi^2\right), \tag{6}$$

where M is the mass of the scalar field φ which satisfies the Klein-Gordon equation

$$g^{ij}\varphi_{;ij} + M^2\varphi = 0. \tag{7}$$

Here a comma and a semicolon indicate ordinary and covariant differentiation respectively and φ is a function of time t. For the action (1), the Lagrangian density of perfect fluid and scalar fields are defined as (Harko et al. 2011; Sharif and Nawazish 2017)

$$L_m = -p_m, \qquad L_{\varphi} = \frac{1}{2}\left(M^2\varphi^2 - \dot{\varphi}^2\right). \tag{8}$$

Now with the use of Eqs. (4) and (8) we obtain the tensor Θ_{ij} as

$$\Theta_{ij} = -2\mathfrak{T}_{ij} - \frac{g_{ij}}{2}\left(2p_m + \dot{\varphi}^2 - M^2\varphi^2\right). \tag{9}$$

Generally, the field equations also depend through the tensor Θ_{ij}, on the physical nature of the matter field. Hence in the case of $f(R, T)$ gravity, depending on the nature of the matter source, we obtain several theoretical models corresponding to each choice of $f(R, T)$ given by (Harko et al. 2011)

$$f(R, T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T). \end{cases} \tag{10}$$

Assuming the function $f(R, T)$ as

$$f(R, T) = R + 2f(T) \tag{11}$$

where $f(T)$ is an arbitrary function of the trace of stress energy tensor of matter (perfect fluid), we get the gravitational field equations of $f(R, T)$ gravity from Eq. (3) as

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi\mathfrak{T}_{ij} - 2(\mathfrak{T}_{ij} + \Theta_{ij})f'(T) + f(T)g_{ij} \tag{12}$$

where the prime denotes differentiation with respect to the argument.

3 Metric and explicit field equations

Here we consider spatially homogeneous and anisotropic Bianchi type-*III* metric of the form

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)e^{-2\alpha x}dy^2 - C^2(t)dz^2, \tag{13}$$

where $A(t)$, $B(t)$ and $C(t)$ represent the scale factors of the universe. Here $\alpha \neq 0$ is a constant which can be set equal to unity. Also, we can obtain Bianchi type-*I* metric by choosing $\alpha = 0$. However the underlying geometry of Bianchi type-*I* and *III* metrics are completely different.

We assume the particular choice of the function (Harko et al. 2011)

$$f(T) = \lambda T, \qquad \lambda = constant. \tag{14}$$

Now using comoving coordinates and Eqs. (5)–(6) the explicit form of $f(R, T)$ gravity field equations (12) and the Klein-Gordon equation, for the metric (13), take the form:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = p_m(8\pi + 3\lambda) - \lambda\rho_m + 4\pi\left(\dot{\varphi}^2 - M^2\varphi^2\right) \tag{15}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = p_m(8\pi + 3\lambda) - \lambda\rho_m + 4\pi\left(\dot{\varphi}^2 - M^2\varphi^2\right) \tag{16}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2}$$
$$= p_m(8\pi + 3\lambda) - \lambda\rho_m + 4\pi\left(\dot{\varphi}^2 - M^2\varphi^2\right) \tag{17}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2}$$
$$= -\rho_m(8\pi + 3\lambda) + \lambda p_m - (4\pi + 2\lambda)\dot{\varphi}^2 - 4\pi M^2\varphi^2 \tag{18}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0. \tag{19}$$

Klein-Gordon equation is given by

$$\ddot{\varphi} + \dot{\varphi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + M^2\varphi = 0 \tag{20}$$

where an overhead dot denotes differentiation with respect to cosmic time t.

The following are the geometrical and physical parameters to be used in finding the solution of the $f(R, T)$ field equations for the Bianchi type-*III* space-time given by Eq. (13). The volume V and average scale factor $a(t)$ of the Bianchi type-*III* space time are defined as

$$V = \sqrt{-g} = ABC, \qquad a(t) = (ABC)^{\frac{1}{3}} \tag{21}$$

Anisotropic parameter A_h is given by

$$A_h = \frac{1}{3} \sum_{i=1}^{3} \left(\frac{H_i - H}{H} \right)^2 \tag{22}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's parameters and $H = \frac{1}{3}(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C})$ is mean Hubble's parameter.

Expansion scalar (θ) and shear scalar (σ^2) are defined as

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{23}$$

$$\sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij}$$
$$= \frac{1}{3} \left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC} \right) \tag{24}$$

where

$$\sigma_{ij} = \frac{1}{2} \left(u_{i;\mu} h_j^{\mu} + u_{j;\mu} h_i^{\mu} \right) - \frac{1}{3} \theta h_{ij}, \tag{25}$$

$h_{ij} = g_{ij} - u_i u_j$ is the projection tensor while $u_i = (1, 0, 0, 0)$ is the four-velocity in comoving coordinates. Deceleration parameter is given by

$$q = \frac{d}{dt} \left(\frac{1}{H} \right) - 1. \tag{26}$$

4 Solution and the model

From Eq. (19), we get

$$A = c_1 B \tag{27}$$

where c_1 is an integration constant which can be taken as unity without any loss of generality so that we have

$$A = B \tag{28}$$

In view of Eq. (28), the field equations (15)–(20) yields the following independent equations

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = p_m(8\pi + 3\lambda) - \lambda\rho_m + 4\pi(\dot{\varphi}^2 - M^2\varphi^2) \tag{29}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{1}{A^2} = p_m(8\pi + 3\lambda) - \lambda\rho_m + 4\pi(\dot{\varphi}^2 - M^2\varphi^2) \tag{30}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2}$$
$$= -\rho_m(8\pi + 3\lambda) + \lambda p_m - (4\pi + 2\lambda)\dot{\varphi}^2 - 4\pi M^2\varphi^2. \tag{31}$$

Klein-Gordon equation is given by

$$\ddot{\varphi} + \dot{\varphi} \left(2\frac{\dot{A}}{A} + \frac{\dot{C}}{C} \right) + M^2\varphi = 0 \tag{32}$$

Now Eqs. (29)–(32) are a system of four independent equations in five unknowns A, C, φ, p_m and ρ_m. Hence to find a determinate solution we use the following physically plausible conditions:

- The shear scalar σ^2 is proportional to scalar expansion θ so that we can taken (Collins et al. 1980)

$$C = A^n \tag{33}$$

  where $n \neq 1$ is a constant and preserves the isotropic character of the space-time.
- We use power law for φ as (Singh and Rani 2015)

$$\frac{\dot{\varphi}}{\varphi} = -(n + 2)\frac{\dot{A}}{A}. \tag{34}$$

Here the question of over determinacy is settled by satisfying the field equations. Now from Eqs. (32)–(34), we get the scalar field $\varphi(t)$ as

$$\varphi = \exp\left(\varphi_0 t - \frac{M^2 t^2}{2} + \varphi_1 \right) \tag{35}$$

and the metric potentials are obtained by

$$A = A_1 \exp\left(\frac{\frac{1}{2}M^2 t^2 - \varphi_0 t + \varphi_1}{n + 2} \right)$$
$$C = A_1^n \exp\left(\frac{\frac{n}{2}M^2 t^2 - n\varphi_0 t + n\varphi_1}{n + 2} \right) \tag{36}$$

where A_1, φ_0 and φ_1 are integrating constants. Now the metric (13) with the help of Eq. (36) can be written as

$$ds^2 = dt^2 - \left(A_1^2 \exp\left(\frac{M^2 t^2 - 2\varphi_0 t + 2\varphi_1}{n + 2} \right) \right)$$
$$\times \left(dx^2 + e^{-2x} dy^2 \right)$$
$$- A_1^{2n} \exp\left(\frac{nM^2 t^2 - 2n\varphi_0 t + 2n\varphi_1}{n + 2} \right) dz^2. \tag{37}$$

The behavior of scalar field φ versus cosmic time t is shown in Fig. 1. The scalar field exhibits a rapid increase from very low value, goes towards maximum value and after a short interval of time rapidly approaches zero. The rapid increasing behavior is because of the exponential potential and it is quite similar to cosmological scaling solutions obtained by Copeland et al. (2006) and the behavior of the quintessence scalar field described by Sharif and Jawad (2012), Sharif and Rani (2013) and Chattopadhyay and Debnath (2011).
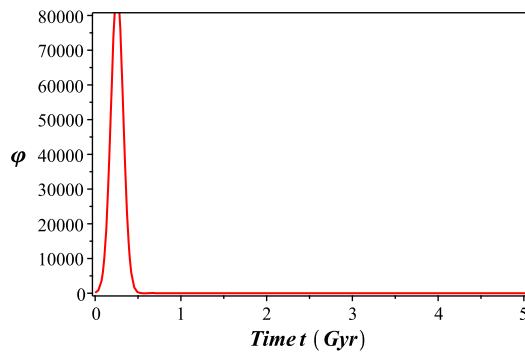
Fig. 1 Plot of massive scalar field φ versus cosmic time t for $M = 14$, $\varphi_0 = 50$, $n = 0.95$, $\varphi_1 = 5$

5 Cosmological parameters and discussion

Equation (37) along with Eq. (35) represents Bianchi type-*III* universe with attractive massive scalar fields in $f(R, T)$ modified theory of gravity along with the following physical and geometrical parameters which are very important in the discussion of cosmology.

Spatial volume

$$V(t) = A_1^{n+2} \exp\left(\frac{1}{2}M^2t^2 - \varphi_0 t + \varphi_1\right). \tag{38}$$

Average scale factor is given by

$$a(t) = \left(A_1^{n+2} \exp\left(\frac{1}{2}M^2t^2 - \varphi_0 t + \varphi_1\right)\right)^{\frac{1}{3}}. \tag{39}$$

The mean Hubble parameter is

$$H = \frac{1}{3}\left(M^2 t - \varphi_0\right). \tag{40}$$

The scalar expansion is

$$\theta = M^2 t - \varphi_0. \tag{41}$$

The shear scalar is

$$\sigma^2 = \frac{(M^2 t - \varphi_0^2)^2}{2}\left(\frac{n^2 + 2}{(n+2)^2} - \frac{1}{3}\right). \tag{42}$$

The average anisotropic parameter is

$$A_h = \frac{9}{(M^2 t - \varphi_0)}$$
$$\times \left\{2\left\{\frac{1}{3} - \frac{A_1}{n+2}\exp\left(\frac{\frac{1}{2}M^2t^2 - \varphi_0 t + \varphi_1}{n+2}\right)\right\}^2 + \left\{\frac{1}{3} - \frac{nA_1}{n+2}\exp\left(\frac{\frac{1}{2}M^2t^2 - \varphi_0 t + \varphi_1}{n+2}\right)\right\}^2\right\}. \tag{43}$$

Now using Eqs. (35) and (36) in the field equations (15)–(20) we get the isotropic pressure p_m as

$$p_m = \frac{1}{3(8\pi + 3\lambda)^2 - 3\lambda^2}\left\{(8\pi + 3\lambda)\right.$$
$$\times \left\{\frac{4}{A_1}\left(\frac{A_1 M^2}{n+2} + \frac{A_1(M^2 t - \varphi_0)^2}{(n+2)^2}\right)\right.$$
$$\left. + 2\left(\frac{n^2(M^2 t - \varphi_0)^2}{(n+2)^2}\left(1 + \frac{nM^2}{n+2}\right)\right)\right\}$$
$$+ 8\pi\left(\frac{(M^2 t - \varphi_0)^2}{(n+2)^2}(2n+1)\right.$$
$$\left. - \frac{1}{A_1^2}\left(\exp\left\{\frac{-M^2t^2 + 2\varphi_0 t + 2\varphi_1}{n+2}\right\}\right)\right)$$
$$\left. + 24\pi M^2\left(\exp\{M^2t^2 - 2\varphi_0 t + 2\varphi_1\}\right)(4\pi + \lambda)\right\} \tag{44}$$

energy density ρ_m can be obtained as

$$\rho_m = \frac{1}{3(8\pi + 3\lambda)^2 - 3\lambda^2}\left\{\frac{4\lambda}{A_1}\left\{\frac{A_1 M^2}{n+2} + \frac{A_1(M^2 t - \varphi_0)^2}{(n+2)^2}\right\}\right.$$
$$- (24\pi + 8\lambda)\left\{\frac{(M^2 t - \varphi_0)^2(2n+1)}{(n+2)^2}\right.$$
$$\left. - \frac{1}{A_1^2}\exp\left\{\frac{-M^2t^2 + 2\varphi_0 t - 2\varphi_1}{n+2}\right\}\right\}$$
$$+ 2\lambda\left\{\frac{n^2(M^2 t - \varphi_0)^2}{(n+2)^2} + \frac{M^2 n}{n+2}\right\}$$
$$- \left(24\pi M^2(4\pi + \lambda) - 6(M^2 t - \varphi_0)^2\right)$$
$$\times \left(3\lambda^2 + 12\lambda\pi + 16p^2\right))$$
$$\left. \times \left(\exp\{M^2t^2 - 2\varphi_0 t + 2\varphi_1\}\right)\right\} \tag{45}$$

The behavior of energy density ρ_m and isotropic pressure p_m are shown in Figs. 2 and 3 respectively. It can be observed that the energy density ρ_m is positive whereas the pressure p_m is negative throughout the evolution of the model. This should be the case because we are considering $f(R, T)$ gravity which has been formulated to explain accelerated expansion of the universe. Also, the rapid change in the behavior of these parameters (p_m and ρ_m) is because of the exponential expansion of the model.

5.1 Deceleration parameter

The sign of deceleration parameter (q) shows whether the model accelerates or decelerates. The positive sign of q indicates standard decelerating model whereas the negative sign $-1 < q < 0$ corresponds to accelerated expansion of
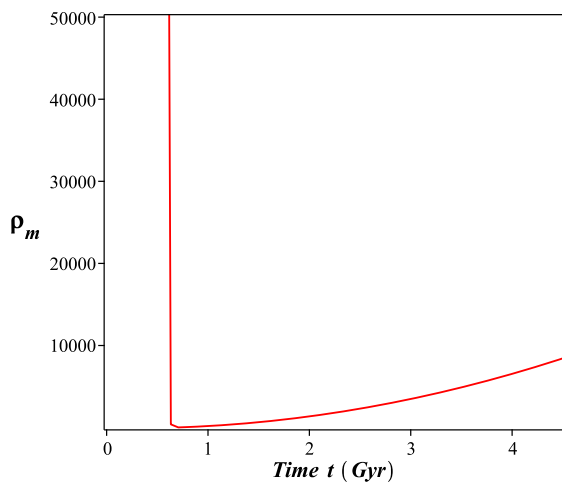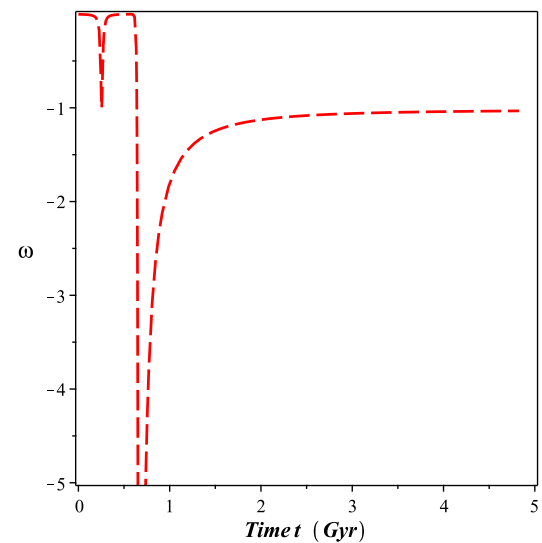
Fig. 2 Plot of energy density ρ_m versus cosmic time t for $M = 14$, $\varphi_0 = 50$, $n = 0.95$, $\varphi_1 = 5$, $A_1 = 55$, $\lambda = -13$
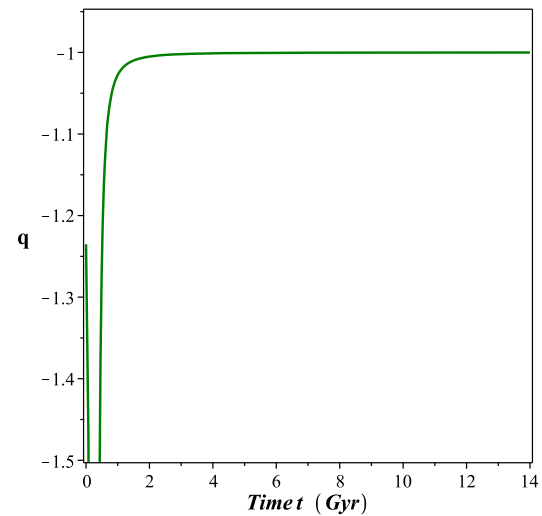
Fig. 4 Plot of deceleration parameter versus cosmic time t for $M = 14$ and $\varphi_0 = 50$
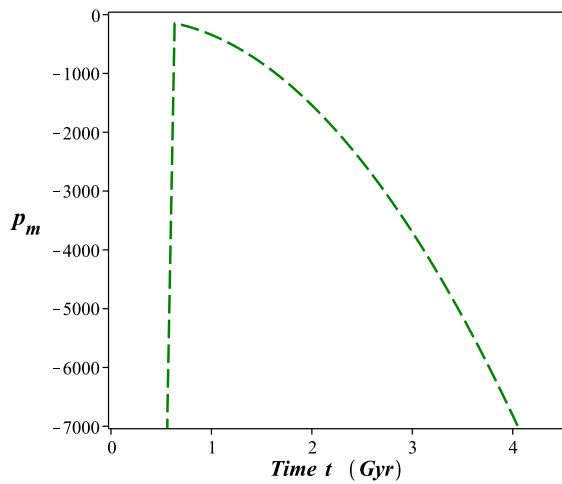
Fig. 3 Plot of pressure versus cosmic time t for $M = 14$, $\varphi_0 = 50$, $n = 0.95$, $\varphi_1 = 5$, $A_1 = 55$, $\lambda = -13$

Fig. 5 Plot of EoS parameter versus cosmic time t for $M = 14$, $\varphi_0 = 50$, $n = 0.95$, $\varphi_1 = 5$, $A_1 = 55$, $\lambda = -13$

the universe. If $q = -1$, the universe exhibits exponential expansion and super-exponential expansion if $q < -1$. In our model, the deceleration parameter q is given by

$$q = -1 - \frac{3M^2}{(M^2 t - \varphi_0)^2}. \tag{46}$$

In Fig. 4, we show the evolution of deceleration parameter q versus cosmic time t. It is observed that the deceleration parameter varies in negative region and hence the evolution of our model is in accelerating phase at present epoch. This behavior is in good agreement with the modern observations. Also, we observe that we have a super-exponential ($q < -1$) model at initial epoch and finally approaching exponential expansion ($q = -1$).

5.2 Equation of state parameter

The equation of state (EoS) parameter is defined as follows

$$\omega = \frac{p_m}{\rho_m} \tag{47}$$

where pressure p_m and energy density ρ_m are given in Eqs. (44) and (45) respectively.

We plot EoS parameter versus cosmic time as shown in Fig. 5. It can be observed that the EoS parameter ω starts from quintessence dark energy era ($-1 < \omega < -1/3$) and goes towards phantom dark energy ($\omega < -1$) era by evolving the vacuum dark energy era ($\omega = -1$). It is also observed that after some finite time t the model approaches to ΛCDM
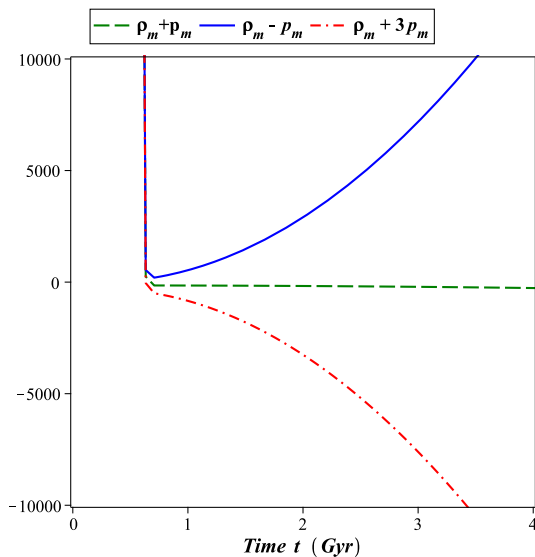
Fig. 6 Plot of energy conditions versus cosmic time t for $M = 14$, $\varphi_0 = 50$, $n = 0.95$, $\varphi_1 = 5$, $A_1 = 55$, $\lambda = -13$

model ($\omega = -1$). Recent studies (Spergel et al. 2003; Riess et al. 2007; Eisenstein et al. 2005; Astier et al. 2006; Bamba et al. 2012) indicate that the model approaches to ΛCDM served as an excellent models to describe the cosmological evolution.

5.3 Energy conditions

It is well known that the energy conditions play an important role in the study of recent scenario of accelerating expansion of the universe. Hence, we now analyze the energy conditions for our massive scalar field model in $f(R,T)$ modified theory of gravity. The energy conditions are given, respectively, by:

- Null energy conditions (NEC): $\rho_m + p_m > 0$
- Weak energy conditions (WEC): $\rho_m \geq 0$, $\rho_m + p_m \geq 0$
- Dominant energy condition (DEC): $\rho_m \geq 0$, $\rho_m \pm p_m \geq 0$
- Strong energy conditions (SEC): $\rho_m + 3p_m > 0$, $\rho_m + p_m \geq 0$.

The violation of NEC and WEC immediately leads to the violation of the other energy conditions. This in turn leads to the reduction of energy density with the expansion of the universe. On the other hand the violation of SEC results in the accelerated expansion of the universe. Within the framework of modified theories of gravitation, these energy conditions have been widely investigated (Santos et al. 2007; Sadeghi et al. 2012; Jawad et al. 2013). The plot of NEC for our model versus cosmic time t is shown in Fig. 6. We see that throughout evolution $\rho_m + p_m < 0 \Longrightarrow \omega < -1$. This indicates the violation of the NEC as well as the remaining energy conditions. For our model SEC are violated and hence we have an accelerating model of the universe.

6 Concluding remarks

Dark energy is one of the attracting and interesting subjects of modern cosmology which leads to accelerated expansion of the universe. In order to explain this cosmic acceleration, various approaches have been adopted through many dynamical dark energy models and modified theories of gravitation. The purpose of this paper is to discuss this phenomenon by studying the dynamics of Bianchi type-*III* perfect fluid cosmological model in the presence of massive scalar field in $f(R,T)$ modified theory of gravity. We have obtained the deterministic solution of the field equations which leads to the varying deceleration parameter. We have concluded our results as follows:

- The scale factors and volume of the model admit constant value at initial epoch i.e., at $t = 0$ and then start increasing exponentially with cosmic time approaching to very large values as $t \to \infty$. Also, the model does not have any type of initial singularity.
- The mean Hubble parameter and expansion scalar are constant at initial epoch indicating homogeneous expansion of the universe and we have accelerated expansion with evolution.
- The scalar field exhibits a rapid increase from very low value, goes towards maximum value and after a short interval of time rapidly approaches zero (Fig. 1). The rapid increasing behavior is because of the exponential potential and it is quite similar to cosmological scaling solutions obtained by Copeland et al. (2006) and the behavior of the quintessence scalar field described by many authors (Chattopadhyay and Debnath 2011; Sharif and Jawad 2012; Sharif and Rani 2013).
- The energy density ρ_m is positive whereas the pressure p_m is negative throughout the evolution of the model (Figs. 2 and 3). This should be the case because we are considering $f(R,T)$ gravity which has been formulated to explain accelerated expansion of the universe. Also, the rapid change in the behavior of all these parameters (φ, p_m and ρ_m) is because of the exponential expansion of the model.
- It is observed that the deceleration parameter varies in negative region and hence the evolution of our model is in accelerating phase at present epoch (Fig. 4). This behavior is in good agreement with the modern observations.
- We have observed that the model evolves from quintessence dark energy era ($-1 < \omega < -1/3$) and goes towards phantom dark energy ($\omega < -1$) era by crossing the phantom divided line or the vacuum dark energy era ($\omega = -1$). It is also observed that after some finite time t the model approaches to ΛCDM model ($\omega = -1$). Recent studies (Spergel et al. 2003; Riess et al. 2007; Eisenstein et al. 2005; Astier et al. 2006; Bamba et al. 2012) indicate that the model approaches to ΛCDM

served as an excellent models to describe the cosmological evolution. Hence our model is in good agreement with these observations.

- The study of energy conditions shows that the NEC and SEC are violated and hence we have a model representing accelerated expansion of the universe.

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