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ORIGINAL ARTICLE

Structure scalars of spherically symmetric dissipative fluids with $f(G,T)$ gravity

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Abstract The aim of this study is to analyze the role of modification of gravity on some dynamical properties of spherically symmetric relativistic systems. In this settings, the mathematical modeling of scalar variables associated with the shearing viscous dissipative anisotropic spherical stars is explored. We assume that the non-static diagonally symmetric spherical structure is coupled with a relativistic matter content in the presence of $f(G,T)=\alpha G^n+\beta\ln[G]+\lambda T$ gravity. After using Misner–Sharp mass function, we have made correspondence between metric scale factors, tidal forces and structure parameters. We have adopted Herrera's technique for the orthogonally breaking down of the Riemann tensor, in order to formulate modified forms of structure scalars. The role of these invariants is then explored in the evolutionary properties of radiating spheres. The parameters responsible for the outbreak of inhomogeneities are being examined in the presence and absence of constant $f(G,T)$ terms. It is inferred that the evolutionary phases of the spherical interiors can be well studied via extended versions of scalar variables.

Keywords Gravitation · Structure scalars · Relativistic dissipative fluids

1 Introduction

From the last recent decades, the remarkable endeavors in the understanding of our cosmic evolution have been seen in literature. Generally, it is believed that almost 95% energy distribution of our cosmos constitutes enigmatic unknown components known as dark energy (DE) and dark matter (DM). Their nature, as well as properties, are still unknown, thus indicating that a radically huge window of relativistic astrophysics needs to explore. The existence of DM constituent is firmly established from the current sound large-scale astrophysical data. It is almost a general belief that a mysterious force produced by DE is a reason behind the recent acceleration of our expanding cosmos. The source of the recently observed acceleration in the cosmic expansion is a debatable issue in this era of modern cosmology (Nojiri and Odintsov 2006; Bamba et al. 2012a). The possible explanation for such dark cosmic sector was given by the cosmological constant proposed by Einstein. Nevertheless, the tuning of the expected amplitude of this constant is a hard task for reconciling the quantum features of the vacuum space. Other possible platforms for understanding the nature and properties of dark sector terms include modified gravity theories (MGTs). Such theories are based on the modification of the gravitational component of the Einstein–Hilbert action (EHA) (for details, please see Capozziello and De Laurentis 2011; Bamba et al. 2012b, 2013; Yousaf et al. 2016a,b; Nojiri and Odintsov 2007, 2008).

Nojiri and Odintsov (2007) reviewed a few attractive candidates of MGTs and found few cosmic models that have passed the solar system tests. They also discussed in detail the late time phases of our expanding cosmos in an account of dark sector terms with an inhomogeneous state equations. Yousaf and Bhatti (2016a) studied the effects of some models of MGT on the occurrence of stellar interiors and concluded that dark sector terms due to MGT probably to support relatively super-massive more compact structures than that in GR. Apart from the notable $f(R)$ theory (in which the Ricci scalar R is replaced with its generic func-

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tion in EHA), it could be worthwhile to include the amalgams of the curvature originating terms, like Riemann tensor ($R_{\gamma\epsilon\mu\nu}$), Ricci tensor ($R_{\gamma\epsilon}$) and R in EHA. This led the theoretical physicists to develop $f(G)$ theory, where $G = R - 4R_{\gamma\epsilon}R^{\gamma\epsilon} + R_{\gamma\epsilon\alpha\beta}R^{\gamma\epsilon\alpha\beta}$ initiated firstly by Nojiri and Odintsov (2005). Such MGTs could lead us to realize inflationary and accelerating transition stages of our cosmos. Recently, this gravitational model is extended by admitting degrees of freedom mediating from the trace of energy-momentum tensor (T) in action of $f(G)$ gravity and widely known as $f(G,T)$ gravity. The motivation for including such terms stem from the justification of including T correction in the usual $f(R)$ theory by Harko et al. (2011).

Houndjo (2012) studied some dynamical perspectives of our expanding universe in the matter dominated era with the help of $f(R,T)$ gravity theoretical models. Bamba et al. (2012b) analyzed our expanding and acceleratory universe with the help of some modified gravity models. Further, one can study the gravitational aspects of the instabilities within the system of compact stellar models through modified gravity (Capozziello et al. 2011, 2012; Astashenok et al. 2013). Yousaf along with his group examined the pace of collapse for the planar (Bhatti et al. 2017a,b), spherical (Sharif and Yousaf 2015a; Yousaf 2017a; Yousaf et al. 2017d) and cylindrical (Yousaf and Bhatti 2016b) systems in the backgrounds of modified gravity. Ilyas et al. (2017) found reasonable stable spherical stellar models with the help of exponential and quadratic $f(R,T)$ models. Recently, Moraes et al. (2017) explored the hydrostatic state of strange stars in order to investigate their stable regimes with $f(R,T) = R + 2\lambda T$ model. Recently, Bhatti et al. (2018) performed computational simulations to check the stable regions of some strange stars with the help of logarithmic $f(G,T)$ gravity. The theoretical formations of wormholes (Sahoo et al. 2017a; Bambi et al. 2016) and some analytical cosmological solutions are mentioned in (Sahoo et al. 2017b; Shamir and Sadiq 2018; Yousaf 2017b). Olmo and Rubiera-Garcia (2011, 2015) discussed the dynamical properties of some stellar structures within the background of modified gravity with Palatini approach. Herrera et al. explored the problem of cylindrical (Di Prisco et al. 2007) and spherical (Herrera and Santos 2005) fate of collapse with the help of matching conditions. Mishra et al. (2017) analyzed the role of anisotropic pressure on the interesting phenomenon of reconstruction in modified gravity. Sharif and Yousaf (2014) extended such analysis by including the dark source terms mediated by the modified gravity. Recently, Yousaf et al. (2017b,c) found some viable and theoretically well-consistent matching condition for the joining of interior cylindrically symmetric metric with an outer region of Einstein–Rosen bridge.

The investigation for exploring the reason behind the emergence of inhomogeneous energy density (IED) has attracted many researchers not only in the field of GR but also in modified gravity. A tremendous work on gravitational collapse was rendered by Oppenheimer and Snyder (1939) and his collaborators. Hawking and Israel (1979) found a specific relation between tidal forces and matter variables for exploring the factors involved in IED over the spherical structure. Herrera et al. (1998) analyzed the formation of naked singularity (NS) with the help of factors of IED and pressure anisotropy in the matter distribution of the spherical system. Virbhadra et al. (1998), Virbhadra and Ellis (2002) presented a mathematical criterion under which one can analyze the difference between NS and black holes formation as a resultant of stellar collapse.

Herrera et al. (2004) used Einstein field equation for the dissipative spheres and calculated and calculated a peculiar relation among IED, tidal forces and pressure anisotropy. Further, Herrera et al. (2011a) found Raychaudhuri (also called as expansion evolution equation (EEE)) and shear evolution equation (SEE) with the help of well-known structure scalars. Such scalars can be calculated from the orthogonal breaking down of the Riemann curvature tensor. Herrera et al. (2009) firstly found these scalar quantities for the case of spherical system and after that many people extended their work by including various interesting physical parameters in the analysis. Sharif and Yousaf (2015b) modified their analysis by including the dark source terms mediating from late-time acceleratory modified gravity model. Bhatti along with his team Bhatti and Yousaf (2017), Bhatti et al. (2017c) found the effects of fourth order and second order gravity models on the existence and maintenance of IED over the surface of regular collapsing structures. Moreover, Herrera et al. (2011b) and Herrera (2017) introduced the notion of tilted congruences and calculated the corresponding equations for discussing the pace of gravitational collapse observed by an observer associated with the noncomoving reference frame. Yousaf et al. (2017a) extended their analysis by including correction coming from Palatini $f(R)$ gravity.

The phenomenon of gravitational implosion begins when the hydrostatic equilibrium of a stellar self-gravitating structure is disturbed. If during evolutionary phases of a celestial object, the gas pressure is inadequate to reinforce the gravitational attractive forces then, the star structure will enter into the collapse window. The resultant of this process gives compact structures, like black hole, neutron star or white dwarf, etc. It worthy to note that no body can experience this phenomenon unless it experiences an irregular and inhomogeneous state of energy density. Thus, it clearly points out that in the study of stellar collapse, one must know the factors responsible for creating the inhomogeneous state. The aim of this work is to explore inhomogeneity factors through set of modified versions of structure scalars in a specific $f(G, T) = \alpha G^n + \beta \ln[G] + \lambda T$ gravity model.

Joshi and Singh (1995) explored the final stages of gravitational collapse of the collection of non-interacting inhomogeneous particles and found that the end state of the spherically symmetric body is a black hole, if the dimensionless parameter coming in the solution has a specific range. Pinheiro and Chan (2011) studied the collapse rate of inhomogeneous dissipative and shearing anisotropic spherically symmetric systems. They, after comparing their results with the literature found the impact of inhomogeneous data sets of energy density on gravitational collapse. Sharma (2014) has found the significant contribution of inhomogeneous initial data of fluid distribution on the collapse rate. They found the relatively fast size shrinking process of the heat radiating star body.

A larger number of possibilities has been seen in the books and works published in the field of relativistic astrophysicist, to incorporate the dark sector terms on the study of the structure formation of the universe. In this direction, the impact of $f(G, T)$ terms could provide an effective platform to resolve DE and DM problem. Shamir and Ahmad (2018a) performed the analytical approach to investigate the existence of alternative to the black hole structure (gravastar) in $f(G, T)$ gravity. Furthermore, the same authors Shamir and Ahmad (2018b) evaluated extended the hydrostatic equation for self-gravitating systems and found relativity more stable star structures due to $f(G, T)$ terms. Therefore, it could be worthwhile to understand the physical causes of irregularity factors of dissipative spherically symmetric systems in $f(G, T)$ gravity.

We use the analytical approach provided by the previously published paper of Herrera et al. (2011a). We would like to explore the factors responsible for creating IED on the radiating anisotropic spherical system with the well-known modified gravity toy model. We consider a particular choice of $f(G, T)$ corrections coming from $f(G, T) = \alpha G^n + \beta \ln[G] + \lambda T$. The paper is organized as below. The coming section is devoted to providing describe some fundamental formalism to understand $f(G, T)$ gravity as well as spherical dissipative viscous matter configurations. Section 3 describes the formation of $f(G, T)$ structure scalars after the orthogonal decomposing of the Riemann tensor with linear T and $\alpha G^n + \beta \ln[G]$ Gauss–Bonnet terms. We shall also express three notable equations, i.e., SEE, EEE and Weyl differential equation (WEE) with the help of modified structure scalars in this gravity. Further, the Sect. 4 examines the IED factors for the case of smooth dust relativistic ball with recent G and T choices. The last section will describe our results.

2 Spherical viscous spherical system and $f(G, T)$ gravity

The modified version of EHA in the background of $f(G, T)$ gravity can be given as

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left[\frac{R}{2} + f(G, T) \right] + S_M \left(g^{\mu\nu}, \psi \right), \qquad (1)$$

where R, g are the traces of Ricci and metric tensors, respectively. Further, S_M is the matter action and κ^2 stands for the coupling constant. We shall take this to be unity in our calculations. The quantity G can be specified through the Ricci scalar (R), tensor ($R_{\mu\nu}$) and the Riemann tensors ($R_{\mu\nu\alpha\beta}$) as

$$G = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R.$$

In our paper, we shall write f_X notation for the derivations of the subsequent quantities with the quantity X. The quantity T appearing in Eq. (1) can be defined generically, through matter Lagrangian L_m, as

$$T = g^{\gamma\delta} T_{\gamma\delta}, \quad \text{with } T_{\gamma\delta} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{\gamma\delta}}, \qquad (2)$$

which further provides

$$T_{\gamma\delta} = g_{\gamma\delta} L_m - \frac{2\partial L_m}{\partial g_{\gamma\delta}}. \qquad (3)$$

The δ variations of the above equation gives rise to

$$\frac{\delta T_{\gamma\delta}}{\delta g^{\mu\nu}} = \frac{\delta g_{\gamma\delta}}{\delta g^{\mu\nu}} L_m - \frac{2\partial^2 L_m}{\partial g^{\mu\nu} \partial g^{\gamma\delta}} + g_{\gamma\delta} \frac{\partial L_m}{\partial g^{\mu\nu}}. \qquad (4)$$

Furthermore, the metric tensor variations of Eq. (1) along with Eq. (4), yield the $f(G, T)$ field equations as

$$G_{\gamma\delta} \equiv R_{\gamma\delta} - \frac{1}{2} R g_{\gamma\delta} = T^{\mathit{eff}}_{\gamma\delta}, \qquad (5)$$

where

$$\begin{aligned} T^{\mathit{eff}}_{\gamma\delta} &= \kappa^2 T_{\gamma\delta} - (T_{\gamma\delta} + \Theta_{\gamma\delta}) f_T(G, T) + \frac{1}{2} g_{\gamma\delta} f(G, T) \\ &\quad - \big(2R R_{\gamma\delta} - 4 R^{\varepsilon}_{\gamma} R_{\varepsilon\beta} - 4 R_{\gamma\varepsilon\delta\eta} R^{\varepsilon\eta} \\ &\quad + 2 R^{\varepsilon\eta\mu}_{\gamma} R_{\delta\varepsilon\eta\mu} \big) f_G(G, T) - \big(2R g_{\gamma\delta} \nabla^2 - 2R \nabla_\gamma \nabla_\delta \\ &\quad - 4 R_{\gamma\delta} \nabla^2 - 4 g_{\gamma\delta} R^{\varepsilon\eta} \nabla_\varepsilon \nabla_\eta + 4 R^{\varepsilon}_{\gamma} \nabla_\delta \nabla_\varepsilon \\ &\quad + 4 R^{\varepsilon}_{\delta} \nabla_\gamma \nabla_\varepsilon + 4 R_{\gamma\varepsilon\delta\eta} \nabla^\varepsilon \nabla^\eta \big) f_G(G, T), \end{aligned}$$

is the effective energy-momentum tensor for $f(G, T)$ gravity. Such modified theories can be treated as among the excellent candidates for toy model gravities This could assists to have some hidden insights about the inflationary and late time cosmic evolution. In the above equation, $\nabla^2 \equiv \nabla_\gamma \nabla^\gamma$ with ∇_γ stands for covariant derivations and

$$\Theta_{\gamma\delta} = g^{\mu\nu} \frac{\delta T_{\mu\nu}}{\delta g_{\gamma\delta}}. \tag{6}$$

Equations (6) and (4) provide

$$\Theta_{\gamma\delta} = g_{\gamma\delta} L_m - 2T_{\gamma\delta} - 2g^{\mu\nu} \frac{\partial^2 L_m}{\partial g^{\gamma\delta} \partial g^{\mu\nu}}.$$

The trace of $f(G,T)$ field equations (5) gives

$$T + R - (\Theta + T) f_T + 2G f_G + 2f - 2R\nabla^2 f_G + 4R_{\gamma\delta}\nabla^\gamma \nabla^\delta f_G = 0.$$

In this paper, we wish to test GR results at cosmological scales with the help of $f(G,T)$ theory. We also wish to check the influences of heat flux vector (q_γ), anisotropic pressure $\Pi \equiv P_r - P_\perp$ and radiation density (ε) in the dynamical features of spherical stars. Therefore, we consider the usual matter form with the stress-energy tensor as follows

$$T_{\lambda\nu} = \mu V_\lambda V_\nu + P_\perp h_{\lambda\nu} + \Pi \chi_\lambda \chi_\nu - 2\eta \sigma_{\lambda\nu} + \varepsilon l_\lambda l_\nu + q(\chi_\nu V_\lambda + \chi_\lambda V_\nu), \tag{7}$$

where ξ^β, V_β and l^β are four vectors, which under non-tilted reference frame obey

$$\chi^\nu \chi_\nu = 1, \quad V^\nu V_\nu = -1, \quad \chi^\nu V_\nu = 0,$$
$$l^\nu V_\nu = -1, \quad V^\nu q_\nu = 0, \quad l^\nu l_\nu = 0,$$

relations. Moreover, η is the coefficient of the scalar associated with the shear tensor $\sigma_{\gamma\delta}$, while projection tensor $(h_{\gamma\delta})$ that can be expressed as $h_{\gamma\delta} = g_{\gamma\delta} + V_\gamma V_\delta$.

It has been noticed that the modified gravity could provide very interesting results about the evolution of the universe. In this respect, it has been seen that one can examine the existence of a finite-time future singularity through Ricci squared terms that was proposed firstly by Abdalla et al. (2005). Furthermore, the early-time cosmic acceleration, elimination of the future singularity, or the inflationary phase and many other phenomenological issue (Kobayashi and Maeda 2009; Sharif and Yousaf 2016) can be dealt through modified gravity models. In this direction, we consider the $f(G,T)$ corrections in separate G and T forms as follows

$$f(G,T) = f(G) + g(T), \tag{8}$$

that provide some T corrections in the already developed $f(G)$ background that was firstly introduced and discussed by Nojiri and Odintsov (2005). Here, we take linear $g(T)$ function, i.e., $g(T) = \lambda T$, then the above equation turns out to be

$$f(G,T) = f(G) + \lambda T,$$

with $\lambda \in \mathbb{R}$, where $\mathbb{R}$ indicates the set of real numbers. We consider the logarithmic as well as power law Gauss–Bonnet with α, n and β as constant terms. Such model has been found to compatible with cosmographic parameters (Setare and Mohammadipour 2012). We consider (Schmidt 2011)

$$f(G) = \alpha G^n + \beta G \log[G]. \tag{9}$$

Motivated from the predictions of M-/string theory in the coupling of scalar field and the Gauss–Bonnet term, one can consider such theories as a reliable tool to understand the existence of non-singularities in the early time cosmic evolutions. One can also consider such a motivations to examine the late-time cosmologies with an effective formulations of Gauss–Bonnet dark energy models (Mavromatos and Rizos 2000).

We consider that the energy momentum tensor represented in Eq. (7) is the gravitational source of the following diagonal non-static irrotational spherical system as

$$ds^2 = B^2(t,r)dr^2 - A^2(t,r)dt^2 + C^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{10}$$

where A, B and C are assumed to be greater than zero. This system under comoving reference frame determines the following definitions

$$V^\nu = \frac{1}{A}\delta^\nu_0, \qquad \chi^\nu = \frac{1}{C}\delta^\nu_1,$$
$$l^\nu = \frac{1}{A}\delta^\nu_0 + \frac{1}{B}\delta^\nu_1, \qquad q^\nu = q(t,r)\chi^\nu.$$

The scalar quantities associated with expansion and shear tensors are found to be

$$\sigma A = \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right), \qquad \Theta_1 A = \left(\frac{\dot{B}}{B} + \frac{2\dot{C}}{C}\right),$$

where overdot indicates $\frac{\partial}{\partial t}$, while we shall describe $\frac{\partial}{\partial r}$ for radial differentiation. The $f(G,T)$ equations of motion for the systems (7) and (10) are

$$G_{00} = A^2\left[\mu + \varepsilon + \varepsilon\lambda - \frac{\alpha}{2}(1-n)G^n - \frac{\beta}{2}G - \frac{\lambda T}{2} - \frac{\varphi_{00}}{A^2}\right], \tag{11}$$

$$G_{01} = BA\left[-(1+\lambda)(q+\varepsilon) + \frac{\varphi_{01}}{BA}\right], \tag{12}$$

$$G_{11}=B^2\left[\mu\lambda+(1+\lambda)\left(P_r+\varepsilon-\frac{4}{3}\eta\sigma\right)+\frac{\alpha}{2}(1-n)G^n+\frac{\beta}{2}G+\frac{\lambda T}{2}-\frac{\varphi_{11}}{B^2}\right], \quad (13)$$

$$G_{22}=C^2\left[(1+\lambda)\left(P_\perp+\frac{2}{3}\eta\sigma\right)+\mu\lambda+\frac{\alpha}{2}(1-n)G^n+\frac{\beta}{2}G+\frac{\lambda}{2}T-\frac{\varphi_{22}}{C^2}\right], \quad (14)$$

where one can found $G_{\gamma\delta}$ from Herrera et al. (2011a). A peculiar form of the matter 4-velocity is defined as

$$U=D_TC=\frac{\dot{C}}{A}. \quad (15)$$

The matter quantity $\mathfrak{m}$ within the geometry of spherically symmetric spacetime can be given through formalisms provided by Misner–Sharp as follows (Misner and Sharp 1964)

$$\mathfrak{m}(t,r)=\frac{C}{2}\left(1+\frac{\dot{C}^2}{A^2}-\frac{C'^2}{H^2}\right), \quad (16)$$

whose variations with respect to their arguments can be expressed via Eqs. (11)–(13) and (15) as

$$D_T\mathfrak{m}=\frac{-1}{2}\left[U\left\{(1+\lambda)\left(\bar{P}_r-\frac{4}{3}\eta\sigma\right)+\lambda\mu+\frac{\alpha}{2}(1-n)G^n+\frac{\beta}{2}G+\frac{\lambda}{2}T+\frac{\varphi_{11}}{H^2}\right\}+E\left\{(1+\lambda)\bar{q}-\frac{\varphi_{01}}{BA}\right\}\right], \quad (17)$$

$$D_C\mathfrak{m}=\frac{C^2}{2}\left[\bar{\mu}+\lambda\varepsilon-\frac{\alpha}{2}(1-n)G^n-\frac{\beta}{2}G-\frac{\lambda T}{2}+\frac{\varphi_{00}}{A^2}-\frac{U}{E}\left\{\frac{\varphi_{01}}{AB}-(1+\lambda)\bar{q}\right\}\right], \quad (18)$$

where over bar notation means $\bar{H}=h+\varepsilon$. Another form of the mass function $\mathfrak{m}$ can be given by

$$\mathfrak{m}=\frac{1}{2}\int_0^C C^2\left[\bar{\mu}+\lambda\varepsilon-\frac{\alpha}{2}(1-n)G^n-\frac{\beta}{2}G-\frac{\lambda T}{2}+\frac{\varphi_{00}}{A^2}+\frac{U}{E}\left\{\frac{\varphi_{01}}{BA}+(1+\lambda)\bar{q}\right\}\right]dC, \quad (19)$$

where

$$E\equiv\frac{C'}{H}=\left[1+U^2-\frac{2\mathfrak{m}(t,r)}{C}\right]^{1/2}. \quad (20)$$

Equations (17)–(20) provide

$$\frac{3\mathfrak{m}}{C^3}=\frac{3\kappa}{2C^3}\int_0^r\left[\bar{\mu}+\lambda\varepsilon-\frac{\alpha}{2}(1-n)G^n-\frac{\beta}{2}G-\frac{\lambda T}{2}+\frac{\varphi_{00}}{A^2}+\frac{U}{E}\left\{(1+\lambda)\bar{q}+\frac{\varphi_{01}}{BA}\right\}C^2C'\right]dr, \quad (21)$$

which has expressed radiating shearing matter parameters, for instance spherical mass, energy density, heat conduction with $f(G,T)$ extra degrees of freedom. It is notable that the Weyl tensor can be decomposed into its magnetic and electric constituents labeled respectively by $H_{\alpha\beta}$ and $E_{\alpha\beta}$. Their definitions are found to be

$$H_{\alpha\beta}=\frac{1}{2}\epsilon_{\alpha\gamma\eta\delta}C^{\eta\delta}{}_{\beta\rho}V^\gamma V^\rho=\tilde{C}_{\alpha\gamma\beta\delta}V^\gamma V^\delta,$$

$$E_{\alpha\beta}=C_{\alpha\phi\beta\varphi}V^\phi V^\varphi,$$

in which $\epsilon_{\lambda\mu\nu\omega}\equiv\sqrt{-g}\eta_{\lambda\mu\nu\omega}$ along with $\eta_{\lambda\mu\nu\omega}$ as a Levi-Civita symbol. The electric part of the Weyl tensor can be expressed through V_γ and its scalar $\mathcal{E}$ as

$$E_{\lambda\nu}=\left[\chi_\lambda\chi_\nu-\frac{g_{\lambda\nu}}{3}-\frac{1}{3}V_\lambda V_\nu\right]\mathcal{E},$$

where

$$\mathcal{E}=\left[\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)-\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right]\frac{1}{2A^2}-\frac{1}{2C^2}-\left[-\left(\frac{A'}{A}-\frac{C'}{C}\right)\left(\frac{C'}{C}+\frac{B'}{B}\right)+\frac{C''}{C}-\frac{A''}{A}\right]\frac{1}{2B^2}. \quad (22)$$

The scalar $\mathcal{E}$ can be associated with power law and logarithmic $f(G,T)$ terms as

$$\mathcal{E}=\frac{1}{2}\left[\bar{\mu}+\lambda\varepsilon-(1+\lambda)(\bar{\Pi}-2\eta\sigma)-\frac{\alpha}{2}(1-n)G^n-\frac{\beta}{2}G-\frac{\lambda T}{2}-\frac{\varphi_{00}}{A^2}+\frac{\varphi_{11}}{B^2}-\frac{\varphi_{22}}{C^2}\right]-\frac{3}{2C^3}\int_0^r C^2\left[\bar{\mu}+\lambda\varepsilon-\frac{\alpha}{2}(1-n)G^n-\frac{\beta}{2}G-\frac{\lambda T}{2}+\frac{\varphi_{00}}{A^2}+\frac{U}{E}\left\{(1+\lambda)\bar{q}-\frac{\varphi_{01}}{BA}\right\}\right]C'dr, \quad (23)$$

in which $\bar{\Pi}$ is defined as $\bar{\Pi}\equiv\bar{P}_r-P_\perp$.

3 Structure scalars and $f(G,T)$ gravity

The aim of this section is to present the effects of modifications of GR on the computations of structure scalars that would be obtain from the orthogonal decomposition of Riemann tensor. We shall consider our analysis to test GR consequences at large scales with the help of $\alpha G^n+\beta G\ln[G]+\lambda T$ corrections. We shall also calculate three important differential equations, i.e., WDE, SEE and EEE via $f(G,T)$ structure scalars for the dissipative spherically symmetric spacetimes. After orthogonal breaking down of the Riemann tensor, we have come up with the following couple of tensorial quantities

$$X_{\alpha\beta}={}^{*}R^{*}_{\alpha\gamma\beta\delta}V^{\gamma}V^{\delta}=\frac{1}{2}\eta^{\varepsilon\rho}{}_{\alpha\gamma}R^{*}_{\epsilon\rho\beta\delta}V^{\gamma}V^{\delta},\\ Y_{\alpha\beta}=R_{\alpha\gamma\beta\delta}V^{\gamma}V^{\delta}, \tag{24}$$

where the right, both and left sterics on the quantities describe right, double and left duals, respectively. We would like to mention that such tensor equations of $X_{\alpha\beta}$ and $Y_{\alpha\beta}$ are apparently found to be same as found in GR by Herrera et al. (2004, 2009). These tensors could be used not only to unveil many hidden aspects of gravitational collapse see for instance (Herrera et al. 2004) but also in the modeling of the many stellar objects. The above tensors can be written alternatively through field equations as

$$\begin{aligned}X_{\gamma\delta}&=X^{(m)}_{\gamma\delta}+X^{(D)}_{\gamma\delta}\\&=\frac{1}{3}\left[\bar{\mu}+\lambda\varepsilon-\frac{\alpha}{2}(1-n)G^{n}-\frac{\beta}{2}G-\frac{\lambda T}{2}+\frac{\psi_{00}}{A^{2}}\right]h_{\gamma\delta}\\&\quad-\frac{1}{2}\left[(1+\lambda)(\bar{\Pi}-2\eta\sigma)-\frac{\psi_{11}}{B^{2}}+\frac{\psi_{22}}{C^{2}}\right]\\&\quad\times\left(\chi_{\gamma}\chi_{\delta}-\frac{1}{3}h_{\gamma\delta}\right)-E_{\gamma\delta},\end{aligned} \tag{25}$$

$$\begin{aligned}Y_{\gamma\delta}&=Y^{(m)}_{\gamma\delta}+Y^{(D)}_{\gamma\delta}\\&=\frac{1}{6}\left[\bar{\mu}+\lambda\varepsilon+3\mu\lambda+(1+\lambda)(3P_{r}-2\bar{\Pi})-\frac{\psi_{00}}{A^{2}}\right.\\&\quad\left.-\frac{\psi_{11}}{B^{2}}+\frac{2\psi_{22}}{C^{2}}+\frac{\alpha}{2}(1-n)G^{n}+\frac{\beta}{2}G+\frac{\lambda T}{2}\right]h_{\gamma\delta}\\&\quad+\frac{1}{2f_{R}}\left[(1+\lambda)(\bar{\Pi}-2\eta\sigma)-\frac{\psi_{11}}{B^{2}}+\frac{\psi_{22}}{C^{2}}\right]\\&\quad\times\left(\chi_{\gamma}\chi_{\delta}-\frac{1}{3}h_{\gamma\delta}\right)-E_{\gamma\delta}.\end{aligned} \tag{26}$$

Through trace (denoted with subscript T) and trace-less (labeled with subscript TF) components, these can be written as

$$X_{\gamma\delta}=\frac{1}{3}\mathrm{Tr}Xh_{\gamma\delta}+X_{\langle\gamma\delta\rangle}, \tag{27}$$

$$Y_{\gamma\delta}=\frac{1}{3}\mathrm{Tr}Yh_{\gamma\delta}+Y_{\langle\gamma\delta\rangle}, \tag{28}$$

where

$$X_{\langle\gamma\delta\rangle}=h^{\nu}_{\gamma}h^{\mu}_{\delta}\left(X_{\nu\mu}-\frac{1}{3}\mathrm{Tr}Xh_{\nu\mu}\right), \tag{29}$$

$$Y_{\langle\gamma\delta\rangle}=h^{\nu}_{\gamma}h^{\mu}_{\delta}\left(Y_{\nu\mu}-\frac{1}{3}\mathrm{Tr}Yh_{\nu\mu}\right). \tag{30}$$

By making use of Eqs. (23)–(26), we obtain

$$\begin{aligned}\mathrm{Tr}X\equiv X_{T}&=\left\{\bar{\mu}+\lambda\varepsilon-\frac{\alpha}{2}(1-n)G^{n}-\frac{\beta}{2}G-\frac{\lambda T}{2}\right.\\&\quad\left.-\frac{\lambda}{2}T+\frac{\hat{\psi}_{00}}{A^{2}}\right\},\end{aligned} \tag{31}$$

$$\begin{aligned}\mathrm{Tr}Y\equiv Y_{T}&=\left\{\bar{\mu}+\lambda\varepsilon+3\mu\lambda+3(1+\lambda)\bar{P}_{r}-\frac{\hat{\psi}_{11}}{B^{2}}\right.\\&\quad+\alpha(1-n)G^{n}+\beta G-\frac{\hat{\psi}_{00}}{A^{2}}\\&\quad\left.-2(1+\lambda)\bar{\Pi}-\frac{2\hat{\psi}_{22}}{C^{2}}-\lambda T\right\},\end{aligned} \tag{32}$$

where X_{TF} and Y_{TF} stand for the trace-free components of the tensors $X_{\alpha\beta}$ and $Y_{\alpha\beta}$, respectively (for details, please see Herrera et al. 2009). These tensors, $X_{\langle\alpha\beta\rangle}$ and $Y_{\langle\alpha\beta\rangle}$ can be expressed in an alternative way as follows

$$X_{\langle\gamma\delta\rangle}=X_{TF}\left(\chi_{\gamma}\chi_{\delta}-\frac{1}{3}h_{\gamma\delta}\right), \tag{33}$$

$$Y_{\langle\gamma\delta\rangle}=Y_{TF}\left(\chi_{\gamma}\chi_{\delta}-\frac{1}{3}h_{\gamma\delta}\right), \tag{34}$$

With the help of Eqs. (11)–(15), (28) and (29), it follows that

$$X_{TF}=-\mathcal{E}-\frac{1}{2}\left\{(\lambda+1)(-2\sigma\eta+\bar{\Pi})+\frac{\varphi_{22}}{C^{2}}-\frac{\varphi_{11}}{H^{2}}\right\}, \tag{35}$$

$$Y_{TF}=\mathcal{E}-\frac{1}{2}\left\{(\bar{\Pi}-2\eta\sigma)(\lambda+1)+\frac{\varphi_{22}}{C^{2}}-\frac{\varphi_{11}}{H^{2}}\right\}. \tag{36}$$

Equations (23) and (36) could provide Y_{TF} in the form

$$\begin{aligned}Y_{TF}&=\frac{1}{2}\left(\bar{\mu}+\varepsilon\lambda-\frac{\alpha}{2}(1-n)G^{n}-\frac{\beta}{2}G-\frac{\lambda T}{2}\right.\\&\quad\left.-2(1+\lambda)(\bar{\Pi}-4\eta\sigma)-\frac{\varphi_{00}}{A^{2}}+\frac{2\varphi_{11}}{H^{2}}-\frac{2\varphi_{22}}{C^{2}}\right)\\&\quad-\frac{3}{2C^{3}}\int_{0}^{r}\frac{C^{2}}{1+2R\lambda T^{2}}\left[\bar{\mu}-\frac{\alpha}{2}(1-n)G^{n}-\frac{\beta}{2}G\right.\\&\quad\left.-\frac{\lambda}{2}T+\varepsilon\lambda+\frac{\varphi_{00}}{A^{2}}+\frac{U}{E}\left\{(1+\lambda)\bar{q}+\frac{\varphi_{01}}{AB}\right\}C^{2}C'\right]dr.\end{aligned} \tag{37}$$

It would be worth while to manipulate our set of equations through arbitrarily defined dagger variables as

$$\mu^{\dagger}\equiv\bar{\mu}-\frac{\varphi_{00}}{A^{2}},\qquad P^{\dagger}_{r}\equiv\bar{P}_{r}-\frac{\varphi_{11}}{H^{2}}-\frac{4}{3}\eta\sigma,$$

$$P^{\dagger}_{\perp}\equiv P_{\perp}-\frac{\varphi_{22}}{C^{2}}+\frac{2}{3}\eta\sigma,$$

$$\Pi^{\dagger}\equiv P^{\dagger}_{r}-P^{\dagger}_{\perp}=\Pi-2\eta\sigma+\frac{\varphi_{22}}{C^{2}}-\frac{\varphi_{11}}{B^{2}}.$$

In view of the dagger variables, Eqs. (31), (32), (35) and (36) boil down to

$$X_{TF}=\frac{3}{2C^3}\int_0^r\left[\left\{\mu^\dagger-\frac{\alpha}{2}(1-n)G^n-\frac{\beta}{2}G-\frac{\lambda T}{2}+\lambda\varepsilon+\left(\hat{q}(\lambda+1)+\frac{\varphi_q}{BA}\right)\frac{U}{E}\right\}C^2C'\right]dr-\frac{1}{2}\left[\mu^\dagger-\frac{\alpha}{2}(1-n)G^n-\frac{\beta}{2}G-\frac{\lambda T}{2}+\lambda\varepsilon\right], \quad (38)$$

$$Y_{TF}=\frac{1}{2}\left[\mu^\dagger-\frac{\alpha}{2}(1-n)G^n-\frac{\beta}{2}G-\frac{\lambda T}{2}+\varepsilon\lambda-2\lambda\times\left(\frac{\varphi_{11}}{H^2}-\frac{\varphi_{22}}{C^2}\right)\right]-\frac{3}{2C^3}\int_0^r\left[\left\{\mu^\dagger-\frac{\alpha}{2}(1-n)G^n-\frac{\beta}{2}G-\frac{\lambda T}{2}+\lambda\varepsilon+\left(\hat{q}(\lambda+1)+\frac{\varphi_q}{BA}\right)\frac{U}{E}\right\}C^2C'\right]dr, \quad (39)$$

$$Y_T=\frac{1}{2}\left[(1+3\lambda)\mu^\dagger-2\lambda\varepsilon+3(1+\lambda)P_r^\dagger-2\Pi^\dagger(1+\lambda)+\frac{\alpha}{2}(1-n)G^n+\frac{\beta}{2}G+\frac{\lambda}{2}T+\lambda\left(2\frac{\varphi_{22}}{C^2}+\frac{\varphi_{11}}{B^2}+3\frac{\varphi_{00}}{A^2}\right)\right], \quad (40)$$

$$X_T=\mu^\dagger-\frac{\alpha}{2}(1-n)G^n-\frac{\beta}{2}G-\frac{\lambda T}{2}+\varepsilon\lambda. \quad (41)$$

These equations are known as $f(G,T)$ structure scalars with $\alpha G^n+\beta G\log[G]+\lambda T$ corrections. These mathematical quantities could be used to understand the rate of gravitational collapse, fluctuations in IED as well as to analyze the role of Tolman mass function in various dynamical properties of the self-gravitating systems. Here, we would like to express EEE widely known as called Raychaudhuri equation through Y_T as

$$-(Y_T)=\frac{1}{3}\left(2\sigma^{\alpha\beta}\sigma_{\alpha\beta}+\Theta^2\right)+V^\alpha\Theta_{;\alpha}-a^\alpha_{;\alpha}. \quad (42)$$

Though this equation appears to be same as in GR but here Y_T is the $f(G,T)$ structure scalar, that contains extra degrees of freedom from the logarithmic modified model. In this way, Y_{TF} has been invoked in the mathematical modeling of EEE as

$$\mathcal{E}-\frac{1}{2}\left\{(\bar{\Pi}-2\eta\sigma)(\lambda+1)+\frac{\varphi_{22}}{C^2}-\frac{\varphi_{11}}{H^2}\right\}=Y_{TF}=a^2+\chi^\alpha a_{;\alpha}-\frac{aC'}{BC}-\frac{1}{3}\left(2\Theta\sigma+\sigma^2\right)-V^\alpha\sigma_{;\alpha}. \quad (43)$$

With the help of Eqs. (21)–(21), the WDE for system of viscous and shearing locally anisotropic spherical system turns out to be

$$\left[X_{TF}+\frac{1}{2}\left(\mu^\dagger-\frac{\alpha}{2}(1-n)G^n-\frac{\beta}{2}G-\frac{\lambda T}{2}\right)\right]'=-X_{TF}\frac{3C'}{C}+\frac{1}{2}(\Theta-\sigma)\left(qB(\lambda+1)+\frac{\varphi_q}{A}\right). \quad (44)$$

It can be seen from the above equation that in the configurations of WDE, the $f(G,T)$ structure scalar, X_{TF}, has pivotal role. The solution of the above equation would present X_{TF} as a factor of controlling inhomogeneous matter density in the background of relativistic spheres in $f(G,T)$ gravity.

4 Dust ball with constant G and T

The purpose of this section is to understand the gravitational effects with $\alpha G^n+\beta G\log[G]+\lambda T$ gravity on the existence as well as the evolution of non-interacting stellar particles. In order to have some our results more simplified, we shall evaluate the corresponding structure scalars along with SEE, WDE and EEE in the background of constant G and T (denoted with tilde notations). In this respect, the matter quantity for the dust ball are found to be

$$\mathfrak{m}=\frac{1}{2}\int_0^r\mu C^2dC-\frac{\lambda R^2T^2}{2\{1+2R\lambda T^2\}}\int_0^r C^2C'dr, \quad (45)$$

while the Weyl scalar is evaluated as follows

$$\mathcal{E}=\frac{1}{2C^3}\int_0^r\mu'C^3dr-\frac{\alpha}{2}(1-n)\tilde{G}^n-\frac{\beta}{2}\tilde{G}-\frac{\lambda T}{2}. \quad (46)$$

From the above two equations, one can write through field equations as

$$\frac{3\mathfrak{m}}{C^3}=\frac{1}{2}\left[\mu-\frac{1}{C^3}\int_0^r\mu'C^3dr\right]-\frac{\alpha}{2}(1-n)\tilde{G}^n-\frac{\beta}{2}\tilde{G}-\frac{\lambda T}{2}. \quad (47)$$

The complete set of four scalar variables with constant G and T gravity are found to be

$$\tilde{X}_T=\mu-\frac{\alpha}{2}(1-n)\tilde{G}^n-\frac{\beta}{2}\tilde{G}-\frac{\lambda T}{2}, \quad (48)$$

$$\tilde{Y}_{TF}=-\tilde{X}_{TF}=\mathcal{E}, \quad (49)$$

$$\tilde{Y}_T=\frac{1}{2}\left[\mu+\alpha(1-n)\tilde{G}^n+\beta\tilde{G}+\lambda T\right]. \quad (50)$$

From these equations, one can see that $f(G,T)$ terms are involved in only trace parts of $X_{\gamma\delta}$ and $Y_{\gamma\delta}$, while their trace-free parts are directly associated with the Weyl scalar (tidal forces) produced by system of non-interacting particles. With the help of field equations as well as Eqs. (45)–(50), the WDE has been found to be

$$\left[\frac{\mu}{2}+\frac{\alpha}{2}(1-n)\tilde{G}^n+\frac{\beta}{2}\tilde{G}+\frac{\lambda}{2}\tilde{T}+\tilde{X}_{TF}\right]'=-\frac{3}{C}\tilde{X}_{TF}C'. \tag{51}$$

This is a WDE for the scalar $\tilde{X}_{TF}$. On solving this equation, we found that the $f(G,T)$ scalar, i.e., $\tilde{X}_{TF}$ is fully involved in controlling the appearance of irregularities on the homogeneous distribution of relativistic dust cloud, under some conditions. It is noticed that μ would be the function of time, whenever $\tilde{X}_{TF}$ is zero along with the vanishing of $f(G,T)$ dark source terms. This indicates that GR results can be recovered by making $f(G,T)\to R$. This indicates that GR results are valid and at cosmological scales, these results are being influenced by dark source terms mediated from the modified gravity. As the behavior of such terms are non-attractive in nature, therefore its expected that such corrections would produce resistance against the variations as well as stability of IED. Moreover, the couple of differential equations, i.e., EEE and SEE are found to be

$$V^\alpha\Theta_{;\alpha}+\frac{2}{3}\sigma^2+\frac{\Theta^2}{3}-a^\alpha_{;\alpha}$$
$$=\frac{1}{2}\left[\mu+\alpha(1-n)\tilde{G}^n+\beta\tilde{G}+\lambda\tilde{T}\right]=-\tilde{Y}_T, \tag{52}$$

$$V^\alpha\sigma_{;\alpha}+\frac{\sigma^2}{3}+\frac{2}{3}\sigma\Theta=-\mathcal{E}=-\tilde{Y}_{TF}. \tag{53}$$

This indicates that SEE and EEE are being formulated through modified versions of $\tilde{Y}_{TF}$ and $\tilde{Y}_T$. Thus, one can understand various theoretical and physically viable aspects of these differential equations through $\tilde{Y}_{TF}$ and $\tilde{Y}_T$.

5 Conclusions

This paper aims to analyze the effects of modifications of GR on some dynamical features of radiating spherical systems. The present work could be treated as test beds for GR at large scales through one of the modified gravity theories. Such kind of approach could be considered to test GR at cosmological scales. In this context, we have taken the diagonally symmetric relativistic spheres which are coupled with dissipative shearing viscous anisotropic fluid distributions. It is further assumed that dissipation from the matter content is free streaming and diffusion degrees. In other words, we have carrying out of dissipation without scattering. After calculating the corresponding field as well as dynamical equations, we have expressed the Misner Sharp mass function in terms of these. This has assisted us to relate Weyl scalar with matter and structural spherical variables with $f(G,T)$ corrections. This relation has peculiar importance in the modeling of stellar structures.

It has been analyzed that the orthogonal breaking down of the Riemann tensor could bring very effective tool to study the reasons behind the emergence of inhomogeneities in the initially regular spheres. In this direction, we have followed the approach developed by Herrera et al. (2009) and able to formulate couple of tensorial quantities namely, $X_{\mu\nu}$ and $Y_{\mu\nu}$ in the background of $f(G,T)$ gravity. We noticed that both of their trace and trace-less components might have direct relevance in few fundamental properties of adiabatic and non-adiabatic spheres. It is seen that one of the $f(G,T)$ structure scalars. i.e., like Weyl scalar, $\mathcal{E}$, X_{TF} has very important role in the existence and evolution of regularity on the surface of relativistic spheres. Further, we have expressed the WDE, SEE and EEE in terms of these scalar variables. It is well established that EEE (or alternatively called as Raychaudhuri equation) could assist one to study the Penrose–Hawking singularity theorems, exact solutions of gravitational field equations. As, we have expressed EEE in view of Y_T, $f(G,T)$ structure scalar, therefore structure scalars could be used to discuss singularity formation in various stellar compact structures like, Kerr, Schwarzschild, Reissner–Nordström, Kerr–Newman metrics etc.

The expressions of SEE and WDE are being expressed in terms of the trace free parts of $X_{\mu\nu}$ and $Y_{\mu\nu}$ tensors. From the leading results of SEE and WDE, it is noticed that one can visualize the pictures of these differential equations through $f(G,T)$ structure scalars, including the Newman–Penrose formalism with $f(G,T)$ gravity corrections. In order to see more deeply the role of modified scalar variables, we have then discussed the case of non-interacting relativistic particles with constant choices of G and T. Here, we noticed that two $f(G,T)$ scalars are controlling the states of conformal flatness, while X_{TF} (that contains modified scalar corrections) is controlling inhomogeneities on the dust ball. All of our studied properties of spherical systems match the GR-results in the classical limits (Herrera et al. 2011a).

The obtained can be summarized as below.

1. In the orthogonal splitting phenomenon of spherical system, the number of structure scalars has been found to be four in number even in modified $f(G,T)$ gravity. These are X_T, X_{TF}, Y_T and Y_{TF}. These scalars were found to be eight in number in the case of cylindrical spacetime (Yousaf and Bhatti 2016a).
2. The scalar Y_T has a direct connection in the formulation of Raychaudhuri equation as described by Eq. (42). Thus, this scalar could be used to understand the Newman–Penrose formalism in $f(G,T)$ gravity.
3. The one of the $f(G,T)$ scalar Y_{TF} has been found to play an important role in understanding the shearing effects of spherically symmetric radiative spacetime as seen by Eq. (43).

4. The $\alpha G^n + \beta \ln[G] + \lambda T$ dark source terms tend to produce hindrances in the working of X_{TF} and thereby making the system more stable.
5. The scalar X_{TF} is an inhomogeneity factor once the system experiences negligible effects of dissipative and pressure components. This can be analyzed from WDE Eq. (51).

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