

Particle creation in the framework of $f(G)$ gravity

R. Rashidi¹  · F. Ahmadi¹ · M.R. Setare²

Received: 3 April 2018 / Accepted: 23 August 2018 / Published online: 29 August 2018
© Springer Nature B.V. 2018

Abstract In this paper, we study the problem of massless particle creation in a flat, homogeneous and isotropic universe in the framework of $f(G)$ gravity. The Bogolyubov coefficients are calculated for the accelerating power-law solutions of the model in a matter dominated universe, from which the total number of created particle per unit volume of space can be obtained. It is proved that the total particle density always has a finite value. Therefore, the Bogolyubov transformations are well-defined and the Hilbert spaces spanned by the vacuum states at different times are unitarily equivalent. We find that the particles with small values of the mode k are produced in the past and particles with large values of k are produced only in the future. The negative pressure resulting from the gravitational particle creation is also determined. It is then argued that this pressure even in the presence of energy density and thermal pressure may affect significantly the cosmic expansion.

Keywords Particle creation in curved spacetime · $f(G)$ gravity · Negative pressure

1 Introduction

The cosmological observations developed in the last two decades indicate that the universe is undergoing a phase

of cosmic acceleration started after the matter domination (Abazajian et al. 2004, 2005; Riess et al. 1998; Perlmutter et al. 1999; Astier et al. 2006; Spergel et al. 2003, 2007). It seems that some unknown energy components (dark energy) with negative pressure are responsible for this late-time acceleration (Copeland et al. 2006; Cai et al. 2010). The simplest model which successfully explains the observational data is the Λ CDM (Λ -cold dark matter) model (Perivolaropoulos 2006; Jassal et al. 2005; Bahcall et al. 1999). But in this model, the key question about the origin of the dark energy remains unanswered. Since the observed value of the cosmological constant (as the density of dark energy) is very small in comparison with the predicted vacuum energy of matter fields, it is not possible to attribute the dark energy directly to the quantum vacuum energy. The origin and the nature of dark energy is still a mystery and its existence is beyond the domain of the standard model of particle physics and general relativity (Padmanabhan 2003; Peebles and Ratra 2003; Sahni and Starobinsky 2006).

Recently, an alternative approach to accommodate dark energy is modifying the general theory of relativity on large scales. The motivation for modifying the gravitational part of the Einstein equation is not restricted to solve the cosmological problems. In fact, general relativity is not a renormalizable theory, and consequently to quantize the gravitational fields conventionally, the Einstein-Hilbert action needs to be supplemented by higher order curvature terms (Utiyama and DeWitt 1962; Stelle 1977). Also, in string theory and when quantum corrections are taken into account, the effective gravitational action at low energy level admits higher order curvature invariants (Birrell and Davies 1982; Fulling 1989; Buchbinder et al. 1992; Vilkovisky 1992).

Among these theories, scalar-tensor theories (Perrotta et al. 2000; Boisseau et al. 2000), $f(R)$ gravity (Kleinert and Schmidt 2002; Capozziello et al. 2003; Nojiri and Odintsov

✉ R. Rashidi
reza.rashidi@sru.ac.ir

F. Ahmadi
fahmadi@sru.ac.ir

M.R. Setare
rezakord@ipm.ir

¹ Department of Physics, Shahid Rajaei Teacher Training University, Lavizan, Tehran 16788, Iran

² Department of Science, Payame Noor University, Bijar, Iran

2009), DGP braneworld gravity (Dvali et al. 2000) and string-inspired theories (Gross and Sloan 1987; Charmousis and Dufaux 2002; Davis 2003) are studied extensively. Also, another theory in this context is scalar-Gauss-Bonnet gravity which is closely related to the low-energy string effective action. In this proposal, the current acceleration of the universe may be caused by mixture of scalar phantom and (or) potential/stringy effects (Nojiri et al. 2005a, 2006b; Carter and Neupane 2006a,b). The coexistence of matter dominated and accelerating power law solutions for this theory has already been shown (Goheer et al. 2009a). It is also seen that the Gauss-Bonnet gravity is less constrained than $f(R)$ gravity (De Felice and Tsujikawa 2009).

On the other hand, as was first pointed out by Zeldovich (1970) and Hu (1982), the process of matter creation in an expanding universe may phenomenologically be equivalent to effective negative bulk pressure. Therefore, in this context, the present accelerating stage may have two origins: the negative pressure resulting from the gravitational particle creation and the higher order terms of the gravitational sector.

In these connections, the process of matter creation in an expanding universe has been extensively discussed in the last five decades (Parker 1968; Grib and Mamayev 1969; Pavlov 2001). The first thorough treatment of particle production by an external gravitational field was given by Parker (1968, 1969, 1971, 1972, 1973). However, the self-consistent macroscopic formulation of the matter creation process was put forward by Prigogine et al. (1989) and Calvao et al. (1992).

In flat space-time, Poincare invariance is a guide which generally allows to identify a unique vacuum state for the theory. However, in curved space-time, we do not have the Poincare symmetry. The absence of Poincare symmetry in curved space-time leads to the problem of the definition of particles and vacuum states. The problem may be solved by using the method of the diagonalization of instantaneous Hamiltonian by a Bogolyubov transformation, which leads to finite results for the number of created particles (Grib and Mamayev 1969; Pavlov 2001). In this direction, some works have been done in the context of modified gravity (Pereira et al. 2010, 2011; Setare and Houndjo 2013).

In the present work, we investigate the particle production in a $f(G)$ theory for a flat and matter dominated universe. It is proved that the total particle density always has a finite value. Therefore, the Bogolyubov transformations are well-defined and the Hilbert spaces spanned by the vacuum states at different times are unitarily equivalent (Mukhanov and Winitzki 2007). The negative pressure resulting from the gravitational particle creation is also obtained for adiabatic processes, i.e. the processes in which the entropy per particle remains constant. In this case the entropy production density is entirely due to the increase of the number of particles (Prigogine et al. 1989; Calvao et al. 1992; Zimdahl and Pavon

1993; Zimdahl et al. 2001). We show that, even if the higher order curvature terms of the Gauss-Bonnet gravity are ignored, this pressure can alone explain the present accelerating expansion. This result indicates that the pressure of the particle creation even in the presence of energy density and thermal pressure may affect significantly the cosmic expansion. Obviously, to reach a self-consistent model at least in a semiclassical framework one should take the back reaction effect of the particle creation into account, i.e. the gravitational equations and the particle creation equations must be solved simultaneously. But, since the coupling between the gravitational background and the density and pressure of particle creation is very complicated, it may be difficult. Then the result of the present paper might be viewed as the first approximation of the particle creation effect.

2 Field equations

We consider the following $f(G)$ action which describes Einstein's gravity coupled with perfect fluid plus a function of the Gauss-Bonnet term (Nojiri and Odintsov 2005b; Nojiri et al. 2006a)

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2k^2} R + f(G) + L_m \right], \tag{1}$$

where $k^2 = 8\pi G_N$ and the Gauss-Bonnet invariant is defined as follows

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}. \tag{2}$$

By varying the action with respect to $g_{\mu\nu}$, it follows that

$$\begin{aligned} 0 = & \frac{1}{2k^2} \left(-R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}R \right) + T^{\mu\nu} + \frac{1}{2}g^{\mu\nu}f'(G) \\ & - 2f_G R R^{\mu\nu} + 4f_G R_\rho^\mu R^{\nu\rho} - 2f_G R^{\mu\rho\sigma\tau} R_{\rho\sigma\tau}^\nu \\ & - 4f_G R^{\mu\rho\sigma\nu} R_{\rho\sigma} + 2(\nabla^\mu \nabla^\nu f_G)R - 2g^{\mu\nu}(\nabla^2 f_G)R \\ & - 4(\nabla_\rho \nabla^\mu f_G)R^{\nu\rho} - 4(\nabla_\rho \nabla^\nu f_G)R^{\mu\rho} + 4(\nabla^2 f_G)R^{\mu\nu} \\ & + 4g^{\mu\nu}(\nabla_\rho \nabla_\sigma f_G)R^{\rho\sigma} - 4(\nabla_\rho \nabla_\sigma f_G)R^{\mu\rho\nu\sigma}, \end{aligned} \tag{3}$$

where $f_G = f'(G)$ and $f_{GG} = f''(G)$. By using the metric of Friedmann-Robertson-Walker (FRW), we can obtain the first FRW equation

$$-\frac{3}{k^2}H^2 + Gf_G - f(G) - 24\dot{G}H^3 f_{GG} + \rho_m = 0, \tag{4}$$

where an over-dot denotes derivative with respect to time t and Hubble parameter H is defined by $H = \frac{\dot{a}}{a}$. Also, using the equation of state $P = w\rho_m$, the energy conservation law can be expressed as

$$\dot{\rho}_m + 3H(1+w)\rho_m = 0, \tag{5}$$

where ρ_m is the matter density. Now, by assuming an exact power-law solution for the field equations as follows

$$a(t) = bt^c, \tag{6}$$

where c and b are positive real numbers, the Friedmann equation is exchanged as

$$0 = \frac{4}{c-1}G^2 f_{GG} + Gf_G - f_G - \frac{G^{\frac{1}{2}}}{k^2} \left(\frac{3c}{8(c-1)} \right)^{\frac{1}{2}} + \rho_0 \left(\frac{G}{24c^3(c-1)} \right)^{\frac{3}{4}c(1+w)}. \tag{7}$$

This is a differential equation for the function $f(G)$ in G space. The general solution of this equation is obtained as

$$f(G) = -\frac{1}{2} \left[\sqrt{\frac{6c(c-1)}{k^4(c+1)^2}} G^{\frac{1}{2}} + A_{cw} G^{\frac{3}{4}c(1+w)} \right], \tag{8}$$

where

$$A_{cw} = \frac{8\rho_0(c-1)(1382c^9(c-1)^3)^{-\frac{1}{4}c(1+w)}}{4 + c[3c(w+1)(w + \frac{4}{3}) - 15w - 19]}. \tag{9}$$

As we see that a real valued solution for $f(G)$ requires the values $c \leq 0$ or $c \geq 1$. In Rastkar et al. (2012), it is shown that only the case $c > 1$ leads to an accelerating universe. Also, in order to avoid divergency in the Gauss-Bonnet term we have to keep c and ω away from the values for which A_{cw} diverges according to the following equation

$$4 + c \left[3c(w+1) \left(w + \frac{4}{3} \right) - 15w - 19 \right] = 0. \tag{10}$$

For the case of matter dominated universe ($w = 0$), it must be supposed $c > 1$ and $c \neq \frac{19}{8} + \frac{\sqrt{345}}{8}$ (by noting (10)). The Hubble parameter $H = \frac{\dot{a}}{a} = \frac{\dot{c}}{c}$ determines the actual age of universe as $t_0 = \frac{c}{H_0}$ that is the order of 10^9 years. Before studying the particle creation process, it is convenient to obtain the scale factor $a(t)$ in terms of the conformal time η . The conformal time is defined as

$$\eta \equiv \int^t \frac{dt'}{a(t')}, \tag{11}$$

then we have

$$a(\eta) = \frac{B}{(-\eta)^{\frac{c}{c-1}}}, \quad B = [b(c-1)^c]^{\frac{1}{1-c}}, \tag{12}$$

where $-\infty < \eta < 0$. The early universe (the past) corresponds to $\eta \rightarrow -\infty$ and the late universe (the future) corresponds to $\eta \rightarrow 0$. To calculate the density of particles per mode, we should determine the parameter b in the numerator of the scale factor $a(\eta)$. So, let us choose b such that

$a(t_0) \equiv a_0 = 1$, for $t_0 = \frac{c}{H_0}$, i.e., the scale factor is normalized to unity for the present time. To satisfy this condition we must have $b = (\frac{H_0}{c})^c$. Using (11) we get the value of the present conformal time as $\eta_0 = -\frac{c}{c-1} \frac{1}{H_0}$. Therefore, the scale factor becomes

$$a(\eta) = \frac{(-\eta_0)^{\frac{c}{c-1}}}{(-\eta)^{\frac{c}{c-1}}}. \tag{13}$$

Although with the power-law solutions the evolution of the universe is basically restricted, these solutions help us to find some quantities analytically. But it is not the only motivation behind this choice. In fact, since these solutions are corresponding to the scaling solutions in $f(G)$ framework (Uddin et al. 2009), they play an important role in cosmology. They can be regarded as approximations to more realistic models and provide a framework for establishing the behavior of more general cosmological solutions (Uddin et al. 2009). Also, it has been proved that the scaling solutions are global attractors for a large class of cosmological models (Copeland et al. 1998; Nunes and Mimoso 2000). Therefore, the choice of such solutions is particularly relevant because in the Friedmann-Robertson-Walker backgrounds, they typically represent asymptotic or intermediate states in the full phase-space of the dynamical system representing all possible cosmological evolutions (Goheer et al. 2009b). They then enable us to determine the asymptotic behavior and stability of a particular cosmological background (Liddle and Scherrer 1999; Rubano and Barrow 2001; Copeland et al. 2005; Tsujikawa and Sami 2004; Steinhardt et al. 1999).

In the following sections, we are going to study the particle creation process in this model.

3 Scalar particle creation in $f(G)$ theory

Generally, the field equation for the study of scalar particle creation in a spatially flat Friedmann-Robertson-Walker geometry can be written as (Mukhanov and Winitzki 2007)

$$\mathcal{X}''(\mathbf{x}, \eta) - \nabla^2 \mathcal{X}(\mathbf{x}, \eta) + \left(m^2 a^2(\eta) - \frac{a''(\eta)}{a(\eta)} \right) \mathcal{X}(\mathbf{x}, \eta) = 0, \tag{14}$$

where the prime denotes derivative with respect to the conformal time η and ∇^2 is the Laplacian. Replacing the mode expansion

$$\mathcal{X}(\mathbf{x}, \eta) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2}} [a_k^- \mathcal{X}_k^*(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + a_k^+ \mathcal{X}_k(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}}] \tag{15}$$

in the field equation (14) implies the decoupled equations of motion for the modes $\mathcal{X}_k(\eta)$,

$$\mathcal{X}_k'' + \omega_k^2(\eta)\mathcal{X}_k(\eta) = 0, \tag{16}$$

with

$$\omega_k^2(\eta) = k^2 + m_{eff}^2 \quad \text{and} \quad m_{eff}^2 = m^2 a^2(\eta) - \frac{a''(\eta)}{a(\eta)}, \tag{17}$$

where \mathcal{X}_k is the Fourier mode of the wave associated to the energy of the particle through the frequency ω_k , m_{eff} represents an effective mass of the particle and the prime denotes derivative with respect to the conformal time η .

Here, the quantization can be carried out by imposing equal-time commutation relations for the scalar field \mathcal{X} and its canonically conjugate momentum $\Pi \equiv \mathcal{X}'$, namely $[\mathcal{X}(x, \eta), \Pi(y, \eta)] = i\delta(x - y)$, and by implementing secondary quantization in the so-called Fock representation. After convenient Bogolyubov transformations, one obtains the transition amplitudes for the vacuum state and the associated spectrum of the produced particles in a non-stationary background (Grib et al. 1994; Mukhanov and Winitzki 2007). Usually, the calculations of particle production deal with comparing the particle number at asymptotically early and late times, or with respect to the vacuum states defined in two different frames and do not involve any loop calculation. Since (16) is a second order differential equation, we obtain two independent solutions.

To quantize the scalar field $\mathcal{X}(x, \eta)$ in the standard fashion by introducing the equal-time commutation relations $[\mathcal{X}(\mathbf{x}, \eta), \mathcal{X}'(\mathbf{y}, \eta)] = i\delta(\mathbf{x} - \mathbf{y})$, each mode solution \mathcal{X}_k must be normalized for all times according to

$$W_k(\eta) \equiv \mathcal{X}_k'(\eta)\mathcal{X}_k^*(\eta) - \mathcal{X}_k(\eta)\mathcal{X}_k'^*(\eta) = 2i. \tag{18}$$

If the vacuum state is defined as the lowest-energy eigenstate of the instantaneous Hamiltonian at time η , the mode decomposition $\mathcal{X}_k(\eta)$ corresponding to this vacuum state should satisfy the following conditions at time η (Mukhanov and Winitzki 2007)

$$\mathcal{X}_k(\eta) = \frac{e^{i\lambda}}{\sqrt{\omega_k(\eta)}}, \quad \mathcal{X}_k'(\eta) = ie^{i\lambda}\sqrt{\omega_k(\eta)}, \tag{19}$$

where λ is an arbitrary real number. A mode function satisfying the above conditions defines a creation and annihilation set of operators \hat{a}_k^\pm and then the vacuum state as the instantaneous lowest-energy state is the state annihilated by \hat{a}_k^- . In addition, the instantaneous Hamiltonian is diagonal in the eigenbasis of the occupation number operators $\hat{N}_k = \hat{a}_k^+ \hat{a}_k^-$. But ω_k is not time-independent in a time-dependent gravitational background. Therefore, the mode function selected by the conditions (19) is time-dependent. It means that the vacuum states at different times differ from

each other. However, these instantaneous vacuum states at different time are related by the Bogolyubov coefficients. If the mode function $\mathcal{X}_k(\eta)$ satisfies the conditions (19) at the initial time η_i and if we suppose that the physical state is the instantaneous vacuum state corresponding to this mode function, then a straightforward calculation shows that the final expression for the number density of created particles in the k mode at time $\eta > \eta_i$ is (Grib et al. 1994; Mukhanov and Winitzki 2007)

$$N_k(\eta) = \frac{1}{4|\omega_k(\eta)|} |\mathcal{X}_k'(\eta)|^2 + \frac{|\omega_k(\eta)|}{4} |\mathcal{X}_k(\eta)|^2 - \frac{1}{2}. \tag{20}$$

The proper density of particles per mode is given by

$$n_k(\eta) = \frac{N_k(\eta)}{a^3(\eta)}, \tag{21}$$

and the total number density of created particles is obtained by integrating overall the modes

$$n(\eta) = \int n_k(\eta) d^3\mathbf{k}. \tag{22}$$

It is worthwhile to note that the Bogolyubov transformation is well-defined only if the total particle density (22) is finite. If this is not the case, the final vacuum state is not expressible as the normalized linear combination of the initial vacuum state and the excited states derived from it. In other words, the two Hilbert spaces spanned by these two vacuum states and their excited states are not unitarily equivalent.

4 The creation of massless particles

Here, by using (12), we find $\frac{a''}{a} = [\frac{c(2c-1)}{(c-1)^2(-\eta)^2}]$ that does not depend on the parameter B . Equation (16) for mode function in the case of a massless particle ($m = 0$) becomes

$$\mathcal{X}_k''(\eta) + \left[k^2 - \left(\frac{c(2c-1)}{(c-1)^2} \right) \frac{1}{\eta^2} \right] \mathcal{X}_k(\eta) = 0. \tag{23}$$

According to the normalization condition (18), the solution of this equation is given by

$$\begin{aligned} \mathcal{X}_k(\eta) &= \sqrt{\frac{\pi|\eta|}{2}} [J_\nu(k|\eta) + iY_\nu(k|\eta)] \\ &= \sqrt{\frac{\pi|\eta|}{2}} H_\nu^{(1)}(k|\eta), \quad \nu \equiv \sqrt{\frac{1}{4} + \frac{c(2c-1)}{(c-1)^2}}, \end{aligned} \tag{24}$$

where J_ν and Y_ν are respectively the Bessel functions of the first and second kind and H_ν is the Hankel function. The solution (24) satisfies the lowest-energy conditions (19) at the initial time ($\eta \rightarrow -\infty$) and has the correct asymptotic behaviour of the form

$$\mathcal{X}_k \rightarrow \frac{1}{\sqrt{k}} \exp(ik\eta + \delta), \tag{25}$$

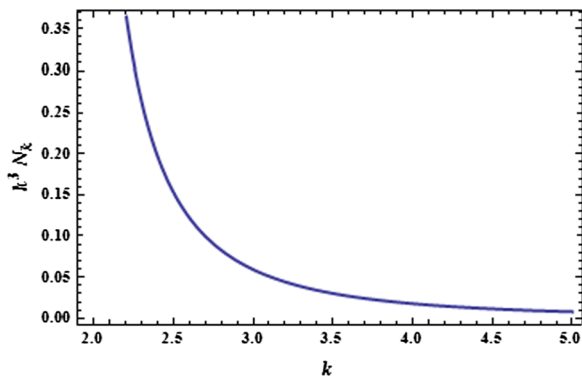


Fig. 1 This figure shows $k^3 N_k(\eta)$ as a function of the mode k with the parameters $c = \frac{10}{3}$ and $\eta = 1$. It shows that $k^3 N_k(\eta)$ tends to zero at $k \rightarrow \infty$. This asymptotic behaviour is necessary to prove that the total number density of created particles is finite

where δ is a phase. This corresponds to plane waves for the modes k in the past. To calculate the spectrum of massless particles created during the evolution of the universe by (20), we have to firstly show that the total density of created particles (22) has a finite value at all times. To show this one should prove that $N_k(\eta)$ tends to zero faster than k^{-3} at $k \rightarrow \infty$. Employing the asymptotic expansion of the Hankel functions, i.e.

$$H_\nu^{(1)}(k|\eta|) = \sqrt{\frac{2}{\pi k|\eta|}} \exp\left\{i\left[k|\eta| - \left(\nu + \frac{1}{2}\right)\frac{\pi}{2}\right]\right\} \times (P_\nu(k|\eta|) + iQ_\nu(k|\eta|)), \tag{26}$$

where

$$P_\nu(z) + iQ_\nu(z) = \sum_{r=0}^{\infty} \frac{\Gamma(\nu + r + \frac{1}{2})}{r! \Gamma(\nu - r + \frac{1}{2})} (-2iz)^{-r}, \tag{27}$$

it is not difficult to see that the first non-vanishing term of $N_k(\eta)$ at $k \rightarrow \infty$ is of order k^{-4} . Thus, $N_k(\eta)$ tends to zero faster than k^{-3} at $k \rightarrow \infty$. Substituting solution (24) into the relation (20) and setting $c = 10/3$ and $\eta = 1$, we plot $k^3 N_k(\eta)$ as a function of k in Fig. 1. It shows that $k^3 N_k(\eta)$ tends to zero at $k \rightarrow \infty$.

On the other hand, for each k there is a critical value of η for which the number density of created particles grows abruptly and diverges. Mathematically this occurs because in the expression (20) the frequency $\omega_k(\eta)$ appears in the denominator, and when $k^2 = \frac{c(2c-1)}{\eta^2(c-1)^2}$ we have $\omega_k(\eta) \rightarrow 0$. Physically, the significance of this divergence is that for values of $k^2 < \frac{c(2c-1)}{\eta^2(c-1)^2}$, the frequency ω_k^2 becomes negative, consequently the state of minimum energy and the quantum vacuum are not well-defined in these cases, and the creation ceases for these values. Therefore, the total number density of particle should be determined as

$$n(\eta) = \frac{1}{a^3(\eta)} \int_{k^2 = \frac{c(2c-1)}{\eta^2(c-1)^2}}^{\infty} N_k(\eta) d^3\mathbf{k}. \tag{28}$$

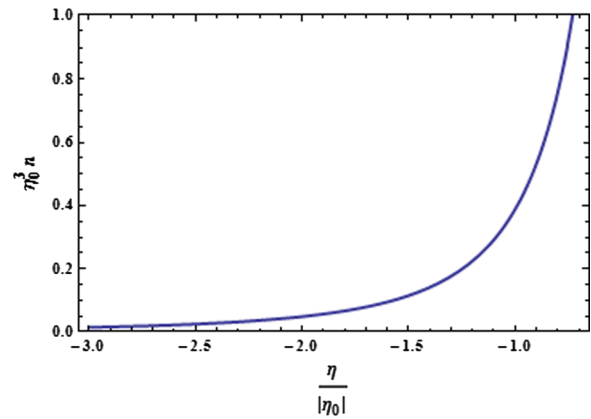


Fig. 2 The figure presents the dependence of the dimensionless total number density of particles $\eta_0^3 n(\eta)$ on $\frac{\eta}{|\eta_0|}$

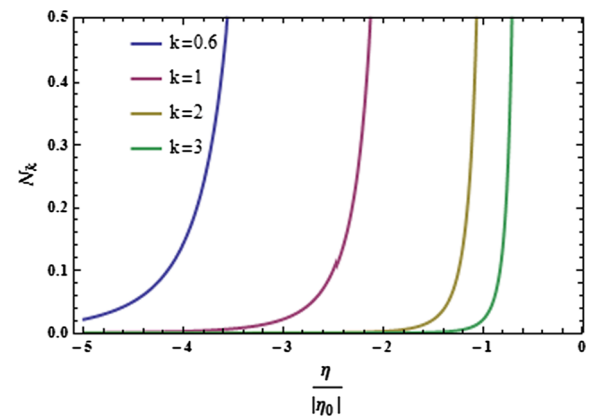


Fig. 3 This figure shows the evolution of density of created particle N_k for four different values of the mode k with the parameter $c = \frac{10}{3}$ (for comparing with results of Pereira et al. 2011, we take $c = \frac{10}{3}$)

Although the first term of $N_k(\eta)$, i.e. $\frac{|\mathcal{X}'_k(\eta)|^2}{4|\omega_k(\eta)|} = \frac{|\mathcal{X}'_k(\eta)|^2}{4\sqrt{k^2 - \frac{c(2c-1)}{\eta^2(c-1)^2}}}$, tends to infinity at $k^2 \rightarrow \frac{c(2c-1)}{\eta^2(c-1)^2}$, its integral is finite because the following integral is convergent:

$$\int_a^{b>a} \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(\frac{b + \sqrt{b^2 - a^2}}{a}\right). \tag{29}$$

Then the total number density of particles has a finite value at all times. In Fig. 2, the dimensionless total number density of particles defined as $\eta_0^3 n(\eta)$ is displayed versus $\frac{\eta}{|\eta_0|}$.

Now, using (20) and (24), we can calculate the evolution of particle density N_k for each mode k . Figure 3 shows the density of massless particle created during the evolution of the universe as a function of $\frac{\eta}{|\eta_0|}$ for different values of mode k . The value $\frac{\eta}{|\eta_0|} = -1$ represents the present time. In the past ($\eta \rightarrow -\infty$) the number density is zero for all modes, but it grows throughout evolution.

By using this figure, as mentioned earlier, we can see that for each k there is a critical value of the conformal time

η for which the number density of created particles grows abruptly and diverges. For $k = 1$ this occurs in the past when $\frac{\eta}{|\eta_0|} \approx -2.05$, for $k = 2$ we have $\frac{\eta}{|\eta_0|} \approx -1.02$, very close to the present time. But for $k = 3$ this will only occur in the future, $\frac{\eta}{|\eta_0|} \approx -0.7$. A value of $k < 1$ is also shown in this figure.

5 Pressure of particle creation

It is not difficult to show that the interactions wherein particle number conservation violated, including the gravitational particle creation, may lead to an effective negative pressure (Prigogine et al. 1989). From the first law of thermodynamics and Euler’s relation for an open system in which the particle number N is time dependent, it follows that

$$d(\rho V) + PdV - \frac{\rho + P}{n}d(nV) - TNd\left(\frac{S}{N}\right) = 0, \quad (30)$$

where $\rho = E/V$ and $n = N/V$. The above relation is known as the Gibbs relation. The Gibbs relation can be also written as

$$d\rho - \frac{\rho + P}{n}dn = nTd\left(\frac{S}{N}\right). \quad (31)$$

If the entropy per particle S/N is constant, i.e. the entropy production is entirely due to the increase of the number of particles, the Gibbs relation implies that

$$d(\rho V) + PdV - \frac{\rho + P}{n}d(nV) = 0, \quad (32)$$

or equivalently

$$\dot{\rho} = (\rho + P)\frac{\dot{n}}{n}, \quad (33)$$

where an over-dot denotes the derivative with respect to time. Now, by defining a supplementary pressure P_c

$$P_c = -\frac{\rho + P}{n} \frac{d(nV)}{dV}. \quad (34)$$

Equation (32) can be rewritten as

$$d(\rho V) = -(P + P_c)dV. \quad (35)$$

It means that the creation of matter corresponds to a supplementary pressure P_c which must be considered as a part of the total pressure P_t entering into the matter part of the gravitational equations (Prigogine et al. 1989; Calvao et al. 1992; Zimdahl and Pavon 1993; Zimdahl et al. 2001), i.e.

$$P_t = P + P_c. \quad (36)$$

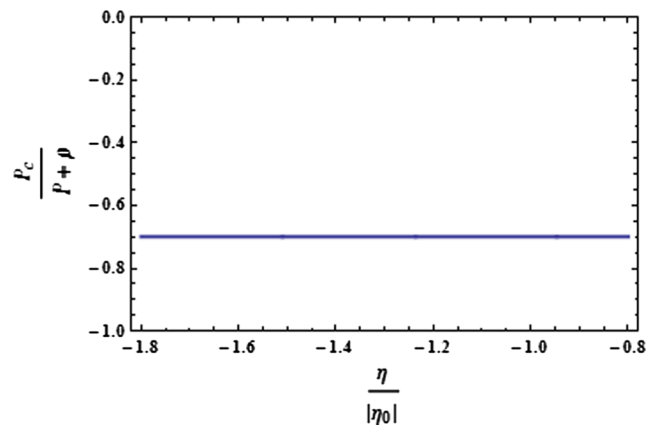


Fig. 4 This figure shows the evolution of $P_c/(\rho + P)$ as a function of $\frac{\eta}{|\eta_0|}$. It is proved that this function has a constant value

In the case of an isotropic and homogeneous universe, one can set $V = a^3(t)$, then

$$P_c = -(\rho + P)\left(\frac{\dot{n}}{3Hn} + 1\right). \quad (37)$$

Before determining the pressure of particle creation, it should be noted that the distribution $\omega_k(\eta)n_k(\eta)$ as a function of k does not correspond to a thermodynamic equilibrium state. Thus, similar to the Gamow condition, we should assume that the transition rate to an equilibrium state is faster than the particle production rate and the expansion rate of the universe. Using (13) and (37), in Fig. 4 the behavior of $\frac{P_c}{\rho + P}$ as a function of $\frac{\eta}{|\eta_0|}$ is displayed.

It shows that the pressure of particle creation has a negative value as it is expected. It also seems that $\frac{P_c}{\rho + P}$ has a constant value. This constancy is not an amazing result because from (20), (23), (24) and (28) it follows that $n(\frac{\eta}{\alpha}) = \alpha^3 n(\eta)$ for all $\eta < 0$ and $\alpha > 0$. Then it is not difficult to prove that $n(\eta) \propto \eta^{-3}$, which implies $\frac{\dot{n}}{3Hn} = \text{const.}$

To compare the effect of this negative pressure on the acceleration of expansion with the effect of energy density and thermal pressure which always have positive values, we can for simplicity neglect the contribution of the higher order curvature terms of the Gauss-Bonnet gravity. So it is enough to restrict ourselves to general relativity. In this case, the Einstein equations yield (Lima et al. 2008)

$$8\pi G_N \rho = 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2}, \quad (38)$$

and

$$8\pi G_N (P + P_c) = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2}, \quad (39)$$

which imply

$$2\frac{\ddot{a}}{a} = -8\pi G_N \left(P_c + P + \frac{\rho}{3}\right). \quad (40)$$

Taking $P = \rho/3$, from (37) and Fig. 4, it follows that the right hand side of above equation has a positive value and therefore, $\ddot{a} > 0$. This result indicates that the pressure of the particle creation even in the presence of energy density and thermal pressure may affect significantly the cosmic expansion.

Obviously, to reach a self-consistent model at least in a semiclassical framework one should take the back reaction effect of the particle creation into account, i.e. the gravitational equation (4) and the particle creation equation (23) must be solved simultaneously. But, since the coupling between the gravitational background and the density and pressure of particle creation is very complicated, it may be difficult. Then the result of the present paper might be viewed as the first approximation of the particle creation effect.

6 Conclusions

We have investigated the problem of massless particle creation in a $f(G)$ theory for a matter dominated universe. We have assumed an exact power-law solution for the scale factor of universe, which leads us to an accelerated expanding universe. The amount of particles created with $k < 1$ is steadily increasing in the past, although the creation of such modes stop abruptly in the past (Fig. 3). This shows that in the past a huge amount of particles with low k were created. These results perfectly agree with one of that obtained in studying quantum effect in the context of $f(R)$ gravity (Pereira et al. 2011). It has been also proved that the total particle density always has a finite value. Therefore, the Bogolyubov transformations are well-defined and the Hilbert spaces spanned by the vacuum states at different times are unitarily equivalent. In addition, we have shown that the pressure of particle creation has negative value as it is expected. This pressure even in the presence of energy density and thermal pressure may affect significantly the cosmic expansion.

References

Abazajian, K., et al. (SDSS Collaboration): *Astron. J.* **128**, 502 (2004). [arXiv:astro-ph/0403325](#)

Abazajian, K., et al. (SDSS Collaboration): *Astron. J.* **129**, 1755 (2005). [arXiv:astro-ph/0410239](#)

Astier, P., et al.: *Astron. Astrophys.* **447**, 31 (2006). [arXiv:astro-ph/0510447](#)

Bahcall, N.A., Ostriker, J.P., Perlmutter, S., Steinhardt, P.J.: The cosmic triangle: revealing the state of the universe. *Science* **284**, 1481 (1999)

Birrell, N.D., Davies, P.C.W.: *Quantum Fields in Curved Space*. Cambridge University Press, Cambridge (1982)

Boisseau, B., Esposito-Farese, G., Polarski, D., Starobinsky, A.A.: *Phys. Rev. Lett.* **85**, 2236 (2000)

Buchbinder, I.L., Odintsov, S.D., Shapiro, I.L.: *Effective Actions in Quantum Gravity*. IOP Publishing, Bristol (1992)

Cai, Y.-F., Saridakis, E.N., Setare, M.R., Xia, J.-Q.: *Phys. Rep.* **493**, 1 (2010)

Calvao, M.O., Lima, J.A.S., Waga, I.: *Phys. Lett. A* **162**, 223 (1992)

Capozziello, S., Cardone, V.F., Carloni, S., Troisi, A.: *Int. J. Mod. Phys. D* **12**, 1969 (2003). [arXiv:astro-ph/0307018](#)

Carter, B.M.N., Neupane, I.P.: *Phys. Lett. B* **638**, 94 (2006a)

Carter, B.M.N., Neupane, I.P.: *J. Cosmol. Astropart. Phys.* **0606**, 004 (2006b)

Charmousis, C., Dufaux, J.F.: *Class. Quantum Gravity* **19**, 4671 (2002). [arXiv:hep-th/0202107](#)

Copeland, E.J., Liddle, A.R., Wands, D.: *Phys. Rev. D* **57**, 4686 (1998). [arXiv:gr-qc/9711068](#)

Copeland, E.J., Lee, S.J., Lidsey, J.E., Mizuno, S.: *Phys. Rev. D* **71**, 023526 (2005). [arXiv:astro-ph/0410110](#)

Copeland, E.J., Sami, M., Tsujikawa, Sh.: *Int. J. Mod. Phys. D* **15**, 1753 (2006)

Davis, S.C.: *Phys. Rev. D* **67**, 024030 (2003). [arXiv:hep-th/0208205](#)

De Felice, A., Tsujikawa, Sh.: *Phys. Lett. B* **675**, 1 (2009)

Dvali, G., Gabadadze, G., Porrati, M.: *Phys. Lett. B* **485**, 208 (2000). [arXiv:hep-th/0005016](#)

Fulling, S.A.: *Aspects of Quantum Field Theory in Curved Spacetime*. Cambridge University Press, Cambridge (1989)

Goheer, N., Goswami, R., Dunsby, P., Ananda, K.: *Phys. Rev. D* **79**, 121301(R) (2009a)

Goheer, N., Larena, J., Dunsby, P.K.S.: *Phys. Rev. D* **80**, 061301 (2009b). [arXiv:0906.3860 \[gr-qc\]](#)

Grib, A.A., Mamayev, S.G.: *Yad. Fiz.* **10**, 1276 (1969). English transl.: *Sov. J. Nucl. Phys.* **10**, 722 (1970)

Grib, A.A., Mamayev, S.G., Mostepanenko, V.M.: *Vacuum Quantum Effects in Strong Fields*, Friedmann Laboratory Publishing. St. Petersburg (1994)

Gross, D.J., Sloan, J.H.: *Nucl. Phys. B* **291**, 41 (1987)

Hu, B.L.: *Phys. Lett. A* **90**, 375 (1982)

Jassal, H., Bagla, J., Padmanabhan, T.: *Phys. Rev. D* **72**, 103503 (2005). [arXiv:astro-ph/0506748](#)

Kleinert, H., Schmidt, H.J.: *Gen. Relativ. Gravit.* **34**, 1295 (2002)

Liddle, A.R., Scherrer, R.J.: *Phys. Rev. D* **59**, 023509 (1999). [arXiv:astro-ph/9809272](#)

Lima, J.A.S., Silva, F.E., Santos, R.C.: *Class. Quantum Gravity* **25**, 205006 (2008)

Mukhanov, V.F., Winitzki, S.: *Introduction to Quantum Effects in Gravity*. Cambridge University Press, Cambridge (2007)

Nojiri, S., Odintsov, S.D., Sasaki, M.: *Phys. Rev. D* **71**, 123509 (2005a)

Nojiri, S., Odintsov, S.D.: *Phys. Lett. B* **631**, 1 (2005b)

Nojiri, S., Odintsov, S.D., Gorbunova, O.G.: *J. Phys. A* **39**, 6627 (2006a)

Nojiri, S., Odintsov, S.D., Sami, M.: *Phys. Rev. D* **74**, 046004 (2006b)

Nojiri, S., Odintsov, S.D.: *AIP Conf. Proc.* **1115**, 212 (2009). [arXiv:0810.1557](#)

Nunes, A., Mimoso, J.P.: *Phys. Lett. B* **488**, 423 (2000). [arXiv:gr-qc/0008003](#)

Padmanabhan, T.: *Phys. Rep.* **380**, 235 (2003)

Parker, L.: *Phys. Rev. Lett.* **21**, 562 (1968)

Parker, L.: *Phys. Rev.* **183**, 1057 (1969)

Parker, L.: *Phys. Rev. D* **3**, 346 (1971)

Parker, L.: *Phys. Rev. Lett.* **28**, 705 (1972)

Parker, L.: *Phys. Rev. D* **7**, 976 (1973)

Pavlov, Yu.V.: *Theor. Math. Phys.* **126**, 92 (2001). [arXiv:gr-qc/0012082](#)

Peebles, P.J.E., Ratra, B.: *Rev. Mod. Phys.* **75**, 559 (2003)

Pereira, S.H., Bessa, C.H.G., Lima, J.A.S.: *Phys. Lett. B* **690**, 103 (2010). [arXiv:0911.0622](#)

- Pereira, S.H., Aguilar, J.C.Z., Romao, E.C.: Massless particle creation in a $f(R)$ expanding universe (2011). [arXiv:1108.3346](#)
- Perivolaropoulos, L.: Accelerating Universe: Observational Status and Theoretical Implications (2006). [arXiv:astro-ph/0601014](#)
- Perlmutter, S., et al. (Supernova Cosmology Project Collaboration): *Astrophys. J.* **517**, 565 (1999). [arXiv:astro-ph/9812133](#)
- Perrotta, F., Baccigalupi, C., Matarrese, S.: *Phys. Rev. D* **61**, 023507 (2000)
- Prigogine, I., et al.: *Gen. Relativ. Gravit.* **21**, 767 (1989)
- Rastkar, A.R., Setare, M.R., Darabi, F.: *Astrophys. Space Sci.* **337**, 487 (2012). [arXiv:1104.1904](#)
- Riess, A.G., et al. (Supernova Search Team Collaboration): *Astron. J.* **116**, 1009 (1998). [arXiv:astro-ph/9805201](#)
- Setare, M.R., Houndjo, M.J.S.: *Can. J. Phys.* **91**, 168 (2013). [arXiv:1111.2821](#)
- Rubano, C., Barrow, J.D.: *Phys. Rev. D* **64**, 127301 (2001). [arXiv:gr-qc/0105037](#)
- Sahni, V., Starobinsky, A.: *Int. J. Mod. Phys. D* **15**, 2105 (2006)
- Spergel, D.N., et al. (WMAP Collaboration): *Astrophys. J. Suppl.* **148**, 175 (2003). [arXiv:astro-ph/0302209](#)
- Spergel, D.N., et al.: *Astrophys. J. Suppl.* **170**, 377 (2007). [arXiv:astro-ph/0603449](#)
- Steinhardt, P.J., Wang, L.M., Zlatev, I.: *Phys. Rev. D* **59**, 123504 (1999). [arXiv:astro-ph/9812313](#)
- Stelle, K.S.: *Phys. Rev. D* **16**, 953 (1977)
- Tsujikawa, S., Sami, M.: *Phys. Lett. B* **603**, 113 (2004). [arXiv:hep-th/0409212](#)
- Uddin, K., Lidsey, J.E., Tavakol, R.: *Gen. Relativ. Gravit.* **41**, 2725 (2009). [arXiv:0903.0270](#)
- Utiyama, R., DeWitt, B.S.: Renormalization of a classical gravitational field interacting with quantized matter fields. *J. Math. Phys.* **3**, 608 (1962)
- Vilkovisky, G.A.: Effective action in quantum gravity. *Class. Quantum Gravity* **9**, 895 (1992)
- Zeldovich, Y.B.: *Pis'ma Zh. Eksp. Teor. Fiz.* **12**, 443 (1970). English transl.: *J. Exp. Theor. Phys.* **12**, 307 (1970)
- Zimdahl, W., Pavon, D.: *Phys. Lett. A* **176**, 57 (1993)
- Zimdahl, W., Schwarz, D.J., Balakin, A.B., Pavon, D.: *Phys. Rev. D* **64**, 063501 (2001)