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ORIGINAL ARTICLE

The nearly periodic fluctuations of blazars in long-term X-ray light curves

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Abstract By means of structure function method, we have analyzed 20 blazars observed by the Rossi X-Ray Timing Explorer (RXTE) to search for possible variability period. There are the year-like periodic values in 11 of these blazars. The year-like periodicities in AO 0235+164, Mrk 501, 3C 273 and PKS 1510-089 are also found by the discrete correlation function method, the Jurkevich method and the power spectrum method. AO 0235+164, Mrk 501 and 3C 273 have possible nearly periodic values of about 370 days, 630 days and 370 days, respectively. PKS 1510-089 has two nearly periodic components, about 230 and 330 days. The year-like periodicities could be explained in the framework of a binary black hole system. The turbulence behind a shock may also give rise to the year-like periodicities, dominated by the dominant eddies' turnover times. Also, the year-like periodicities may be from the interplay of a shock with essentially helical structures wrapping around a relativistic jet.

Keywords BL Lacertae objects: general · Galaxies: active · Methods: statistical · X-rays: galaxies · X-rays: general

1 Introduction

Blazars belong to one extreme sub-category in active galactic nuclei (AGNs). Their light curves often show some striking features, e.g., high luminosity, high polarization, large amplitude and fast variation, etc. The dominant radiation in these objects usually is believed to originate from relativistic jets which are very strongly Doppler boosted along the direction of observer's sight at angles equal or less than 10° (Ulrich et al. 1997). In general, blazars are divided into two categories, BL Lacertae (BL Lac) objects and flat spectrum radio quasars (FSRQs) (Urry and Padovani 1995), in which the former usually has no emission lines or the full width at half-maximum (FWHM) ≤ 5 Å, the latter usually has obvious emission lines. The light curves of blazars contain abundant physical information, e.g., the sizes and locations of the emission regions.

Almost all blazars show violent flux variability in which the time scale varies from a few minutes to many years. Some light curves of blazars show possible periodic or quasi-periodic behaviors. Many astronomers have done the periodic research of blazars in radio and optical band. The quasi-periodic behavior of OJ 287 about 12 yrs has been confirmed by many astronomers (Sillanpää et al. 1998; Kidger et al. 1992; Fan et al. 2010 and reference therein). A possible quasi-periodic variations of 5–6 years was indicated for AO 0235+164 in the radio and optical bands (Raiteri et al. 2001). Yuan and Fan (2011) analyzed the radio spectral index of blazars 2251+158 which show two strong periodicity of 6.3 ± 1.1 yrs and 3.8 ± 1.2 yrs. Wang (2014) analyzed the historical light curves of AO 0235+164 in optical band and found a possible quasi-periodic variations of 2.5–3 yrs (Wang 2014). Five major optical outbursts of S5 0716+714 from 1995 to 2007 seem to take on periodic variation of 3.0 ± 0.3 yrs (Raiteri et al. 2003). Zhang et al. (2010) found a possible periodic variation of 1211 days of S5 0716+714 in optical band. Wang and Su (2016) analyzed the historical light curve of the blazar J1359+4011 in 15 GHz, and suggested a intrinsic variations of 120–130 days.

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Some blazars also present possible quasi-periodic behaviors in X-ray band. The X-ray light curve of 3C 273 observed by X-ray satellite XMM-Newton demonstrated a periodic behavior with a variability timescale of about 3.3 ks (Espaillat et al. 2008). Rani et al. (2009) reported nearly periodic variations of blazars AO 0235+164 and 1ES 2321+419 in X-ray band by analyzing the data series of All-Sky Monitor(ASM), which have about 17.7 days and 420 days, respectively (Rani et al. 2009). The narrow line Seyfert 1 galaxy, RE J1034+396, strongly indicated near 1 hour periodic behavior during 91 ks observation by XMM-Newton (Gierliński et al. 2008).

We collect the observed data of blazars by All Sky Monitor and Proportional Counter Array (PCA) on RXTE which covers all the data series from 30 December 1995 to 5 January 2012 above 16 years, and select the sources in which observed data points is above 1000. We seek possible quasi-periodic values in these light curves by means of structure function method, discrete correlation function method and J-K method. Comparing with the result of Rani et al. (2009), our works adopt the entire observed data of All Sky Monitor above 16 years. More data series is conducive to look for more possible periodic value and increase the reliability.

In Sect. 2, we discuss the data series of 20 blazars in X-ray about 16 yrs. The analytical methods and results are in Sect. 3. The discussion and conclusion are in Sect. 4 and Sect. 5, respectively.

2 Data

We adopt 1 day average value of X-ray fluxes in 2–10 keV. The data series of BL Lac, 3C 273 and PKS 1510-089 are from "RXTE AGN Timing and Spectral Database" observed by PCA in RXTE,[1] and the other are from the ASM on the RXTE satellite[2] for the 20 blazars listing in Table 1, in which the columns represent order number, name of source, type, redshift, time span, the number of data points and the result of period from column 1 to column 7, respectively. These data series cover the whole observing period from 1 January 1996 to 5 January 2012. These objects are all included in the research objects of Nieppola et al. (2006) and Rivers et al. (2013). A detailed description of the ASM and the processing method of observed data are presented in Levine et al. (1996).

We find some source flux counts are negative value in many days of observations, indicating that the flux is below the detection threshold of instrument ASM on RXTE. Such negative flux counts are deleted in our analysis procedure.

3 Analysis method and results

3.1 Structure function method

Structure function method (SF) is a classical method which is used to analyze the periodicity and timescales firstly introduced by Simonetti et al. (1985). It is suitable to handle data gaps in the time series and analyze uneven data series appropriately. We adopt the first-order SF method defined as

$$D^{(1)}(k) = \frac{1}{N^{(1)}(k)} \sum_{i=1}^{N} w(i)w(i+k)\big[f(i+k) - f(i)\big]^2, \tag{1}$$

where k is the time lag, and $N^{(1)}(k) = \sum_{i=1}^{N} w(i)w(i+k)$. If there is a data in the measurement, the weighting factor is 1, or else is 0. The square of uncertainty of SF is $\sigma_f^{(1)} = \frac{8\sigma_{f\delta}^2}{N^{(1)}(k)D_f^{(1)}(k)}$, where $\sigma_{f\delta}$ is measured noise variance.

The possible analytical results usually are divided into three aspects as follows: (1) If the SF figure present an upward trend and no plateau, it means the timescale is longer than the time span of data series; (2) If the SF figure rise and then appear a dip, follow that rise again, the dip is usually considered as a possible periodic value and the turning point is usually considered as variability timescale; (3) If there are one or more plateaus in the SF figure, these may mean one or more timescales in the light curve and one or more dips corresponding to more possible periodic values.

Four blazars may have intrinsic periodicity in the SF and are listed in Fig. 1. The upper left panel shows AO 0235+164 has a possible period of about 50 days and a modulated period of 102.2 days. This period of 50 days is slightly larger than the period of 17 days in Rani et al. (2009). The periods close to 1 yr and the modulated results are artificial, may be caused by windowing effects from the satellite. The upper right panel of Mrk 501 indicates a less obvious period of about 48.2 days and a harsh period of 559.2–638.2 days. 102.2 days and 151.3 days may be the double and triple periods of 48.2 days, respectively. 1181.8 days may be the double period of 559.2–638.2 days. The lower left panel of 3C 273 presents a possible intrinsic period of 195.5 days. The lower right panel of PKS 1510-089 shows a possible period of 244.4 days. All the results are listed in Table 1, which presents sequence number, source name, type of the source, redshift, time spanning, number of data point and possible period from column 1 to column 7, respectively. The possible period and its error are obtained from the average and standard deviation of two more dips in SF, but the 195.5 days of 3C 273 only have one single dip so error can't be quoted. The majority of the periods are very close to 1 yr. These results may also be caused by windowing effects deriving from the satellite and may not be taken

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[1] http://cass.ucsd.edu/~rxteagn/.

[2] http://xte.mit.edu/XTE.html.

Table 1 The period of 20 blazars by structure function method

No	Source name	Type	Redshift	MJD(1)–MJD(N)	Data points	Period/days
1	BL Lac	BL Lac	0.0686	50645.05–55926.28	1384	367.6 ± 7.7
2	AO 0235+164	BL Lac	0.9400	50088.38–55927.18	2627	50.6 ± 0.6, 369.3 ± 6.3
3	Mrk 501	BL Lac	0.0337	50087.33–55921.83	4340	49.9 ± 1.5, 559.2–638.2
4	Mrk 421	BL Lac	0.0300	50143.27–55926.50	1182	352.7 ± 15.2
5	OQ 530	BL Lac	0.1526	50092.36–55922.71	3110	No obvious
6	3C 66A	BL Lac	0.4440	50093.10–55927.11	3050	No obvious
7	ON 231	BL Lac	0.1020	50091.12–55928.72	2429	372.6 ± 3.4
8	1ES 2344+514	BL Lac	0.044	50087.36–55926.73	3532	No obvious
9	1ES 2321+419	BL Lac	0.0590	50087.36–55926.73	3209	No obvious
10	3C 371	BL Lac	0.0508	50091.11–55927.12	3295	No obvious
11	OJ 287	BL Lac	0.3056	50087.42–55929.81	2626	358.3 ± 2.9
12	ON 325	BL Lac	0.1300	50092.30–55929.69	2460	370.7 ± 5.6
13	PG 1553+113	BL Lac	0.3600	50088.23–55791.61	3324	358.0 ± 18.9
14	PKS 0735+178	BL Lac	0.4500	50087.39–55929.61	2643	364.2 ± 1.1
15	S5 0454+844	BL Lac	1.3400	50088.36–55927.11	3170	No obvious
16	S5 0716+714	BL Lac	0.3000	50087.37–55926.47	3352	No obvious
17	S5 2007+777	BL Lac	0.3420	50087.36–55927.12	3236	No obvious
18	3C 454.3	FSRQ	0.8590	50091.19–55909.16	2926	369.1 ± 3.5
19	3C 273	FSRQ	0.1583	50115.06–55926.56	1960	195.5, 381.3 ± 7.2
20	PKS 1510-089	FSRQ	0.3610	50430.66–55926.01	1317	212.6 ± 19.5

into account as the possible period of blazars (Rani et al. 2009).

Emmanoulopoulos et al. (2010) suggested SF may give rise to incorrect verdict of timescales or periodicity. Hence, we further analyze the possible periodicity of time series by two different analyzing methods, discrete correlation function (DCF) method and J-K method, again.

3.2 DCF method

The DCF method is a classical method for analyzing correlation between two time series, and was first introduced by Edelson and Krolik (1988). Hufnagel and Bregman (1992) further gave the error estimate. It is also suitable for unevenly data and no interpolation. The computing process is divided into three steps as follows. The first step is to calculate the unbinned DCF value between two time series, and the formula is

$$UDCF_{ij}=\frac{(a_i-\bar{a})(b_j-\bar{b})}{\sqrt{(\sigma_a^2-e_a^2)(\sigma_b^2-e_b^2)}},\qquad(2)$$

where a_i and b_j are the radiant flux in data series a and b, respectively. $\bar{a}$ and $\bar{b}$ are the mean value of data series a and b, respectively. σ_a and σ_b are their variances. The second step is to calculate the DCF among these data points which have the same lag τ

$$DCF(\tau)=\frac{1}{M}\Sigma UDCF_{ij}(\tau),\qquad(3)$$

where M is the number of data pairs in bin. The third step is to calculate the standard deviation in each bin which is defined as

$$\sigma_{DCF}=\frac{1}{M-1}\left\{\Sigma\left[UDCF_{ij}-DCF(\tau)\right]^2\right\}^{1/2}.\qquad(4)$$

The DCF method is usually used for analyzing correlation and time lag between different time series in AGN (Raiteri et al. 2003, and references therein). When a and b are the same data series, there is an obvious peak at zero lag position which shows no lag between the two data series. The other strong peaks may correspond to possible periodicities. Figure 2 shows the analytical results of DCFs for the four blazars which may have periodic components. No obvious period of 0235+164 are found in the upper left panel. One obvious quasi-period of 276.0 days of Mrk 501 is found in the upper right panel of Fig. 2, in which the 540.2 days may correspond to double period of 276.0 days. The DCF of 3C 273 in the lower left panel shows a high peak about 201.2 days and the other peaks about 370.1 days. 571.1 days and 759.1 days may correspond to possible periodic components of about 201.2 days. The DCF of PKS 1510-089 in the lower right panel shows a possible periodic component of about

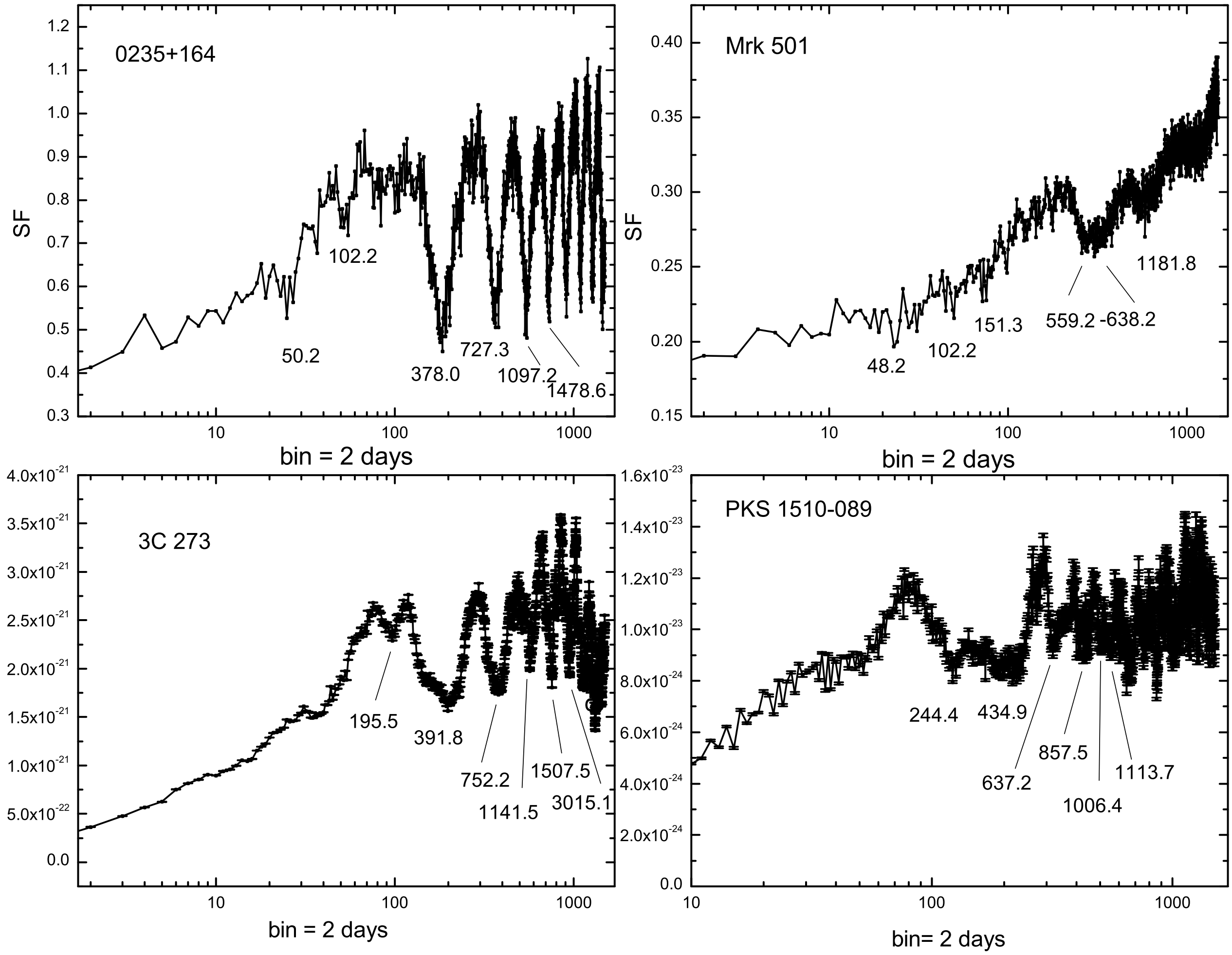

Fig. 1 The analytical results of periodicity by structure function method

198.1 days, which is roughly broad, the other peaks about 340.1 days, 554.6 days, 656.6 days and 864.8 days may be the periodic modulation of 198.1 days in the error range.

3.3 J-K method

In order to further confirm the periodic components which are found by SF and DCF, we utilize the Jurkevich method to analyze the possible periodicity again (Jurkevich 1971). It is also suited for analyzing uneven time series. It tests based on the expected mean square deviation. The time series are folded according to the trial period and then divided into m groups according to their phases bins, and the total variances V_m^2 of each bin are calculated. The minimum value of V_m^2 indicates a possible period. Kidger et al. (1992) introduced a method to judge the authenticity of period value which depends on the formula $f = \frac{1-V_m^2}{V_m^2}$, in which V_m^2 has been normalized (Kidger et al. 1992). If f is large than 0.5, it means a strong periodicity; if f is small than 0.25, it implies a weak periodicity. The analytical results of four blazars by J-K method are listed in Fig. 3. The upper left panel for 0235+164 shows a possible period of 364.8 days, and the other values of 732.5 days, 1091.6 days, 1458.6 days and 1858.4 days which may be the periodic modulation of 364.8 days. But the factor f is only about 0.11, it means the periodic signal of 364.8 days is weak. The upper right panel for Mrk 501 shows a possible period of about 575.7 days, but the dip is roughly broad and the factor f is only about 0.25. The lower left panel for 3C 273 indicates a possible period of 371.4 days. The other values of 734.5 days, 1147.3 days, 1494.0 days and 1857.0 days may be the possible periodic components of about 371.4 days in which the factor f of 371.4 days is about 0.19. The lower right panel for PKS 1510-089 shows a possible period of 338.0 days in which the factor f is 0.15. The other values of 338.0 days, 676.1 days, 1089.0 days and 1361.0 days may be the periodic modulation of 338.0 days. All the factors f of four possible periods are less than or equal to 0.25, hence the periodic signals are weak. The FWHMs of the dips in the V_m^2 are treated as errors

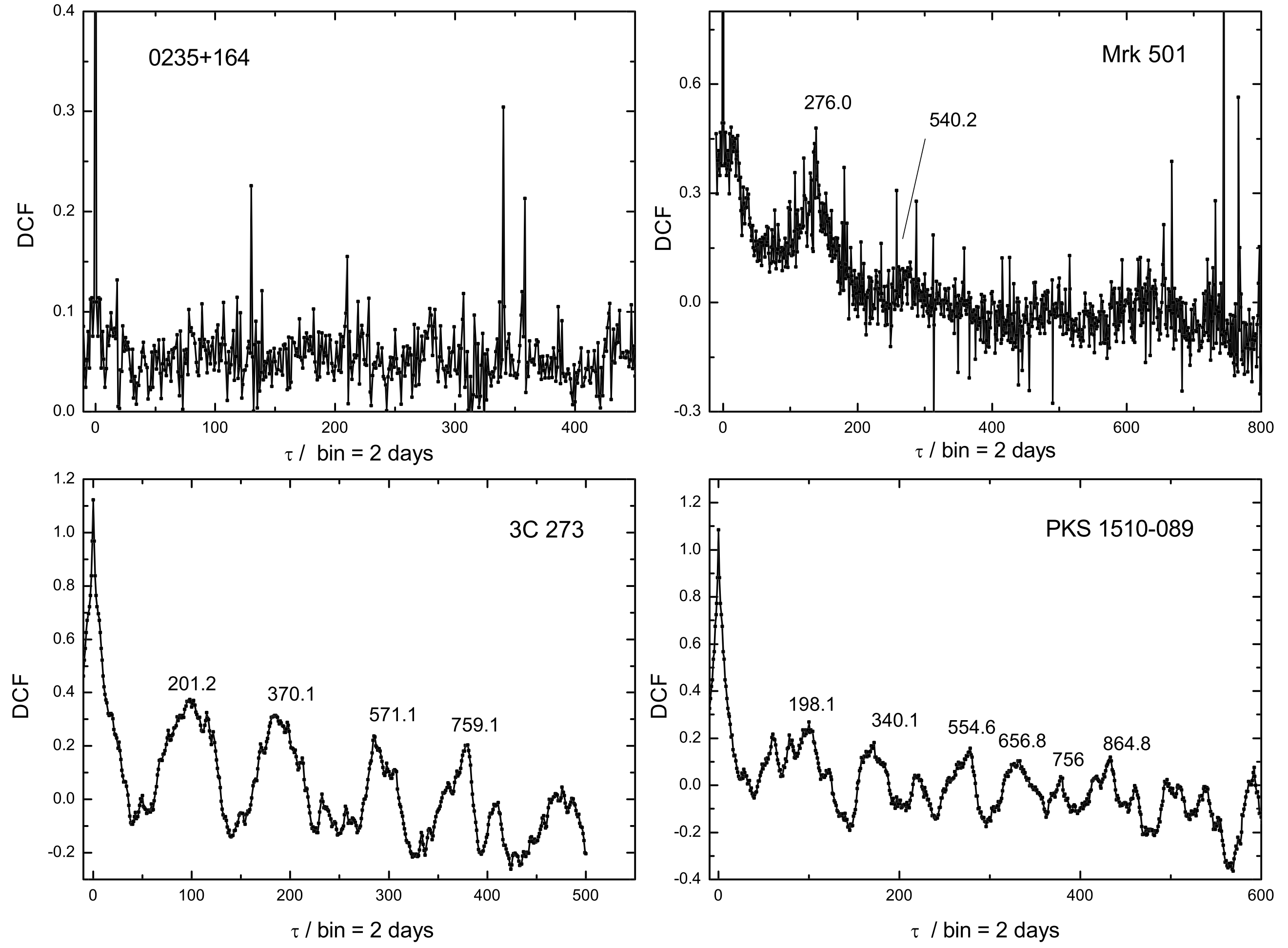

Fig. 2 The analytical results of periodicity by discrete correlation function method

Table 2 The possible periodic results by three analytical methods

Object	SF period/day	DCF period/day	J-K period/day
0235+164	50.6 ± 0.6	no obvious	364.8 ± 12.1 ($f = 0.11$)
	369.3 ± 6.3		
Mrk 501	49.9 ± 1.5	276.0	575.7 ± 68.4 ($f = 0.25$)
	559.2–638.2	540.2	
3C 273	195.5	201.2	371.4 ± 25.5 ($f = 0.19$)
	381.3 ± 7.2		
PKS 1510-089	212.6 ± 19.5	198.1	218.7 ± 14.7 ($f = 0.07$)
		340.1	338.0 ± 22.0 ($f = 0.15$)

of possible periods which are collected in the fourth column in Table 2.

These four blazars with plausible periods are listed in the first column of Table 2. The results of SF, DCF and J-K are listed in column 2, 3 and 4, respectively. The period of 369.3 ± 6.3 days of 0235+164 by SF and J-K may be caused by windowing effects arising from the satellite, another value 50.6 ± 0.6 found by SF is no obvious in DCF and J-K method. The 575.7 ± 68.4 days of Mrk 501 by J-K method are identified by SF about 559.2–638.2 days, 540.2 days by DCF. The period of 195.5 days of 3C 273 by SF is identified by the DCF about 201.2 days. The 371.4 ± 25.5 days of 3C 273 by the J-K may correspond to double period of about 196.0 days. The 212.6 ± 19.5 days of PKS

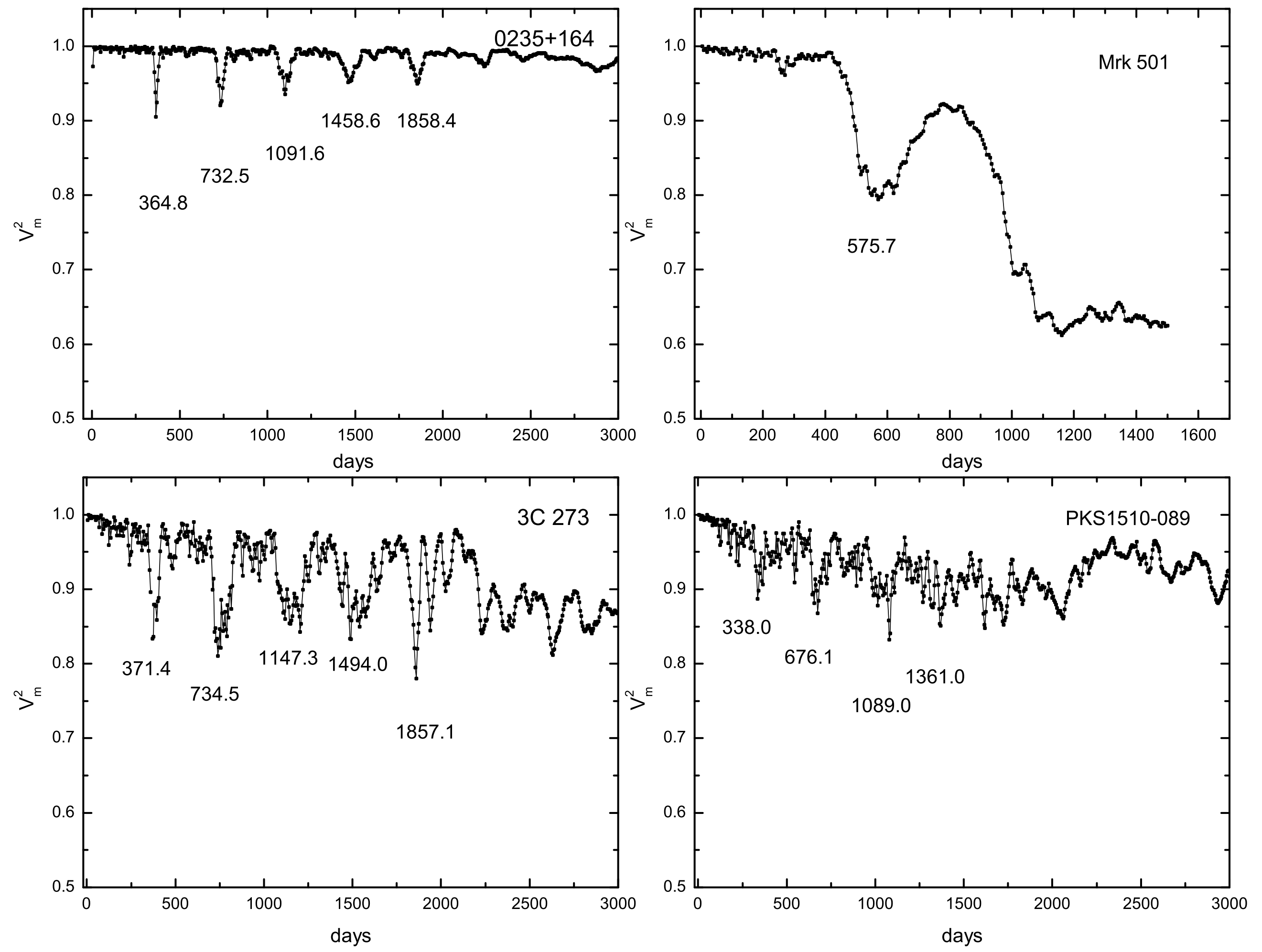

Fig. 3 The analytical results of periodicity by J-K method

1510-089 found by the SF is identified by the 198.1 days in the DCF, and a weak periodic signal of 218.7 ± 14.7 days found by the J-K method, another possible period of 340.1 days found by DCF and identified by the 338.0 ± 22.0 days. Found by the J-K method for PKS 1510-089.

4 Discussion

Sandrinelli et al. (2016) searched for quasi-periodicities on year-like timescales in the light curves of six blazars in the near-infrared-optical and γ-ray bands. The reality of periodicities was carried out by measuring a significance against the background noise (Sandrinelli et al. 2016, references therein). Thus, we follow Sandrinelli et al. (2016) to check the periodicities listed in Table 2 by using the power spectra of X-ray light curves. First, 3C 273 has a periodicity of 370 days with a significance $>99\%$ in four X-ray bands (see Fig. 4). This periodicity is consistent with those obtained in the SF and J-K methods (see Table 2). So, the periodicity of 370 days seems to be reliable in the X-ray light curves for 3C 273. There is a periodicity of 370 days in the 2–10 keV bands for 0235+164, and this periodicity has a significance $>99\%$ (see Fig. 5a). The periodicity of 370 days confirms those results derived by the SF and J-K methods. In addition, a periodicity of 51 days has a significance $>99\%$, and this periodicity is consistent with that found by the SF method (see Table 2). PKS 1510-089 has periodicities of 333 and 227 days with a significance $>90\%$ in 2–10 keV bands, and has periodicities of 326 and 230 days with a significance $>99\%$ in 2–4 keV bands (see Figs. 5c and 5d). These periodicities are consistent with those derived by the SF, DCF and J-K methods (see Table 2). Also, there is a periodicity of 637 days with a significance $>99\%$ in 2–10 keV bands for Mrk 501 (see Fig. 5b). This periodicity is consistent with those derived by the SF and J-K methods. The periodicities derived by the power spectrum method confirm some results listed in Table 2.

Some physical mechanisms may explain the possible periodicities at year-like timescales, such as the binary black

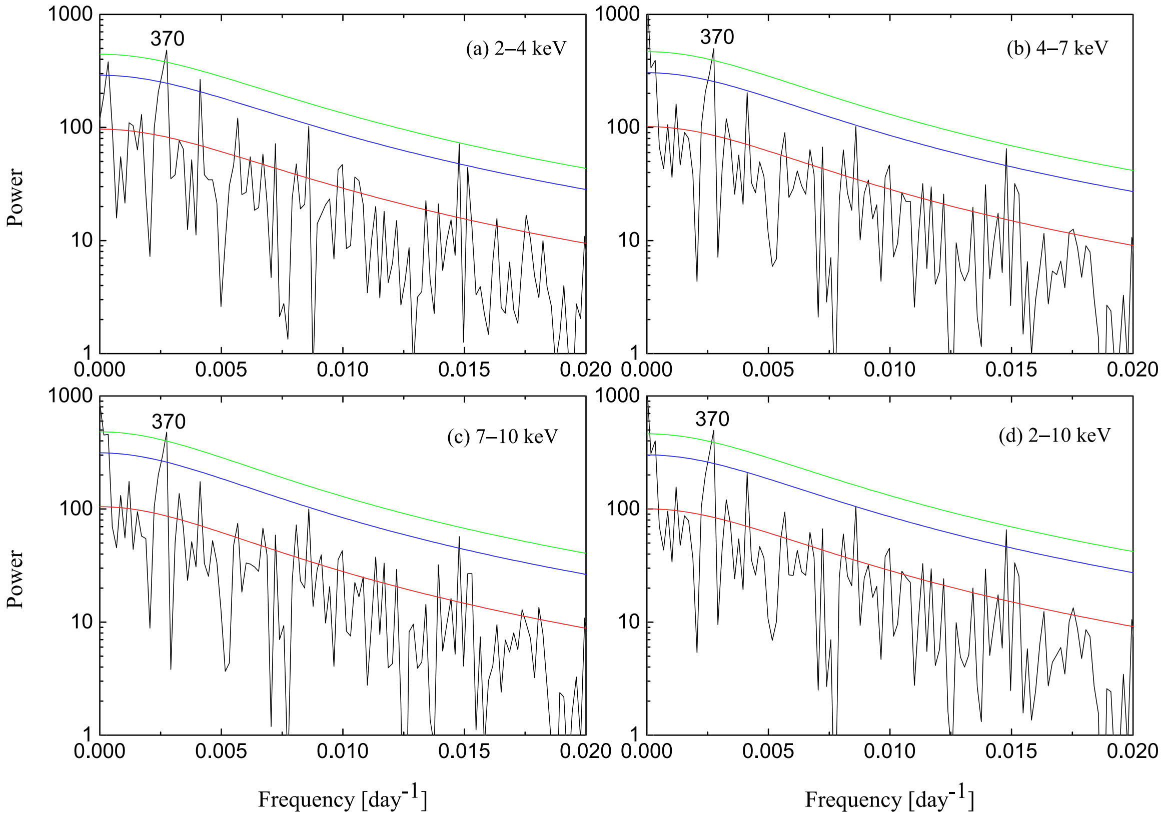

Fig. 4 Bias-corrected power spectra of 3C 273 in X-ray bands. The power is the output obtained by the procedure of Schulz and Mudelsee (2002, REDFIT) normalized with the variance. Curves in each panel, starting from the bottom, are the theoretical red-noise spectrum, and the 95% and 99% χ^2 significance levels, respectively (the *red*, *blue* and *green* ones). Periods in days corresponding to the prominent peaks are marked

hole model and the precession model (e.g. Sandrinelli et al. 2016). The binary black hole model was firstly taken into account to interpret the quasi-periodic variability of OJ 287 in optical band (Lehto and Valtonen 1996). The observed year-like timescale periodicity could be related to the orbit of the secondary black hole, which could destabilize the accretion flow onto the primary black hole and modulate the accretion rate, and as a consequence, the luminosity of the active nucleus (Sandrinelli et al. 2016). Xie et al. (2002, 2004, 2005) studied the periodic optical variations of PKS 1510-089. Xie et al. (2002) explained the period 336 ± 14 days of the quasi-periodic optical flux minimum by the binary black hole model. Xie et al. (2008) found the radio variation periodicity similar to the optical ones. The possible periodicities of ∼330–340 days in the X-ray bands for PKS 1510-089 are consistent with those found in Xie et al. (2002, 2004, 2005, 2008). The wiggling milliarcsecond radio jet observed in this object is taken as further evidence for the binary black hole system, and the "coupling" of the periodicity in the light curves and the helicity in the radio jet could be interpreted in the framework of a binary black hole system (Wu et al. 2005). The precession in the inner jet could produce the periodic variations of blazars. Romero et al. (2000) presented a binary black hole model for the central engine of 3C 273, where the rapid precession is tidally induced in the primary accretion disk inner region by a secondary black hole in a non-coplanar orbit. The inner jet will be aligned with the inner accretion disk of the primary black hole. This model could explain the year-like periodicity.

The periodic variability of blazars at long-term timescales could be from shocks propagating down relativistic jets pointing close to the line of sight (e.g., Scheuer and Readhead 1979; Marscher and Gear 1985; Wagner and Witzel 1995). The short-term variability and intra-day variability could arise from shocks passing through turbulence behind a shock (Marscher et al. 1992). The dominant eddy in the turbulent flows would generate transient quasi-periodic fluctuations. Regions in different distances behind the shock will produce radiation in different wavelengths. Periodic emission variations could be from combinations of relativistic shocks and essentially helical structures, easily induced by magnetohydrodynamical instabilities in magnetized jets (e.g., Camenzind and Krockenberger 1992; Hardee and Rosen 1999). Because the Doppler boosting is extremely sensitive to the viewing angle, substantial changes of jet

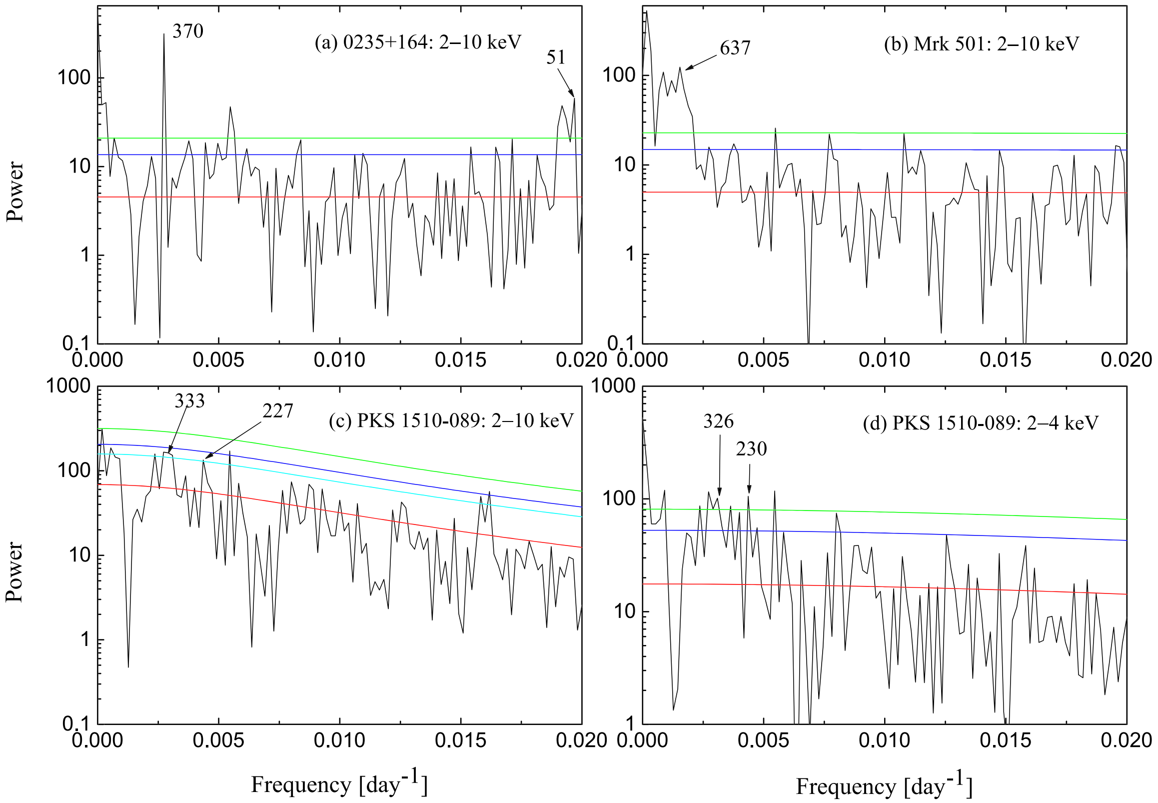

Fig. 5 Same as Fig. 4, but for 0235+164, Mrk 501 and PKS 1510-089. *Cyan line* denotes the 90% χ^2 significance level in plot (**c**)

emission in the radio and optical bands will be observed by us at a small angle to the jet axis when the emission effectively swing past us (e.g., Gopal-Krishna and Wiita 1992; Camenzind and Krockenberger 1992). This situation may also occur in the X-ray band. Fan et al. (2013) calculated the lower limit of gamma-ray Doppler factor of a Fermi blazar samples including Mrk 501, 3C 273 and PKS 1510-089. The lower limits are 2.83, 4.75 and 4.03 for Mrk 501, 3C 273 and PKS 1510-089, respectively (Fan et al. 2013). Because of the appropriate Doppler factor of the jet in the three objects, the observed substantial and nearly periodic fluctuations in their X-ray light curves may be caused by the interplay of a relativistic shock with successive twists in a non-axisymmetric jet structure.

5 Conclusions

We collect the data series of 20 blazars in 2–10 keV covering the whole observing period of the ASM and PCA instrument on the RXTE satellite. We find possible periodicities by structure function method. The periodic values in 11 of these blazars are close to 1 year. These periodicities may be attributed to windowing effects arising from the satellite. The year-like periodicities in AO 0235+164, Mrk 501, 3C 273 and PKS 1510-089 are also found by the DCF, J-K, and power spectrum methods. The year-like periodicities in these four objects could be explained in the framework of a binary black hole system, where the rapid precession is tidally induced in the primary accretion disk inner region by the secondary black hole in a non-coplanar orbit. AO 0235+164, Mrk 501 and 3C 273 have possible nearly periodic values of about 370 days, 630 days and 370 days, respectively. PKS 1510-089 has two nearly periodic components, about 230 and 330 days. There are another possible explanations for these quasi-periodicities at the year-like timescales. One possible case is the turbulence behind a shock, and the quasi-periods could be dominated by the dominant eddies' turnover times. The other is the interplay of a shock with a relativistic jet with essentially helical structure.

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