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ORIGINAL ARTICLE

Evidence for quasiperiodicity in orbital period modulation of WW Cygni

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Abstract The Algol type binary system WW Cygni is known to display an intricate orbital period variability phenomenon which consists in the presence of an alternating component superposed on a secular trend. The alternating component clearly deviates from a strictly periodic pattern. In the present paper we studied WW Cyg using a data set consisting of 309 times of primary minimum, free of outliers, covering a time base of 113.8 yr. We investigated these timing data using periodicity detection methods relying on both amplitude spectrum and enhanced self-correlation (ESC) methods through Monte Carlo simulations. The problem of estimation of the statistical significance in the ESC method was further developed. The multiperiodicity of the $O-C$ residuals was investigated via iterative prewhitening using three types of models consisting of polynomial (P) + multiperiodic (MP) terms, P + light travel time effect (LTTE) terms, and P + MP terms + LTTE terms. The interpretation of the results led us to the following two hypotheses: (i) a quasiperiodic modulation at a time scale of about 46 yr induced by the cyclic magnetic activity occurring in the cool secondary component of WW Cyg, or (ii) the coexistence of a periodic modulation with a period of 50.41 yr related to the presence of an unseen compact stellar object of at least 2.22 $M_\odot$ in the system and a quasiperiodic modulation at a time scale of about 19.7 yr, caused by the mechanism (i).

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1 Introduction

The variability of the Algol type binary system WW Cygni (HIP 98814, HD 227457, BD 41°3595) was discovered by Ceraski (1904). The binary consists of a hot B8 primary component, and a cool G9: lobe-filling secondary component (Yoon et al. 1994; Zavala et al. 2002). Previous spectral classifications of this binary system were B8 + G: (Struve 1946) and B7(V) + G8(VI) (Hall and Wawrukiewicz 1972). Zavala et al. (2002) estimated from their CCD observations a spectral type K2 V for the secondary component of WW Cyg. Two years later, relying on the CCD observations made through a Cousins I-band filter with the HAT-5 telescope from the Hungarian-made Automated Telescope network (HATnet), Hartman et al. (2004) argued that the secondary component of this system should have an even later spectral type, i.e. later than K3. Güdel and Elias (1996) included WW Cyg among PELAs (Peculiar Emission-Line Algols) (see also Elias and Güdel 1993). However, as Mennickent et al. (2016) recently noted, "*there is no reported evidence for a W Serpentis classification*".

The orbital period variability of this system is known since the study of Graff (1922). Blažko (1924) established periodic ephemerides involving periodicities of 18.2 and 17.3 yr, while the corresponding amplitudes were 0.0140 and 0.0105 d, respectively. Later, Prager (1933) proposed a similar ephemeris, but taking into account a longer period of 38.5 yr, with an amplitude of 0.025 d. Dugan and Wright (1939) also mentioned the possibility to describe the $O-C$ residuals of WW Cyg using a sinusoidal term similar with that given by Prager, but with a smaller amplitude. The discussion on the periodic character of the orbital period variation of WW Cyg was later resumed by Nason and Moore (1951) who found a period yet longer of 45.4 yr, the corresponding amplitude being of 0.026 d. They also invoked the apsidal motion as a possible cause of this alternating period change. Whitney (1957) examined the timing data covering a relatively short time base of about 11 yr and found a possible orbital period variation with a period of about 6.8 yr, and an amplitude of 0.008 d. Wood and Forbes (1963) computed the elements of a cubic ephemeris for WW Cyg. Note that it is well-known that a third order polynomial model is able to describe a piece of a sinusoid with a length of about one cycle (e.g. Skillman and Patterson 1988, Applegate and Shaham 1994). Hall and Wawrukiewicz (1972) remarked that a strictly periodic model may describe the $O-C$ residuals for timing data recorded earlier than 1960, but it fails to fit the more recent data. Thus, they concluded that the orbital period variation of WW Cyg is not sinusoidal. Consequently, in order to describe the more and more complicated shape of the curve, they hypothesized the existence of seven abrupt period changes in this binary system. Finally, Zavala et al. (2002) emphasized the presence of a secular parabolic trend in the $O-C$ residuals, which corresponds to a linear increase of the orbital period of WW Cyg. Then, they analysed the alternating pattern of the detrended residuals (see Fig. 5 in Zavala et al. 2002), but only by computing their Fourier spectrum. They found an orbital period modulation phenomenon with a periodicity of about 56.4 yr, and amplitude of 0.0187 d. Due to the fact that the eccentricity of the system is zero (Struve 1946; Hall and Wawrukiewicz 1972), the hypothesis of apsidal motion has been rejected. Thus, Zavala et al. (2002) considered as a possible cause of this modulation the light travel time effect (LTTE) caused by the presence of an unseen companion. They performed some numerical tests concerning the possible amplitude of the first harmonic of the above periodicity, but they did not succeed to obtain a representation of the $O-C$ curve via LTTE on an elliptical orbit. Consequently, they also rejected this mechanism as an explanation of the orbital period modulation of WW Cyg. Moreover, they noted the lack of strict periodicity of the $O-C$ residuals obtained after removing a strictly periodic component with the period of 56.4 yr. Therefore, they concluded that the existence of magnetic cycles within the secondary component of the system may explain the observed variability displayed by the $O-C$ residuals (see also Zavala et al. 2004, Zavala 2005).

In this paper we focused our attention on the analysis of both long-term and short-term alternating variability of the detrended $O-C$ residuals of WW Cyg. The standard interpretation of the physical causes of such a behaviour appearing in the timing data of eclipsing binaries takes into account the two well-known mechanisms: the LTTE, and the cyclic magnetic activity (Hall 1989, 1990). The orbits of WW Cyg components' being circular, the apsidal motion can be rejected as a possible cause for the observed orbital period modulation. The differential diagnosis between the above two mechanisms may be approached by taking into account their signatures in both time and frequency domains.

In the *time domain*, LTTE produces periodic harmonic (sinusoidal) $O-C$ curves for circular orbits, and arbitrary periodic $O-C$ curves in case of eccentric orbits. In the latter situation, the shape of the $O-C$ curves is determined by the values of the eccentricity and longitude of the periastron (e.g. Kopal 1978). In the *frequency domain*, the signature of LTTE in case of circular orbits is a peak situated at the frequency which corresponds to the orbital frequency, while for eccentric orbits it consists of an equidistant structure of peaks corresponding to the fundamental (orbital) frequency and its harmonics (Kopal 1978).

The signature of the orbital period modulation due to the cyclic magnetic activity in the *time domain* consists of quasiperiodic $O-C$ curves (e.g. Warner 1988, Hall 1990, Applegate 1992, Richman et al. 1994, Lanza et al. 1998, Watson and Marsh 2010, Völschow et al. 2016).

By analogy with the extensively studied case of the quasiperiodic light curves of dwarf novae (e.g. Patterson et al. 1977, Robinson and Nather 1979, Patterson 1981, Middleditch 1982, Lewin et al. 1988, van der Klis 1989, Larsson 1992; see also Deeming 1975), in the *frequency domain*, one may expect to find the presence of broad 'humps' or 'bumps' (i.e. band spectra) in the amplitude spectra of the respective $O-C$ residuals.

In this paper we model the data with a series of functions (polynomial, (multi)periodic, LTTE, and their linear combinations) and apply amplitude spectrum analysis for periodicity detection. Furthermore, we use the enhanced self-correlation (ESC) method (Crăciun et al. 2015) which is a development of the self-correlation (SC) method proposed by Percy et al. (1981), suitable not only for the analysis of periodic, but also for quasiperiodic and irregular phenomena. In this study we present another, more straightforward approach of the estimation of the statistical significance of each of the features appearing in the ESC diagram (ESCD) and consequently, of the variability at the involved time scales. Thus, the aim of our analysis methodology is to emphasize the signatures of both periodic and quasiperiodic modulations possibly involved in the orbital period variability of WW Cyg.

2 Observational data

The present study of the orbital period variation of WW Cyg relies on 368 times of primary minimum light [253 visual (v), 78 photographic (pg), 4 photoelectric (pe), and 33 ccd] collected from the Lichtenknecker-Database of the BAV (http://www.bav-astro.de/LkDB) and '*$O - C$ Gateway. Database of times of minima (E) and maxima (RR)*' of Variable Star Section of Czech Astronomical Society (http://var.astro.cz/ocgate/index.php). We also include the times of primary minimum light given by Zavala et al. (2002), Hoffman et al. (2006), and that published by Samolyk (2015a,b).

The preliminary stages of the $O - C$ curve analysis and modelling involves the fitting of: (i) cubic polynomial ephemeris (P3), and (ii) an ephemeris consisting of an arbitrary periodic model (including the fundamental frequency together with its first two harmonics) superposed on a cubic polynomial trend (P3 + MP1;3) (see Sect. 3 below for the details concerning notation). During these data processing stages, 26 outlying times of primary minima (11 v, 13 pg, 1 pe, and 1 ccd) have been rejected in an iterative way, using a $k\sigma$ ($k > 3.0$) criterion (see also Pop and Vamoş 2013).

The rejected timing data have the following structure, according to the observational technique: 4.35% of the v data, 16.67% of the pg data, and 5.41% of the pe + ccd data. The relatively high percentage of the outlying pg data is understandable taking into account both the integrator character of photographic data and the length of the respective integrating time.

Thus, we obtained 342 times of primary minima. Their number was further reduced by averaging the observed minimum light times corresponding to the same event (primary minimum). The final data set consists of 309 data points, covering a time base of about 113.8 yr. The following linear ephemeris has been established

$$t_n = HJD2436466.3276(44) + 3.3177436(12)n. \qquad (1)$$

The increase of the observational time base allowed us to find that the long term behaviour of the timing data of WW Cyg may be described by a cubic ephemeris, the standard deviation of the $O - C$ residuals being $\sigma_{res} = 0.01292$ d. The coefficient of the third degree term is statistically significant according to Student's t test, the null hypothesis being rejected at a significance level lower than 10^{-9}. The values of the parameters of the cubic ephemeris will be refined within the frame of the more complex models considered in the following sections.

3 Data analysis methods

3.1 Periodicity and multiperiodicity analysis

The methodology applied for the analysis of the timing data on WW Cyg is the same as the one used in the previous papers of Pop (1996), Pop et al. (2003, 2011), Pop and Vamoş (2012). The methodology used for periodicity detection relies on the analysis of the amplitude spectrum of the detrended $O - C$ residuals. The statistical significance of the spectral peaks was estimated using Monte Carlo simulations involving 50,000 artificial data sets; these allow us to build up null hypotheses using Gaussian noise, bootstrap resampling, and random permutations (Pop 2005, 2007, which relies on the approach of Kuschnig et al. 1997, Pop and Vamoş 2007, Pop et al. 2010). Note that similar approaches of periodicity detection through Monte Carlo simulations were successfully used, e.g., by Frescura et al. (2008), Paunzen et al. (2013), and Mikulášek et al. (2015), too. Finally, we estimated the confidence levels for which the null hypothesis, that the highest peak in the amplitude spectrum is caused by chance, can be rejected. This approach is possible due to the preliminary standardization of the analysed data. Thus, unlike the original method of Kuschnig et al. (1997), the considered test statistic is not altered by the mean noise level in the considered frequency range. The periodicity detection and the data modelling were applied in the frame of an iterative prewhitening.

In order to perform the fitting of the timing data on WW Cyg through non-linear least squares method, we considered the following ephemeris models:

(i) Ephemeris consisting of the superposition of a *polynomial trend* (P[K]) and a *multiperiodic term* ($MP[L; M_1, M_2, \ldots, M_L]$) (see Pop 1996, 1999b, Pop et al. 2011)

$$t_n = t_0 + \sum_{k=1}^{K} \tau_k n^k + \sum_{l=1}^{L} \sum_{m=1}^{M_l} \tau_{lm} \sin(\Omega_{lm} n + \Phi_{lm}), \qquad (2)$$

in which t_0 is the initial epoch, $\tau_1 = P_p$, the orbital period (the *primary* period), τ_k ($k = 1, \ldots K$) are the coefficients of the polynomial term, τ_{lm}, and Φ_{lm} ($l = 1, \ldots, L; m = 1, \ldots, M_l$) are the amplitudes and phases of the periodic terms, $\Omega_{lm} = m\Omega_l$, $\Omega_l = 2\pi f_{0l} = 2\pi f_{sl}/f_p = 2\pi P_p/P_{sl}$, P_{sl} ($l = 1, \ldots, L$) being the *secondary* periodicities detected in the original $O - C$ curve. From the viewpoint of its time behaviour, the *multiperiodic term* may contain either periodic harmonic components (i.e. $M_l = 1$) or arbitrary periodic components. According to their physical interpretation in case of WW Cyg, the strictly sinusoidal components may be induced by the LTTE related to unseen companions revolving on circular orbits. Depending on their *structural properties* which are functions only on the orbital eccentricity (e) and on the longitude of the periastron (ω) (Pop 1998, 1999a, see also Kopal 1978, Borkovits and Hegedüs 1996, Konacki and Maciejewski 1996, 1999a,b), some of the arbitrary periodic components may also be caused by the LTTE related to unseen companions, but revolving on elliptical orbits. The keystone of this approach is that, using the well-known expansion of the elliptic motion, it may be immediately shown that the ratios between the harmonics' amplitudes and the amplitude of the fundamental frequency ($\tau_{l2}/\tau_{l1}, \tau_{l3}/\tau_{l1}, \ldots$), are functions only of e and of ω. Furthermore, using the expansions of the Bessel functions according to the powers of e, we can evaluate the above amplitude ratios at the desired precision level. The above references supply us methods to obtain first guesses of the values of the orbital parameters.

(ii) Ephemeris consisting of the superposition of a *polynomial trend* (P[K]) and a single LTTE term related to the elliptical orbit case (e.g. Paparó et al. 1988, Pop 1998, Borkovits et al. 2016),

$$t_n = t_0 + \sum_{k=1}^{K} \tau_k n^k - \tau_0 e \sin\omega + \tau_0\big(\sqrt{1-e^2}\cos\omega\sin E_n + \sin\omega\cos E_n\big), \tag{3}$$

where $\tau_0 = a \sin i / c$, $a \sin i$ is the projected semi-major axis of the absolute orbit of the barycentre of the close binary around the barycentre of the hypothetical triple system, i is the inclination of the normal to orbit plane with respect to the line of sight, c is the speed of light, and E_n is the eccentric anomaly. The E_n values were calculated by numerically solving Kepler's equation using Danby's method (see Murray and Dermott 1999, p. 35) which relies on Taylor series expansion and which displays a quadratic convergence.

(iii) Ephemeris consisting of the superposition of a *polynomial trend* (P[K]), a given number of LTTE terms corresponding to N_b hypothetical companions (LTTE[N_b]) on elliptical orbits, and a *multiperiodic term* (*MP*[L; M_1, $M_2, \ldots, M_L$]), assuming that the mutual gravitational perturbations between these unseen companions may be neglected (e.g. Hinse et al. 2012, 2014)

$$t_n = t_0 + \sum_{k=1}^{K} \tau_k n^k + \sum_{k=1}^{N_b}\Big[-\tau_{0k} e_k \sin\omega_k + \tau_{0k}\Big(\sqrt{1-e_k^2}\cos\omega_k \sin E_{nk} + \sin\omega_k \cos E_{nk}\Big)\Big] + \sum_{l=1}^{L}\sum_{m=1}^{M_l} \tau_{lm}\sin(\Omega_{lm} n + \Phi_{lm}). \tag{4}$$

In this case, the *multiperiodic term* may include strictly sinusoidal terms (i.e. $M_l = 1$) possibly related to LTTE on circular orbits and arbitrary periodic terms caused by mechanisms other than LTTE on elliptical orbits. If one detects the presence of an additional periodic component, the values of the input parameters for this model are determined through an intermediary model fitting. The initial timing data were corrected for the third order polynomial terms (2) and for the contribution of the LTTE. We then fitted a P1 + MP1;1 model (see Eq. (2)) in which the frequency value is the one previously detected in the residuals obtained after removing the P[K] + LTTE model. Then, the values of the parameters of the periodic component together with those of the P[K] + LTTE model are taken as first guesses for the P[K] + LTTE1 + MP1;1 model fitting. Additional periodic components may be included after performing this intermediary step. At this point, the presence of additional harmonics should be investigated; these might be the signature of an unseen companion on an elliptical orbit.

3.2 Quasiperiodicity analysis through ESC method

Finally, the other method used was the ESC method proposed by Crăciun et al. (2015). It relies on the SC method proposed by Percy et al. (1981) and further described by Percy et al. (1993, 2002, 2003, 2004, 2006a,b) (see also Percy and Mohamed 2004, Percy and Palaniappan 2006, Percy et al. 2009, Percy and Terziev 2011, Terziev et al. 2011). This analysis together with that of the amplitude spectrum provides us with complementary information (the so-called '*variability profile*') useful in order to discriminate between periodic, quasiperiodic, and irregular variability phenomena. It is also useful because it works with unevenly spaced data.

A strictly periodic phenomenon is recognized in its (E)SCD by the presence of a typical pattern consisting of equidistant minima (and also maxima) covering the whole range of δt values between 0 and the observations time base (see Crăciun et al. 2015 and the above quoted papers of Percy et al.). On the contrary, in the case of a quasiperiodic phenomenon, due to the lack of strictly periodicity, the respective (E)SCD displays specific features:
(i) the number of minima is lower than in the case of strictly periodicity. As Percy noted in the above quoted papers, a higher degree of irregularity involves a lower coherence of the analysed signal;
(ii) the profile of minima and maxima appearing in the (E)SCD are distorted;
(iii) the distance between successive minima is variable. Additional distortions of the shape of the (E)SCD is induced by the finite character of the observational data, their non-equidistant sampling, and by the presence of noise.

Crăciun et al. (2015) presented a methodology based on the ESC function which may be applied in order to estimate the statistical significance of a deterministic signal, possibly hidden in the final $O-C$ residuals. In the present study we tackled the problem of estimation of the statistical significance of a (quasi)periodicity existing in a given observational data set in a slightly different way. The presence of a more or less regular variability phenomenon is emphasized in the (E)SCD by the presence of alternating minima and maxima. The estimation of the involved time-scale (period or quasiperiod) relies on the estimation of the position of the

Table 1 Parameters of the cubic + multiperiodic (P3 + MP3;2,1,1) ephemeris for the timing data of WW Cyg

$t_0 = HJD\ 2436466.23595(50)$			
$\tau_1 \equiv P_p = 3.31775570(28)$ dc^{-1}			
$\tau_2 = 6.243(31) \times 10^{-9}$ dc^{-2}			
$\tau_3 = -2.72(14) \times 10^{-13}$ dc^{-3}			
$f_{01} = 0.0001845(11)$	$P_{s1} = 49.16(30)$ yr	$\tau_{11} = 0.01842(52)$ d	$\Phi_{11} = 0.469(25)$ rad
$2f_{01} = 0.0003695$	$P_{s1}/2 = 24.58$ yr	$\tau_{12} = 0.00844(44)$ d	$\Phi_{12} = 0.118(52)$ rad
$f_{02} = 0.000518(50)$	$P_{s2} = 17.55(17)$ yr	$\tau_{21} = 0.00460(36)$ d	$\Phi_{21} = 5.41(11)$ rad
$f_{03} = 0.000874(61)$	$P_{s3} = 10.393(73)$ yr	$\tau_{31} = 0.00316(36)$ d	$\Phi_{31} = 5.25(13)$ rad
$\sigma_{res} = 0.00428$ d			

first minimum and, if it is the case, of the distance between successive minima (see the above quoted papers of Percy et al.). Consequently, our approach is focused on the estimation of the statistical significance of the different features appearing in the ESCD. One can state that a given (quasi)periodicity is statistically significant if at least one minimum in the ESCD used to estimate it is statistically significant. In the case of the presence of additional minima, they also have to be statistically significant. This means that it is not the result of the interaction between the random observational noise (the null hypothesis H_0) and the data sampling. As in the paper of Crăciun et al. (2015) we applied this statistical test through Monte Carlo simulations in which we assumed that under H_0 there is no deterministic component and the observational noise is Gaussian white noise.

Through this method we may identify those regions in the ESCD which have a significantly different profile than that produced by a time series consisting of purely Gaussian white noise having the same sampling and standard deviation as the observational data. The occurrence of such situations may be interpreted as an indicator of the existence of a deterministic component which may be hidden in the observational noise.

For each set of $O-C$ residuals obtained during the iterative prewhitening process we applied the following analysis method:
(i) the input data are standardized;
(ii) the *observed* ESCD is computed by considering binning of Δt values between 10 and 100;
(iii) we generated 50,000 time series under H_0, having the same sampling as that of the $O-C$ residuals and consisting of Gaussian white noise with zero mean and unit standard deviation;
(iv) for each of these artificial time series the ESCD is computed;
(v) during the numerical simulations the *observed* and *computed* ESC values (the $\Delta(O-C)$ values) are compared and counted, and thus a value of the confidence level ($Prob$) for rejecting H_0 is estimated. The confidence level value is computed relying on the upper-tail (lower-tail) test if the 'observed' ESC value is lower (higher) than the median of the corresponding ESC values computed from numerically generated data. In Figs. 8–12 below we displayed only the confidence level values between 95% and 100%.

4 Data analysis and results

4.1 Classical $O-C$ analysis and hypothesis of a third companion

4.1.1 Polynomial + multiperiodic ephemeris

For the first approach of modelling the run of the $O-C$ residuals, we took into account the ephemeris model given in Eq. (2), which has a high degree of generality. The improved values of the parameters of the adopted P3 + MP3;2,1,1 model are displayed in Table 1, while Figs. 1 and 2 illustrate the evolution of the prewhitening process and the run of this final model among the observed data points. The first three periodicities (having the preliminary values resulted from the respective amplitude spectra 52.12 yr, 17.57 yr, and 10.41 yr) detected during the iterative prewhitening procedure proved to be statistically significant, the three null hypotheses mentioned in the previous section being rejected at confidence levels of 100%. Let us note that, because of the complexity of the P3 residuals both in time and frequency domains, the removal of the longest periodicity through non-linear least squares fitting required an arbitrary periodic model including the fundamental frequency together with its first harmonic.

Examining the three frequencies listed in Table 1, we found that the values of their ratios, $f_{02}/f_{01} = 2.81(27)$ and $f_{03}/f_{01} = 4.74(37)$, seem to be even marginally suggestive of the following frequency relations: $f_{02} = 3f_{01}$ and $f_{03} = 5f_{01}$. Although the hypothesis of the arbitrary periodic nature of this orbital period modulation phenomenon (including the fundamental frequency together with its four harmonics) could be tempting, we note the lack of the third

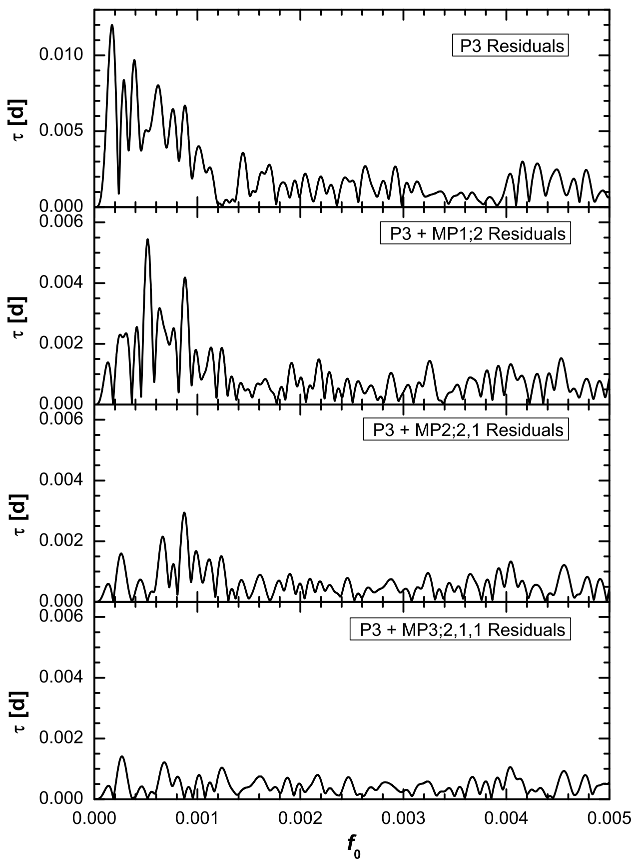

Fig. 1 Amplitude spectra corresponding to different stages of the $O-C$ curve prewhitening of WW Cyg considering models consisting of a cubic trend and different multiperiodic type components

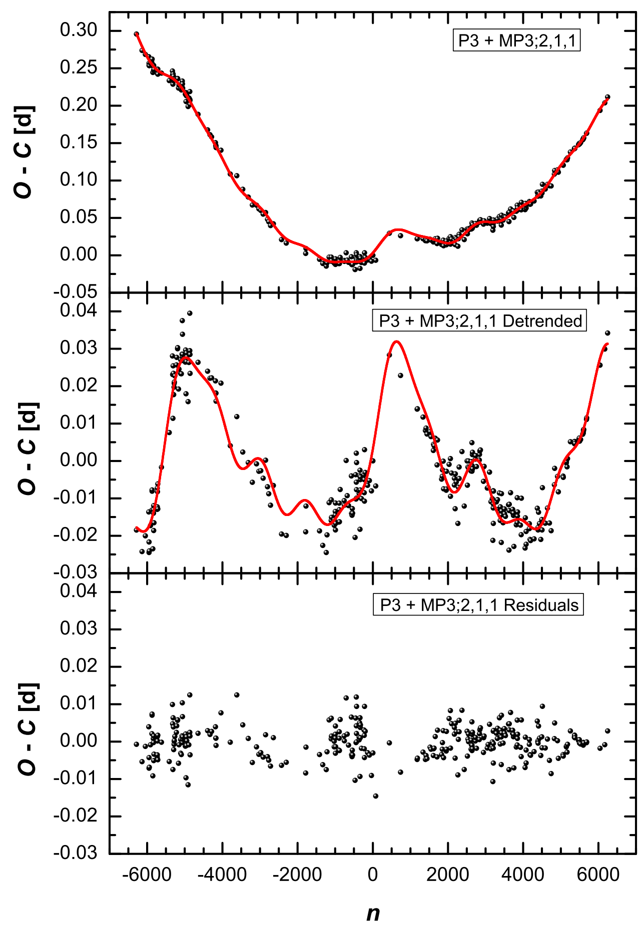

Fig. 2 Multiperiodic model superposed on a cubic polynomial trend of the $O-C$ curve of WW Cyg

harmonic, i.e. $4f_{01}$. Moreover, the estimation of the involved standard errors implicitly assumes the Gaussianity of the observational errors, while the $O-C$ residuals obviously display a heteroskedastic character. It is related to the different nature of the used observational techniques, to their evolution during the observational time base, and also to the progress of the data processing methods. Because of these two arguments, we rejected at this moment the hypothesis of an arbitrary periodic modulator signal involving a relatively high number of detectable harmonics.

From the viewpoint of a physical interpretation, the adopted multiperiodic model supplies substantiation for two scenarios: (i) three independent mechanisms, two of them having their frequencies in some resonance with the first one, i.e. 3 : 1 and 5 : 1, and (ii) two independent mechanisms, the first one displaying an arbitrary periodic variability (including the fundamental frequency and its first two harmonics), the second one being in some 5 : 1 resonance with the first mechanism. The amplitude spectrum of the final P3 + MP3;2,1,1 residuals displayed no statistically significant peaks.

4.1.2 *Polynomial + multiperiodic + LTTE ephemeris*

The first harmonic of the longest periodicity found in the previous model indicates a non-sinusoidal shape of the $O-C$ curve. Its physical significance might be related to the occurrence of a LTTE due to the presence of a third object in the system. Having in view this hypothesis and the above P3 + MP3;2,1,1 ephemeris (see also Table 1), we estimated first guesses for the values of the orbital elements of the absolute orbit of the WW Cyg eclipsing binary around the wide system barycentre relying on a P3 + MP1;3 ephemeris (see also the second scenario in the above subsection). A P3 + LTTE ephemeris was then established through non-linear least squares fitting. As it can be seen from Fig. 3, the LTTE on a highly eccentric orbit (see Table 2) is able to supply a good description of the long term variation (50.41 yr) in the $O-C$ residuals (see Fig. 3 and Table 2). In addition to the parameters mentioned in Sect. 3, we also included in Table 2 the corresponding values of the expected semi-amplitude of the radial velocity curve (K_{rad}) and that of the mass function ($f(M)$).

The analysis of the obtained residuals led to the detection of three additional periodicities of 17.85 yr, 12.32 yr, and 33.89 yr. For the first periodicity, the respective confidence levels were 100%. For the second and the third periodicities, the three considered null hypotheses were rejected at high confidence levels (99.968%, 99.966%, 99.980%, and

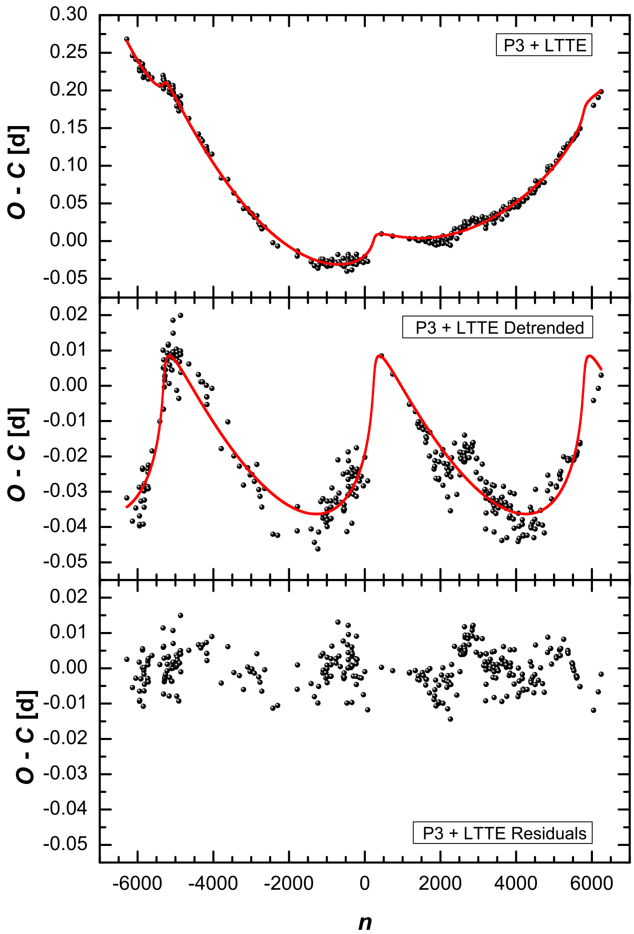

Fig. 3 LTTE model superposed on a cubic polynomial trend of the $O - C$ curve of WW Cyg

99.926%, 99.938%, 99.934%, respectively), but higher than the threshold of 99.9% indicated by Kuschnig et al. (1997). We remark that the three periodicities are in good agreement with those included in the P3 + MP3;2,1,1 model. The values of the parameters of the final P3 + LTTE1 + MP3;1,1,1 ephemeris are listed in Table 3, while the evolution of the amplitude spectrum shape during the iterative prewhitening and the run of the *observed* and *computed* $O - C$ curves are displayed in Figs. 4 and 5, respectively. According to our statistical tests, the P3 + LTTE1 + MP3;1,1,1 residuals do not show significant periodicities.

For a better evaluation of the models consisting of the superposition of a LTTE term and a multiperiodic one, we displayed in Fig. 6 the amplitude spectra of the corresponding residuals. Examining these spectra we remark the appearance of some peaks with higher amplitude despite the successive removal of each of the three low amplitude signals led to an overall decrease of the amplitude level in the residuals' spectra. Such a behaviour indicates the inadequacy of the considered multiperiodic model which consists of the superposition of the three periodic harmonic components.

Let us consider the hypothesis of the presence of an unseen companion of WW Cyg corresponding to each of the detected periodicities both in the LTTE and LTTE1 + MP3;1,1,1 models. In order to get some estimates of the orbital and physical parameters of each of the hypothetical companions, we took into account the physical parameters of WW Cyg as derived by Mezzetti et al. (1980). According to their Tables II and III, the components of this system have masses of 6.2 and 1.9 $M_\odot$, the radii of 4.2 and 5.1 $R_\odot$, and luminosities of 573.03 and 19.05 $L_\odot$, while the inclination angle is about 88.8°. The separation between the component stars may be estimated as 18.81 $R_\odot$, or 0.0875 AU. With these values and with those of the orbital elements given in Tables 2 and 3, we computed the mass (M_3) and the semi-major axis of the absolute orbit (a_3) of the respective hypothetical companions around the system barycentre for different values of the inclination angle (Fig. 7). Within the hypothesis of the coplanarity of the absolute orbits of the close

Table 2 Parameters of the P3 + LTTE ephemeris for the timing data of WW Cyg. For $i = 88.8°$ we also listed the respective values of M_3, a_{12}, and a_3

$t_0 = HJD\ 2436466.2563(21)$		
$\tau_1 \equiv P_p = 3.31775454(29)\ \mathrm{dc}^{-1}$		
$\tau_2 = 6.295(33) \times 10^{-9}\ \mathrm{dc}^{-2}$		
$\tau_3 = -2.06(12) \times 10^{-13}\ \mathrm{dc}^{-3}$		
$f_{01} = 0.00018020(79)$	$P_{s1} = 50.41(22)$ yr	
$\tau_{01} = 0.0369(51)$ d	$a \sin i = 6.38(88)$ AU	
$e = 0.879(42)$	$\omega = 25.427(64)°$	
$\Omega_{s1} = 3.413(15) \times 10^{-4}\ \mathrm{rad\,d}^{-1}$	$T_p = HJD\ 2437270(89)$	
$K_{rad} = 7.9(1.7)\ \mathrm{km\,s}^{-1}$	$f(M) = 0.102(42)\ M_\odot$	
$M_3 = 2.22\ M_\odot$	$a_{12} = 6.38$ AU	$a_3 = 23.32$ AU
$\sigma_{res} = 0.00525$ d		

Table 3 Parameters of the P3 + LTTE1 + MP3;1,1,1 ephemeris for the timing data of WW Cyg. For $i = 88.8°$ we also listed the respective values of M_3, a_{12}, and a_3

$t_0 = HJD$ 2436466.2517(24)			
$\tau_1 \equiv P_p = 3.31775522(26)$ dc^{-1}			
$\tau_2 = 6.421(32) \times 10^{-9}$ dc^{-2}			
$\tau_3 = -2.32(12) \times 10^{-13}$ dc^{-3}			
$f_{01} = 0.00017964(88)$		$P_{s1} = 50.56(25)$ yr	
$\tau_{01} = 0.0319(40)$ d		$a \sin i = 5.53(69)$ AU	
$e = 0.821(52)$		$\omega = 28.809(60)°$	
$\Omega_{s1} = 3.402(17) \times 10^{-4}$ rad d^{-1}		$T_p = HJD$ 2437258(112)	
$K_{rad} = 5.7(1.0)$ km s^{-1}		$f(M) = 0.066(25)$ M$_\odot$	
$M_3 = 1.87$ M$_\odot$		$a_{12} = 5.52$ AU	$a_3 = 23.91$ AU
$f_{02} = 0.0005166(52)$	$P_{s2} = 17.58(18)$ yr	$\tau_{21} = 0.00398(47)$ d	$\Phi_{21} = 4.40(12)$ rad
	$a \sin i = 0.690(81)$ AU	$K_{rad} = 1.17(14)$ km s^{-1}	$f(M) = 1.06(37) \times 10^{-3}$ M$_\odot$
	$M_3 = 0.43$ M$_\odot$	$a_{12} = 0.69$ AU	$a_3 = 13.12$ AU
$f_{03} = 0.0007321(97)$	$P_{s3} = 12.41(16)$ yr	$\tau_{31} = 0.00259(41)$ d	$\Phi_{31} = 2.38(20)$ rad
	$a \sin i = 0.449(71)$ AU	$K_{rad} = 1.08(17)$ km s^{-1}	$f(M) = 5.9(2.8) \times 10^{-4}$ M$_\odot$
	$M_3 = 0.35$ M$_\odot$	$a_{12} = 0.45$ AU	$a_3 = 10.47$ AU
$f_{04} = 0.0002715(81)$	$P_{s4} = 33.5(1.0)$ yr	$\tau_{41} = 0.00260(46)$ d	$\Phi_{41} = 2.77(18)$ rad
	$a \sin i = 0.449(80)$ AU	$K_{rad} = 0.400(71)$ km s^{-1}	$f(M) = 8.1(4.3) \times 10^{-5}$ M$_\odot$
	$M_3 = 0.18$ M$_\odot$	$a_{12} = 0.45$ AU	$a_3 = 20.55$ AU
$\sigma_{res} = 0.00422$ d			

pair ('12') and that of the companion ('3') (for $i = 88.8°$), we computed the values of M_3, a_{12} (the semi-major axis of the absolute orbit of the close pair), and a_3. Obviously, the orbits which correspond to the three periodic harmonic modulations are circular. We also listed these values in Tables 2 and 3. Relying on the results displayed in Fig. 7 and in Tables 2 and 3, one can formulate some remarks concerning the nature and plausibility of the respective companions as well as their observational evidence.

The hypothetical third body associated to the longest periodicity (50.41 yr) in the P3 + LTTE model would have a minimum mass a little higher than that of the secondary component of the eclipsing binary system. Taking into account the estimations of the maximum mass of neutron stars, i.e. 2.5 M$_\odot$ (e.g. Chamel et al. 2013), we found that the corresponding threshold value for inclination angle is about 64.6°. Consequently, for $i \geq 64.6°$ the unseen companion would be a neutron star, while for $i < 64.6°$, it would be a black hole. In the case of coplanar orbits ($i = 88.8°$) the companion would be a neutron star.

According to SIMBAD Astronomical Database, there is no X-Ray object or radio source in the proximity of WW Cygni. This may be understood by the absence of a mass accretion phenomenon by the compact object. Furthermore, the lack of observational evidence for the presence of any visual companions is in good agreement with the hypothesis of a compact companion.

The hypothetical companions corresponding to the other three shorter periodicities have lower mass values (upper panel of Fig. 7). Within the coplanarity hypothesis, they would be red dwarfs stars. For low enough values of i these companions may also be compact objects.

We investigated the stability of the hypothetical triple system corresponding to the longest periodicity, i.e. 50.41 years, involved in the P3 + LTTE ephemeris. We took into account the two body approximation using the simple stability criterion of Szebehely and Zare (1977). Thus, assuming as above the coplanarity of the orbits of the inner close binary system (WW Cyg) and of the wide system (WW Cyg + companion) we found that the respective hypothetical triple systems (i.e. for the considered M_3 values, see Fig. 7) are stable either for direct or retrograde motion, but "more stable" in the case of direct motion.

The interpretation of the P3 + LTTE1 + MP3;1,1,1 model in terms of the LTTE hypothesis implies that WW Cyg would be a sextuple system. Taking into account the values of the orbital elements listed in Table 3, the following conclusions can be drawn:

(i) the periastron distance of the absolute orbit of the massive companion ($P_{s1} = 50.56$ yr) is about 4.28 AU, while

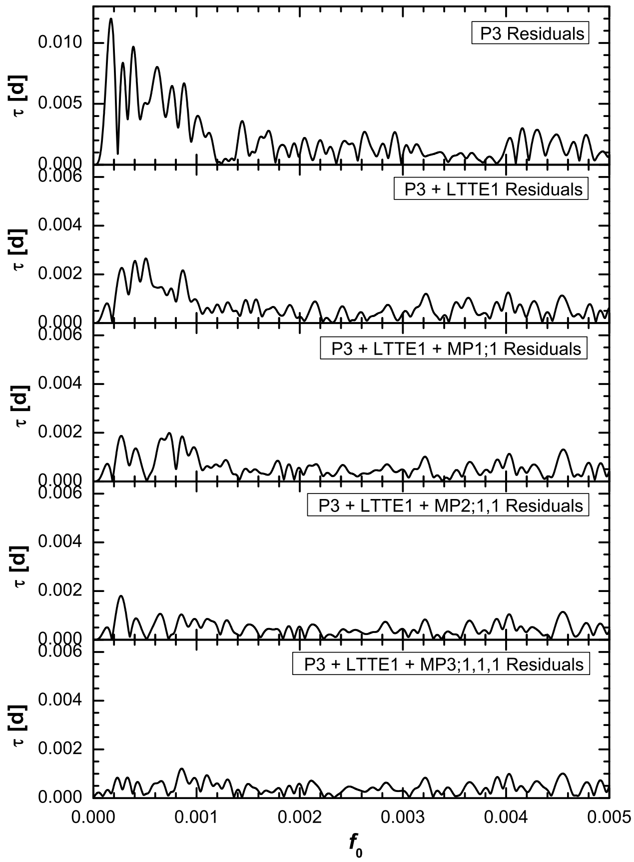

Fig. 4 Amplitude spectra corresponding to different stages of the $O-C$ curve prewhitening of WW Cyg using models consisting of a cubic trend and different LTTE + multiperiodic type components

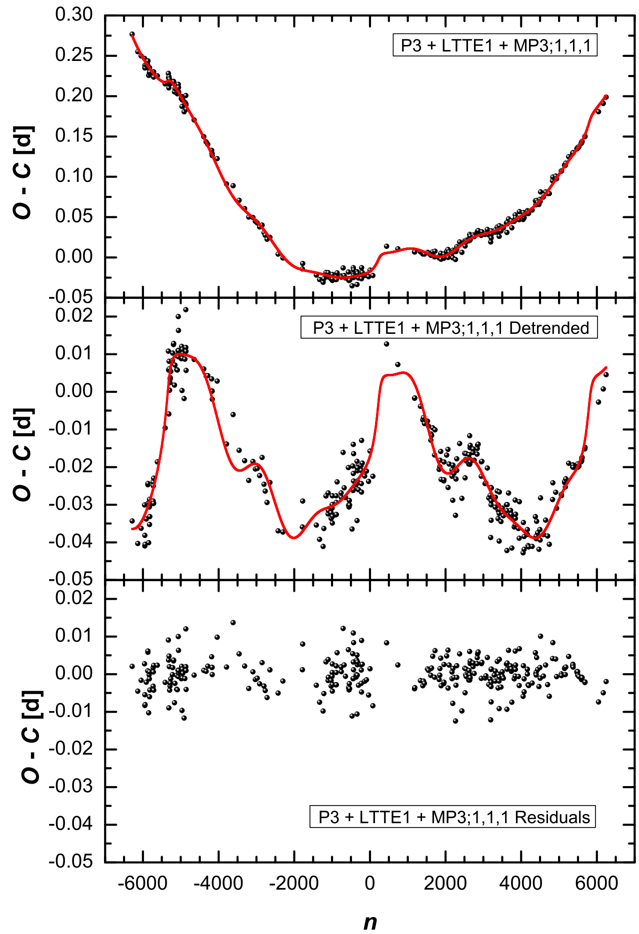

Fig. 5 LTTE + multiperiodic model superposed on a cubic polynomial trend of the $O-C$ curve of WW Cyg

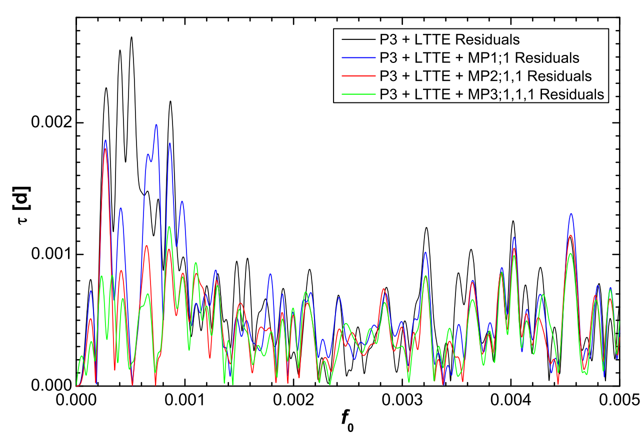

Fig. 6 Superposed amplitude spectra of the $O-C$ residuals obtained after removing models containing LTTE terms

the orbit radii of the three low mass companions are significantly higher: 13.12 AU, 10.47 AU, and 20.50 AU, respectively. It means that the elliptical orbit intersects the three circular orbits;

(ii) the hypothetical sextuple system does not fulfil the requirements to be a hierarchical multiple system (e.g. Roy 2005);

(iii) the direct consequence of the previous conclusion is that the model consisting of the superposition of four independent LTTE components is not adequate for modelling the observed orbital period variability phenomenon.

Consequently, we may conclude that the scenario involving the hypothesis of a stable sextuple system is not plausible.

4.2 Searching for quasiperiodicity signatures using the ESCD method

Although the two multiperiodic models previously established supplied good descriptions of the $O-C$ residuals, they have some limits which must be taken into account. According to their nature, this type of models implicitly assumes the additivity of the involved periodic signals (e.g. Pop 1996), but we have no certainty that this is also true for the actual mechanism(s) operating in the considered system (Sterken 2003). Consequently, even if multiperiodic models are able to accurately describe the obser-

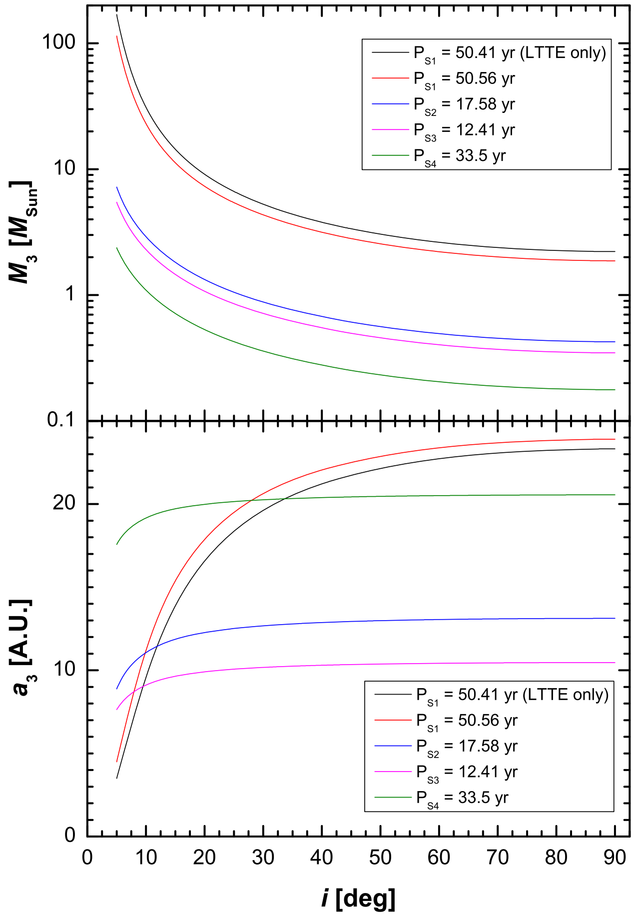

Fig. 7 The values of the mass and semi-major axis of the hypothetical third body absolute orbit for each of the periodicities involved in the P3 + LTTE and P3 + LTTE1 + MP3;1,1,1 model and for different inclination angle values

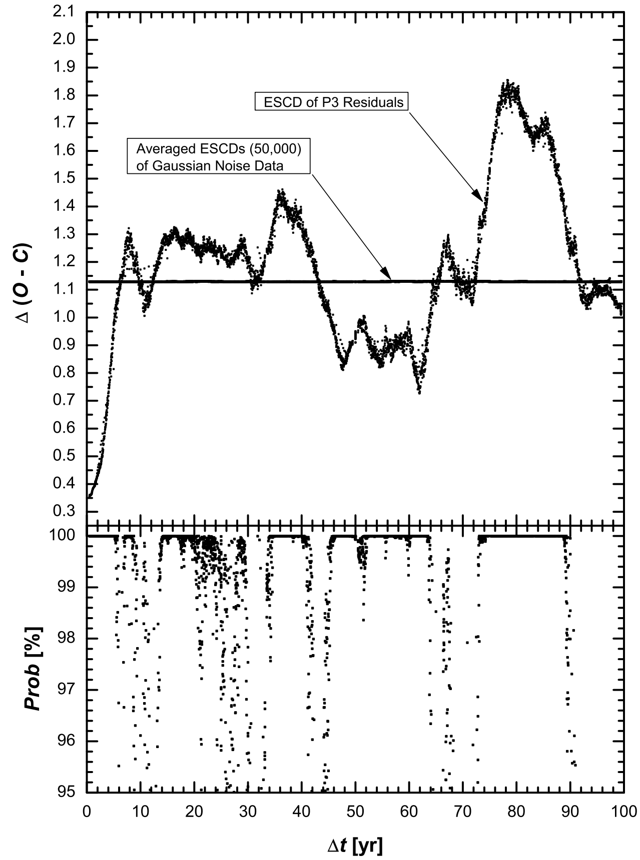

Fig. 8 ESCD analysis of the P3 residuals of WW Cyg timing data

vational data, their use for prediction goals is rather doubtful (Buchler et al. 1996; Sterken 2003).

The examination of the amplitude spectra of the residuals obtained in different stages of the prewhitening reveals an interesting specific feature which may be useful for the physical interpretation of the orbital period variability of WW Cyg. Especially in the case of P3 + LTTE residuals, but also in those of P3 + MP1;2 and P3 residuals, the shape of the amplitude spectrum in the low frequency range (i.e. between dimensionless frequency values of 0 and 0.0012) resembles that of a band spectrum. As discussed in the Introduction, this may be the signature of a quasiperiodic variability phenomenon.

We applied the ESC analysis as described in Sect. 3.2 to some of the residuals obtained during the above described model fitting procedures. The respective ESCDs are presented in Figs. 8–12. Each of these figures contains in the upper panel the corresponding *observed* ESCD and the ESCD obtained by averaging the 50,000 time series consisting in Gaussian noise. The averaged ESCD is situated at the level $\Delta(O-C) = 2\sigma/\sqrt{\pi} \approx 1.128$ (see Crăciun et al. 2015), with standardized $O-C$ residuals, i.e. $\sigma = 1$. The lower panel of each figure displays the value of the confidence level for rejecting the null hypothesis for each Δt value. A possible error source in estimating the values of the confidence level is the non-Gaussianity of the timing data noise. Fortunately, in our case, the departure from the Gaussian character is small. For this reason the estimated values of the confidence levels in the case of amplitude spectra (see Sect. 4.1) are in very good agreement for all the three considered null hypotheses. The following features related to the quasiperiodicity of the computed ESCDs were analysed: the persistence of minima, the levels of minima and maxima, and the behaviour of ESC shape within the coherence time scale (i.e. the interval of Δt values in which the persistence of minima occurs).

We interpreted these results as follows:

(i) P3 residuals (Fig. 8) display a high amplitude variability featured by a relatively deep minimum centred on a time scale of 54.8 yr which is split into three local minima at the following Δt values: 47.7 yr, 54.7 yr, and 62.0 yr;

(ii) P3 + MP1;2 (Fig. 9) residuals reveal a quasiperiodic variability at a time scale of about 17.7 yr with relatively

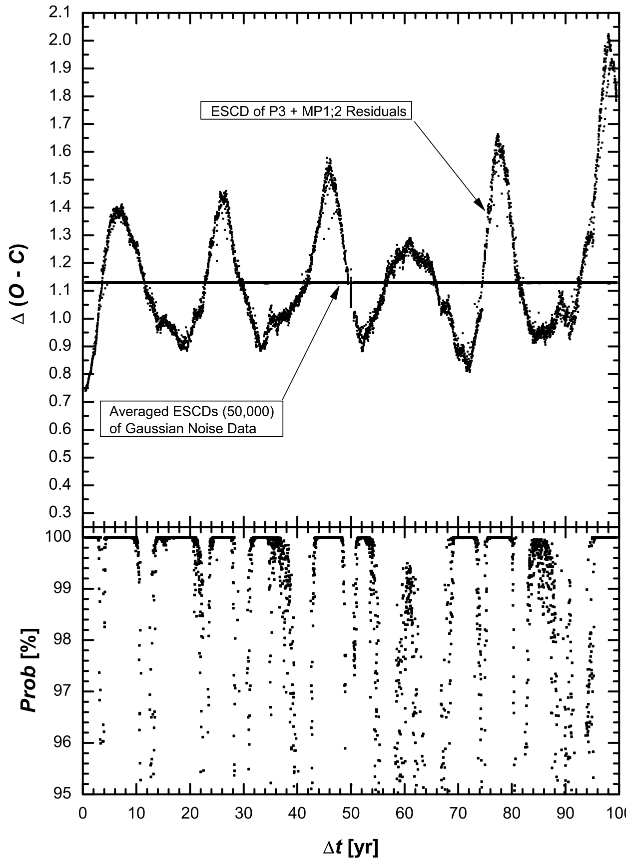

Fig. 9 ESCD analysis of the P3 + MP1;2 residuals of WW Cyg timing data

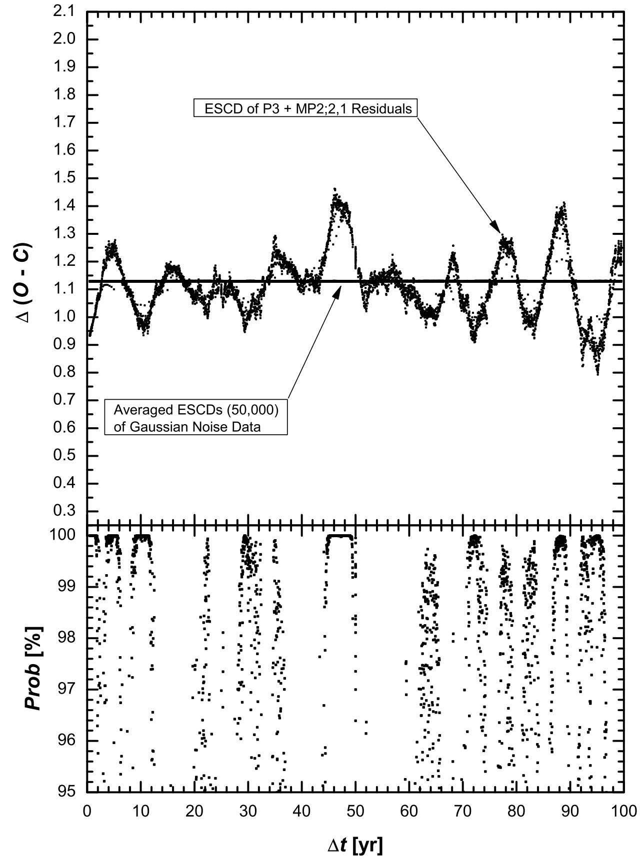

Fig. 10 ESCD analysis of the P3 + MP2;2,1 residuals of WW Cyg timing data

high coherence. The respective $O - C$ curve seems to display cycle-to-cycle and amplitude variability phenomena;
(iii) P3 + MP2;2,1 residuals (Fig. 10) display a somewhat coherent quasiperiodic variability at a time scale of about 10.7 yr, featured by cycle-to-cycle and amplitude variability, but also by some amount of irregular variability;
(iv) P3 + MP3;2,1,1 (Fig. 11) residuals reveal the presence of a statistically significant, low amplitude, deterministic component with low coherence apparently involving the following time scales: 6.5 yr, 10.5 yr, and 25.3 yr;
(v) P3 + LTTE residuals (Fig. 12) present a quasiperiodic variability at a time scale of about 19.7 yr with relatively low coherence. Both cycle-to-cycle and irregular variability seem to be involved in the $O - C$ curve.

The ESCD analysis applied to the considered $O - C$ residuals obtained during the modelling process of WW Cyg timing data refined the results of the amplitude spectra analysis. The main result is that all these residuals proved to display more or less coherent quasiperiodic type variability. Amplitude and cycle-to-cycle variability phenomena are also present. Most of the involved time scale values are in good agreement with those obtained in Sect. 4.1. In all cases, the coherence time scale is shorter than the time base of the analysed data.

The results of both Fourier and ESCD analyses emphasized that the above models consisting of a (linear) superposition of periodic components may be considered just a first approximation for the description of the timing data on WW Cyg. Moreover, these results revealed the quasiperiodic character of the orbital period modulation phenomenon displayed by this eclipsing binary system.

4.3 Magnetic fields, matter distribution and orbital period variability

There is a wealth of observational data on the presence of magnetic fields in the secondaries of Algol systems; the complex interaction occurring in the system may be described as follows: since the orbital period is less than four days, the material leaving the secondary does not form an accretion disk around the primary, but is rather heterogeneously distributed in space, not necessarily confined to the central plane and with no clear symmetry with respect to it (Richards et al. 2012; see also Richards and Albright 1999). Since in astrophysical conditions, the frozen in flux condition holds, one can argue that matter flow is an indicator of magnetic field topology. This is what observations find,

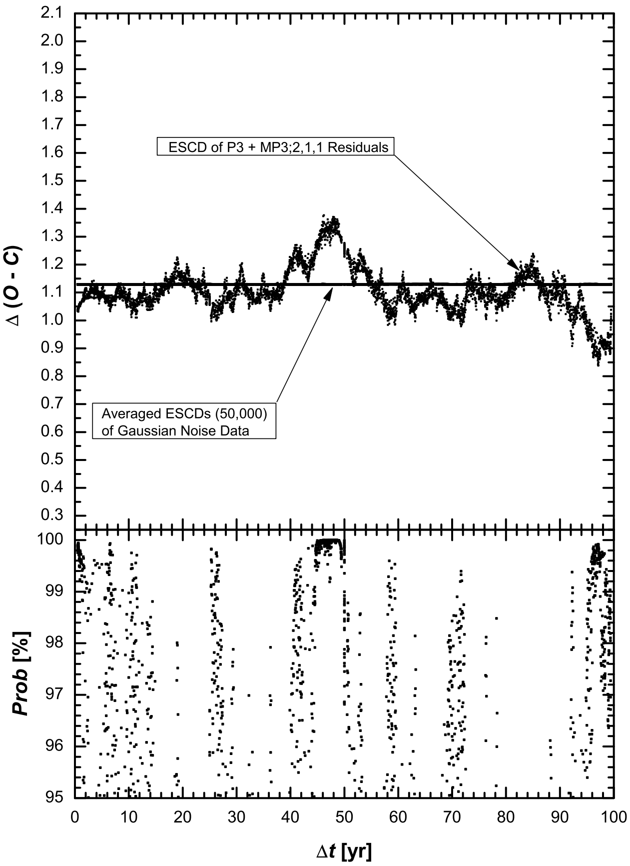

Fig. 11 ESCD analysis of the P3 + MP3;2,1,1 residuals of WW Cyg timing data

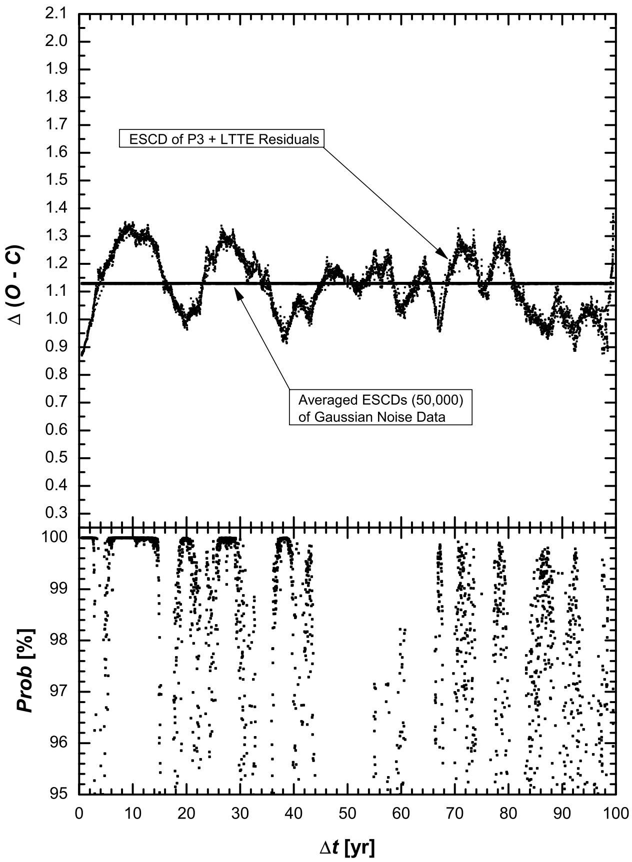

Fig. 12 ESCD analysis of the P3 + LTTE residuals of WW Cyg timing data

i.e., Peterson et al. (2010) report a persistent asymmetric magnetic field structure in Algol. Even more, observations complicate the picture of the magnetized secondary plus some non-active primary by clearly showing that the magnetic field of the secondary is coupled to the accretion structures around the primary (Retter et al. 2005). It is found that coronal loops on the secondary are oriented towards the centre of mass and this may be a result of the magnetic topology in which a magnetically threaded accretion structure around Algol A interacts with Algol B magnetic field, causing reconnection (Peterson et al. 2010). Also, superimposed on the dynamo process, nonconservative mass transfer should be considered, e.g. Moss et al. (2002).

The magnetic field topology is dynamic: a discussion of the time dependent magnetic properties displayed by Algol secondaries can be found in Richards and Albright (1993). There is observational data confirming the existence of both a large scale and a small scale magnetic field. Moreover, this magnetic field is dynamic and asymmetric. Since from the point of view of currently accepted morphological models, Algol and WW Cyg have the same type of matter distribution during mass transfer according to their position in the $r-q$ diagram (Lubow and Shu 1975; see also Plavec 1989, Richards and Albright 1999), we will carry on with making some educated comments regarding the effects of a magnetic field on the orbital period variability as displayed by the $O-C$ curve of WW Cyg, based on results derived from Algol observational data.

One tool used to relate the variability of the $O-C$ curve to magnetic field is through the Applegate (1992) framework. We have populated Table 4 with parameters derived from the $O-C$ of WW Cyg in case of the P3 + MP3;2,1,1 model (see also Table 1). We used the following usual notations: A_{O-C}—the peak-to-peak amplitude of the oscillation in the $O-C$ curve, $\Delta P_p/P_p$—the relative amplitude of the orbital period modulation, ΔQ—the variation of the quadrupole moment, ΔJ—the angular momentum transfer, $\Delta\Omega/\Omega$—the variable part of the differential rotation, ΔE—the energy required to transfer the angular momentum, ΔL_{RMS}—the *RMS* luminosity variation, Δm—the bolometric magnitude difference relative to the mean light level calculated from ΔL_{RMS}, using Eq. (4) in the paper of Kim et al. (1997), and B—the mean subsurface magnetic field. In order to compute the values of these parameters we used as input values those given or derived from the paper of Mezzetti et al. (1980). We obtained an estimation of the moment of inertia of the shell which determines

Table 4 Interpretation within P3 + MP3;2,1,1 of the periodicities detected in the timing data of WW Cyg through the theory of Applegate (1992)

P_s (yr)	A_{O-C} (d) $\Delta P_p/P_p$	ΔQ ($\mathrm{g\,cm^2}$) ΔJ ($\mathrm{g\,cm^2\,s^{-1}}$)	$\Delta\Omega/\Omega$ ΔE (erg)	ΔL_{RMS} ($L_\odot$, L_2)	Δm (mmag) B (kG)
49.16	0.04481 7.84×10^{-6}	-5.63×10^{51} 4.36×10^{48}	4.18×10^{-3} 1.20×10^{42}	0.631 0.033	±1.23 3.44
17.55	0.00920 4.51×10^{-6}	-3.24×10^{51} 2.51×10^{48}	2.41×10^{-3} 3.96×10^{41}	0.585 0.031	±1.14 4.37
10.39	0.00632 5.23×10^{-6}	-3.76×10^{51} 2.91×10^{48}	2.79×10^{-3} 5.33×10^{41}	1.329 0.070	±2.59 6.12

the quadrupole moment, $I_S = 3.17\times10^{55}\ \mathrm{g\,cm^2}$. In the case of the second model we took into account the three short periodic components of the model.

The effect of magnetic activity on the orbital period of eclipsing binary systems may be seen through the modulation of eclipse timing data by migrating starspots investigated by Kalimeris et al. (2002), Watson and Dhillon (2004), Tran et al. (2013), and Balaji et al. (2015). Unfortunately, these studies refer to types of binary systems other than Algols: contact or near-contact binaries and detached eclipsing binary systems consisting of a low-mass main sequence star and a white dwarf, where the Applegate mechanism is weak. Even in these conditions, although the amplitudes of the short-term modulations are in agreement with the predicted order of magnitude, the respective periodicities/time scales are much longer than the expected ones. Consequently, at this moment we can assume that this mechanism is rather unlikely to cause the orbital period modulation observed in WW Cyg.

The detailed analysis of the $O-C$ curve and the impossibility to fit well-behaved curves to it down to an agreeable uncorrelated noise level and the persistence of low amplitude and low coherence deterministic components, points in the same direction. We infer that the unaccounted-for features of the residuals in both the amplitude spectrum and ESCDs of the $O-C$ curves could be an indicator of the time dependent magnetic topology.

5 Discussion and concluding remarks

The present study of the orbital period variability of WW Cyg relied on a time base of about 113.8 yr. This allowed us to establish that the secular trend showed by the $O-C$ curve of this eclipsing binary may be described by a cubic polynomial, unlike Zavala et al. (2002) which found a parabolic trend. Such behaviour may be a consequence of the non-conservative mass transfer in the system (e.g. Rafert 1982). The values of the parameters of this cubic trend were further refined by considering more complex models involving periodic components (see Tables 1–3).

Having in view the intricate alternating run of the $O-C$ residuals obtained after removing the already emphasized parabolic and cubic trends, we approached their analysis and modelling, by taking into account a multiperiodic model. The input frequencies were determined rigorously through amplitude spectrum analysis, while their statistical significance was estimated using Monte Carlo simulations. We found the following three periodicities (according to the decreasing order of their amplitudes): 49.16 yr (together with its first harmonic, i.e. 24.58 yr), 17.55 yr, and 10.393 yr.

The main goal of the studies on orbital period variation phenomena in eclipsing binary systems is to identify or to make at least some inferences concerning the possible causes which might explain the observed features of the respective $O-C$ curve. Due to the non-sinusoidal character of the longest periodicity (i.e. the presence of its first harmonic), we interpreted it to be the result of a modulation via the LTTE induced by the presence of an unseen companion, the respective orbital periodicity being 50.41 yr. Obviously, this model is unable to describe the complexity of the $O-C$ curve of WW Cyg. It is interesting to note that the P3 + MP1;2 and P3 + LTTE residuals revealed the presence of a band spectrum in the low frequency range. Assuming the orbits coplanarity, for the estimated inclination angle ($i = 88.8°$), we found the mass of the hypothetical unseen companion to be about 2.22 $M_\odot$, i.e. a little higher than the mass of the secondary component of WW Cyg. Such an object would be a neutron star. For inclination angles lower than 64.6°, the unseen companion would be a black hole. In the given conditions the presumed triple system proved to be stable, according to the criterion given by Szebehely and Zare (1977).

Taking into account the already proved multiperiodicity of this $O-C$ curve, we considered for further refinements the superposition of LTTE and some low amplitude periodic components. We were able to establish a final model consisting in the following four periodicities (in the order of their detection): 50.56 yr (LTTE), 17.58 yr, 12.41 yr, and 33.5 yr. However, the evolution of some details in the amplitude spectra of the successive residuals revealed the inadequacy of taking into account the three additional frequencies.

According to Crăciun et al. (2015) the ESCD analysis is very sensitive to the presence of a deterministic (either periodic or quasiperiodic) component in a noisy series. We proposed here another approach of the estimation of the statistical significance of all values of the *observed* ESCD of a given data set, relying on Monte Carlo simulations with Gaussian white noise. It also represents an additional way to remove the deficiency of the SC (and also of the ESC) method mentioned by Percy et al. (2006b) which consists in the lack of an estimation of the statistical significance of the detected (quasi)periodicity/time scale. In the present study we applied the ESCD analysis together with the associated statistical test for the detection and the investigation of quasiperiodicity in the orbital period modulation phenomenon in the eclipsing binary system WW Cygni, in which the magnetic activity cycles are expected to be present.

The ESCD analysis of the different $O - C$ residuals resulted during establishing the P3 + MP3;2,1,1 and P3 + LTTE models revealed the presence of more or less quasiperiodic variability involving amplitude and cycle-to-cycle variability phenomena.

The interpretation of the above time and frequency features in terms of a quasiperiodic modulation related to cyclic (in fact, alternating, neither strictly sinusoidal, nor arbitrarily periodic) magnetic activity of the secondary component, could be more plausible.

Moreover, the hypothesis of a quasiperiodic modulation caused by the cyclic magnetic activity would satisfy the simplicity requirement of Ockham's Razor: a single physical mechanism would explain all the frequencies involved in the observed band spectrum. Consequently, at this stage, we may formulate the following two hypotheses concerning the physical mechanisms involved in the alternating orbital period modulation of WW Cyg: (i) a quasiperiodic modulation at a dominant time scale of about 49.16 yr induced by the magnetic cycles which occurs in the cool secondary component of the system, and (ii) the coexistence of two modulating mechanisms: periodic modulation (with a period of 50.41 yr) of the orbital period due to the presence of an unseen companion with a relative high mass (at least comparable with that of the secondary component) and a quasiperiodic modulation (with a dominant time scale of about 19.7 yr) caused, as in the previous scenario, by the magnetic cycles of the secondary star in the system. Unlike the paper of Zavala et al. (2002), our study proves, both qualitatively and quantitatively, the existence of a quasiperiodic modulation in the orbital period of the binary system WW Cyg.

To discriminate between the above two scenarios requires a differential diagnosis which should decide whether the longest periodicity is caused by the presence of an unseen companion or not. This is not a trivial task and it requires some extra information. The case of the Algol systems AB Cas and Y Cam are relevant from this viewpoint (Khaliullin and Khaliullina 2012, 2013). From this perspective, an important goal for the future observational studies of WW Cyg is to investigate the photometrical stability of the primary component of the system. The detection of pulsations of this star will allow to test the dynamical situation of WW Cyg using the approach of the above quoted authors. Therefore, new high precision and high temporal resolution multicolour and spectroscopic observations are required.

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