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ORIGINAL ARTICLE

Effects of anisotropy on interacting ghost dark energy in Brans-Dicke theories

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Abstract In this work we concentrate on the ghost dark energy model within the framework of the Brans-Dicke theory in an anisotropic Universe. Within this framework we discuss the behavior of equation of state, deceleration and dark energy density parameters of the model. We consider the squared sound speed and quest for signs of stability of the model. We also probe observational constraints by using the latest observational data on the ghost dark energy models as the unification of dark matter and dark energy. In order to do so, we focus on observational determinations of the Hubble expansion rate (namely, the expansion history) $H(z)$. Then we evaluate the evolution of the growth of perturbations in the linear regime for both ghost DE and Brans-Dicke theory and compare the results with standard FRW and ΛCDM models. We display the effects of the anisotropy on the evolutionary behavior the ghost DE models where the growth rate is higher in this models. Eventually the growth factor for the ΛCDM Universe will always fall behind the ghost DE models in an anisotropic Universe.

Keywords Bianchi type I · Ghost dark energy · Brans-Dicke theories · Growth factor · Instability of theory

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1 Introductions

Accelerating expansion of the Universe (Reiss et al. 1998; Perlmutter et al. 1999) can be demonstrated either by a missing energy component which can be usually called "dark energy" (DE) with an exotic equation of state (EoS), or by modifying the underlying theory of gravity on large scales. The other models of DE have been discussed widely in literature considering a cosmological constant (Peebles and Ratra 2003), a canonical scalar field (quintessence) (Caldwell et al. 1998), a phantom field, which is a scalar field with a negative sign of the kinetic term (Nojiri and Odintsov 2003; Khodam-Mohammadi et al. 2012), or the combination of quintessence and phantom in a unified model named quintom (Sadeghi et al. 2008). Ghost dark energy (ghost DE), is another dynamical DE model suggested in Urban and Zhitnitsky (2010), Ohta (2011). It is supposed to exist to solve the $U(1)$ problem in low-energy effective theory of QCD, has attracted a lot of interests in recent years (Witten 1979; Veneziano 1979; Nath and Arnowitt 1981; Kawarabayashi and Ohta 1980), though it is completely decoupled from the physical sector (Kawarabayashi and Ohta 1980). There are some DE models where the ghost plays the role of DE (see, e.g., Piazza and Tsujikawa 2004) and becomes a real propagating physical degree of freedom subjected to some severe constraints. They have explored a DE model with a ghost scalar field in the context of the runaway dilaton scenario in low-energy effective string theory and addressed the problem of vacuum stability by implementing higher-order derivative terms and shown that a cosmologically viable model of "phantomized" DE can be constructed without violating the stability of quantum fluctuations. However, the Veneziano ghost is not a physical propagating degree of freedom and the corresponding GDE model does not violate unitarity causality or gauge invariance and other important features of renormalizable quantum field theory, as advocated in Zhitnitsky (2010, 2011), Holdom (2011).

Scalar-tensor theory provide the most natural generalizations of General Relativity (GR) by presenting additional fields. In this theory, the field equations are even more complex then in GR. One the simplest of the scalar tensor is the Brans-Dicke (BD) theory of gravity which was proposed by Brans and Dicke (1961). BD theory involves a scalar field and is perhaps the most viable alternative theory to Einstein's general theory. The BD theory has passed experimental tests in the solar system (Bertotti et al. 2003). Since the ghost DE model have a dynamic behavior it is more reasonable to consider this model in a dynamical framework such as BD theory. One can find more features of BD cosmology in Alavirad and Sheykhi (2014), Ebrahimi and Sheykhi (2011a), Saaidi et al. (2012).

Recent experimental data as well as theoretical arguments support the existence of anisotropic expansion phase, which evolves into an isotropic one. It forces one to study evolution of the Universe with the anisotropic background. Bianchi type I (BI) model is among the simplest models with anisotropic spacetime. Many authors (Saha 2006a,b; Pradhan and Singh 2004; Shamir 2010; Yadav and Saha 2012; Pradhan and Pandey 2006) explored BI model in different aspects. In Aluri et al. (2013) it was studied the evolution of BI Universe containing different types of anisotropic matter sources. Some exact anisotropy solutions have been also investigated in this BD theory (Sharif and Waheed 2012; Ram 1983; Farasat Shamir and Ahmad Bhatti 2012). Recently, Hossienkhani (2016), Fayaz (2016) investigated the holographic and new agegraphic DE models in a sense of BI model by considering a Brans-Dicke framework in which there is a non-minimal coupling between the scalar field. Consequently, it would be worthwhile to explore anisotropic DE models in the context of BD theory. In this work we study the evolution of the Hubble parameter, squared sound speed and growth of perturbations in ghost DE of BD theory. The ghost DE model is considered as a dynamical DE model with varying EoS parameter which can dominate the Hubble flow and influence the growth of structure in the Universe. Here we consider the interacting case of ghost DE model in BI model.

This paper is outlined as follows. In Sect. 2 we give a full introduction about the BI equations in the context of interacting ghost DE model in an anisotropic Universe and describe the evolution of background cosmology in this model. In Sect. 3 we discuss the linear evolution of perturbations in ghost DE cosmology of BI model. Section 4, we write down the BI equations in BD theory and the efforts have been made to constrain this ghost DE model by using cosmological observations. Finally we conclude in Sect. 5.

2 Metric and ghost dark energy model

We consider a class of homogeneous and anisotropic models where the line component is of the Bianchi type I,

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2, \tag{1}$$

where $A(t)$, $B(t)$ and $C(t)$ are the scale factors which describe the anisotropy of the model and the average expansion scale factor $a(t) = (ABC)^{1/3}$. It reduces to the FLRW case when $A(t) = B(t) = C(t) = a(t)$. Defining the time-like hypersurface-orthogonal vector $u = \partial/\partial t$, we can define the average Hubble scalar, H, and the shear, $\sigma_{\mu\nu}$, as follows:

$$H = \frac{1}{3}u^{\mu}_{;\mu}, \qquad \sigma_{\mu\nu} = u_{(\mu;\nu)} - H\delta_{\mu\nu}. \tag{2}$$

The contribution of the interaction with the matter fields is given by the energy momentum tensor which, in this case, is defined as

$$T^{\mu}_{\nu} = (\rho + p)u^{\mu}u_{\nu} - pg^{\mu}_{\nu}, \tag{3}$$

where ρ and p characterized by the energy density and pressure of cosmic fluid, respectively. We assume that the Universe is filled with isotropic fluid, and the isotropic fluid is characterized by the EoS $p = \omega\rho$, where ω is not necessarily constant. The signature used in this article is $(+---)$. The 4-velocity of the comoving particles is u^{μ}, $u^{\mu} = (1, 0, 0, 0)$ and $u^{\mu}u_{\mu} = 1$. This 4-velocity is orthogonal to the surfaces of spatial homogeneity. Therefore, we can write the following Einstein's field equations for BI model (Hossienkhani and Pasqua 2014; Saaidi and Hossienkhani 2011):

$$3H^2 - \sigma^2 = \frac{1}{M_p^2}(\rho_m + \rho_\Lambda), \tag{4}$$

$$3H^2 + 2\dot{H} + \sigma^2 = -\frac{1}{M_p^2}(p_m + p_\Lambda), \tag{5}$$

$$R = -6(\dot{H} + 2H^2) - 2\sigma^2, \tag{6}$$

where dot represents derivative with respect to t. Let us take the average Hubble parameter and the shear scalar σ^2 as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{7}$$

$$2\sigma^2 = \sigma_{\mu\nu}\sigma^{\mu\nu} = \left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - 3H^2. \tag{8}$$

According to the ghost DE model (Ohta 2011; Borges and Carneiro 2005), the energy density of the DE defined by:

$$\rho_\Lambda = \alpha H = \Lambda^3_{QCD}H, \tag{9}$$

where α is a constant and Λ_{QCD} is QCD mass scale. With $\Lambda_{QCD} \sim 100$ MeV and $H \sim 10^{-33}$ eV, $\Lambda^3_{QCD}H$ gives the right order of magnitude $\sim 3\times 10^{-3}$ eV for the observed density of DE (Ohta 2011). The dimensionless energy density of various components are

$$\Omega_m = \frac{\rho_m}{\rho_{cr}}, \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} = \frac{\alpha}{3M_p^2 H}, \tag{10}$$

where $\rho_{cr} = 3M_p^2 H^2$, so the first BI equation can be written as

$$\Omega_m + \Omega_\Lambda = 1 - \frac{\sigma^2}{3H^2}. \tag{11}$$

We shall take that the shear scalar can be described based on the average Hubble parameter, $\sigma^2 = \sigma_0^2 H^2$, where σ_0^2 is a constant. So, Eq. (11) lead to

$$\Omega_m + \Omega_\Lambda = 1 - \Omega_{\sigma 0}, \quad \text{with } \Omega_{\sigma 0} = \frac{\sigma_0^2}{3}, \tag{12}$$

where $\Omega_{\sigma 0}$ is the anisotropy parameter. We use Eq. (9) for the energy density of the DE component and insert it into (4) in order to obtain the Hubble parameter in ghost DE cosmologies

$$H = \sqrt{\left(\frac{\alpha}{6M_p^2(1-\Omega_{\sigma 0})}\right)^2 + \frac{\rho_{m0}a^{-3}}{3M_p^2(1-\Omega_{\sigma 0})}} + \frac{\alpha}{6M_p^2(1-\Omega_{\sigma 0})}. \tag{13}$$

In terms of the dimensionless energy density $\Omega_{m0} = \rho_{m0}/(3H_0^2 M_p^2)$ and redshift parameter $z = 1/a - 1$, the above Hubble equation becomes

$$H = \frac{H_0}{2(1-\Omega_{\sigma 0})}\Big[1 - \Omega_{m0} - \Omega_{\sigma 0} + \sqrt{(1-\Omega_{m0}-\Omega_{\sigma 0})^2 + 4\Omega_{m0}(1-\Omega_{\sigma 0})(1+z)^3}\Big]. \tag{14}$$

In the ΛCDM model Hubble's parameter is $H = H_0\big(\frac{\Omega_{m0}(1+z)^3+\Omega_\Lambda}{1-\Omega_{\sigma 0}}\big)^{\frac{1}{2}}$ and the EoS of DE is fixed to be $\omega_\Lambda = -1$. For model such as wCDM (with the constant EoS w), it is $H = H_0\big(\frac{\Omega_{m0}(1+z)^3+(1-\Omega_{m0}-\Omega_{\sigma 0})(1+z)^{3(1+w)}}{1-\Omega_{\sigma 0}}\big)^{\frac{1}{2}}$. The Hubble constant H_0 in (14) is taken as 72 km/s Mp/c, in the whole discussion. Another the Hubble constant measurements, $H_0 = 73.8 \pm 2.4$ km s^{-1} Mpc^{-1} from Riess et al. (2011), $H_0 = 73 \pm 3$ km s^{-1} Mpc^{-1} from the combination WMAP (Spergel et al. 2007), and the other with $H_0 = 68 \pm 4$ km s^{-1} Mpc^{-1} from a median statistics analysis of 461 measurements of H_0 (Chen et al. 2003; Gott et al. 2001). Here we assume that a interaction term (Q) exists between DE and DM components. Hence the energy conservation equations read as:

$$\dot\rho_\Lambda + 3H\rho_\Lambda(1+\omega_\Lambda) = -Q, \tag{15}$$

$$\dot\rho_m + 3H\rho_m = Q, \tag{16}$$

where ω_Λ is the DE EoS parameter. The interaction term is given by the coupling constant $Q = 3b^2 H(\rho_m + \rho_\Lambda)$ (Wang et al. 2005; Sen and Pavón 2008). Differentiating Eq. (4) with respect to the redshift, we obtain

$$\frac{dH}{dz} = \frac{3H}{2(1+z)}\frac{\Omega_\Lambda(z)}{1-\Omega_{\sigma 0}}\big(1 + r + \omega_\Lambda(z)\big), \qquad r = \frac{1-\Omega_\Lambda(z)-\Omega_{\sigma 0}}{\Omega_\Lambda(z)}. \tag{17}$$

Combining Eqs. (9) and (17) with the continuity equation given in Eq. (15), the EoS parameter for ghost DE model is

$$\omega_\Lambda(z) = \frac{1-\Omega_{\sigma 0}}{-2+\Omega_\Lambda(z)+2\Omega_{\sigma 0}}\left(1 + \frac{2b^2}{\Omega_\Lambda(z)}(1-\Omega_{\sigma 0})\right). \tag{18}$$

We now calculate the equation of motion for the energy density of DE in ghost DE model. Now we determine the evolution of Ω_Λ. Taking the derivative of (10) and using the definition $\dot\Omega_\Lambda = -H(z)(1+z)\Omega'_\Lambda(z)$, we get

$$\Omega'_\Lambda(z) = -\frac{3\Omega_\Lambda(z)}{1+z}\left(\frac{-1+\Omega_{\sigma 0}+\Omega_\Lambda(z)+b^2(1-\Omega_{\sigma 0})}{-2+\Omega_\Lambda(z)+2\Omega_{\sigma 0}}\right), \tag{19}$$

where the prime denotes the derivative with respect to the redshift z. We can determine the deceleration parameter (q) as $q = -1 + \frac{1+z}{H}\frac{dH}{dz}$ as follow. Using Eqs. (17), (18) and in the presence of interaction the deceleration parameter is obtained by

$$q(z) = \frac{1}{2} + \frac{3}{2}\left(\frac{\Omega_\Lambda(z)+2b^2(1-\Omega_{\sigma 0})}{-2+\Omega_\Lambda(z)+2\Omega_{\sigma 0}}\right), \tag{20}$$

where $\Omega_\Lambda(z)$ is given by Eq. (19). The speed of sound c_s^2 is defined as.[1] The energy conservation equation $T^{\mu\nu}_{;\mu}$ yields $\delta\ddot\rho = c_s^2\nabla^2\delta\rho(t,x)$ (Peebles and Ratra 2003).

$$c_s^2 = \frac{\dot p_\Lambda}{\dot\rho_\Lambda} = \frac{\rho_\Lambda}{\dot\rho_\Lambda}\dot\omega_\Lambda + \omega_\Lambda. \tag{21}$$

Now by computing $\dot\omega_\Lambda$ and using Eqs. (9), (15) and (18) which reduces to

$$c_s^2 = \frac{2(-1+\Omega_{\sigma 0})}{\Omega_\Lambda(-2+2\Omega_{\sigma 0}+\Omega_\Lambda)^2}\big(\Omega_\Lambda(1-\Omega_{\sigma 0}-\Omega_\Lambda) + b^2(\Omega_{\sigma 0}-1)(-4+4\Omega_{\sigma 0}+3\Omega_\Lambda)\big). \tag{22}$$

[1] In the perturbation theory we presume a small fluctuation in the context of the energy density and we want to observe whether the perturbation will grow or collapse. In the linear perturbation factor, the perturbed energy density is $\rho(t,x) = \rho(t) + \delta\rho(t,x)$, with the unperturbed energy density $\rho(t)$.

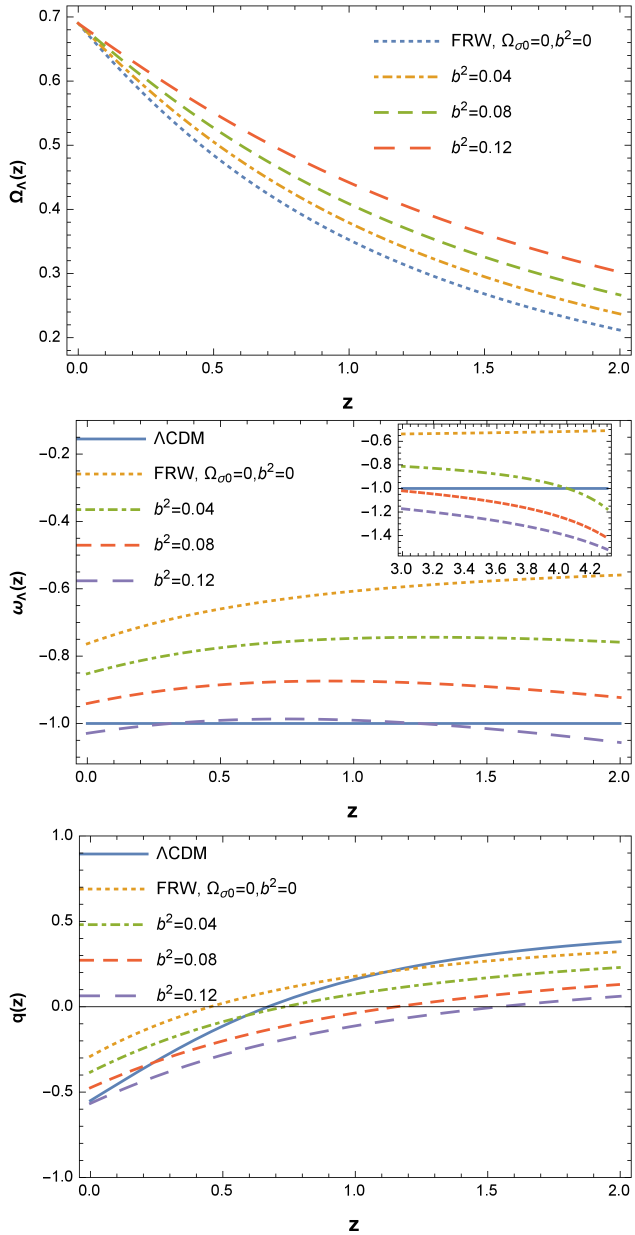

Fig. 1 *Upper panel*: the redshift evolution of the density parameters $\Omega_\Lambda(z)$. *Middle panel*: the evolution of EoS parameter $\omega_\Lambda(z)$. *Lower panel*: the deceleration parameter $q(z)$ as a function of cosmic redshift z for the different parameter b^2. Here we take $\Omega_\Lambda^0 = 0.69$ and $\Omega_{\sigma 0} = 0.001$

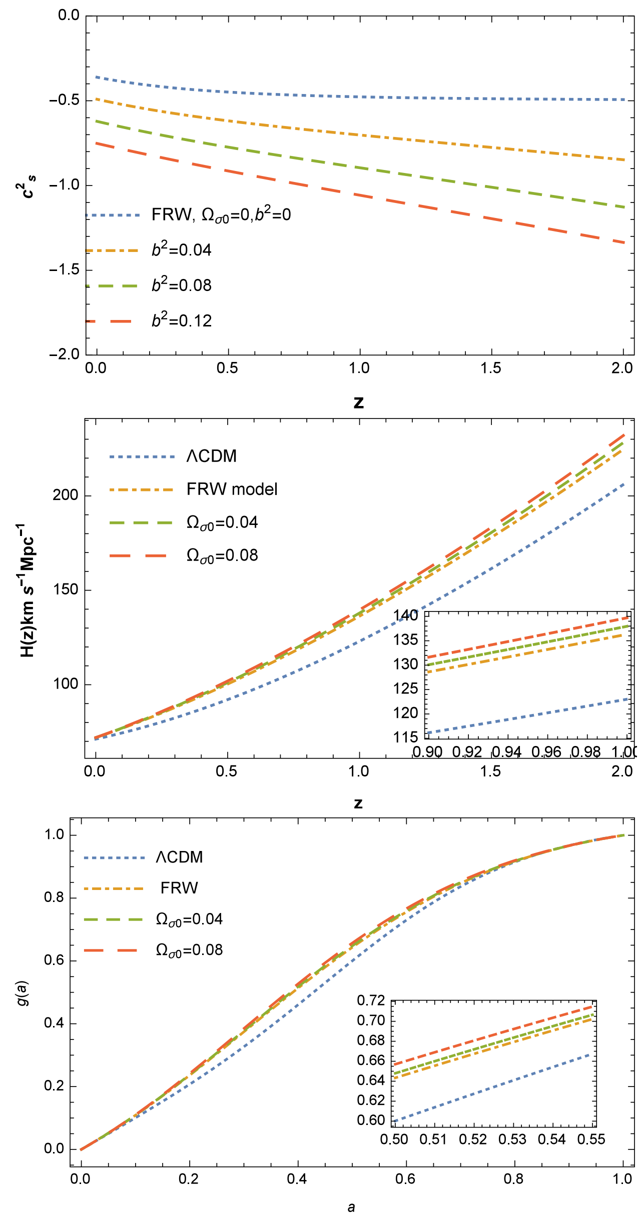

Fig. 2 *Upper panel*: the evolution of squared sound speed c_s^2 as a function of cosmic redshift z for the different parameter b^2 with $\Omega_\Lambda^0 = 0.69$ and $\Omega_{\sigma 0} = 0.001$. *Middle panel*: evolution of the Hubble parameter $H(z)$ as a function of cosmic redshift z for the different parameter $\Omega_{\sigma 0}$ with $b^2 = 0.1$, $\Omega_{m0} = 0.277$ and $H_0 = 72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. *Lower panel*: time evolution of the growth factor as a function of the scale factor for the different cosmological models and comparing to the ΛCDM and FRW models. Auxiliary parameters are the same as shown in the *middle panel*

It may be mentioned that for causality and stability under perturbation it is required to satisfy the inequality condition $0 < c_s^2 < 1$ (Lixin et al. 2012).

In Fig. 1 we show the energy density of DE component Ω_Λ (upper panel), the evolution of the EoS parameter ω_Λ (middle panel), the deceleration parameter $q(z)$ (lower panel) while Fig. 2 indicates the squared sound speed c_s^2 (upper panel) and Hubble parameter H (middle panel) as a function of the cosmic redshift z for various choices of b^2 and $\Omega_{\sigma 0}$ parameters, with the best fitting values of the model, the FRW ghost DE and ΛCDM models. In the case of the ghost DE model we have assumed the present values: $\Omega_{\sigma 0} = 0.001$, $\Omega_\Lambda^0 = 0.69$ and $H_0 = 72$ km/s Mp/c. Also for the case of ΛCDM model it is $\Omega_\Lambda^0 = 0.7$ and $\Omega_{m0} = 0.3$. From Fig. 1 we see that for the case of $b^2 < 0.12$, the EoS parameter for ghost DE model is always bigger than $\omega_\Lambda = -1$ and remains in the quintessence regime, i.e.,

$\omega_\Lambda > -1$ while for $b^2 \geq 0.12$, we see that ω_Λ of the ghost DE can cross the phantom divide. In the limiting case of the FRW Universe, $\omega_\Lambda(z)$ without interaction ($b^2 = 0$) varies from $\omega_\Lambda > -1$ to $\omega_\Lambda = -1$ which is similar to freezing quintessence model (Wei and Cai 2008) while in the presence of the interaction the situation is changed. Recent studies have constructed $q(z)$ taking into account that the strongest evidence of accelerations happens at redshift of $z \sim 0.2$. In order to do this, they have set $q(z) = 1/2(q_1 z + q_2)/(1+z)^2$ to reconstruct it and then they have obtained $q(z) \sim -0.31$ by fitting this model to the observational data (Gong and Wang 2006). Under these conditions and considering bottom panel of Fig. 1, we give the present value of the deceleration parameter for the interacting ghost DE with $b^2 = 0.12$ is $q_0 \simeq -0.56$, is significantly smaller than $q_0 \sim -0.54$ for the ΛCDM cosmological model (Daly et al. 2008), as expected (see also Fig. 1). Also, the accelerating expansion begins at $z = 0.74^{+0.40+0.78}_{-0.00-0.28}$, which is earlier than what the ΛCDM model predicts. Graphical analysis of c_s^2 shows that our theory could be unstable in FRW and BI models as shown in upper panel of Fig. 2. Furthermore, we would see that the non interacting ghost DE in FRW is more stable than the interacting ghost DE in an anisotropic Universe. It is also interesting to see how our models when compared with recent measurements of the Hubble parameter performed with the ΛCDM model. This comparison is done in Fig. 2 (middle panel), where we plot the evolution of $H(z)$ depends on the value of the $\Omega_{\sigma 0}$ parameter for the ghost DE and ΛCDM model considered in this work. It was observed that the Hubble parameter are bigger in these models compared to the ΛCDM model. Also, we can see that for the biggest value, the $\Omega_{\sigma 0}$ parameter is taken, and the biggest value of the Hubble expansion rate $H(z)$ is gotten.

3 Linear perturbation theory in ghost DE

The coupling between the dark components could significantly affect not only the expansion history of the Universe, but also the evolutions of the density perturbations, which would change the growth history of cosmic structure. The linear growth of perturbations for the large scale structures is derived from matter era, by calculating the evolution of the growth factor $g(a)$ in ghost DE models and compare it with the solution found for the ΛCDM model. The differential equation for $g(a)$ is given by (Pace et al. 2010, 2014; Percival 2005)

$$g''(a) + \left(\frac{3}{a} + \frac{E'(a)}{E(a)}\right) g'(a) - \frac{3}{2}\frac{\Omega_{m0}}{a^5 E^2(a)} g(a) = 0, \tag{23}$$

for the prime denoting the derivative with respect to $\ln a$ and $E(z) = H/H_0$ is the evolution of dimensionless Hubble parameter. For a non interacting DE model, by using Eqs. (14), (17) and (19), we solve numerically Eq. (23) for studying the linear growth with ghost DE in an anisotropic Universe. Then we compare the linear growth in the ghost DE model with the linear growths in the ΛCDM and FRW models. To evaluate the initial conditions, since we are in the linear regime, we take that the linear growth factor has a power law solution, $g(a) \propto a^n$, with $n > 1$, then the linear growth should grow in time. In bottom panel of Fig. 2 we show the evolution of the linear growth factor $g(a)$ as a function of the scale factor. In the ghost DE model with $\Omega_{\sigma 0} \neq 0$, the growth factor evolves proportionally to the scale factor, as expected. In the FRW model ($\Omega_{\sigma 0} = 0$), the growth factor evolves more slowly compared to the BI model because the FRW model dominates in the late time Universe. In the case of ΛCDM, $g(a)$ evolves more slowly than in the ghost DE of FRW model since the expansion of the Universe slows down the structure formation.

4 Bianchi type I field equations and ghost dark energy in Brans-Dicke theory

The BD theory with self-interacting potential is described by the action (Arik and Calik 2006; Cataldo et al. 2001; Ebrahimi and Sheykhi 2011b)

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{8\omega_0}\phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) + f(\phi) L_m \right), \tag{24}$$

where R is the Ricci scalar curvature, ϕ is the BD scalar field with a potential $V(\phi)$. The chameleon field ϕ is non-minimally coupled to gravity, ω_0 is the generic dimensionless parameter of the BD theory. The matter Lagrangian L_m represents the perfect fluid matter. In the limiting case $f(\phi) = 1$, we obtain the original BD theory. In the limiting of BD theory and $V(\phi) = 0$, the gravitational field equations derived from the action (24) with respect to the metric is

$$\phi G_{\mu\nu} = -8\pi T^m_{\mu\nu} - \frac{\omega_0}{\phi}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g^{\mu\nu} \phi_{,\gamma}\phi^{,\gamma} \right) - \phi_{;\mu;\nu} + g_{\mu\nu} \Box \phi, \tag{25}$$

and

$$\Box \phi = \alpha' T^{m\gamma}_{\gamma}, \tag{26}$$

respectively, where $\alpha' = \frac{8\pi}{2\omega_0 + 3}$ and $T^{m\gamma}_{\gamma} = g^{\mu\nu} T^m_{\mu\nu}$ is the trace of (3) and $\Box = \nabla^\alpha \nabla_\alpha$ in which the operator ∇_α represents covariant derivative. The gravitational field equations derived from the variation of the action (24) with respect to BI metric is (Hossienkhani 2016; Fayaz 2016)

$$\frac{\phi^2}{4\omega_0}(3H^2 - \sigma^2) - \frac{1}{2}\dot{\phi}^2 - \frac{3H}{2\omega_0}\phi\dot{\phi} = \rho_\Lambda + \rho_m, \tag{27}$$

$$\frac{-1}{4\omega_0}(2\dot{H}+3H^2+\sigma^2)\phi^2-\frac{1}{2}\left(1+\frac{1}{\omega_0}\right)\dot{\phi}^2$$
$$+\frac{H}{\omega_0}\phi\dot{\phi}-\frac{1}{2\omega_0}\phi\ddot{\phi}=p_\Lambda, \qquad (28)$$

and the dynamical equation for the scalar field is

$$\ddot{\phi}+3H\dot{\phi}-\frac{1}{2\omega_0}(3\dot{H}+6H^2+\sigma^2)\phi=0. \qquad (29)$$

As above, Eqs. (27), (28) and (29), are 3 independent equation which having unknown parameters such as ϕ, H and σ. To solve them we take $\sigma^2=\sigma_0^2H^2$ and $\phi=\phi_0a^\epsilon$ (Riazi and Nasr 2000), where ϵ is any integer, implies that $\dot{\phi}=\epsilon H\phi$. So, Eq. (27) lead to

$$1+2\epsilon-\frac{2}{3}\omega_0\epsilon^2-\frac{\sigma_0^2}{3}=\frac{4\omega_0}{3H^2\phi^2}(\rho_\Lambda+\rho_m). \qquad (30)$$

We also define the dimensionless density fractions

$$\Omega_m=\frac{\rho_m}{\rho_{cr}}=\frac{4\omega_0\rho_m}{3\phi^2H^2},$$
$$\Omega_\Lambda=\frac{\rho_\Lambda}{\rho_{cr}}=\frac{4\omega_0\rho_\Lambda}{3\phi^2H^2}=\frac{4\omega_0\alpha}{3\phi^2H}, \qquad (31)$$

where $\rho_{cr}=\frac{3\phi^2H^2}{4\omega_0}$. Therefore, Eq. (30) give

$$\Omega_\Lambda+\Omega_m=1+2\epsilon-\frac{2}{3}\omega_0\epsilon^2-\Omega_{\sigma 0}. \qquad (32)$$

In the following, we take the time derivative of (30), after using (32), so

$$\frac{H'(z)}{H}=\frac{3}{2(1+z)}\left(1+\frac{2}{3}\epsilon+\frac{\Omega_\Lambda\omega_\Lambda}{1+2\epsilon-\frac{2}{3}\omega_0\epsilon^2-\Omega_{\sigma 0}}\right). \qquad (33)$$

For the case of $\epsilon=0$, the above equation reduce to (17). Combining (9) with (15) and (33), we obtain the EoS parameter in BD theory as

$$\omega_\Lambda(z)=\frac{1+2\epsilon-\frac{2}{3}\omega_0\epsilon^2-\Omega_{\sigma 0}}{-2(1+2\epsilon-\frac{2}{3}\omega_0\epsilon^2-\Omega_{\sigma 0})+\Omega_\Lambda(z)}\times\left(1-\frac{2\epsilon}{3}+\frac{2b^2(1+2\epsilon-\frac{2}{3}\omega_0\epsilon^2-\Omega_{\sigma 0})}{\Omega_\Lambda(z)}\right). \qquad (34)$$

The solar-system experiments give the lower bound for the value of ω_0 to be $\omega_0>40000$ (Ohta 2011). However, when probing the larger scales, the limit obtained will be weaker than this result. It was shown (Acquaviva and Verde 2007) that ω_0 can be smaller than 40000 on the cosmological scales. Also, Sheykhi et al. (2013) obtained the result for the value of ϵ is $\epsilon<0.01$. The ghost DE model in BD framework has an interesting feature compared to the ghost DE model in BI Universe. In the case of $b^2=0$, the EoS parameter of in the BD framework, requiring condition $\omega_\Lambda<-1$ leads to $(1+2\epsilon-\frac{2}{3}\omega_0\epsilon^2-\Omega_{\sigma 0})(3+2\epsilon)<3\Omega_\Lambda$. In the following, we will determine the density parameters of DE. For this purpose by taking the derivative of (31) as $\dot{\Omega}_\Lambda=-\Omega_\Lambda H(\frac{\dot{H}}{H^2}+2\epsilon)$ and using relation $\dot{\Omega}_\Lambda=-H(z)(1+z)\Omega'_\Lambda(z)$, it follows that

$$\Omega'_\Lambda(z)=-\frac{3\Omega_\Lambda}{1+z}\left(\frac{\Omega_\Lambda-1-2\epsilon+\frac{2}{3}\omega_0\epsilon^2+\Omega_{\sigma 0}}{-2(1+2\epsilon-\frac{2}{3}\omega_0\epsilon^2-\Omega_{\sigma 0})+\Omega_\Lambda(z)}\right)\times\left(1-\frac{2}{3}\epsilon+b^2\left(1+2\epsilon-\frac{2}{3}\omega_0\epsilon^2-\Omega_{\sigma 0}\right)\right). \qquad (35)$$

Now, the deceleration parameter in BD theory is obtained as

$$q(z)=\frac{1}{2}+\epsilon+\frac{3\Omega_\Lambda}{-2(1+2\epsilon-\frac{2}{3}\omega_0\epsilon^2-\Omega_{\sigma 0})+\Omega_\Lambda(z)}\times\left(1-\frac{2}{3}\epsilon+\frac{b^2(1+2\epsilon-\frac{2}{3}\omega_0\epsilon^2-\Omega_{\sigma 0})}{\Omega_\Lambda(z)}\right), \qquad (36)$$

where Ω_Λ is given by Eq. (35). A same steps as the previous section can be followed to obtain the squared sound speed c_s^2 for BD theories. In this concept, Eq. (21) lead to

$$c_s^2=\frac{\gamma}{3\Omega_\Lambda(2\gamma+\Omega_\Lambda)^2}\Bigg[(2\gamma+\Omega_\Lambda)(6b^2\gamma+\epsilon'\Omega_\Lambda)+\frac{(\gamma(\epsilon'+3b^2)+\epsilon'\Omega_\Lambda)(\epsilon'\Omega_\Lambda^2+12\gamma b^2(\gamma+\Omega_\Lambda))}{\gamma(-3+3b^2-2\epsilon)-3\Omega_\Lambda}\Bigg], \qquad (37)$$

where $\gamma=-1-2\epsilon+\frac{2}{3}\omega_0\epsilon^2+\Omega_{\sigma 0}$ and $\epsilon'=-3+2\epsilon$. In Fig. 3 we have depicted the energy density of DE component $\Omega_\Lambda(z)$ and the energy density of DM $\Omega_m(z)$ (upper panel), the redshift evolution of EoS $\omega_\Lambda(z)$ as a function of both z and ϵ in middle and lower panel while the parameter ϵ versus the anisotropy parameter is plotted in Fig. 4. Figure 5 indicates that the deceleration parameter (upper panel) and the squared sound speed (middle panel) as a function of the cosmic redshift z for various choice of parameter b^2 in BD theory. In the case of the ghost DE of BD theory we choose the model parameter as $\Omega_{\sigma 0}=0.001$, $\Omega_\Lambda^0=0.69$, $\epsilon=0.003$ and $\omega_0=10^4$. The upper part of Fig. 3 indicates that at the late time $\Omega_\Lambda\to 0.7$ while for the case of the energy density of DM $\Omega_m\to 0.3$, which is similar to the behavior of the original ghost DE in previous section. From Fig. 3 (middle) we observe that for $b^2=\Omega_{\sigma 0}=0$, the EoS parameter of BD theory translates the Universe from low quintessence region towards high quintessence region. But for $0<b^2<0.12$, ω_Λ increases from phantom region at early times and approaches to quintessence region at late

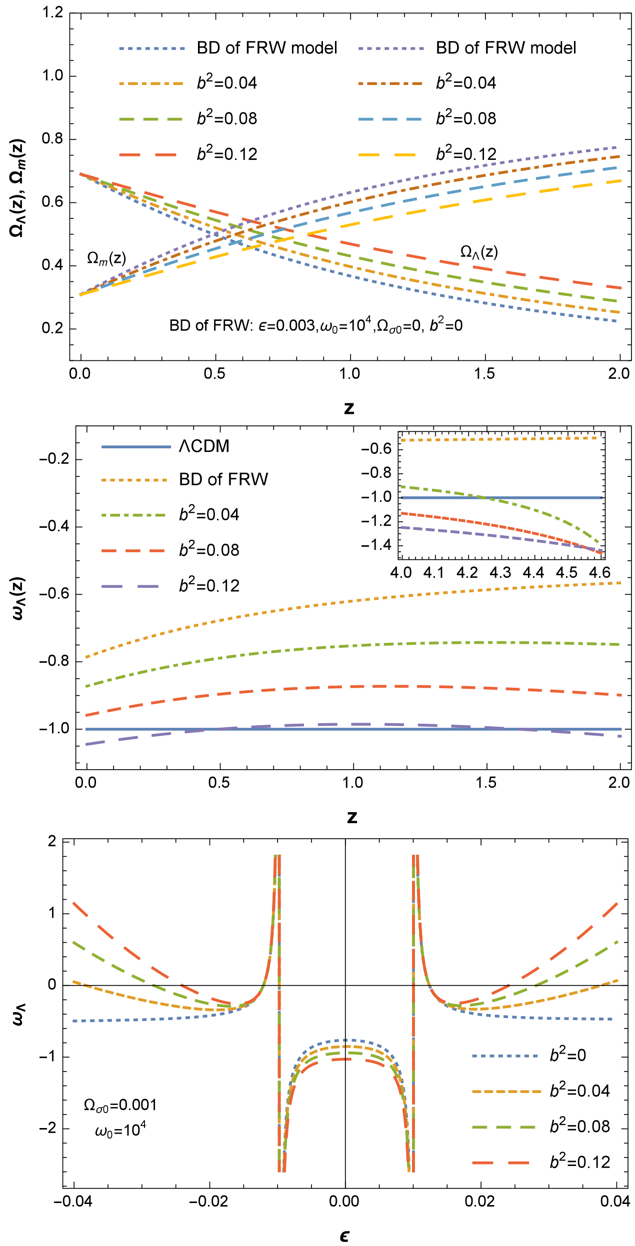

Fig. 3 *Top panel*: DE density parameters Ω_Λ and Ω_m for the interacting ghost DE of BD theory with different b^2. The evolutionary trajectories of ω_Λ for the interacting ghost DE with $\epsilon = 0.003$ for different values b^2 as a function of cosmic redshift z (*middle panel*) and in terms of ϵ (*lower panel*). Here we choose $\omega_0 = 10^4$, $\Omega_\Lambda^0 = 0.69$ and $\Omega_{\sigma 0} = 0.001$

times. Also from Fig. 3 we see that for $b^2 \geq 0.12$, ω_Λ of the interacting ghost DE in BD theory can cross the phantom divide and eventually the Universe approaches low phantom phase of expansion at late time. The lower of Fig. 3 indicates that one can generate a phantom-like behavior provided $-0.01 < \epsilon < 0.01$ which this point is completely compatible with Sheykhi et al. (2013). For a better insight, we plotted ϵ against the anisotropy parameter as shown in Fig. 4. The sweet spot is estimated to be $z = 1$.

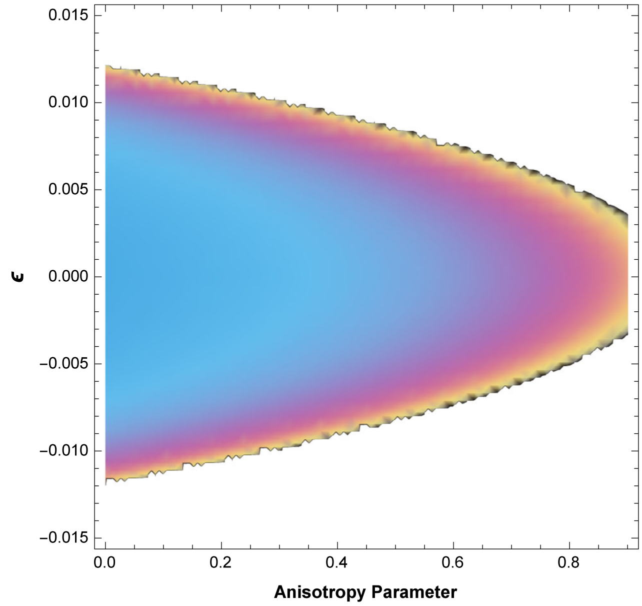

Fig. 4 The best fits of ϵ with anisotropy parameter for the interacting GDE model. The results given by current only are $z = 1$, $H_0 = 72$ km/s/Mpc, $b^2 = 0.1$, and $\Omega_{m0} = 0.277$

We figure out that the behavior of the deceleration parameter for the best-fit Universe is quite different from the ΛCDM cosmology as shown in Fig. 5 (upper panel). We can also see that the best fit values of transition redshift and current deceleration parameter with ghost DE of BD theory are $z = 2.13^{+0.84+1.28}_{-0.00-0.55}$ and $q_0 = -1.32^{+00+0.10}_{-0.07-0.17}$ which is matchable with the observations (Ishida et al. 2008) while for the case of ΛCDM, where $z \sim 0.67$ and $q_0 = -0.54$. We can see that increasing b^2 decreases the value of $q(z)$. The evolution of c_s^2 against z is plotted in Fig. 5 (middle panel) for different values of the coupling parameter b^2. The figure reveals that c_s^2 is always negative and thus, as the previous case, a background filled with the interacting ghost DE seems to be unstable against the perturbation. This implies that we cannot obtain a stable ghost DE dominated Universe in BD theory, which are in agreement with Hossienkhani (2016), Fayaz (2016), Myung (2007), Kim et al. (2008). One important point is the sensitivity of the instability to the coupling parameter b^2. The larger b^2, leads to more instability against perturbations.

In the following, we study the capability of the $H(z)$ measurements in constraining DE models in BD theory. The evolution of Hubble parameter $H(z)$ in ghost DE model with BD theory is obtained by using Eqs. (9) and (30) as follows

$$H = \frac{H_0}{-2\gamma}\Big(-\Omega_{m0} - \gamma + \sqrt{(-\gamma - \Omega_{m0})^2 - 4\gamma\Omega_{m0}(1+z)^3}\Big). \tag{38}$$

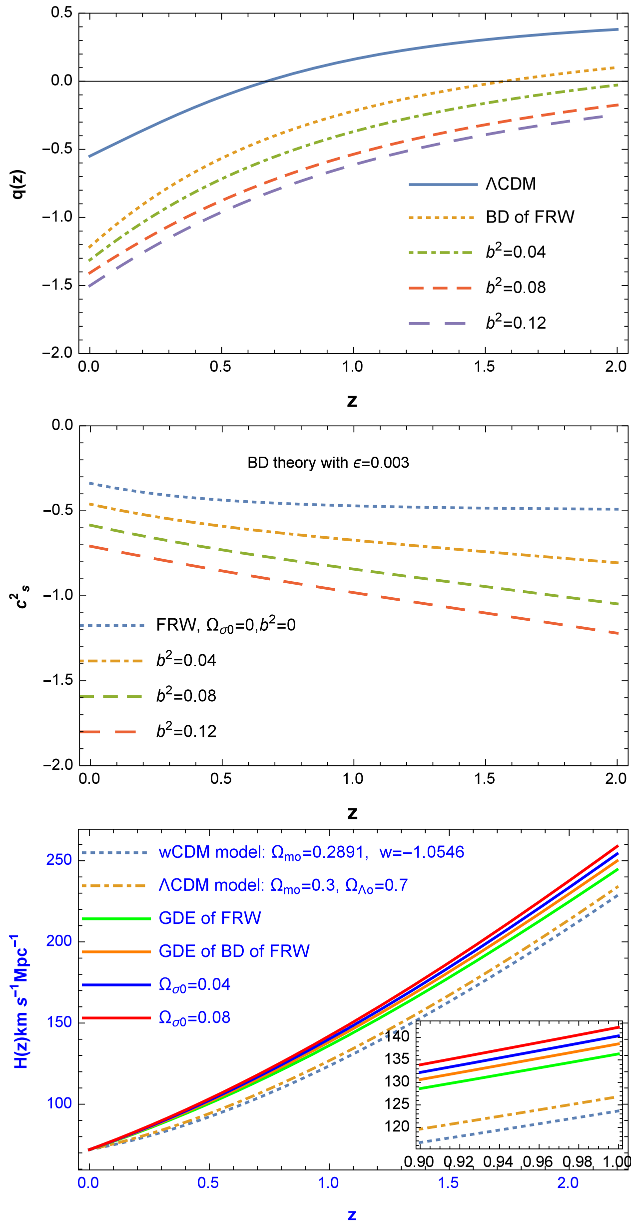

Fig. 5 *Top panel*: the evolution of $q(z)$ in terms of z for the interacting ghost DE of BD theory with different b^2. *Middle panel*: the evolution of c_s^2 as a function of cosmic redshift z for the different parameter b^2 with $\Omega_\Lambda^0 = 0.69$ and $\Omega_{\sigma 0} = 0.001$. *Lower panel*: Hubble expansion parameter in terms of redshift for the different parameter $\Omega_{\sigma 0}$ with $b^2 = 0.1$, $\Omega_{m0} = 0.277$, $H_0 = 72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\epsilon = 0.003$ and $\omega_0 = 10^4$

The behavior of the Hubble parameter is similar to that of the matter density parameters (Ω_m), which is expected because DE comes to dominate the evolution of the Hubble parameter only at very low redshift. We choose three specific DE models as representatives of cosmological models in order to make the analysis. They are the ΛCDM, wCDM, ghost DE of BD theory in BI (FRW) models. We consider to use the SGL+CBS (the strong gravitational lensing, the cosmic microwave background, baryon acoustic oscillations and type Ia supernova) data to constrain the wCDM and ghost DE models and we take $\Omega_{m0} = 0.2891$ and $w = -1.0546$ (Cui et al. 2015). In fact we can also see the lower panel of Fig. 5 that in a BI model although ghost DE model performs a little poorer than ΛCDM model, but it performs better than the ghost DE in BD theory. Also, from this figure we can understand the Hubble parameter in ghost DE of BD theory in BI is bigger than the ghost DE of FRW, ΛCDM and wCDM models. The larger the Hubble expansion rate $H(z)$ is taken, the bigger the anisotropy parameter $\Omega_{\sigma 0}$ can reach. Therefore, from the above analysis, we will figure out that both the parameters, b^2 and $\Omega_{\sigma 0}$, can impact the cosmic expansion history in the interacting ghost DE of BD theory in BI model.

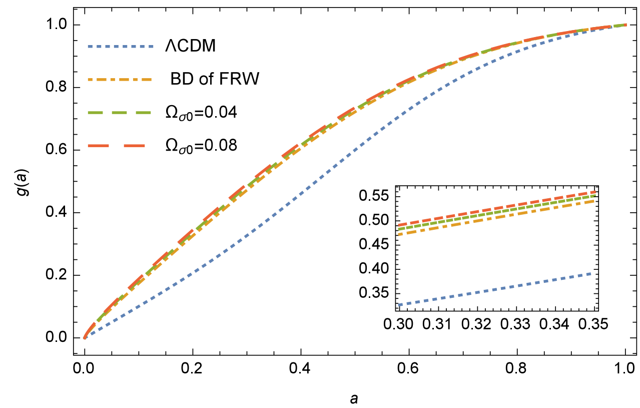

Fig. 6 Evolution of growth function $g(a)$ in terms of a for the different $\Omega_{\sigma 0}$ and comparing to the ΛCDM and FRW models in ghost DE of BD theory. The rest information is the same as that of mentioned in Fig. 5

In Fig. 6 we illustrate the effects of anisotropy on the growth factor in ghost DE of BD theory for the DE models considered in this work, as compared to the ΛCDM model. Generally, the ΛCDM model observe less growth compared to the ghost DE of BD theory in an anisotropic Universe. Therefore the growth factor $g(a)$ for the ΛCDM Universe will always fall behind the ghost DE models.

The theoretical distance modulus $\mu_{th}(z)$ is defined as (Wang et al. 2016)

$$\mu_{th}(z) = 5 \log_{10} \frac{d_L(z)}{Mpc} + 25, \tag{39}$$

where $d_L(z) = (1+z)\int_0^z H^{-1}(z')dz'$ is the luminosity distance. The structure of the anisotropies of the CMB radiation depends on two eras in cosmology, such as last scattering and today. We can also measure $d_L(z)$ through the Hubble parameter by using the Eq. (14). Figure 7 presents the distance modulus with the best fit of our model and the best fit of the ΛCDM model. From Fig. 7 we can observe the Universe is accelerating expansion. In all, current data are unable to discriminate between the popular ΛCDM, FRW and our interaction models.

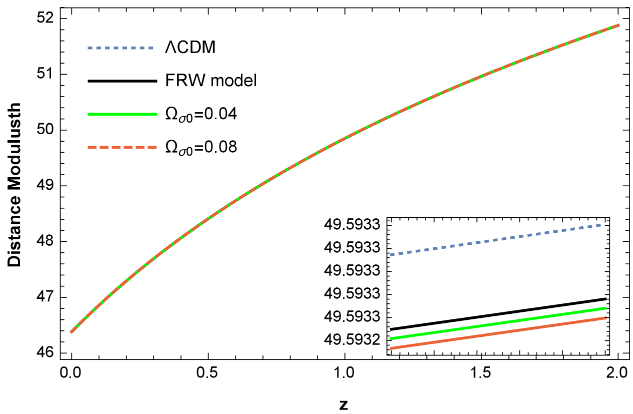

Fig. 7 Distance modulus for the best fit model $\Omega_{m0} = 0.277$, $H_0 = 72$ km/s/Mpc, $b^2 = 0.1$, and the ΛCDM model, $\Omega_{m0} = 0.3$, $H_0 = 72$ km/s/Mpc and $\Omega^0_\Lambda = 0.7$

5 Conclusion

In this work we studied the linear evolution of structure formation in interacting ghost DE models within the framework of Brans-Dicke theory. We start our analysis by studying the effects of anisotropy on the background expansion history of the growth factor. We obtained the evolution of density parameter Ω_Λ, the equation of state parameter ω_Λ, the deceleration parameter q and the squared sound speed c_s^2 for both the ghost DE and Brans-Dicke theory with respect to the cosmic redshift function. At first, the EoS parameter of the ghost DE and BD theory models in the case of $b^2 < 0.12$, cannot cross the phantom divide while it for $b^2 \geq 0.12$ can cross the phantom divide line. Beside, increasing of the anisotropy and the interaction parameter is increased the phantomic. Then, the evolution of the interacting ghost DE density parameter in BD theory is depend on the anisotropy density parameter $\Omega_{\sigma 0}$ and the coupling constant b^2. On the basis of the above considerations, it seems reasonable to investigate an anisotropic Universe, in which the present cosmic acceleration is followed by a decelerated expansion in an early matter dominant phase. In other words, it indicates that the values of transition scale factor and current deceleration parameter are $z = 0.74^{+0.40+0.78}_{-0.00-0.28}$ and $q_0 = -0.37^{+00+0.08}_{-0.09-0.19}$ for the case of ghost DE, $z = 2.13^{+0.84+1.28}_{-0.00-0.55}$ and $q_0 = -1.32^{+00+0.10}_{-0.07-0.17}$ for the case of ghost DE with BD theory while for the case of ΛCDM model, $z = 0.67$ and $q_0 = -0.54$ which is consistent with observations (Gong and Wang 2006; Myung 2007; Kim et al. 2008). However, for the interacting ghost DE shows signs of instability (due to negativity of adiabatic squared sound speed c_s^2) with (without) BD theory. In this case the frequency of the oscillations becomes purly imaginary and the density perturbations will grow with time.

Then, we analyzed $H(z)$ and compare the results with observational data. We found that, by choosing appropriate values of constant parameters, we figure out our model has more agreement with observational data than ΛCDM. Furthermore, we show that in anisotropic Universe with the ghost DE of BD theory, the Hubble parameter is bigger than the ghost DE of FRW, ΛCDM and wCDM models. It was observed that the larger the Hubble expansion rate $H(z)$ is taken, the bigger the anisotropy parameter $\Omega_{\sigma 0}$ can reach. Finally the effects of anisotropy on the growth of structures in linear regime is investigated and we compared the linear growth in the ghost DE and BD theory with the linear growth in the FRW and ΛCDM models which in the ΛCDM, the growth factor evolves more slowly compared to the ghost DE of FRW in BD theory because the cosmological constant dominates in the late time Universe. Also, in the ghost DE of FRW in BD theory, the growth factor evolves more slowly compared to the ghost DE models in an anisotropic Universe. Therefore due to BD theory the growth factor $g(a)$ for the ΛCDM Universe will always fall behind the ghost DE models in an anisotropic Universe.

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