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ORIGINAL ARTICLE

Relativistic compact anisotropic charged stellar models with Chaplygin equation of state

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Abstract This paper presents a new model of static spherically symmetric relativistic charged stellar objects with locally anisotropic matter distribution together with the Chaplygin equation of state. The interior spacetime has been matched continuously to the exterior Reissner–Nordström geometry. Different physical properties of the stellar model have been investigated, analyzed, and presented graphically.

Keywords General relativity · Relativistic astrophysics · Exact solution · Anisotropic fluid sphere · Charged fluid sphere · Compact stars · Relativistic stars · Equation of state

1 Introduction

The study of static charge fluid sphere is an interesting topic to the researchers. Bonnor (1960, 1965) proposed that a spherical body carrying certain modest electric charge density can remain in equilibrium under its own gravitational and electric repulsion. Stettner (1973) considered a model of homogeneous distribution of matter with a net surface charge. He showed that a fluid sphere of uniform density with modest surface charge is more stable without charge. According to Bekenstein (1971) gravitational attraction may be balanced by electrostatic repulsion due to electric charge and pressure gradient. Joshi (1993) proposed that Einstein–Maxwell solutions are also important to study the cosmic censorship hypothesis and the formation of naked singularities. The presence of charge affects the values for redshifts, luminosities, and maximum mass for stars. Komathiraj and Moharaj (2007) obtained new classes of exact solutions to the Einstein–Maxwell system of equations for a charged sphere with a particular choice of the electric field intensity and one of the gravitational potentials and found exact solutions to the Einstein–Maxwell field equations corresponding to a static spherically symmetric gravitational potential in terms of hypergeometric functions. A good collection of Einstein–Maxwell solutions, satisfying a variety of criteria for physical acceptability was studied in (Ivanov 2002). Thomas et al. (2005), Tikekar and Thomas (1998), Paul and Tikekar (2005) described that charged relativistic spheres may be used to model core-envelope stellar configuration where the core consists an isotropic fluid and the envelope comprises an anisotropic fluid. Charged, self-gravitating anisotropic fluid spheres have been investigated in general relativity was studied by Bonnor (1960). Varela et al. (2010) proposed a model of charged anisotropic matter with linear or nonlinear equation of state. The model was obtained by Krori–Barua (KB) ansatz. Rahaman et al. (2010) obtained a new model of singularity-free solutions for anisotropic charged fluids with Chaplygin equation of state by using KB metric. From the investigation of Ruderman (1972) the pressure inside the highly compact astrophysical objects like X-ray pulsar, Her-X-1, X-ray buster 4U 1820-30, millisecond pulsar SAX J 1804.4-3658, PSR J1614-2230, LMC X-4 etc. having the core density beyond the nuclear density ($\sim 10^{15}\ \mathrm{g\,cm^{-3}}$) becomes anisotropy in nature, i.e., the pressure can be decomposed into two parts; radial pressure P_r and transverse pressure P_t. The difference, denoted by $\Delta = P_t - P_r$, measures the anisotropy is called the *anisotropic factor*. The existence of solid core, in presence of type 3A superfluid (Kippenhahn et al. 2012),

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rotation, magnetic field, mixture of two fluid, existence of external field etc. are reasonable for it. Local anisotropy in self-gravitating systems were extensively studied by Herrera and Santos (1997). Bhar et al. (2015) have studied the behavior of static spherically symmetric relativistic objects with locally anisotropic matter distribution considering the Tolman VII form for the gravitational potential g_{rr} in curvature coordinates together with the linear relation between the energy density and the radial pressure.

In 1998, according to the discovery of High-z Supernova Search Team the expansion of our universe is accelerating. Dark energy is one of the suitable hypotheses to explain this. $P=\omega\rho$ with $\omega<0$ is called the dark energy equation of state, ω being the equation of state parameter. A two dimensional Brans–Dicke star model with exotic matter and dark energy was studied by Jun (2009). Chan et al. (2011) have proposed a model of the dark energy star consisting of four regions and by analyzing the model they conclude that for a static solution at least one of the regions must be constituted by dark energy. Lobo (2006) has given a model of a stable dark energy star by assuming two spatial types of mass function: one is of constant energy density and the other mass function is a Tolman–Whitker mass. He showed that the system is stable under a small linear perturbation. Inspired by this work, Bhar and Rahaman (2015) proposed a new model of dark energy star consisting of five zones, namely, the solid core of constant energy density, the thin shell between core and interior, an inhomogeneous interior region with anisotropic pressures, a thin shell, and the exterior vacuum region. They have also discussed the stability condition under a small linear perturbation. $P=-B/\rho^{\alpha}$ is generally called the Chaplygin equation of state where the α and B are positive constants and $0\leq\alpha\leq 1$. The general properties of a spherically symmetric body described through the generalized Chaplygin equation of state was studied by Bernardini and Bertolami (2005). The modified Chaplygin gas equation of state for the radial pressure is described by $P=A\rho-B/\rho^{\alpha}$. Here A, B, and α are constant parameters. If we take $\alpha=1$ then it gives generalized Chaplygin equation of state. In a very recent work one of us (Bhar 2015) obtained a new model of anisotropic star by using Finch–Skea ansatz by using generalized Chaplygin equation of state. Inspired by all of these previous works in this present paper we want to present a model for charged anisotropic star admitting generalized Chaplygin equation of state.

As a continuation of our previous work we intend to develop some new analytical relativistic anisotropic stellar models by using a particular type of metric function together with the generalized Chaplygin equation of state. Our analysis depends on several mathematical key assumptions. The form of metric potential ensures that the metric function is nonsingular, continuous, and well behaved in the interior of the star. This is one of the desirable features for the model on physical grounds. The solutions obtained in this work are expected to provide simplified but easy to mathematically analyzed stellar models with nonzero super-high surface density which could reasonably model the stellar core of a bare strange quark star by satisfying applicable physical boundary conditions.

Our paper is organized as follows. In Sect. 2 the interior spacetime and Einstein–Maxwell field equations are described. The solution of the field equations are given in Sect. 3 In the next section, 4, physical acceptability conditions are described. We match our interior spacetime to the exterior Reissner–Nordström spacetime in Sect. 5. Some physical properties have been discussed in Sects. 6–10 and finally some concluding remarks are made in Sect. 11.

2 Einstein–Maxwell field equations

To describe the interior of a static spherically symmetric distribution of matter the line element can be taken in the standard form as (Tolman 1939, Oppenheimer and Volkoff 1939)[1],

$$ds^2=e^{\nu(r)}dt^2-e^{\lambda(r)}dr^2-r^2\left(d\theta^2+\sin^2\theta d\phi^2\right), \tag{2.1}$$

where λ and ν are the functions of radial coordinate r only.

Let us further assume that the matter distribution inside the compact star is *locally* anisotropic in nature whose energy momentum tensor is given by the following:

$$T_\nu^\mu=\begin{pmatrix} \rho+\frac{E^2}{8\pi} & 0 & 0 & 0 \\ 0 & -P_r+\frac{E^2}{8\pi} & 0 & 0 \\ 0 & 0 & -P_t-\frac{E^2}{8\pi} & 0 \\ 0 & 0 & 0 & -P_t-\frac{E^2}{8\pi} \end{pmatrix} \tag{2.2}$$

where ρ is the matter density, P_r and P_t are respectively the radial and the tangential pressure of the fluid distribution. E is the electric field intensity.

Taking $G=1=c$, the Einstein–Maxwell field equations can be written as,

$$\kappa\rho=\frac{1}{r^2}\left[r\left(1-e^{-\lambda}\right)\right]'-E^2, \tag{2.3}$$

$$\kappa P_r=-\frac{1}{r^2}\left(1-e^{-\lambda}\right)+\frac{\nu'}{r}e^{-\lambda}+E^2, \tag{2.4}$$

$$\kappa P_t=\frac{e^{-\lambda}}{4}\left(2\nu''+\nu'^2+\frac{2\nu'}{r}-\nu'\lambda'-\frac{2\lambda'}{r}\right)-E^2, \tag{2.5}$$

and

$$\sigma=\frac{e^{-\lambda}}{4\pi r^2}\left(r^2E\right)'$$

[1] Throughout the work we will use $c=G=1$, except in figures.

where $\kappa = 8\pi$ and $'$ denotes the derivative with respect to the radial coordinate r and σ is the proper charge density.

In analogy to the electrically charged case one usually introduces a quantity $m(r)$ by the following expression:

$$e^{-\lambda} = 1 - \frac{2m(r)}{r} + \frac{q^2}{r^2}. \tag{2.6}$$

If r_Σ represents the radius of the fluid distribution then it can be showed that m is constant $m(r = r_\Sigma) = M$ outside the fluid distribution where M is the gravitational mass. Thus $m(r)$ represents the gravitational mass of the matter contained in a sphere of radius r. Using Eqs. (2.6) and (2.3)–(2.5), respectively, one can arrive at the following:

$$m(r) = \frac{\kappa}{2}\int \rho r^2 dr + \frac{q^2}{2r} + \frac{1}{2}\int \frac{q^2}{r^2}dr, \tag{2.7}$$

$$\nu' = \frac{(\kappa r P_r + 2m/r^2 - 2q^2/r^3)}{(1 - 2m/r + q^2/r^2)}, \tag{2.8}$$

$$\frac{dP_r}{dr} = -\frac{(P_r + \rho)}{2}\nu' + \frac{2(P_t - P_r)}{r}. \tag{2.9}$$

Finally combining (2.8) and (2.9), one gets the anisotropic generalization of well known Tolman–Oppenheimer–Volkoff (TOV) equation of hydrostatic equilibrium for charged stellar configuration (Oppenheimer and Volkoff 1939, Bowers and Liang 1974),

$$\frac{dP_r}{dr} = -\frac{(P_r + \rho)}{2}\frac{(\kappa r P_r + 2m/r^2 - 2q^2/r^3)}{(1 - 2m/r + q^2/r^2)} + \frac{2(P_t - P_r)}{r} \tag{2.10}$$

It is to be noted that the presence of an additional term, $2(P_t - P_r)/r$, represents additional "force" due to the pressure anisotropy, which is directed outward when $P_t > P_r$ and inward when $P_t < P_r$. The existence of repulsive force, $P_t > P_r$, allows the construction of more compact distribution when using anisotropic fluid than when using isotropic perfect fluid, $P_t = P_r$, (León 1987, Gokhroo and Mehra 1994).

From Eqs. (2.4) and (2.5) one gets,

$$\kappa(P_t - P_r) = \frac{e^{-\lambda}}{4}\left(2\nu'' + \nu'^2 + \frac{2\nu'}{r} - \nu'\lambda' - \frac{2\lambda'}{r}\right) + \frac{1}{r^2}\left(1 - e^{-\lambda}\right) - \frac{\nu'}{r}e^{-\lambda} + 2E^2 \tag{2.11}$$

Introducing the transformations

$$x = r^2, \quad Z(x) = e^{-\lambda(r)}, \quad \text{and} \quad y(x) = e^{\nu(r)} \tag{2.12}$$

Eqs. (2.3)–(2.5), and (2.7) take the following forms:

$$\kappa\rho = \frac{1 - Z}{x} - 2\dot{Z} - E^2, \tag{2.13}$$

$$\kappa P_r = 2Z\frac{\dot{y}}{y} - \frac{1 - Z}{x} + E^2, \tag{2.14}$$

$$\kappa P_t = Z\left[\left(\frac{2\ddot{y}}{y} - \frac{\dot{y}^2}{y^2}\right)x + \frac{2\dot{y}}{y}\right] + \dot{Z}\left(1 + x\frac{\dot{y}}{y}\right) - E^2, \tag{2.15}$$

$$m(x) = \frac{\kappa}{4}\int_0^x \sqrt{\omega}\rho(\omega)d\omega, \tag{2.16}$$

where $\dot{}$ denotes the derivative with respect to x.

Introducing $\Delta = P_t - P_r$, the *anisotropic factor*, which measures the pressure anisotropy within the star and combining Eqs. (2.14) and (2.15) one obtains,

$$\kappa\Delta = Z\left(\frac{2\ddot{y}}{y} - \frac{\dot{y}^2}{y^2}\right)x + \dot{Z}\left(1 + x\frac{\dot{y}}{y}\right) + \frac{1 - Z}{x} - E^2. \tag{2.17}$$

To solve Eqs. (2.13)–(2.15) we assume that the radial pressure, P_r, and the matter density ρ are related by the following form:

$$P_r = \alpha_1\rho + \frac{\alpha_2}{\rho}, \tag{2.18}$$

where α_1 and α_2 are constants.

Using Eq. (2.18) together with Eqs. (2.13) and (2.14) we obtain

$$\frac{\dot{y}}{y} = \frac{\alpha_1}{2Z}\left(\frac{1 - Z}{x} - 2\dot{Z} - E^2\right) + \frac{\alpha_2\kappa^2}{2Z}\left(\frac{1 - Z}{x} - 2\dot{Z} - E^2\right)^{-1} + \frac{1 - Z}{2xZ} - \frac{E^2}{2Z}. \tag{2.19}$$

3 Solution of the Einstein–Maxwell field equations

To solve the system of Eqs. (2.19) let us take the following potential

$$Z = \frac{1 + (a - b)x}{1 + ax}, \tag{3.1}$$

where, a and b are constants but $b \neq 0$. Because $b = 0$ leads to

$$\kappa\rho = -E^2 < 0.$$

The expression for the metric potential Z is physically reasonable since it gives a monotonic increasing mass function which is regular at the center of the stellar configuration and at the same time it gives a monotonic decreasing matter density. The same metric potential was used earlier by Lobo (2006) to model a dark energy star and putting $a=b$ we obtain our previous result (Bhar 2015).

We further assume,

$$E^2=\frac{kax}{(1+ax)^2}, \tag{3.2}$$

where, $k>0$. This form of E^2 gives a monotonic increasing electric field regular at the center and positive inside the star. Moreover by putting $k=0$ we regain the uncharged model.

Inserting the Z and the E^2 from Eqs. (3.1)–(3.2) into Eq. (2.19), we obtain,

$$\begin{aligned}\frac{\dot{y}}{y}&=\frac{b(1+3\alpha_1)+a(b-k)(1+\alpha_1)x}{2(1+ax)(1+(a-b)x)}\\&\quad+\frac{\alpha_2\kappa^2}{2}\frac{(1+ax)^3}{(1+(a-b)x)(3b+a(b-k)x)}.\end{aligned} \tag{3.3}$$

Integrating Eq. (3.3), we get,

Case I: $a\neq b\neq k$.

$$\begin{aligned}\ln y&=\frac{k+(2b+k)\alpha_1}{2b}\ln(1+ax)\\&\quad+\frac{(b^2-ak)+(3b^2-a(2b+k)\alpha_1)}{2(a-b)b}\\&\quad\times\ln\big(1+(a-b)x\big)+\frac{\alpha_2\kappa^2}{2}\bigg[\frac{a(3bk-a(b+2k))}{(a-b)^2(b-k)^2}x\\&\quad+\frac{a^2}{2(a-b)(b-k)}x^2\\&\quad-\frac{b^3}{(a-b)^3(-3b^2+a(2b+k))}\ln\big(1+(a-b)x\big)\\&\quad-\frac{(2b+k)^3}{(b-k)^3(3b^2-a(2b+k))}\\&\quad\times\ln\big(3b+a(b-k)x\big)\bigg]+C_1,\end{aligned} \tag{3.4}$$

Case II: $a=b\neq k$.

$$\begin{aligned}\ln y&=\frac{-a+k+(2a+k)\alpha_1}{2a}\ln(1+ax)\\&\quad+\frac{(1+\alpha_1)(a-k)}{2a}x\\&\quad+\frac{\alpha_2\kappa^2}{2}\bigg[\frac{(1+ax)}{6a(a-k)^3}\big((20a^2+23ak+11k^2)\\&\quad-a(a-k)(2a+7k)x+2a^2(a-k)^2x^2\big)\\&\quad-\frac{(2a+k)^3}{a(a-k)^3}\ln\big(3a+a(a-k)x\big)\bigg]+C_2,\end{aligned} \tag{3.5}$$

Case III: $a\neq b=k$.

$$\begin{aligned}\ln y&=\frac{(1+3\alpha_1)}{2}\ln\bigg(\frac{1+ax}{1+(a-k)x}\bigg)\\&\quad+\frac{\alpha_2\kappa^2}{6k}\bigg[\frac{a(a^2-3ak+3k^2)}{(a-k)^3}x\\&\quad+\frac{a^2(2a-3k)}{2(a-k)^2}x^2+\frac{a^3}{3(a-k)}x^3\\&\quad-\frac{k^3}{(a-k)^4}\ln\big(1+(a-k)x\big)\bigg]+C_3,\end{aligned} \tag{3.6}$$

Case IV: $a=b=k$.

$$\begin{aligned}\ln y&=\frac{(1+3\alpha_1)}{2}\ln(1+kx)+\frac{\alpha_2\kappa^2}{6k^2}(1+kx)^4\\&\quad+C_4,\end{aligned} \tag{3.7}$$

where, C_1, C_2, C_3, and C_4 are constants of integration to be determined by using an appropriate physical boundary condition. Therefore Eqs. (2.13)–(2.16), with the help of Eqs. (3.2) and (3.4) become,

$$\kappa\rho=\frac{3b+a(b-k)x}{(1+ax)^2}, \tag{3.8}$$

$$\kappa P_r=\alpha_1\frac{3b+a(b-k)x}{(1+ax)^2}+\alpha_2\kappa^2\frac{(1+ax)^2}{3b+a(b-k)x}, \tag{3.9}$$

$$\kappa P_t=\kappa P_r+\kappa\Delta, \tag{3.10}$$

where

$$\begin{aligned}\kappa\Delta&=\frac{x(1+(a-b)x)}{(1+ax)}\bigg(\frac{\dot{y}}{y}\bigg)^2-\frac{bx}{(1+ax)^2}\frac{\dot{y}}{y}\\&\quad+\frac{a(b-2k)x}{(1+ax)^2}+\frac{x(B_0+B_1x+B_2x^2)}{(1+ax)^2(1+(a-b)x)}\\&\quad+\alpha_2\kappa^2\frac{x(A_0+A_1x+A_2x^2+A_3x^3+A_4x^4)}{(1+ax)(3b+a(b-k)x)^2(1+(a-b)x)},\end{aligned}$$

where

$$\begin{aligned}A_0&=3b^2+a(5b+k),\\A_1&=2a\big(b(b-k)+a(8b+k)\big),\\A_2&=9ab\big(2a^2+b-ab\big),\\A_3&=2a^3\big(4ab-3b^2-ak\big),\\A_4&=a^4(a-b)(b-k),\\B_0&=b^2-a(b+k)+\big(3b^2-a(b+k)\big)\alpha_1,\\B_1&=-2ab(a-b)(1+3\alpha_1),\\B_2&=-a^2(a-b)(b-k)(1+\alpha_1).\end{aligned}$$

4 Physical acceptability conditions

For the well behaved nature of the solution, the following conditions should be satisfied (Abreu et al. 2007):

(i) The metric potentials should be free from singularities inside the radius of the star moreover the fluid sphere should satisfy $e^{\nu(0)}=$ constant, and $e^{-\lambda(0)}=1$.
(ii) The density ρ and pressures P_r, P_t should be positive inside the fluid configuration.
(iii) The radial pressure P_r must be vanishing but the tangential pressure P_t may not necessarily vanish at the boundary $r=r_\Sigma$. However, the radial pressure is equal to the tangential pressure at the center of the fluid sphere, i.e., pressure anisotropy vanishes at the center, $\Delta(0)=0$ (Bowers and Liang 1974, Ivanov 2002) and $\Delta(r=r_\Sigma)=\dfrac{\kappa}{C}P_t(r_\Sigma)>0$ (Böhmer and Harko 2006).
(iv) The radial pressure gradient $dP_r/dr\le 0$ for $0\le r\le r_\Sigma$.
(v) The density gradient $d\rho/dr\le 0$ for $0\le r\le r_\Sigma$.
(vi) A physically acceptable fluid sphere must satisfy the causality conditions, the radial and tangential adiabatic speeds of sound should less than the speed of light. In the unit $c=1$ the causality conditions take the form $0<v_{sr}^2=dP_r/d\rho\le 1$ and $0<v_{st}^2=dP_t/d\rho\le 1$.
(vii) The interior solution should satisfy either
   - strong energy condition (SEC) $\rho-P_r-2P_t\ge 0$, $\rho-P_r\ge 0$, $\rho-P_t\ge 0$ or
   - dominant energy condition (DEC) $\rho\ge P_r$ and $\rho\ge P_t$.
(viii) The charged interior solution should continuously match with the exterior Reissner–Nordström solution.

Conditions (iv) and (v) imply that pressure and density should be maximum at the center and monotonically decreasing towards the surface.

5 Physical boundary conditions

5.1 Mass to radius ratio

The interior solution should match continuously with an exterior Reissner–Nordström solution,

$$ds^2=\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2-\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}dr^2 - r^2\left(d\theta^2+\sin^2\theta d\phi^2\right);\ r\ge r_\Sigma. \tag{5.1.1}$$

This requires the continuity of e^ν and e^λ across the boundary $r=r_\Sigma$,

$$e^{\nu(r_\Sigma)}=e^{-\lambda(r_\Sigma)}=\left(1-\frac{2M}{r_\Sigma}+\frac{Q^2}{r_\Sigma^2}\right), \tag{5.1.2}$$

which sets the compactness parameter,

$$\frac{2M}{r_\Sigma}=\frac{r_\Sigma^2[b+a(b+k)r_\Sigma^2]}{(1+ar_\Sigma^2)^2}. \tag{5.1.3}$$

5.2 Determination of the constant of integration

From Eq. (5.1.2), we get,

$$\begin{aligned}C_1 &= \ln\left(1-\frac{2M}{r_\Sigma}+\frac{Q^2}{r_\Sigma^2}\right)-\frac{k+(2b+k)\alpha_1}{2b}\ln\left(1+ar_\Sigma^2\right)\\ &\quad -\frac{(b^2-ak)+(3b^2-a(2b+k)\alpha_1)}{2(a-b)b}\\ &\quad \times\ln\left(1+(a-b)r_\Sigma^2\right)-\frac{\alpha_2\kappa^2}{2}\left[\frac{a(3bk-a(b+2k))}{(a-b)^2(b-k)^2}r_\Sigma^2\right.\\ &\quad +\frac{a^2}{2(a-b)(b-k)}r_\Sigma^4\\ &\quad -\frac{b^3}{(a-b)^3(-3b^2+a(2b+k))}\ln\left(1+(a-b)r_\Sigma^2\right)\\ &\quad \left.-\frac{(2b+k)^3}{(b-k)^3(3b^2-a(2b+k))}\ln\left(3b+a(b-k)r_\Sigma^2\right)\right].\end{aligned}$$

6 Some features

6.1 Mass function

The mass function within the radius r can be obtained by integrating Eq. (3.8),

$$m(x)=\frac{2a(b-k)x^{3/2}-3k\sqrt{x}}{4a(1+ax)}+\frac{3k}{4a^{3/2}}\arctan\sqrt{ax}. \tag{6.1}$$

6.2 Mass-radius relation

The ratio of mass to the radius of a compact star can not be arbitrarily large. Buchdahl (1959) obtained an absolute constraint of the maximally allowable mass-to-radius ratio (M/r_Σ) for isotropic fluid spheres of the form $2M/r_\Sigma\le 8/9$ (in the units $c=G=1$), which states that for a given radius a static isotropic fluid sphere cannot be arbitrarily massive. Böhmer and Harko (2007) proved that for a compact object with charge, $Q(<M)$, there is a lower bound for the

mass-radius ratio,

$$\frac{3Q^2}{2r_\Sigma^2}\frac{(1+\frac{Q^2}{18r_\Sigma^2})}{(1+\frac{Q^2}{12r_\Sigma^2})}\le\frac{2M}{r_\Sigma}.$$

The upper bound of the mass of charged sphere was generalized by Andréasson (2009) and one proved that

$$\sqrt{M}\le\frac{\sqrt{r_\Sigma}}{3}+\sqrt{\frac{r_\Sigma}{9}+\frac{Q^2}{3r_\Sigma}}.$$

The ratio of mass to radius for our model is given by,

$$u(x)=\frac{2a(b-k)x-3k}{4a(1+ax)}+\frac{3k}{4a^{3/2}}\frac{\arctan\sqrt{ax}}{\sqrt{x}}. \tag{6.2.1}$$

6.3 Surface redshift

The surface redshift z_s of a star is given by

$$z_s=\left(1-\frac{2M}{r_\Sigma}+\frac{Q^2}{r_\Sigma^2}\right)^{-\frac{1}{2}}-1. \tag{6.3.1}$$

7 Construction of physically realistic fluid spheres

7.1 Pressure and density gradients

A straightforward differentiation of the pressure and density Eqs. (3.8)–(3.10) with respect to the auxiliary variable x one obtains the pressure and density gradients respectively,

$$\kappa\frac{d\rho}{dx}=-\frac{a(5b+k)+a^2(b-k)x}{(1+ax)^3}<0, \tag{7.1.1}$$

$$\kappa\frac{dP_r}{dx}=\left(\alpha_1-\frac{\alpha_2}{\rho^2}\right)\kappa\frac{d\rho}{dx}<0, \tag{7.1.2}$$

$$\kappa\frac{dP_t}{dx}=\kappa\frac{dP_r}{dx}+\kappa\frac{d\Delta}{dx}, \tag{7.1.3}$$

where

$$\begin{aligned}\frac{d\Delta}{dx}&=\frac{1+(a-b)(2x+ax^2)}{(1+ax)^2}f^2\\&\quad+2\frac{x(1+(a-b)x)}{(1+ax)}f\frac{df}{dx}-\frac{b(1-ax)}{(1+ax)^3}f\\&\quad-\frac{bx}{(1+ax)^2}\frac{df}{dx}+\frac{a(b-2k)(1-ax)}{(1+ax)^3}\\&\quad+\frac{G(x)}{(1+ax)^3(1+(a-b)x)^2}\\&\quad+\alpha_2\kappa^2\frac{I(x)}{(1+ax)^2(1+(a-b)x)^2(3b+a(b-k)x)^3},\end{aligned}$$

and

$$\begin{aligned}f(x)&=\frac{\dot{y}}{y},\\G(x)&=B_0+(-aB_0+2B_1)x\\&\quad+\big[(a-b)(-2aB_0+B_1)+3B_2\big]x^2\\&\quad+\big[-a(a-b)B_1+(3a-2b)B_2\big]x^3,\\\frac{df}{dx}&=\frac{F(x)}{2(1+ax)^2(1+(a-b)x)^2}\\&\quad+\frac{\alpha_2\kappa^2}{2}\frac{H(x)}{(1+(a-b)x)^2(3b+a(b-k)x)^2},\\F(x)&=b^2-a(b+k)+\big(3b^2-a(5b+k)\big)\alpha_1\\&\quad-2a(a-b)b(1+3\alpha_1)x\\&\quad+\big(a^2(-a+b)(b-k)(1+\alpha_1)\big)x^2,\\H(x)&=3b^2+a(5b+k)\\&\quad+2a\big[b(b-k)+a(8b+k)\big]x\\&\quad+3a^2b(6a-2b-k)x^2\\&\quad+2a^3\big(4ab-3b^2-ak\big)x^3\\&\quad+a^4(a-b)(b-k)x^4,\\I(x)&=3bA_0+\big(a(-b+k)A_0+6bA_1\big)x\\&\quad+\big[a\big(-7ab+5b^2+4ak-2bk\big)A_0\\&\quad+3b\big((2a-b)A_1+3A_2\big)\big]x^2\\&\quad+\big[-3a^2(a-b)(b-k)A_0\\&\quad-a(2a-b)(b-k)A_1\\&\quad+13abA_2-6b^2A_2-akA_2+12bA_3\big]x^3\\&\quad+\big[-2a^2(a-b)(b-k)A_1+3ab(a-b)A_2\\&\quad+20abA_3-9b^2A_3-2akA_3+15bA_4\big]x^4\\&\quad+\big[-a^2(a-b)(b-k)A_2\\&\quad+a\big(8ab-7b^2-2ak+bk\big)A_3\\&\quad+3\big(9ab-4b^2-ak\big)A_4\big]x^5\\&\quad+a\big(13ab-11b^2-4ak+2bk\big)A_4x^6\\&\quad+a^2(a-b)(b-k)A_4x^7.\end{aligned}$$

8 Junction condition

Our solution should satisfy the Darmois conditions on the boundary (Darmois 1927) to avoid the singularity and it requires the continuity of the first and second fundamental form across the boundary. These conditions are equivalent to the conditions imposed by Lichnerowicz (1955) proposed by Bonnor and Vickers (1981). These conditions require the existence of a coordinate system where the metric and all their first derivatives are continuous across the boundary surface. If the second fundamental form is not continuous, then there is a shell on the boundary surface, and the matching is described by the Israel conditions. Using the Darmois–Israel (1966, 1967) formation the surface stresses at the junction boundary is obtained as,

$$\sigma = -\frac{1}{4\pi r_\Sigma}\left[\sqrt{1-\frac{2M}{r_\Sigma}} - \sqrt{\frac{1+(a-b)r_\Sigma^2}{1+ar_\Sigma^2}}\right], \tag{8.1}$$

$$\begin{aligned}\mathcal{P} = \frac{1}{8\pi r_\Sigma}\Bigg[&\frac{1-\frac{M}{r_\Sigma}}{\sqrt{1-\frac{2M}{r_\Sigma}}} - \sqrt{\frac{1+(a-b)r_\Sigma^2}{1+ar_\Sigma^2}} \\ &\times\left(1+\frac{r_\Sigma^2}{2}\frac{b(1+3\alpha_1)+a(b-k)(1+\alpha_1)r_\Sigma^2}{(1+ar_\Sigma^2)(1+(a-b)r_\Sigma^2)}\right. \\ &\left.+\frac{\alpha_2\kappa^2 r_\Sigma^2}{2}\frac{(1+ar_\Sigma^2)^3}{(1+(a-b)r_\Sigma^2)(3b+a(b-k)r_\Sigma^2)}\right)\Bigg]\end{aligned} \tag{8.2}$$

where σ and $\mathcal{P}$ are respectively the surface stress energy and surface pressure. Hence one can match the interior spacetime to the exterior Schwarzschild spacetime in presence of a thin shell.

9 Relativistic adiabatic index and stability

The stability of a relativistic anisotropic sphere is related to the adiabatic index Γ (the ratio of two specific heats) defined by (Chan et al. 1993),

$$\Gamma = \frac{\rho+P_r}{P_r}\frac{dP_r}{d\rho}. \tag{9.1}$$

It is well known that the collapsing condition for a Newtonian isotropic sphere is $\Gamma < 4/3$ (Bondi 1964). For an anisotropic general relativistic sphere the collapsing condition becomes

$$\Gamma < \frac{4}{3} + \left[\frac{4}{3}\frac{(P_{t0}-P_{r0})}{|P'_{r0}|r} + \frac{1}{2}\kappa\frac{\rho_0 P_{r0}}{|P'_{r0}|}r\right]_{\max}, \tag{9.2}$$

where, P_{r0}, P_{t0}, and ρ_0 are the initial radial, tangential, and energy density in static equilibrium satisfying Eq. (2.10). The first and last term inside the square brackets, the anisotropic and relativistic corrections respectively, being positive quantities, increase the unstable range of Γ (Herrera et al. 1979, Chan et al. 1993).

To study the stability of anisotropic stars under the radial perturbations Herrera (1992) introduced the concept of

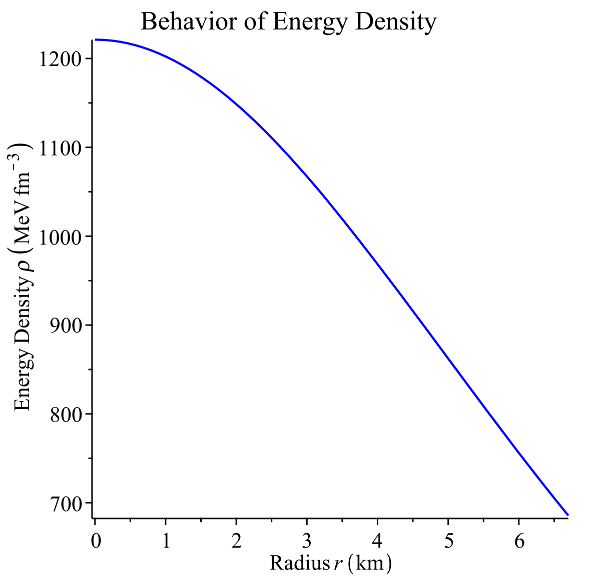

Fig. 1 Behavior of energy density ρ (MeV fm^{-3}) for the anisotropic fluid sphere generated with $a = 0.009$ km^{-2}, $b = 0.1349565242$ km^{-2}, and $r_\Sigma = 6.7$ km

"cracking", breaking of self-gravitating spheres, which results from the appearance of total radial forces of different signs in different regions of the sphere once the equilibrium is perturbed. The occurrence of such a "cracking" may be induced by the local anisotropy of the fluid.

By this concept of cracking Abreu et al. (2007) proved that the region of the anisotropic fluid sphere where $-1 \le v_{st}^2 - v_{sr}^2 \le 0$ is potentially stable but the region where $0 < v_{st}^2 - v_{sr}^2 \le 1$ is potentially unstable.

The radial and tangential speeds of sound of the strange star are obtained as,

$$v_{sr}^2 = \frac{dP_r}{d\rho} = \left(\alpha_1 - \frac{\alpha_2}{\rho^2}\right) < 1, \tag{9.3}$$

$$v_{st}^2 = \frac{dP_t}{d\rho} = \frac{dP_r}{d\rho} + \frac{d\Delta}{d\rho}. \tag{9.4}$$

10 Physical analysis

To generate an anisotropic fluid sphere we set, $a = 0.009$ (km^{-2}), $b = 0.01349565242$ (km^{-2}), $k = 0.0028$ and $\alpha_1 = 0.066$. By fixing the radius of the star is 6.7 km, and by using the condition $P_r(r = r_\Sigma) = 0$ we obtain $\alpha_2 = -5.4 \times 10^{-8}$ These values correspond to the surface density $\rho_s = 1.22 \times 10^{15}$ g cm^{-3} and central density $\rho_c = 2.174 \times 10^{15}$ g cm^{-3}. The central pressure is obtained as 0.88×10^{35} dyne/cm^2. Correspondingly the mass of the star is obtained as $0.97 M_\odot$, which is very close to the observational data of strange star Her X-1 (see Table 1 of Thirukkanesh et al. 2015). For these choices

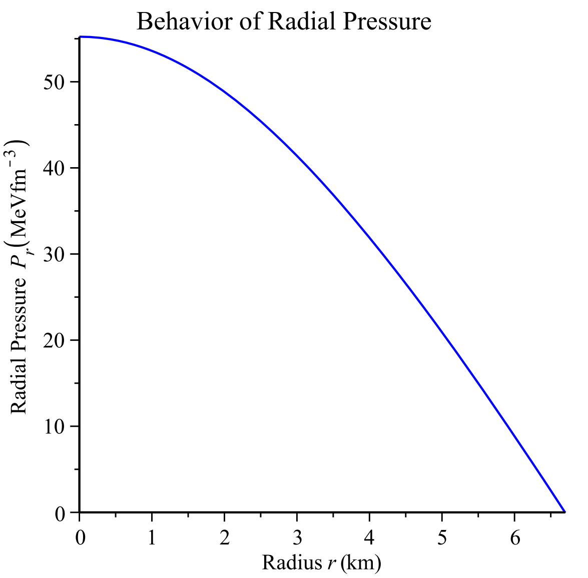

Fig. 2 Behaviors of radial pressure in the unit of MeV fm^{-3} for the stellar configuration as in Fig. 1

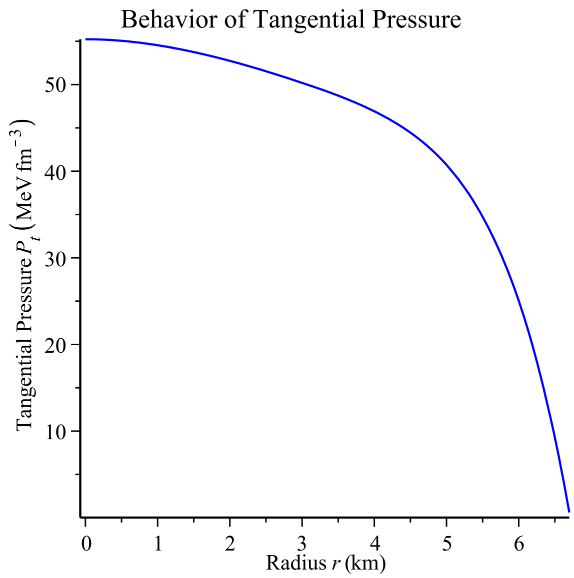

Fig. 4 Behaviors of tangential pressure in the unit of MeV fm^{-3} for the stellar configuration as in Fig. 1

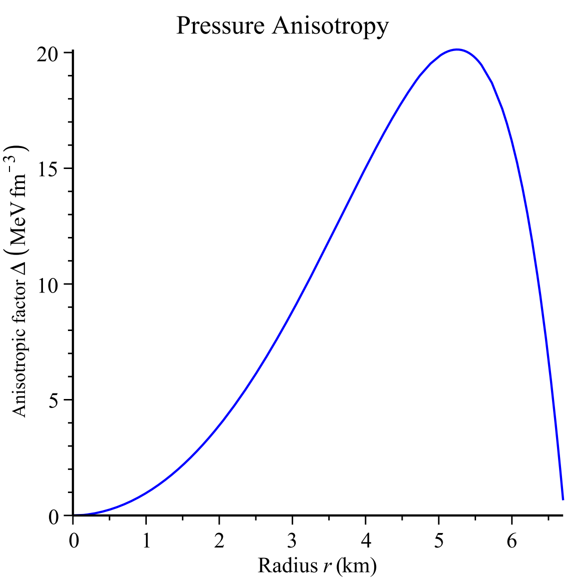

Fig. 3 Behavior of pressure anisotropy Δ in the unit of MeV fm^{-3} for the stellar configuration as in Fig. 1

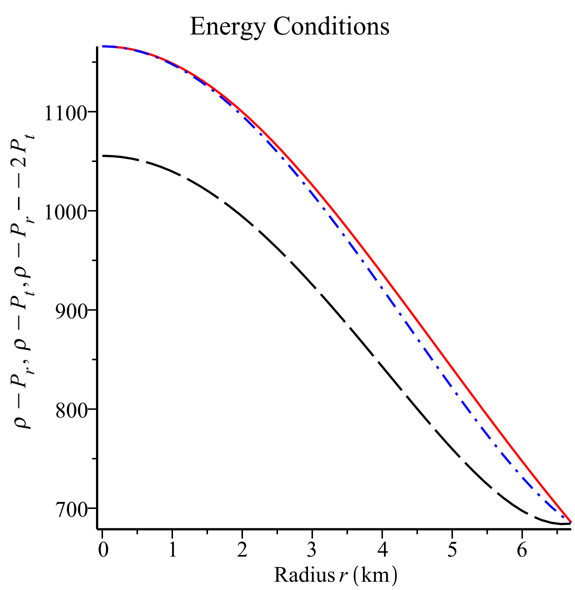

Fig. 5 The energy conditions for the stellar configuration as in Fig. 1. The solid (red) line corresponds to $\rho - P_r$, the dash-doted (blue) line corresponds to $\rho - P_t$, and the long dashed (black) line corresponds to $\rho - P_r - 2P_t$

the maximum value of compactness parameter is obtained $(2M/r_\Sigma)_{\max} = 0.213$, lies in the Böhmer and Harko (2006) and Andréasson (2009) limits. The surface redshift calculated to be $z_s = 0.319$.

The profiles of ρ, P_r, anisotropic factor $\Delta = P_t - P_r$, and P_t are shown in Figs. 1, 2, 3, 4 which show the positivity of those quantities inside the fluid sphere. Figure 3 indicates that $\Delta \geq 0$ for our model. The energy conditions are presented in Fig. 5. The profiles of v_{sr} and v_{st} are presented in Fig. 6, from which it is clear that the speeds are not superluminal for our model sphere and hence the causality conditions are satisfied. The mass-radius relation M–R is given in Fig. 7, which shows that the mass function is monotonically increasing function of r and is positive inside the stellar interior. The profile of $|v_{st}^2 - v_{sr}^2|$ is shown in Fig. 8. From the figure it is clear that $|v_{st}^2 - v_{sr}^2| < 1$,satisfies the condition of Andréasson (2009). The adiabetic index Γ is plotted in Fig. 9 and $\Gamma > 4/3$ for our model and hence stable.

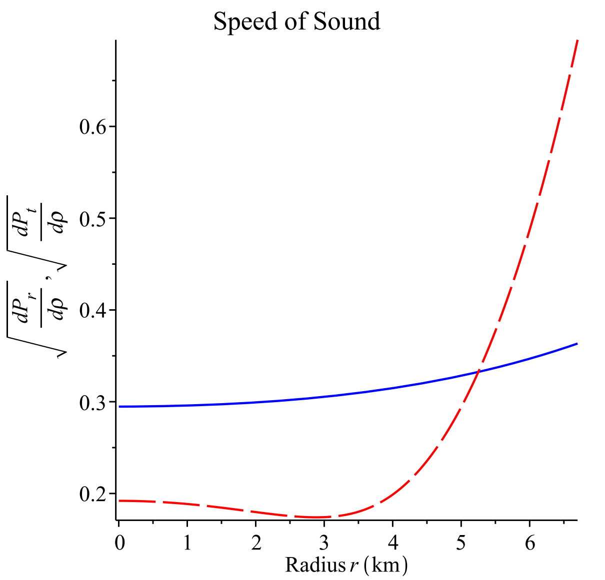

Fig. 6 The adiabatic speeds of sound for the same stellar configuration as in Fig. 1. The solid (blue) line corresponds to $v_{sr} = \sqrt{dP_r/d\rho}$, the long dashed (red) line corresponds to $v_{sr} = \sqrt{dP_t/d\rho}$

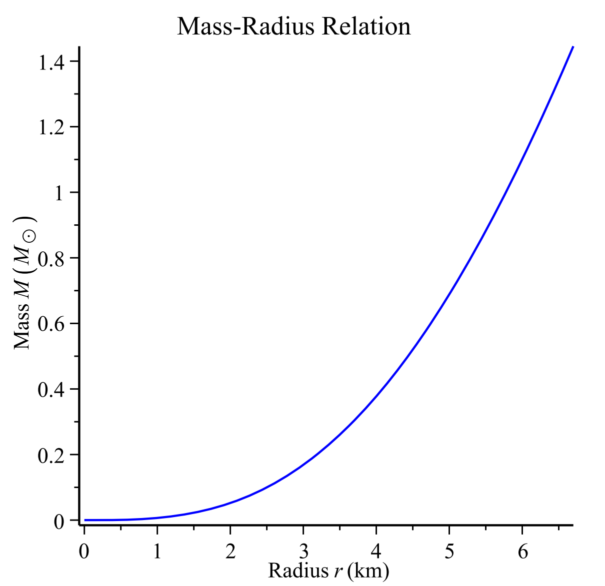

Fig. 7 The mass-radius relation for the same stellar configuration as in Fig. 1

11 Concluding remarks

Under the *ad hoc* assumption on one of the metric potentials $e^{-\lambda}$ together with the Chaplygin equation of state we have solved Einstein–Maxwell field equations and presented a particular simple class of static spherically symmetric anisotropic charged stellar models.

One notable feature of this solution is that one can regain the uncharged model by setting $k = 0$. Another interesting property of our model is that if one puts $\alpha_2 = 0$ and $k = 0$ then one can obtain the model of Lobo (2006) and by

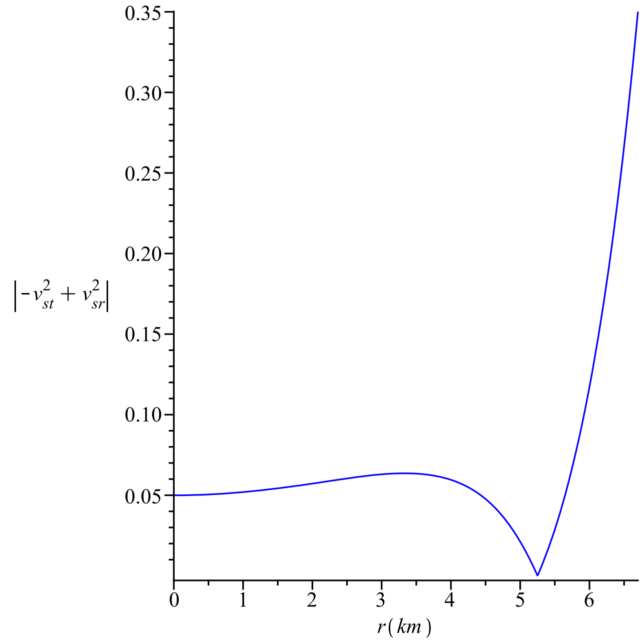

Fig. 8 The difference $|v_{st}^2 - v_{sr}^2|$ for the same stellar configuration as in Fig. 1

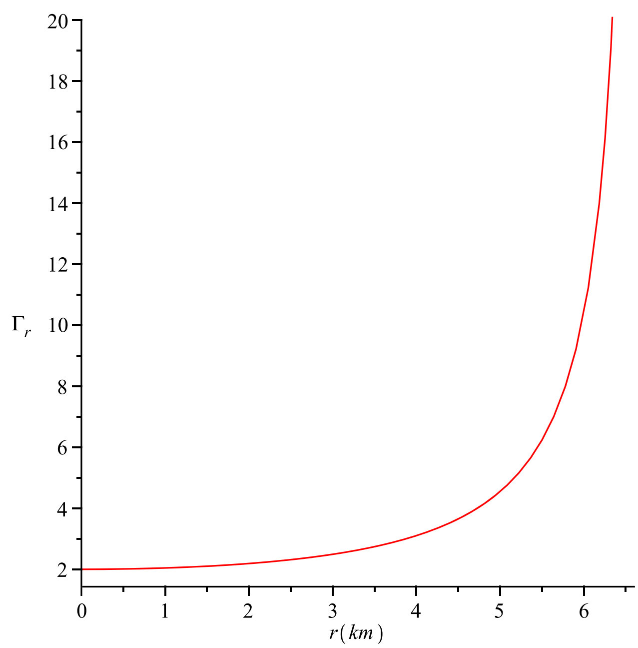

Fig. 9 The adiabetic index Γ for the same stellar configuration as in Fig. 1

setting $a = b$ and $k = 0$ we obtain a model of strange star obtained earlier by Bhar (2015). In the construction of the stellar models we further assumed $P_t > P_r$ ($\Delta > 0$). The stability is examined by the relativistic adiabatic index, and the adiabatic radial and tangential sound speeds. Even though it is yet unclear to what extent this metric can be applied to describe strange quark stars but the stellar models obtained here with such physical features could play a significant role in the description of internal structure of *bare* strange quark stars.

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