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ORIGINAL ARTICLE

Rogue waves in electronegative space plasmas: The link between the family of the KdV equations and the nonlinear Schrödinger equation

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Abstract We examine the likelihood of the ion-acoustic rogue waves propagation in a non-Maxwellian electronegative plasma in the framework of the family of the Korteweg–de Vries (KdV) equations (KdV/modified KdV/Extended KdV equation). For this purpose, we use the reductive perturbation technique to carry out this study. It is known that the family of the KdV equations have solutions of distinct structures such as solitons, shocks, kinks, cnoidal waves, etc. However, the dynamics of the nonlinear rogue waves is governed by the nonlinear Schrödinger equation (NLSE). Thus, the family of the KdV equations is transformed to their corresponding NLSE developing a weakly nonlinear wave packets. We show the possible region for the existence of the rogue waves and define it precisely for typical parameters of space plasmas. We investigate numerically the effects of relevant physical parameters, namely, the negative ion relative concentration, the nonthermal parameter, and the mass ratio on the propagation of the rogue waves profile. The present study should be helpful in understanding the salient features of the nonlinear structures such as, ion-acoustic solitary waves, shock waves, and rogue waves in space and in laboratory plasma where two distinct groups of ions, i.e. positive and negative ions, and non-Maxwellian (nonthermal) electrons are present.

Keywords Rogue waves · Negative ion plasmas · The nonlinear Schrödinger equation · Nonthermal electrons

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1 Introduction

There are several non-linear evolution equations that govern the physics and dynamics of the non-linear structures (Biswas 2009, 2010; Wazwaz 2008, 2015). The solitons arise as a consequence of an exact balance between broadening effects due to wave dispersion and wave steepening effects of non-linearities, which can propagate steadily for a long time. Moreover, another class of solutions, namely breathers, are also of fundamental importance. Breathers propagate steadily, and are localized in either time or space or both of them. Due to their localization properties, breathers have been invoked as models of rogue waves/freak waves/monster waves/ or rogons (Mu *et al.* 2014). The rogue waves (RWs) which have a peak amplitude generally more than twice the significant wave height, appear from nowhere and disappear without a trace (Akhmediev *et al.* 2009a, 2009b).

The family of Korteweg–de Vries (KdV) equations and the non-linear Schrödinger equation (NLSE) along with their variants, have been used to interpret and explore a variety of non-linear phenomena observed in non-linear systems, such as ocean, water tank, space, and astrophysical plasmas as well as in laboratory experiments (Gill *et al.* 2010; Lü 2015). The reductive perturbation technique has been used to derive the KdV family of equations, which describes the evolution of a non-modulated (non-envelope) waves, i.e., a bare pulse with no fast oscillations inside the packet. On the other side, the NLSE governs the dynamics of a modulated (envelope) wavepacket (Lü *et al.* 2015) in a way that the non-linearities are in balance with the wave group dispersion, resulting in the stationary solutions with an envelope structure. The mechanism of the freak waves may arise from the instability of a certain class of initial conditions that tend to grow exponentially and thus have the possibility of increasing up to very high amplitudes, due to modulation instability (Priya *et al.* 2015). Experimental observations of the modulational instability of the monochromatic ion-acoustic wave was reported firstly by Watanabe (1977). Many efforts have been devoted to the study of the structural characteristics and dynamic properties of the rogons. It has been found that the rational solutions of the NLSE can describe the dynamics of the RWs. The Peregrine breather/or Peregrine soliton (Peregrine 1983) which is a localized solution in both space and time is considered as one of the rational solutions of the NLSE. Others solutions associated with freak wave solution of the NLSE include the Akhmediev breather (Akhmediev *et al.* 2009a, 2009b), and the Kuznetsov–Ma breathers (KM) (Kuznetsov 1977). The Akhmediev breather is periodic in space and localized in time, while the KM is periodic in time and localized in space, but the Peregrine breather/rogue wave is localized in both time and space. Both Akhmediev breather and KM solitons become the Peregrine breather in the limiting case (Wang *et al.* 2015).

The generation of very large amplitude and highly energetic RWs in plasmas has been a topic of important research. Moreover, the RWs have been observed in many other fields of science, such as, in fiber optics, optical systems, Bose–Einstein condensates, capillary waves, superfluid helium, atmosphere, and even in astrophysical environments (Rahman *et al.* 2013). Recently, the rogue waves are investigated in a multicomponent plasma and have been experimentally observed and modeled by using the NLSE (Bailung *et al.* 2011; Sharma and Bailung 2013; Pallabi Pathak *et al.* 2016). Moreover, Peregrine- and KM-solitons have been observed in non-linear fiber optics experiments and also modeled by using the NLSE (Kibler *et al.* 2010, 2012). Clearly, we cannot do much if we leave the creation of rogue wave to chance. So, preparing special initial conditions and understanding the features of rogue wave could be useful either to avoid it or to generate highly energetic pulses (El-Labany *et al.* 2012). However, the physical mechanism behind the formation of RWs is still under investigation; observations indicate that they are essentially non-linear objects. Also, they are very steep and their steepness becomes infinite, thus, forming a wall of water in oceans and seas.

The structure of the ion-acoustic (IA) waves is investigated theoretically and experimentally in different plasma models. For example, one of these models is negative ion plasmas; a plasma that includes both negative- and positive-ion species, as well as distributed electrons. Negative ion plasmas are found near the Earth (Massey 1976), in the coma of comet Halley (Chaizy *et al.* 1991), in plasma processing reactors (Gottscho and Gaebe 1986), in neutral beam sources (Bascal and Hamilton 1979), and in low-temperature laboratory experiments (Jacquinot *et al.* 1977). Also, Coates *et al.* (2007) reported the presence of the negative ions in Titan's atmosphere. These ions are presented in mass groups with high densities, and it is expected that these ions have an important role in the ion chemistry and in forming organic-rich aerosols, which are falling on the surface (El-Labany *et al.* 2012). The simultaneous presence of positive and negative ions in a multi-component electron–ion plasma introduces new aspects of the non-linear IA waves (Das and Tagare 1975) (e.g. the coexistence of localized positive and negative potentials) depending upon the ratio between the negative and positive ion number densities. There have been theoretical as well experimental studies on the propagation of ion acoustic structures in negative-ion plasmas. In most of these investigations, the electrons are assumed to be isothermal and therefore they follow the Boltzmann (Maxwellian) distribution (Ruderman *et al.* 2008). However, in the case of deviation from isothermality, the electrons do not follow the Boltzmann distribution; rather, their velocity distribution may be represented by many non-Maxwellian distributions such as, nonthermal distribution. It has been found from both experimental observation and theoretical analysis that the presence of energetic nonthermal electrons (Cairns *et al.* 1995), which occurs in many space plasma situations, particularly, in upper part of ionosphere or lower part of magnetosphere, the aurora acceleration region, and in/around the Earth's bow shock, significantly modifies the basic features of IA structures or introduces new features for them (Mamun and Shukla 2009). Motivated by the different spacecraft/satellite observations. Cairns *et al.* (1995) introduced a distribution function representing the population of the energetic or nonthermal particles, which is able to explain some special features (particularly the existence of rarefactive IA solitons that are observed by the Freja Satellite and Viking spacecraft) of the IA solitary structures. After this pioneer work of Cairns *et al.* (1995), a significant number of the theoretical investigations have been made on the role of this nonthermal distribution of electrons in modifying the basic features of the non-linear IA structures (Mamun 1997).

Numerous theoretical and experimental investigations have been reported for the study of RWs in various plasma systems (Selim *et al.* 2015; Bacha *et al.* 2012; Irfan *et al.* 2014; El-Wakil *et al.* 2014a). By way of example, not exhaustive enumeration, the rogue wave in a collisionless, unmagnetized electronegative plasma with Maxwellian electrons is investigated El-Labany *et al.* (2012). Elwakil *et al.* (2014b) examined the properties of non-linear electron-acoustic rogue waves in an unmagnetized collisionless four-component plasma system consisting of a cold electron fluid, nonthermal electrons, an electron beam and stationary ions. Shalini and Saini (2015) studied the dust ion-acoustic rogue waves in an unmagnetized collisionless plasma system composed of charged dust grains, superthermal electrons and warm ions as a fluid. Rahman *et al.* (2013) investigated the propagation of IA solitary and rogue waves in a two-dimensional ultrarelativistic degenerate warm dense plasma. Moreover, the existence of atmospheric rogue waves is examined by Stenflo and Marklund (2010) using the NLSE.

It may be worth to mention here that the origin and mechanism for the generation of nonthermal particles in space plasma is still a central problem (Elwakil *et al.* 2010). Therefore, it is of practical interest to examine the effect of nonthermal electrons on the properties of the propagation of the freak waves in a positive–negative ion plasma. In this paper, the possibility of the propagation of the ion-acoustic RWs in electronegative plasmas with nonthermal electrons will be investigated. The paper is arranged in the following fashion: In Sect. 2, the basic equations are presented for positive–negative ions plasmas and the evolution equations, i.e. the extended Korteweg–de Vries (EKdV)/Gardner equation and NLSE, are derived. The appearance and disappearance of freak waves (short-lived large-amplitude pulses) in a nonlinear long wave model is examined in the framework of the NLSE derived from the Gardner equation. The analytical and numerical results are discussed in Sect. 3. Finally, the results are summarized in Sect. 4.

2 Model equations and evolution equations

Let us consider the propagation of non-linear ion-acoustic waves in an unmagnetized cold plasma whose constituents are the positive and negative ions fluid, as well as nonthermal electrons. At equilibrium, the quasineutrality condition demands $n_+^{(0)} = n_-^{(0)} + n_e^{(0)}$, where $n_{+i}^{(0)}$, $n_{-i}^{(0)}$, $n_e^{(0)}$ are the unperturbed positive-, negative-ions, and electron number densities, respectively. We focus our attention on the low frequency waves, so, the cold ion species is inertial and the electron inertia is neglected by assuming that the phase speed of the ion-acoustic waves is much smaller (larger) than the electron (ion) thermal speed, i.e. $\upsilon_{th(+)}$ and $\upsilon_{th(-)} \ll \upsilon_{ph} \ll \upsilon_{th(e)}$, where $\upsilon_{th(+)}$, $\upsilon_{th(-)}$, and $\upsilon_{th(e)}$ are, respectively, the positive, negative ion (electron) thermal speed and υ_{ph} is the ion acoustic phase velocity. Without repetition, we will follow the same normalized plasma model used by Sabry *et al.* (2009) for studying the fully nonlinear ion-acoustic solitary waves. Thus, the non-linear dynamics of such electrostatic perturbation mode are governed by the following dimensionless set of equations:

$$\partial_t n_{l+,-} + \partial_x (n_{+,-} u_{+,-}) = 0, \tag{1}$$

$$(\partial_t + u_{+,-}\partial_x) u_{+,-} + \frac{s}{q}\partial_x \phi = 0, \tag{2}$$

and

$$\partial_x^2 \phi = n_e + n_- - n_+, \tag{3}$$

where the subscripts denote the corresponding partial derivatives. In Eqs. (1)–(3), $n_{+,-}$ and $u_{+,-}$ are the ions number densities and ion fluid velocities, respectively, and ϕ is the electrostatic potential. Here, $q(= m_-/m_+)$ is the mass ratio with m_+ and m_- being the positive ion mass and the negative ion mass and $s = +1(-1)$ represents the polarity of the positive (negative) ion fluids. One may also express the neutrality condition as $\mu + \alpha = 1$ where $\alpha(= n_-^{(0)}/n_+^{(0)})$ is the ratio of the negative ions to the positive ions concentration and $\mu(= n_e^{(0)}/n_+^{(0)})$ is the ratio of the electrons to the positive ions concentration.

To model the effects of electron nonthermality, we will pick out the nonthermal Cairns distribution function for the electrons (Cairns *et al.* 1995). Thus, the normalized electron charge density is given by (Pakzad and Tribeche (2010))

$$n_e = \mu\left(1 - \beta\phi + \beta\phi^2\right)\exp(\phi). \tag{4}$$

The macroscopic nonthermality parameter $\beta = 4\delta/(1+3\delta)$, determines the population of energetic nonthermal electrons and it is a measure of nonthermality. The values of β are chosen to lie in the physically important range $0 \le \beta < 0.57$ (corresponding to $0 \le \delta < 0.25$). For reasons of physical plausibility one cannot take large values of β, lest the Cairns distributions in phase space develop bump-on-tail or beam instabilities, which occur for $\beta > 0.57$ (Verheest 2010). In the limiting case $\beta = 0$, the Cairns distribution loses its nonthermal wings and the density of the nonthermal electrons reduces to their well-known Maxwell–Boltzmann counterpart.

We confine ourselves to study the small, but finite, amplitude ion-acoustic excitations in nonthermal plasmas. For this purpose, we will use the reductive perturbation method (Washimi and Taniuti 1966; Mishra *et al.* 2002). According to this method, the following stretched space-time coordinates are used as: $\zeta = \varepsilon(x - \lambda t)$ and $\tau = \varepsilon^3 t$, where ε is a small parameter ($0 < \varepsilon \ll 1$) and λ is the wave propagation. Also, we expand the dynamic quantities in Eqs. (1)–(4) about their equilibrium values as: $\Gamma = \Gamma^{(0)} + \sum_{j=1}^{\infty} \varepsilon^j \Gamma^{(j)}$, where $\Gamma = [n_+, n_-, u_+, u_-, \phi]^T$ and $\Gamma^{(0)} = [1, \alpha, 0, 0, 0]^T$ and T stands for the transpose of the matrix. By substituting the last expansions and stretching in Eqs. (1)–(4) and after tedious calculations one finally obtains an extended Korteweg–de Vries (EKdV)/Gardner equation as

$$\partial_\tau \Phi + \left(A\Phi + C\Phi^2\right)\partial_\zeta \Phi + B\partial_\zeta^3 \Phi = 0, \tag{5}$$

with

$$A = 3B\left(\frac{1}{\lambda^4} - \frac{\alpha}{\lambda^4 q^2} - \frac{\mu}{3}\right),$$

$$B = \frac{\lambda^3 q}{2(q+\alpha)},$$

$$C=\frac{3B}{2}\left(\frac{5}{\lambda^6}-\frac{5\alpha}{(\lambda^6 q)}-\frac{\mu}{3}(1+3\beta)\right),$$
$$\lambda=\sqrt{\frac{q+\alpha}{q+\mu(1-\beta)}},$$

where $\Phi\equiv\phi^{(1)}$ for simplicity.

It is clear from Eq. (5) that at $A=0$, one can get on the critical value of the plasma parameters, which in case gives the parametric region of positive and negative potential of solitary waves. One notes that the coefficient of the nonlinear term A is a function of the negative ion relative concentration α, the nonthermal parameter β, and the mass ratio q for the model under consideration in this manuscript. Thus, to find the parametric regions corresponding to $A=0$, one has to express one (e.g., the critical value of the nonthermality β_c, let us say, β_c) of these parameters in terms of the others (viz. α and q). We will discuss this case, i.e. the nonlinear phenomena at the critical value of the nonthermality β_c, in details below.

It is known that the Gardner Eq. (5) has different nonlinear solutions including solitary wave, shock wave, cnoidal wave, freak wave, etc. Here, our study will be concentrated on the possibility of the ion-acoustic freak waves propagation in the present system. Thus, we examine the modulational instability of a weakly non-linear wavepackets described by the Gardner Eq. (5). First, the dynamics of very weak disturbances should be considered, i.e. the non-linear terms in Eq. (5) are less importance than the dispersive one. This situation produces the NLSE for the envelope of quasi-sinusoidal waves. It is well-known that the behavior of weakly non-linear wavepackets can be analyzed using the NLSE, which can be obtained from the KdV family of equations (Grimshaw *et al.* 2001; Ruderman 2010), using an asymptotic method. Therefore, one considers the solution of Eq. (5) in the form of a weakly modulated sinusoidal wave by expanding Φ, as

$$\Phi=\sum_{m=1}^{\infty}\varepsilon^m\sum_{l=-m}^{m}\Phi_m^{(l)}(X,T)\exp\big(il(k\zeta-\omega\tau)\big), \tag{6}$$

where k and ω are the wavenumber and wave frequency of the carrier ion-acoustic waves. We introduce the new stretched variables X and T as

$$X=\varepsilon(\zeta-v_g\tau)\quad\text{and}\quad T=\varepsilon^2\tau, \tag{7}$$

where v_g represents the group velocity of the basic wave (to be determined later). It is convenient to note that all the perturbed states depend on the fast scales through the phase $(k\zeta-\omega\tau)$, whereas the slow scales (X,T) enter the arguments of the mth harmonic amplitude $\Phi_m^{(l)}$. Here the quantity $\Phi_m^{(l)}$ must be real. Since the variables in Eq. (6) must satisfy the condition that $\Phi_m^{(l)}$ is real, then one must have $\Phi_{-m}^{(l)}=\Phi_m^{(l)*}$, where $(*)$ stands for the complex conjugate. The derivative operators appearing in Eq. (5) are transformed into

$$\left.\begin{aligned}&\partial_\tau\to\partial_\tau-\varepsilon v_g\partial_X+\varepsilon^2\partial_T,\\&\partial_\zeta\to\partial_\zeta+\varepsilon\partial_X.\end{aligned}\right\} \tag{8}$$

Substituting Eqs. (6)–(8) in Eq. (5) and collecting terms of the same powers of ε, one obtains a set of equations corresponding to the different carrier harmonics. During our calculations it is assumed that at $m=0$, then $\Phi_0^{(l)}=0$, because there are no higher harmonic wave components. Moreover, one can see from Eq. (6) that $l=0$ when $m=0$ implies that the plane wave doesn't exist in this case.

From the first-order with the first harmonic $(m,l)=(1,1)$, one can get $i(\omega+Bk^3)\Phi_1^{(1)}=0$, which gives

$$\omega=-Bk^3. \tag{9}$$

The second-order with the first-harmonic $(m,l)=(2,1)$ gives $-i(\omega+Bk^3)\Phi_2^{(1)}-(v_g+3Bk^2)\partial_X\Phi_1^{(1)}=0$, which yields the group velocity v_g as

$$v_g=-3Bk^2\equiv\partial_k\omega. \tag{10}$$

For the second harmonic modes $(m=2=l)$, one gets $iAk\Phi_1^{(1)2}-2i(\omega+4Bk^3)\Phi_2^{(2)}=0$, so one can get $\Phi_2^{(2)}$ as

$$\Phi_2^{(2)}=\frac{A}{6Bk^2}\Phi_1^{(1)2}. \tag{11}$$

From the third harmonic with the zeroth order $(m,l)=(3,0)$, one obtains $v_g\partial_X\Phi_2^{(0)}+A\partial_X(\Phi_1^{(1)}\Phi_1^{(1)*})=0$, which gives $\Phi_2^{(0)}$ as

$$\Phi_2^{(0)}=-\frac{A}{v_g}\big|\Phi_1^{(1)}\big|^2. \tag{12}$$

To the third-order with the first-harmonic $(m,l)=(3,1)$, a compatibility equation is obtained, by imposing the condition of annihilation of secular terms. This leads to the NLSE

$$i\partial_T\psi+\frac{1}{2}P\partial_X^2\psi+Q\,\psi^2\psi^*=0, \tag{13}$$

where, for simplicity, we introduced the notation $\psi=\Phi_1^{(1)}$. The coefficients of the dispersion and non-linear terms are given, respectively, by

$$P=-6Bk\equiv\partial_k^2\omega, \tag{14}$$

and

$$Q=\frac{A^2}{6Bk}-Ck. \tag{15}$$

Equation (13) describes the nonlinear evolution of the modulated amplitude ion-acoustic wave carrier. If we use the derivative expansion method to derive a NLSE form the basic Eqs. (1)–(4), then we can obtain an equation similar to Eq. (13), but with complicated expressions for P and Q. In this case, equation is valid for a carrier wave with an arbitrary wavelength (wavenumber). However, the derivation of the Gardner Eq. (5) has its own value because this equation describes many wave phenomena in plasmas as we mentioned before. After deriving the Gardner equation, it is natural to study modulations of sinusoidal waves described by this equation. For this purpose we derive the NLSE from the Gardner equation. Since the Gardner equation only describes long waves, in this case the NLSE can be only used to describe the nonlinear modulation of long sinusoidal waves (Ruderman *et al.* 2008; Ruderman 2010).

The product PQ is an important quantity as it determines:

(i) the stability property of the ion-acoustic wavepackets, i.e. the wavepacket becomes stable when $PQ < 0$, and unstable when $PQ > 0$;
(ii) the nature of envelope excitations, i.e. bright solitons can propagate in unstable regions, while dark solitons propagate in stable regions;
(iii) the regions of the existence of the rogue waves.

Since the coefficient of the dispersion term P is always negative (see Eq. (14)), the stability of monochromatic ion-acoustic waves is completely determined by the sign of the coefficient of the non-linear term Q. Thus, if $Q < 0$, then the non-linearity and dispersion can balance leading to creating an envelope modulated wave and vice versa. Moreover, the modulational instability is possible when (Grimshaw *et al.* 2001; Ruderman 2010)

$$k > k_c = \frac{|A|}{\sqrt{6BC}}, \tag{16}$$

where k_c is the critical wavenumber and it represents to the boundary between the stable and unstable regions.

3 Breather solutions and numerical results

One of the interesting results here is the modulational instability (MI) of the wavepackets resulting from the generation of the breather solutions in the unstable region, i.e. in the region $PQ < 0$. The modulational instability is considerably examined within the NLSE that describes weakly non-linear and dispersive waves. In this context, the non-linear stage of the MI is described by an exact solution of the NLSE, known as a breather wave. Breather waves have been considered as prototypes of freak waves (Onorato *et al.* 2013). Thus, for $PQ < 0$, the general breather solutions of the NLSE (10) can be written compactly as follows (Kibler *et al.* 2012):

$$\psi(X,T) = \psi_0\left[1 + \frac{2(1-2a)\cosh[bPT] + ib\sinh[bPT]}{\sqrt{2a}\cos(c\zeta) - \cosh[bPT]}\right] \times \exp(iPT), \tag{17}$$

where, $\psi_0 = \sqrt{P/Q}$ and the single governing parameter a determines the physical behavior of the solution through the function arguments $b = \sqrt{2ac}$ and $c = \sqrt{4(1-2a)}$. It is shown, for parametric investigation purposes, all physical information is contained within the coefficients P and Q which are functions of relevant plasma parameters.

In the following, we shall summarize the breather solutions, regarding analytical rogue-wave-like (breather) solutions of the NLSE (13), i.e. discussing briefly their relevance in our current context. It is clear from the general solution (17) that the parameter a determines the kind of the breather wave. For $a < 0.5$, the solution describes the Akhmediev breather (AB), and the parameters a and b are real with physical significance as a modulation frequency and exponential growth and decay rate. The AB is an exact solution of the NLSE; it describes the modulational instability in its non-linear regime; it is periodic in space. Figure 1a demonstrates the growth and decay cycle of the initial weak periodic modulation. For $\lim_{a\to 0.5}\psi(X,T)$, the solution describes the Peregrine soliton, also known as rational solution (rogue wave) corresponding to the low frequency limit of the Akhmediev breather which in this case is localized in both space and time dimensions, i.e. it exhibits nontrivial behavior over a small region of (X,T); see Fig. 1b. Most interestingly, a first experimental observation of Peregrine solitons in plasmas has been reported (Bailung *et al.* 2011; Sharma and Bailung 2013), yet in restricted conditions which are far from being thoroughly understood.

The Peregrine soliton/rogue wave reads (Kibler *et al.* 2012):

$$\begin{aligned}\psi(X,T) &= \psi_0 \lim_{a\to 0.5}\left\{\left[1 + \frac{2(1-2a)\cosh[bPT] + ib\sinh[bPT]}{\sqrt{2a}\cos(c\zeta) - \cosh[b\tilde{T}]}\right] \times \exp(iPT)\right\} \\ &= \psi_0\left[-1 + \frac{4(1+2iPT)}{1+4X^2+4(PT)^2}\right]\exp(iPT).\end{aligned} \tag{18}$$

For $a > 0.5$ as in Fig. 1c, the solution describes the Kuznetsov–Ma (KM) soliton where the parameters a and b become imaginary such that the hyperbolic trigonometric functions in solution (17) become ordinary circular functions and vice-versa. The KM solution is periodic in time only and decreases exponentially in space.

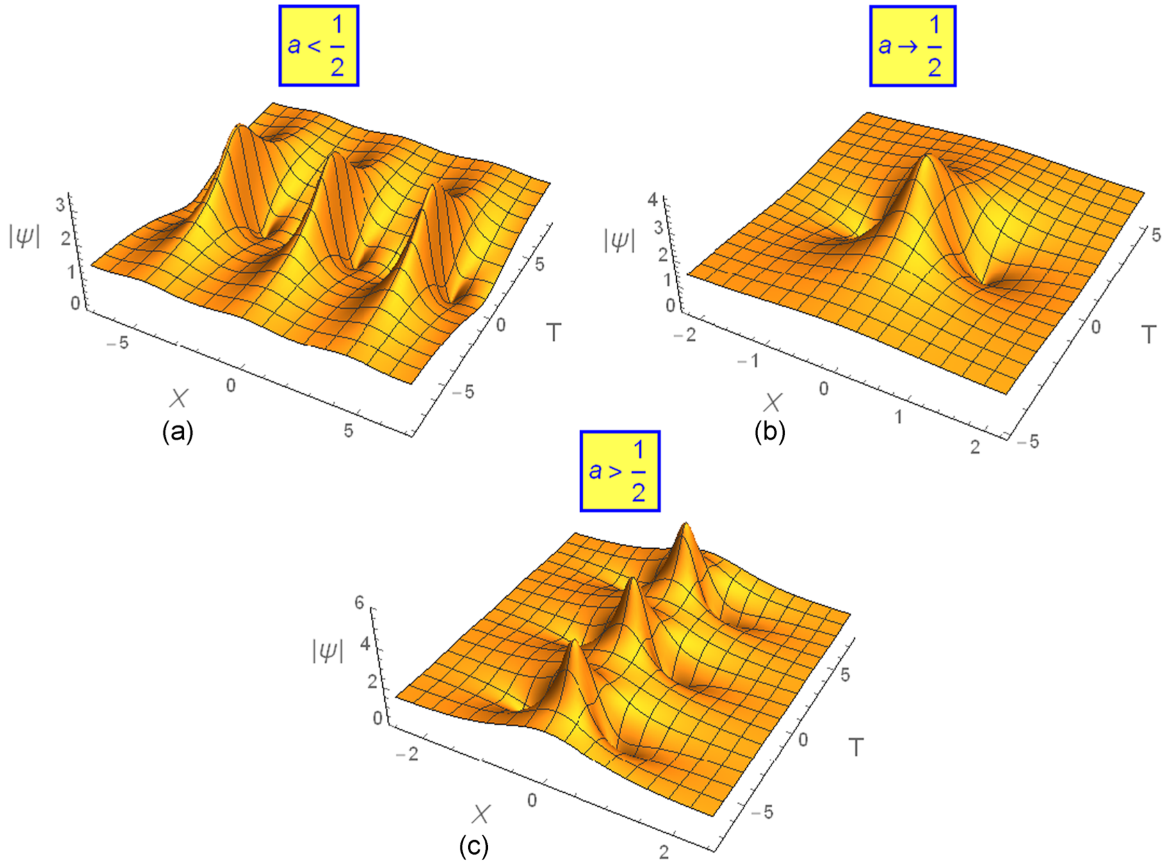

Fig. 1 Comparison between the profiles of the breather solutions (**a**) Akhmediev breather, (**b**) Rogue waves (or Peregrine soliton), and (**c**) Kuznetsov–Ma soliton. Here, $k = \beta = 0.2$ and $\alpha = 0.02$

The scenario of generating rogue waves may be demonstrated as, the instability of a moderate amplitude monochromatic wave leads first to the creation of a chain of solitons, which merge due to inelastic interaction into one soliton of large amplitude. This soliton sucks energy from neighboring waves and becomes unstable and collapses, thus, producing a rogue wave (Dyachenko and Zakharov 2005). The breather solution (17) predicts the concentration of the ion-acoustic wave energy into a small region due to the non-linear properties of the medium. This solution is able to concentrate a significant amount of the ion-acoustic wave energy into a small area in space.

It is interesting to investigate the effects of relevant physical parameters, namely, α, β, and q, of the positive hydrogen and negative oxygen ions $(H^+ - O_2^-)$, the positive hydrogen and negative hydrogen ions $(H^+ - H^-)$, and the positive Argon and negative Flourine ions $(Ar^+ - F^-)$ on the generation and propagation of the ion-acoustic rogue waves in unmagnetized nonthermal plasmas. The mass ratios of these plasmas are, respectively, 32, 1, and 0.475. It is known that the $(H^+ - H^-)$ and $(H^+ - O_2^-)$ plasmas occur in the D- and F-regions of the Earth's ionosphere, whereas the $(Ar^+ - F^-)$ plasma is used to study the ion-acoustic structures in laboratory experiment (Nakamura and Tsukabayashi 1984).

Let us assume that α or β are close to their critical values. In this case we use Eq. (5) with Eqs. (13)–(15) to describe the freak waves that can propagate in the present model. Figure 2 shows how the maximum rogue wave amplitude $\psi_{\text{Max}}[\equiv \psi(0,0) = \sqrt{9P/Q}]$ depends on α, β, and q. It is noteworthy from Figs. 2a and 2b that the critical values of α_c and β_c, i.e. the values of α and β that make the amplitude of the nonlinear phenomena such as rogue waves equals to zero or make the coefficient of the quadratic nonlinear term of Eq. (5) equals to zero; shrink with the enhancement of the nonthermality β and the negative ions relative concentrations α. In contrast, the critical value of α_c shifts to higher values with the increase of q as depicted in Fig. 2c. Moreover, increasing q would lead to the reduction of the value of β_c as shown in Fig. 2d. One can find that for $(\alpha, \beta) < (\alpha_c, \beta_c)$, the rogue wave amplitude ψ_{Max} decreases with the increase of α and β, but for $(\alpha, \beta) > (\alpha_c, \beta_c)$, the rogue wave amplitude ψ_{Max} has opposite behavior. Obviously, by increasing q, the rogue wave amplitude is increased (see Figs. 2c and 2d). Thus, the localized pulses of $(H^+ - O_2^-)$ plasma are much more spiky (taller amplitude) than $(Ar^+ - F^-)$ plasma in-

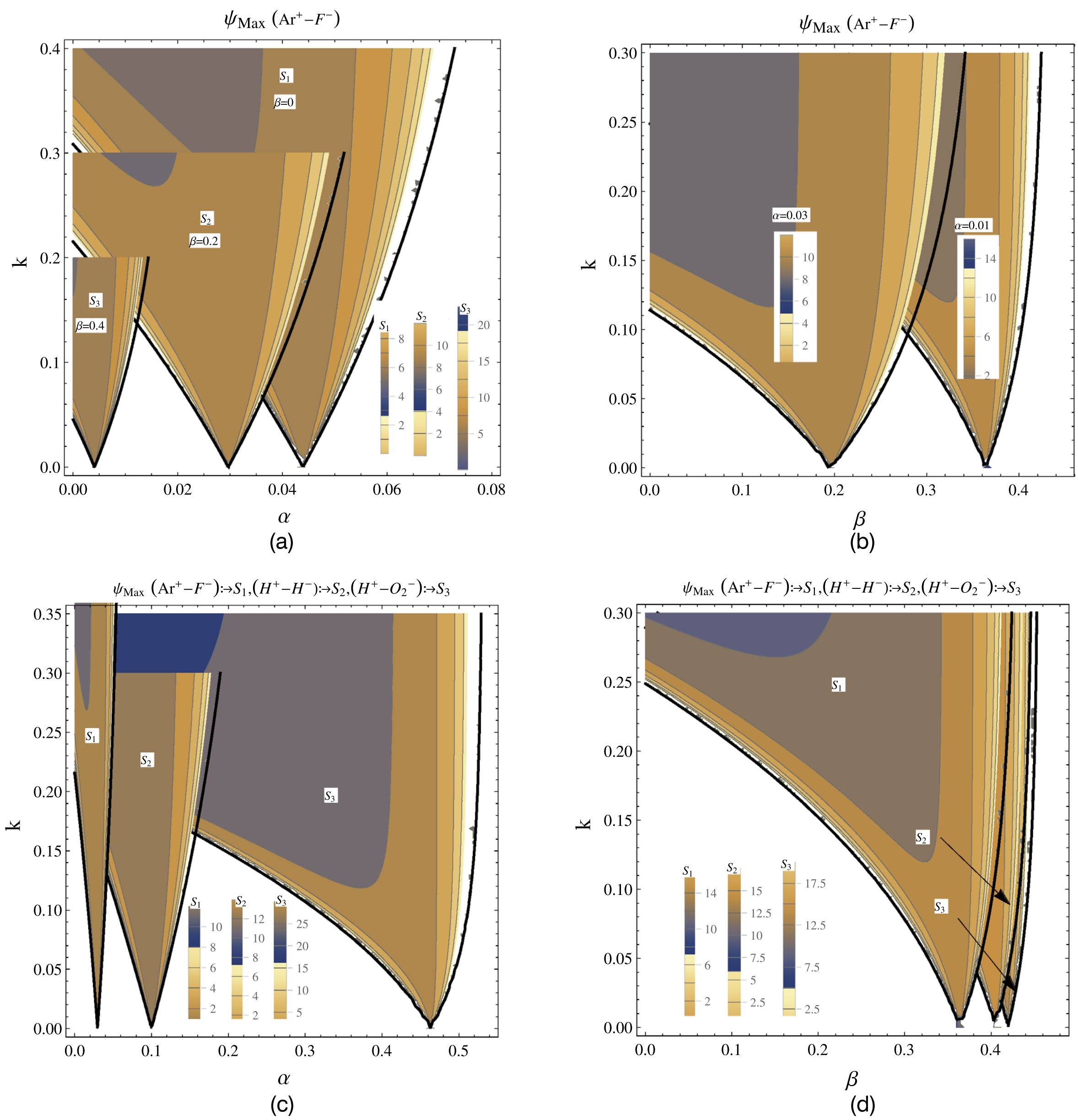

Fig. 2 Maximum amplitude of the rogue wave ψ_{Max} for the EKdV equation is plotted in the plane (**a**) $k-\alpha$ with different values of β, (**b**) $k-\beta$ with different values of α, (**c**) $k-\alpha$ with different values of q, (**d**) $k-\beta$ with different values of q

dicating that q affects on the pulse profile (via, amplitude). Furthermore, the electron nonthermality makes rogue pulses taller, in comparison with the case of Maxwellian electrons (see Fig. 2a). The behavior of higher or lower amplitude of the rogue waves could be explained as follows. Increasing α and β in the range $(\alpha, \beta) < (\alpha_c, \beta_c)$ would lead to the reduction in the non-linearity and thus the rogue wave cannot suck energy from the background waves, which makes the pulses shorter. On the other side, in the range $(\alpha, \beta) > (\alpha_c, \beta_c)$ and with the increase of the mass ratio q, the rogue waves amplitude increases due to the enhancement of the non-linearity and the pulses suck more energy from the background.

It is important to note that the quadratic non-linear term of Eq. (5) vanishes, i.e. $A = 0$ at the critical value of the nonthermal parameter β_c; which is given by

$$\beta_c = 1 - \frac{(\alpha + q)}{\sqrt{3(\alpha - 1)(3\alpha - q^2)}}, \tag{19}$$

and thus, Eq. (5) is reduced to the mKdV equation as

$$\partial_\tau \Phi + C\Phi^2 \partial_\zeta \Phi + B\partial_\zeta^3 \Phi = 0. \tag{20}$$

In this case, the coefficients of the dispersion and non-linear terms of Eq. (13), respectively, are reduced to $P = -6Bk$

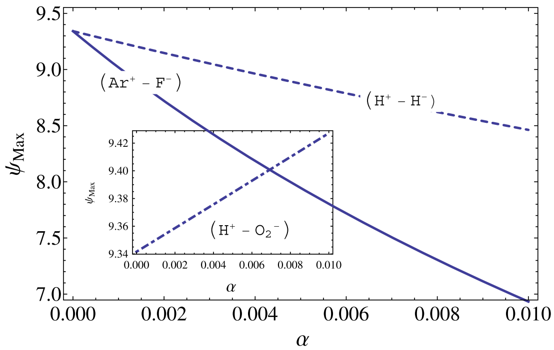

Fig. 3 Maximum amplitude of the rogue wave ψ_{Max} of the mKdV equation vs α and q

and $Q = -Ck$. The plane wave stability in this case depends only on the sign of the coefficient of cubic non-linear term C, i.e. the plane wave becomes stable if $C < 0$ and unstable if $C > 0$, for any value of the carrier wavenumber (see Eq. (16)). To see what happens when β is equal to its critical value β_c, we study numerically the behavior of the ion-acoustic rogue wave amplitude and examine its dependence on α as well as q. It is seen from Fig. 3 that the rogue wave amplitude decreases with the increase of α for the $(Ar^+ - F^-)$ and $(H^+ - H^-)$ plasmas, but for the $(H^+ - O_2^-)$ plasma, the rogue waves amplitude grows up with the growth of α. Therefore, α and q play significant roles in maximizing or minimizing the rogue wave energy. Ruderman *et al.* (2008) and Ruderman (2010) found that the rogue waves exist only for short periods of time and then disappear. Also, they demonstrated that the main role of the quadratic non-linearity is that it decelerates the wave evolution. As a result, the first rogue waves appearing in the case of mixed non-linearity (Gardner equation) are larger than in the case of purely cubic non-linearity (mKdV equation).

Moreover, if Eq. (5) is reduced to the KdV equation, i.e. $C = 0$,

$$\partial_\tau \Phi + A\Phi \partial_\zeta \Phi + B\partial_\zeta^3 \Phi = 0, \tag{21}$$

the coefficients of the dispersion and non-linear terms of the NLSE (13), respectively, become $P = -6Bk$ and $Q = -A^2/P$. It is noted that $P < 0$ and $0 < Q$; which means that for KdV–NLSE, the plane wave is always stable and does not support rogue waves generation.

4 Summary

Summing up, the effects of the nonthermal electrons on the generation and propagation of ion-acoustic rogue waves (IARWs) in a multifluid plasma consisting of positive–negative ions have been investigated. The basic set of equations is reduced to Gardner equation using the reductive perturbation method. To examine the rogue wave generation in the present model, a NLSE was derived from the KdV family of equations (i.e. the KdV-, mKdV-, and EKdV-equation) by using a standard perturbation method. The numerical investigations show that the IARWs need enough space and special initial conditions to be generated. So, we defined precisely the possible regions for the existence of the rogue waves in the laboratory plasma $(Ar^+ - F^-)$ as well as in astrophysics plasmas $(H^+ - H^-)$ and $(H^+ - O_2^-)$. The dependence of the rogue wave profile on the negative ion concentration α, the nonthermality β, and mass ratio q is numerically examined. Firstly, it is observed that the rogue wave amplitude for the EKdV–NLSE is increased with the enhancement of the mass ratio q, i.e. $\psi_{\text{Max}}(Ar^+ - F^-) < \psi_{\text{Max}}(H^+ - H^-) < \psi_{\text{Max}}(H^+ - O_2^-)$. This means that the rogue wave can suck more energy from the background with the increase of the mass ratio q. Moreover, it is found that for $(\alpha, \beta) < (\alpha_c, \beta_c)$, the rogue wave amplitude shrinks with the increase of α and β but for $(\alpha, \beta) > (\alpha_c, \beta_c)$, the rogue wave amplitude grows with the enhancement of α and β. Secondly, for the mKdV–NLSE rogue solution, the rogue wave amplitude of $(Ar^+ - F^-)$ and $(H^+ - H^-)$ plasmas shrinks with the increase of α, but for the $(H^+ - O_2^-)$ plasma, the rogue wave amplitude grows up with the increase of α. Finally, for KdV–NLSE, the plane wave is always stable and does not support rogue waves. The present results can contribute to the in-depth understanding of non-linear excitations such as, the ion-acoustic rogue waves that may appear in the laboratory (Sharma and Bailung 2013; Pallabi Pathak *et al.* 2016), Earth's ionosphere (Nakamura and Tsukabayashi 1984), and Titan's atmosphere (El-Labany *et al.* 2012).

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References

Akhmediev, N., Soto-Crespo, J.M., Ankiewicz, A.: Phys. Lett. A **373**, 2137 (2009a)
Akhmediev, N., Ankiewicz, A., Taki, M.: Phys. Lett. A **373**, 675 (2009b)
Bailung, H., Sharma, S.K., Nakamura, Y.: Phys. Rev. Lett. **107**, 255005 (2011)
Bascal, M., Hamilton, G.W.: Phys. Rev. Lett. **42**, 1538 (1979)
Bacha, M., Boukhalfa, S., Tribeche, M.: Astrophys. Space Sci. **341**, 591 (2012)
Biswas, A.: Nonlinear Dyn. **58**, 345 (2009)
Biswas, A.: Nonlinear Dyn. **59**, 423 (2010)
Cairns, R.A., Mamun, A.A., Bingham, R., Boström, R., Dendy, R.O., Nairns, C.M.C., Shukla, P.K.: Geophys. Res. Lett. **22**, 2709 (1995)

Chaizy, P., Rème, H., Sauvaud, J.A., d'Uston, C., Lin, R.P., Larson, D.E., Mitchell, D.L., Anderson, K.A., Carlson, C.W., Korth, A., Mendis, D.A.: Nature **349**, 393 (1991)
Coates, A.J., Crary, F.J., Lewis, G.R., Young, D.T., Waite, J.H. Jr., Sittler, E.C. Jr.: Geophys. Res. Lett. **34**, L22103 (2007)
Das, G.C., Tagare, S.G.: Plasma Phys. **17**, 1025 (1975)
Dyachenko, A.I., Zakharov, V.E.: Pis'ma Zh. Eksp. Teor. Fiz. **81**, 318 (2005)
El-Labany, S.K., Moslem, W.M., El-Bedwehy, N.A., Sabry, R., Abd El-Razek, H.N.: Astrophys. Space Sci. **338**, 3 (2012) and references therein
El-Wakil, S.A., Abulwafa, E.M., Elhanbaly, A., El-Shewy, E.K.: Astrophys. Space Sci. **353**, 501 (2014a)
Elwakil, S.A., El-hanbaly, A.M., Elgarayh, A., El-Shewy, E.K., Kassem, A.I.: Adv. Space Res. **54**, 1786 (2014b)
Elwakil, S.A., El-Shewy, E.K., Abdelwahed, H.G.: Phys. Plasmas **17**, 052301 (2010)
Gill, T.S., Bains, A.S., Saini, N.S., Bedi, C.: Phys. Lett. A **374**, 3210 (2010)
Gottscho, R.A., Gaebe, C.E.: IEEE Trans. Plasma Sci. **14**, 92 (1986)
Grimshaw, R., Pelinovsky, D., Pelinonsky, E., Talipova, T.: Physica D **159**, 35 (2001)
Irfan, M., Ali, S., Mirza, A.M.: Astrophys. Space Sci. **353**, 1 (2014)
Jacquinot, J., McVey, B.D., Scharer, J.E.: Phys. Rev. Lett. **39**, 88 (1977)
Kuznetsov, E.: Sov. Phys. Dokl. **22**, 507 (1977)
Kibler, B., Fatome, J., Finot, C., Millot, G., Dias, F., Genty, G., Akhmediev, N., Dudley, J.M.: Nat. Phys. **6**, 790 (2010)
Kibler, B., Fatome, J., Finot, C., Millot, G., Genty, G., Wetzel, B., Akhmediev, N., Dias, F., Dudley, J.M.: Sci. Rep. **2**, 463 (2012)
Lü, X.: Nonlinear Dyn. **81**(1), 239 (2015)
Lü, X., Ma, W.X., Yu, J., Lin, F., Khalique, C.M.: Nonlinear Dyn. **82**, 1211 (2015)
Mamun, A.A., Shukla, P.K.: Phys. Rev. E **80**, 037401 (2009)
Mamun, A.A.: Phys. Rev. E **55**, 1852 (1997)
Massey, H.: Negative Ions, 3rd edn., p. 663. Cambridge University Press, Cambridge (1976)
Mishra, M.K., Arora, A.K., Chhabra, R.S.: Phys. Rev. E **66**, 046402 (2002)
Mu, G., Qin, Z., Grimshaw, R.: arXiv:1404.2988 [nlin.SI] (2014)
Nakamura, Y., Tsukabayashi, I.: Phys. Rev. Lett. **52**, 2356 (1984)
Onorato, M., Residori, S., Bortolozzo, U., Montina, A., Arecchi, F.T.: Phys. Rep. **528**, 47 (2013)
Pakzad, H.R., Tribeche, M.: Astrophys. Space Sci. **330**, 95 (2010)
Pallabi Pathak, Sharma, S.K., Nakamura, Y., Bailung, H.: Phys. Plasmas **23**, 022107 (2016)
Peregrine, D.H.: J. Aust. Math. Soc. Ser. B, Appl. Math **25**, 16 (1983)
Priya, N.V., Senthilvelan, M., Lakshmanan, M.: Pramāna **84**, 339 (2015)
Rahman, A.U., Ali, S., Moslem, W.M., Mushta, A.: Phys. Plasmas **20**, 072103 (2013) and references therein
Ruderman, M.S.: Eur. Phys. J. Spec. Top. **185**, 57 (2010)
Ruderman, M.S., Talipova, T., Pelinovsky, E.: J. Plasma Phys. **74**, 639 (2008)
Sabry, R., Moslem, W.M., Shukla, P.K.: Phys. Plasmas **16**, 032302 (2009)
Selim, M.M., Abdelwahed, H.G., El-Attafi, M.A.: Astrophys. Space Sci. **359**, 25 (2015)
Sharma, S.K., Bailung, H.: J. Geophys. Res. Space Phys. **118**, 919 (2013)
Shalini, Saini, N.S.: J. Plasma Phys. **81**, 905810316 (2015)
Stenflo, L., Marklund, M.: J. Plasma Phys. **76**, 293 (2010)
Verheest, F.: Phys. Plasmas **17**, 062302 (2010)
Wazwaz, A.M.: Appl. Math. Comput. **204**, 817 (2008)
Wazwaz, A.M.: Nonlinear Dynamics and Systems Theory, vol. 15, p. 209 (2015)
Watanabe, S.: J. Plasma Phys. **17**, 487 (1977)
Wang, L., Li, M., Qi, F-H., Geng, C.: Eur. Phys. J. D **69**, 108 (2015)
Washimi, H., Taniuti, T.: Phys. Rev. Lett. **17**, 996 (1966)