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LETTER

Effects of quintessence on thermodynamics of the black holes

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Abstract In this letter, we investigate the effects of quintessence on thermodynamics of the Bardeen black hole and compare them with the results of our former paper. Black hole thermodynamic stability can be determined by studying the nature of heat capacity of the system. We use the first-law of thermodynamics to derive the thermodynamic quantities of these black holes and we compare and analyse the results. We plot the variation of mass, temperature and heat capacity as a functions of entropy related to the quintessence. Finally, we study the equation of state of these black holes with quintessence.

Keywords Thermodynamics · Black holes · Quintessence · Schwarzschild · Reissner-Nordström · Bardeen

1 Introduction

General relativity describes that black hole absorbs all the light that hits the horizon, reflecting nothing, just like a perfect black body in thermodynamics. The thermodynamic properties of black holes have received considerable attraction in recent times, as it is hoped that these studies can establish a connection among thermodynamics, gravitation and quantum statistical mechanics and eventually leading to quantum gravity. Black hole thermodynamics is one of the interesting subjects in modern cosmology and is the area of study that seeks to reconcile the laws of thermodynamics with the existence of black hole event horizons which is widely studied in the literature. The seminal connections between black holes and thermodynamics were initially made by Hawking and Bekenstein (Hawking 1975; Bekenstein 1973; Bardeen et al. 1973). Since the area law of the black hole entropy has been discovered, there has been much attention to thermodynamic quantities and thermodynamic phase transitions on various black holes. Black holes behave as thermodynamic objects which emit radiation from the event horizon by using the quantum field theory in curved space-time, named as Hawking radiation with a characteristic temperature proportional to their surface gravity at the event horizon and they have an entropy equal to one quarter of the area of the event horizon in Planck units (Hawking 1975; Bekenstein 1973; Bardeen et al. 1973). It was shown that the Einstein equation can be derived from the first law of thermodynamics by assuming the proportionality of entropy and the horizon area. The relation between the Einstein equation and the first law of thermodynamics has been generalized to the cosmological context. The Hawking temperature, entropy and mass of the black holes satisfy the first law of thermodynamics (Hawking 1975; Bekenstein 1973; Bardeen et al. 1973).

Accelerating expansion of the universe is one of the most recent fascinating results of observational cosmology. To explain the accelerated expansion of the universe, it is proposed that the universe is regarded as being dominated by an exotic scalar field with a large negative pressure called "dark energy" which constitutes about 70 percent of the total energy of the universe (Perlmutter et al. 1999; Riess et al. 1998, 1999; Spergel et al. 2007; Tegmark et al. 2004; Seljak et al. 2005).

In physical cosmology and astronomy, dark energy is an unknown form of energy which is hypothesized to permeate all of space, tending to accelerate the expansion of the universe. Dark energy is the most accepted hypothesis

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to explain the observations indicating that the universe is expanding at an accelerating rate. The nature of dark energy is yet to be understood. There are several cosmological models proposed in which the dominant component of the energy density has negative pressure. One of them is the cosmological constant (Padmanabhan 2003) which corresponds to the case of dark energy with a state parameter $\omega_q = -1$. There are alternative models that are proposed as candidates for dark energy. Most of these models are based on a scalar field. Such scalar field models include but not limited to, quintessence (Carroll 1998), chameleon (Khoury and Weltman 2004), K-essence (Armendariz-Picon et al. 2000), tachyon (Padmanabhan 2002), phantom (Caldwell 2002) and dilaton (Gasperini et al. 2002). Basically, the difference between these models returns to the magnitude of ω_q which is the ratio of pressure to energy density of dark energy and for quintessence $-1 < \omega_q < -\frac{1}{3}$.

Black holes surrounded by quintessence have received considerable attention and their thermodynamics has been intensively investigated. It would be also interesting to know how does the quintessence affect the thermodynamics of black holes. Quintessence as one candidate for the dark energy is defined as an ordinary scalar field coupled to gravity (Copeland et al. 2006). Kiselev (2003) by considering the Einstein's field equations for a black hole charged or not and surrounded by quintessence, derived a new solution related to ω_q and by using this solution, we studied the null geodesics of the Reissner-Nordström and Schwarzschild-anti de Sitter black holes surrounded with quintessence (Malakolkalami and Ghaderi 2015a, 2015b).

This paper is a continuation of our previous paper (Ghaderi and Malakolkalami 2016) which we investigated the thermodynamics of the Schwarzschild and Reissner-Nordström black holes related to the quintessence. Here, in addition to these two black holes, we study the thermodynamic properties of the Bardeen black hole surrounded by quintessence and we compare and analyze the results.

The paper is arranged in the following way. In Sect. 2, we briefly review the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence. In Sect. 3, we discuss the thermodynamic quantities of these black holes and we compare the results with each other. The last section deals with the conclusions of our results.

2 Black holes surrounded by quintessence

In this section we will introduce the black holes surrounded by quintessence derived by Kiselev (2003). His derivation assumed a static spherically symmetric gravitational field with the energy momentum tensor,

$$T_t^t = T_r^r = \rho_q \tag{1}$$

$$T_\theta^\theta = T_\phi^\phi = -\frac{1}{2}\rho_q(3\omega_q + 1) \tag{2}$$

Here, ω_q is the quintessential state parameter which has the range $-1 < \omega_q < -\frac{1}{3}$ and ρ_q is the density of quintessence matter which is always positive and given by,

$$\rho_q = -\frac{c}{2}\frac{3\omega_q}{r^{3(1+\omega_q)}} \tag{3}$$

where c is the positive normalization factor. From Kiselev's investigations (Kiselev 2003) on spherically symmetric solutions for Einstein equations describing black holes surrounded by quintessence with the energy momentum tensor, the metric of the black hole space-time surrounded by quintessence can be written as,

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{4}$$

where

$$f(r) = 1 - \frac{r_g}{r} - \sum_n \left(\frac{r_n}{r}\right)^{3w_n+1} \tag{5}$$

For the Schwarzschild and the Reissner-Nordström black holes with quintessence, $f(r)$ is given by (Kiselev 2003),

$$f(r)_{Sch} = 1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q+1}} \tag{6}$$

$$f(r)_{RN} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{c}{r^{3\omega_q+1}} \tag{7}$$

Here, M and Q are the mass and charge of the black hole.

The Bardeen model describes a regular black hole space-time which satisfying the weak energy conditions (Bardeen 1968). This metric can be formally obtained by coupling Einstein's gravity to a non-linear electrodynamics field (Ayon-Beato and Garcia 2000). For the Bardeen model, $f(r)$ is given by,

$$f(r) = 1 - \frac{2Mr^2}{(r^2 + g^2)^{\frac{3}{2}}} \tag{8}$$

Here, M is the mass of the black hole and g is the magnetic charge of the nonlinear self-gravitating monopole. It can be noted that $f(r)$ asymptotically behaves as,

$$f(r) = 1 - \frac{2M}{r} + \frac{3Mg^2}{r^3} + O\left(\frac{1}{r^5}\right) \tag{9}$$

By plotting the actual and asymptotical forms of the Bardeen black hole we can see that these two forms have the same behavior and using any of them lead to the same results. Then we use the asymptotical form and we assume that the terms of order $O(\frac{1}{r^5})$ and the higher orders can be neglected. By considering $w_n = \frac{2}{3}$, $r_g = 2M$ and $r_n = -(3Mg^2)^{\frac{1}{3}}$, Eq. (5)

leads to the Bardeen metric. Kiselev (2003) showed that, for the black holes surrounded by the quintessence one can add the quintessence term $\frac{c}{r^{3\omega_q+1}}$ to the metric of the black holes. Then, the metric of the Bardeen black hole surrounded by quintessence can be written as,

$$f(r)_{Bar} = 1 - \frac{2M}{r} + \frac{3Mg^2}{r^3} - \frac{c}{r^{3\omega_q+1}} \tag{10}$$

In this paper, we are going to study the thermodynamics of the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence in detail corresponds to the choice of $\omega_q = -\frac{2}{3}$. Then for these black holes we have,

$$f(r)_{Sch} = 1 - \frac{2M}{r} - cr \tag{11}$$

$$f(r)_{RN} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - cr \tag{12}$$

$$f(r)_{Bar} = 1 - \frac{2M}{r} + \frac{3Mg^2}{r^3} - cr \tag{13}$$

The event horizon of the black hole can be found from the following equation,

$$f(r) = 0 \tag{14}$$

This equation will lead us to two event horizons for the Schwarzschild black hole surrounded by quintessence. The inner and outer horizons of this black hole for $8Mc < 1$ is given by,

$$r_{in\text{-}Sch} = \frac{1 - \sqrt{1 - 8Mc}}{2c} \tag{15}$$

$$r_{out\text{-}Sch} = \frac{1 + \sqrt{1 - 8Mc}}{2c} \tag{16}$$

Since the region between these two horizons is significant and we want to compare our results with those obtained for the Schwarzschild black hole surrounded by quintessence and has two horizons, by using the formulas for cubic and quartic functions and having two real roots for $f(r)_{RN}$ and $f(r)_{Bar}$, we can only choose Q^2 and g^2 as follows,

$$Q^2 = \frac{2}{27}\frac{-1 + 9Mc + \sqrt{-(6Mc-1)^3}}{c^2} \tag{17}$$

$$g^2 = \frac{32Mc(1-2Mc) - 3}{192Mc^3} \tag{18}$$

Then $f(r)$ for the Reissner-Nordström and Bardeen black holes can be expressed as,

$$f(r)_{RN} = \frac{-2 + 18Mc + 2\sqrt{-(6Mc-1)^3}}{27c^2r^2} + 1 - \frac{2M}{r} - cr \tag{19}$$

$$f(r)_{Bar} = \frac{32Mc(1-2Mc) - 3}{64c^3r^3} + 1 - \frac{2M}{r} - cr \tag{20}$$

From Eq. (19), the Reissner-Nordström black hole for $6Mc < 1$ has the following horizons,

$$r_{in\text{-}RN} = \frac{(-(6Mc-1)^3)^{\frac{1}{6}}}{-6c} + \frac{6Mc-1}{6c(-(6Mc-1)^3)^{\frac{1}{6}}} + \frac{1}{3c} + \frac{\sqrt{3}}{6c}I\left(-(6Mc-1)^3\right)^{\frac{1}{6}} + \frac{6Mc-1}{(-(6Mc-1)^3)^{\frac{1}{6}}} \tag{21}$$

$$r_{out\text{-}RN} = \frac{(-(6Mc-1)^3)^{\frac{1}{6}}}{3c} - \frac{6Mc-1}{3c(-(6Mc-1)^3)^{\frac{1}{6}}} + \frac{1}{3c} \tag{22}$$

For $8Mc < 1$, Eq. (20) has two real roots which we choose them as horizons of the Bardeen metric. By considering $a = c$, $b = -1$, $h = 2M$, $d = 0$ and $e = -\frac{32Mc(1-2Mc)-3}{64c^3}$, we can obtain the inner and outer horizons of the Bardeen metric as,

$$r_{in\text{-}Bar} = -\frac{b}{4a} + s - \frac{1}{2}\sqrt{-4s^2 - 2p - \frac{q}{s}} \tag{23}$$

$$r_{out\text{-}Bar} = -\frac{b}{4a} + s + \frac{1}{2}\sqrt{-4s^2 - 2p - \frac{q}{s}} \tag{24}$$

where

$$p = \frac{8ah - 3b^2}{8a^2} \tag{25}$$

$$q = \frac{b^3 - 4abh + 8a^2d}{8a^3} \tag{26}$$

$$s = \frac{1}{2}\sqrt{-\frac{2}{3}p + \frac{1}{3a}\left(k + \frac{\Delta_0}{k}\right)} \tag{27}$$

$$k = \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}} \tag{28}$$

$$\Delta_0 = h^2 - 3bd + 12ae \tag{29}$$

$$\Delta_1 = 2h^3 - 9bhd + 27b^2e + 27ad^2 - 72ahe \tag{30}$$

Figure 1 shows the difference between horizons of the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence matter.

By considering $\omega_q = -\frac{2}{3}$, the density of quintessence as a function of c at the event horizon of the black hole can be expressed as,

$$\rho_q = \frac{c}{r_{out}} \tag{31}$$

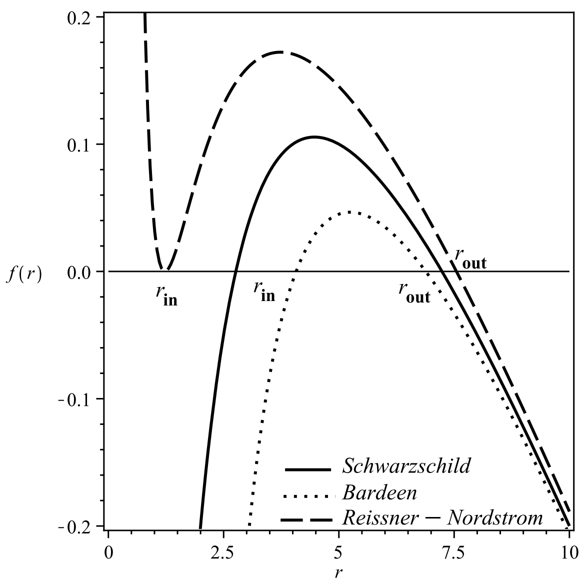

Fig. 1 Horizons of the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence with $M = 1$ and $c = 0.1$

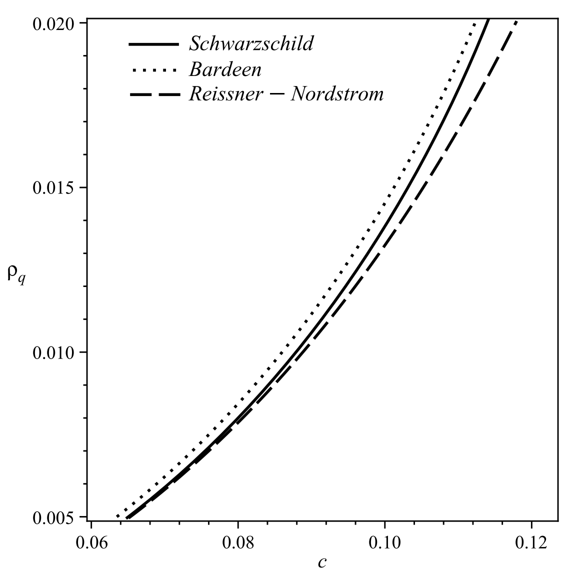

Fig. 2 Density of quintessence ρ_q as a function of c for the black holes surrounded by quintessence with $M = 1$

The density of quintessence as a function of c at the event horizon for the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence matter is shown in Fig. 2. This figure illustrates that the density of quintessence for the Bardeen is higher than for the Schwarzschild and the Reissner-Nordström black holes surrounded by quintessence. From this figure and comparing the density of quintessence for these black holes we find,

$$\rho_{q-Bar} > \rho_{q-Sch} > \rho_{q-RN} \tag{32}$$

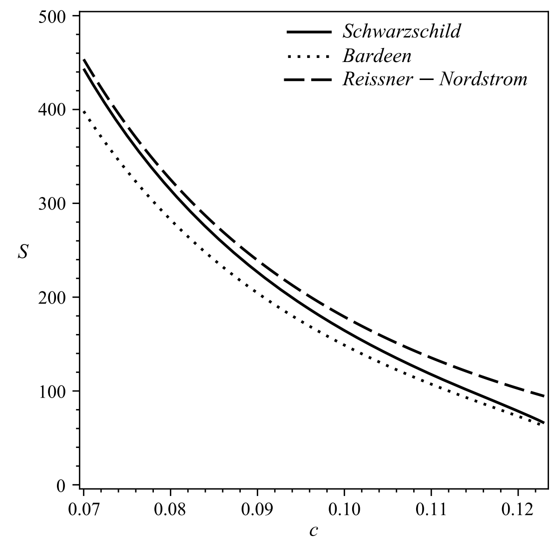

Fig. 3 Entropy as a function of c for the black holes surrounded by quintessence with $M = 1$

3 Thermodynamic quantities of the black holes with quintessence

If the black hole is regarded as a thermal system, it is then natural to apply the laws of thermodynamics; however, a crucial difference from the other thermal systems is that it is a gravitational object whose entropy is identified with the area of the black hole. By using the thermodynamical laws of the black holes, we can derive the thermodynamic properties of the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence. The relation between the Einstein equation and the first law of thermodynamics has been generalized to the cosmological context. In this section we plot the variation of mass, temperature and heat capacity as a functions of entropy related to the quintessence.

The black holes have an entropy equal to one quarter of the area of the event horizon and we know that the entropy can be written as (Hawking 1975; Bekenstein 1973; Bardeen et al. 1973),

$$S = \frac{A}{4} = \frac{4\pi r_{out}^2}{4} = \pi r_{out}^2 \tag{33}$$

The graphs for the entropy as a function of c for the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence matter are given in Fig. 3. The entropy with the same c parameter is more for the Reissner-Nordström in comparison with the Schwarzschild and Bardeen black holes surrounded by quintessence. This result can be expressed as follows,

$$S_{RN} > S_{Sch} > S_{Bar} \tag{34}$$

We can establish the relation between the density of quintessence ρ_q and the entropy of the black hole from (31) and

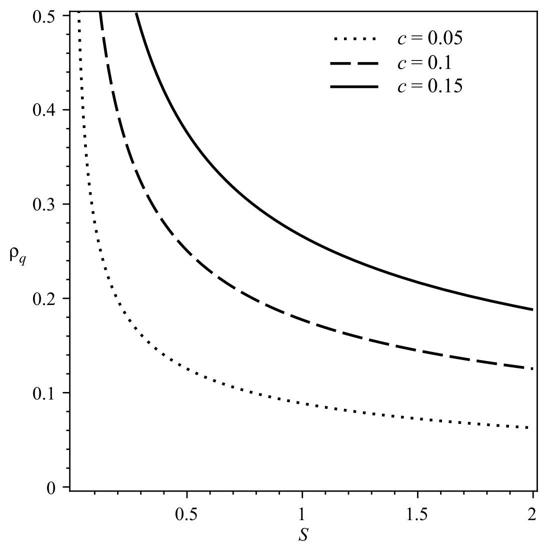

Fig. 4 Behavior of ρ_q as a function of S with the different values of c

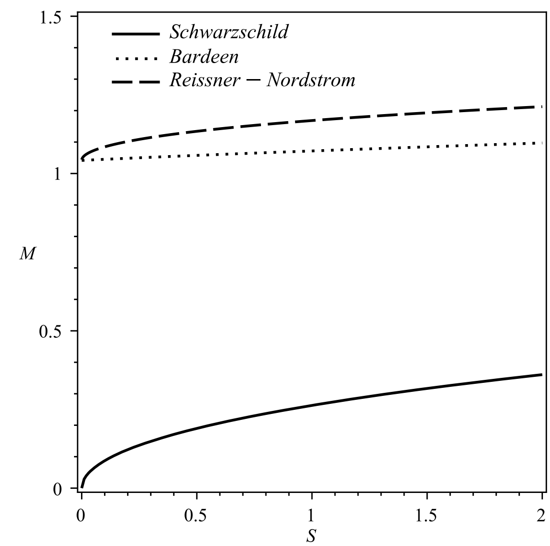

Fig. 5 Variation of M as a function of S for the black holes surrounded by quintessence with $c = 0.12$

(33) as,

$$\rho_q = c\sqrt{\frac{\pi}{S}} \tag{35}$$

Behavior of the density of quintessence ρ_q as a function of entropy with the different values of c is plotted in Fig. 4 which all of them represent decreasing function.

The relation between mass and horizon radius of these black holes with $\omega_q = -\frac{2}{3}$ can be expressed as,

$$M_{Sch} = \frac{1}{2}\left(r_{out} - cr_{out}^2\right) \tag{36}$$

$$M_{RN} = \frac{1}{2}\left(r_{out} + \frac{Q^2}{r_{out}} - cr_{out}^2\right) \tag{37}$$

$$M_{Bar} = \frac{1}{2}\left(r_{out} + \frac{3Mg^2}{r_{out}^2} - cr_{out}^2\right) \tag{38}$$

By using Eqs. (17), (18) and (33), the mass of these black holes as a function of S and c can be written as,

$$M_{Sch} = \frac{1}{2}\left(\sqrt{\frac{S}{\pi}} - c\frac{S}{\pi}\right) \tag{39}$$

$$M_{RN} = RootOf\left(\sqrt{\frac{\pi}{S}}\frac{-1 + 9Xc + \sqrt{-(6Xc-1)^3}}{27c^2} - X + \sqrt{\frac{S}{4\pi}} - c\frac{S}{2\pi} = 0\right) \tag{40}$$

$$M_{Bar} = -\frac{\sqrt{-32\pi Sc^2 + \pi^2 + 64\sqrt{\pi S}Sc^3}}{8\pi c} - \frac{Sc}{\pi} + \frac{1}{4c} \tag{41}$$

In Fig. 5 we have plotted the variation of mass as a function of entropy for the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence. Mass increases as the entropy increases, and it is evident that the horizon area also increases. Since we have an area law of entropy, the increase in area will cause the increase in entropy. From this figure we can see that the mass of the Reissner-Nordström with the same entropy is higher than for the Schwarzschild and Bardeen black holes surrounded by quintessence. In summary, this figure shows that,

$$M_{RN} > M_{Bar} > M_{Sch} \tag{42}$$

Black hole thermodynamic stability can be determined by studying the nature of heat capacity of the system. We can derive the temperature T and the heat capacity C of the black hole from the following equations (Hawking 1975; Bekenstein 1973; Bardeen et al. 1973),

$$T = \frac{\partial M}{\partial S} \tag{43}$$

$$C = T\frac{\partial S}{\partial T} \tag{44}$$

The graphs for the temperature T and heat capacity C as a function of S for the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence matter are given in Figs. 6 and 7. Figure 6 indicates that the temperature with the same S is more for the Schwarzschild in comparison with the Reissner-Nordström and Bardeen black holes surrounded by quintessence. From Fig. 7 we can see that, the heat capacity of the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence is negative and so these black holes are thermodynamically unstable. The heat capacity of the Reissner-Nordström black hole is higher than for the Schwarzschild and Bardeen

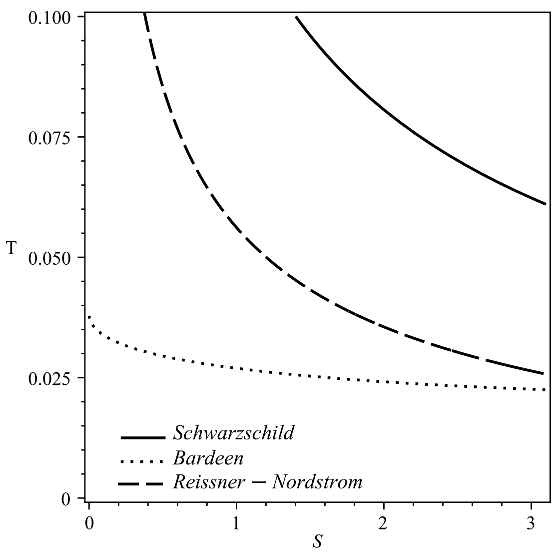

Fig. 6 Temperature T as a function of S for the black holes surrounded by quintessence with $c = 0.12$

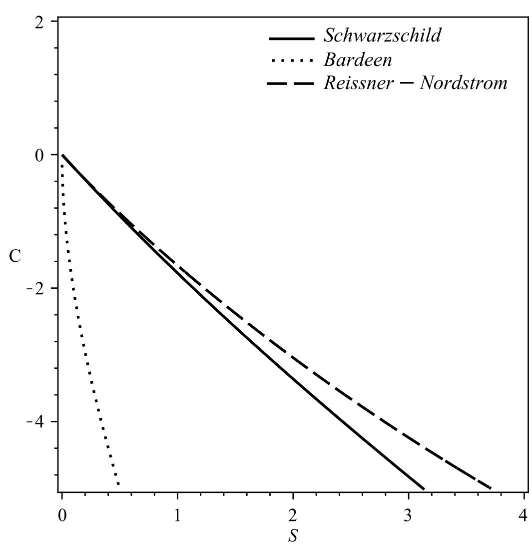

Fig. 7 Heat capacity C as a function of S for the black holes surrounded by quintessence with $c = 0.1$

black holes surrounded by quintessence and it means that the Reissner-Nordström is less unstable in comparison with the Schwarzschild and Bardeen black holes. The results of these figures can be summarized as follows,

$$T_{Sch} > T_{RN} > T_{Bar} \tag{45}$$

$$C_{RN} > C_{Sch} > C_{Bar} \tag{46}$$

The Hawking temperature for these black holes can be also calculated from the following equation,

$$T = \frac{1}{4\pi} \left| \frac{df(r)}{dr} \right|_{r_{out}} \tag{47}$$

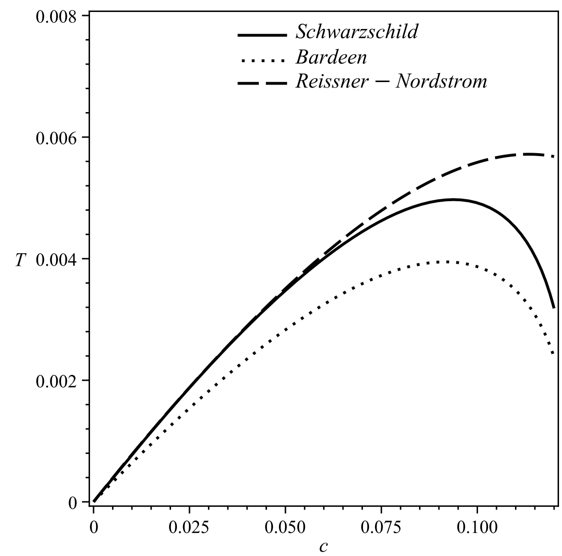

Fig. 8 Hawking temperature as a function of c for the black holes surrounded by quintessence with $M = 1$

The variation of Hawking temperature as a functions of c parameter for the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence matter are shown in Fig. 8. This figure shows that, the temperature with the same c parameter for the Reissner-Nordström black hole is higher than for the Schwarzschild and Bardeen black holes surrounded by quintessence.

The relation between pressure P and c parameter can be written as (Tharanath et al. 2014),

$$P = -\frac{c}{8\pi} \tag{48}$$

In black hole thermodynamics, volume has been considered as a thermodynamic variable. So we find the volume of the black hole thermodynamically and obtain the equation of state. The volume V of the black hole with $\omega_q = -\frac{2}{3}$, is defined as (Tharanath et al. 2014),

$$V = 4\pi r_{out}^2 \tag{49}$$

By using Eqs. (33) and (49), the entropy can be expressed as,

$$S = \frac{V}{4} \tag{50}$$

By considering $c = -8\pi P$, $S = \frac{V}{4}$ and rewriting the equation of temperature T as a function of P and V, we can obtain the equation of state $P - V$ for the Schwarzschild, Reissner-Nordström and Bardeen black holes surrounded by quintessence. Figure 9 shows the $P - V$ isotherms for these black holes. The pressure P with the same volume V is more for the Reissner-Nordström in comparison with the Schwarzschild and Bardeen black holes surrounded by

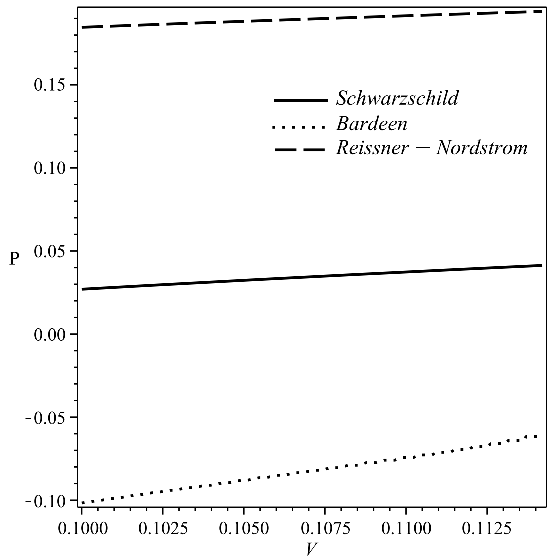

Fig. 9 $P - V$ isotherms for the black holes surrounded by quintessence with $T = 1$

quintessence. From this figure we can see that,

$$P_{RN} > P_{Sch} > P_{Bar} \tag{51}$$

4 Conclusion

We have investigated the effect of quintessence on the thermodynamic properties of the Schwarzschild, Reissner-Nordström and Bardeen black holes. By using the thermodynamical laws of the black holes, we have plotted the variation of mass, temperature and heat capacity as a functions of entropy for these black holes surrounded by quintessence and we compared the results. We showed that the density of quintessence for the Bardeen is higher than for the Schwarzschild and Reissner-Nordström black holes. It is shown that the entropy with the same c parameter is more for the Reissner-Nordström in comparison with the Schwarzschild and Bardeen black holes. We also have shown that the mass and heat capacity of the Reissner-Nordström with the same entropy are higher than for the Schwarzschild and Bardeen, while the temperature with the same S is more for the Schwarzschild in comparison with the Reissner-Nordström and Bardeen black holes. Finally, we have plotted the $P - V$ isotherms for these black holes and we showed that the pressure with the same volume is more for the Reissner-Nordström in comparison with the Schwarzschild and Bardeen black holes. These results can be summarized as follows,

$$\rho_{q-Bar} > \rho_{q-Sch} > \rho_{q-RN} \tag{52}$$

$$S_{RN} > S_{Sch} > S_{Bar} \tag{53}$$

$$M_{RN} > M_{Bar} > M_{Sch} \tag{54}$$

$$T_{Sch} > T_{RN} > T_{Bar} \tag{55}$$

$$C_{RN} > C_{Sch} > C_{Bar} \tag{56}$$

$$P_{RN} > P_{Sch} > P_{Bar} \tag{57}$$

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