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ORIGINAL ARTICLE

Rotating and expanding Bianchi type-IX model in $f(R,T)$ theory of gravity

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Abstract The spatially homogeneous shear-free, rotating and expanding Bianchi type-IX universe has been considered in the presence of perfect fluid in $f(R,T)$ theory of gravity. The exact solution of the field equations has been obtained and the functional form of $f(R,T)=R+2f(T)$ gravity has been reconstructed. The existence of such a solution suggests that the general relativistic shear-free perfect fluid conjecture which claims that a shear-free perfect fluid cannot rotate and expand at the same time, is not valid in this modified theory.

Keywords $f(R,T)$ gravity · Bianchi type-IX model · Shear-free perfect fluid · Rotation

1 Introduction

The cosmological observations suggest that current universe is undergoing an accelerated expansion. The indications of this late time accelerated expansion of the universe is provided by observations from Supernova type-Ia experiments (Perlmutter et al. 1997, 1998, 1999; Riess et al. 1998, 2004), cosmic microwave background (CMB) anisotropies (Bennett et al. 2003; Spergel et al. 2003, 2007), large scale structure (Hawkins et al. 2003; Tegmark et al. 2004; Cole et al. 2005). In view of this, it is now believed that ∼76 % of the cosmic energy density is dark energy (DE). In recent years, as well as introducing exotic energy component in the universe such as quintessence, phantom, tachyon, Chaplygin gas (Padmanabhan 2002, 2008; Bento et al. 2002; Caldwell 2002; Nojiri and Odintsov 2003; Feng et al. 2005), modifying General Relativity (GR) is attracting more attention to explain the late time acceleration and existence of dark energy.

Among the various modifications, one of the most popular modified gravity theory is $f(R)$ theory of gravity (Maartens and Taylor 1994; Rippl et al. 1996; Nojiri and Odintsov 2007; Capozziello and Francaviglia 2008; Sotiriou 2009; Felice and Tsujikawa 2010), which is obtained by modifying Einstein-Hilbert (EH) action by replacing Ricci curvature scalar R with an $f(R)$ function, which is an arbitrary function of R. It was shown that the late time acceleration of the universe can be explained within this modified theory (Carroll et al. 2004).

Recently Harko et al. (2011) have presented a new modified theory of gravity known as $f(R,T)$ gravity. In this theory, EH action is modified by introducing an arbitrary function of the Ricci Scalar R and of the trace of the energy-momentum tensor of T. Several problems have been considered by multiple authors in the $f(R,T)$ theory of gravity (Sharif and Zubair 2012a, 2014a, 2014b; Reddy et al. 2012, 2014; Singh and Sharma 2014; Hossienkhani et al. 2014; Shamir and Raza 2015; Sahoo and Sivakumar 2015).

In this paper, we investigate *shear-free perfect fluid conjecture* in the context of $f(R,T)$ gravity. In contrast to Newtonian theory of gravitation (Narlikar 1963; Senovilla et al. 1998; Heckmann and Schücking 1959), μ and p are being the energy density and the pressure, respectively, this conjecture, which was originated by Treciokas and Ellis (1971), states that a general relativistic shear-free perfect fluid, with $\mu+p\neq 0$, is either non-expanding or non-rotating. There is a wide range of studies that support the consequence using either a particular tetrad or coordinate system or a fully covariant approach in particular cases for instance, dust, spatial homogeneity, incoherent radiation, vorticity and accel-

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eration are parallel, the magnetic part of the Weyl tensor or divergence of electric or magnetic part of Weyl tensor is vanishes, fluid velocity parallel with a conformal Killing vector field, functionally dependent expansion and energy density, Petrov types N and III, ... etc. (Ellis 1967, 2011; King and Ellis 1973; Collins 1984, 1985, 1986, 1988; White and Collins 1984; Carminati 1987, 1988, 1990; Coley 1991; Senovilla et al. 1998; Sopuerta 1998; Van den Bergh 1999; Van den Bergh et al. 2007; Carminati et al. 2009; Herrera et al. 2010, 2014; Slobodeanu 2014), but a general proof or a counter-example has not appeared up till now.

On the other hand, in the context of modified $f(R)$ gravity theory, as far as we know, there are two studies that have dealt with this conjecture. In the first of them, Abebe et al. (2011) have shown that in R^3 gravity, there is at least one physically realistic non-vacuum case (stiff fluid) which is a flat Milne-universe solution can have rotation and expansion simultaneously at the level of linearized perturbation about a Friedmann-Lemaître-Robertson-Walker (FLRW) background. In the second study, Sofuoğlu and Mutuş (2014) have shown that there exist two types of $f(R)$ models in which shear-free rotating Bianchi type-IX universe filled with perfect fluid exhibits always coasting anisotropic expansion like a flat Milne universe. These solutions, respectively, are the first and the second counter-examples that violated the general relativistic shear-free perfect fluid conjecture in $f(R)$ gravity.

Inspired by these situations, in the present paper, to investigate existence of simultaneously rotating and expanding solutions, we have considered a spatially homogeneous rotating Bianchi type-IX metric in the context of $f(R,T)$ modified gravity by following the same method used by Sofuoğlu and Mutuş (2014).

The outline of this paper is as follows: In Sect. 2, we present a brief description of $f(R,T)$ gravity. In Sect. 3, we derive $f(R,T)$ gravity tetrad equations for rotating Bianchi type-IX metric and the solutions of the tetrad equations for the model are obtained. In Sect. 4 conclusions are summarized.

We use the natural units system with $c = 8\pi G = 1$. Latin indices $a, b, c, \dots$ run from 0 to 3 while Greek indices $\mu, \nu, \rho, \dots$ from 1 to 3. (ab) and $\langle ab\rangle$ denotes symmetrization and orthogonal, symmetric, trace-free parts over the indices a and b, respectively.

2 Field equations of $f(R,T)$ gravity

The action of $f(R,T)$ modified theory of gravity is given by Harko et al. (2011)

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\big[f(R,T) + 2L_m\big], \tag{1}$$

where L_m is the matter Lagrangian density. The stress-energy tensor of matter is given by

$$\begin{aligned} T^m_{ab} &= \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ab}} = \mu^m u_a u_b + p^m h_{ab} \\ &\quad + q^m_a u_b + q^m_b u_a + \pi^m_{ab}, \end{aligned} \tag{2}$$

where μ^m, p^m, q^m_a and π^m_{ab} are the energy density, isotropic pressure, heat flux and anisotropic pressure of standard matter, respectively, u_a is the four velocity of observers comoving with the fluid, with $u_a u^a = -1$ and $u^b \nabla_a u_b = 0$, and $h_{ab} \equiv g_{ab} + u_a u_b$ is the standard projection tensor on the rest three-space of the observers.

Variation of the action (1) with respect to the metric tensor g_{ab} leads to the following field equations of $f(R,T)$ gravity

$$\begin{aligned} & f_R(R,T)R_{ab} - \frac{1}{2}f(R,T)g_{ab} + (g_{ab}\Box - \nabla_a\nabla_b)f_R(R,T) \\ &\quad = T^m_{ab} - f_T(R,T)\big(T^m_{ab} + \Theta_{ab}\big), \end{aligned} \tag{3}$$

where $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$, $\Box = g^{ab}\nabla_a\nabla_b$, ∇_a is the covariant derivative, and $\Theta_{ab} = -2T^m_{ab} + g_{ab}L_m - 2g^{mn}\frac{\partial^2 L_m}{\partial g^{mn}\partial g^{ab}}$. In the case of perfect fluid, i.e. $q^m_a = 0 = \pi^m_{ab}$, the stress-energy tensor (2) takes the form

$$T^m_{ab} = \big(\mu^m + p^m\big)u_a u_b + p^m g_{ab}, \tag{4}$$

and the matter Lagrangian can be taken as $L_m = -p^m$. Then Θ_{ab} has the explicit form

$$\Theta_{ab} = -2T^m_{ab} - g_{ab}p^m. \tag{5}$$

Substituting Eq. (5) for Θ_{ab} into Eq. (3), we obtain

$$\begin{aligned} & f_R(R,T)R_{ab} - \frac{1}{2}f(R,T)g_{ab} + (g_{ab}\Box - \nabla_a\nabla_b)f_R(R,T) \\ &\quad = T^m_{ab} + f_T(R,T)\big(T^m_{ab} + g_{ab}p^m\big). \end{aligned} \tag{6}$$

One can write this equation in the form of standard Einstein field equations such as

$$\begin{aligned} G_{ab} &\equiv R_{ab} - \frac{1}{2}Rg_{ab} \\ &= \frac{T^m_{ab}}{f_R(R,T)} + \frac{f_T(R,T)}{f_R(R,T)}\big(T^m_{ab} + p^m g_{ab}\big) \\ &\quad + \frac{1}{f_R(R,T)}\bigg\{\frac{1}{2}\big[f(R,T) - Rf_R(R,T)\big]g_{ab} \\ &\quad - (g_{ab}\Box - \nabla_a\nabla_b)f_R(R,T)\bigg\}, \end{aligned} \tag{7}$$

where G_{ab} is Einstein tensor. If the right hand side of (7) is considered as effective total energy-momentum tensor T^t_{ab}, then it can be written as $G_{ab} = T^t_{ab}$.

It may be noted that when $f(R,T)\equiv f(R)$, Eqs. (7) yield the field equations of $f(R)$ gravity. We mention here that three explicit specification of the functional form of $f(R,T)$ function has been considered in Harko et al. (2011). In this paper we consider the function $f(R,T)$ of the form

$$f(R,T)=R+2f(T). \tag{8}$$

Then the field Eqs. (7) become

$$G_{ab}=T^m_{ab}+2f_T(T)T^m_{ab}+\left[2p^m f_T(T)+f(T)\right]g_{ab}, \tag{9}$$

where

$$T^t_{ab}\equiv T^m_{ab}+2f_T(T)T^m_{ab}+\left[2p^m f_T(T)+f(T)\right]g_{ab}, \tag{10}$$

is being the effective total energy-momentum tensor which can be split, similarly (2), as

$$T^t_{ab}=\mu^t u_a u_b+p^t h_{ab}+q^t_a u_b+q^t_b u_a+\pi^t_{ab}. \tag{11}$$

Here μ^t, p^t, q^t_a and π^t_{ab} are total effective dynamic quantities, namely, effective total energy density, pressure, heat flux, and anisotropic pressure, respectively. On the other hand, dynamic quantities defined by corresponding energy-momentum tensor, as

$$\begin{aligned}&\mu=u^a u^b T_{ab}, \qquad p=\frac{1}{3}h^{ab}T_{ab},\\ &q_a=-h^b{}_a u^c T_{bc}, \qquad \pi_{ab}=h^c{}_{\langle a}h^d{}_{b\rangle}T_{cd}.\end{aligned} \tag{12}$$

Now, using (12) and (4), Eq. (11) gives us the following effective total dynamic quantities of $R+2f(T)$ gravity:

$$\mu^t=\mu^m+2f_T(T)\left(\mu^m-p^m\right)-f(T), \tag{13a}$$

$$p^t=p^m+4f_T(T)p^m+f(T), \tag{13b}$$

$$q^t_a=0, \tag{13c}$$

$$\pi^t_{ab}=0. \tag{13d}$$

On the other hand, in this paper we will use non-linear tetrad evolution and constraint equations instead of components of field equations. In view of this, we shall assume familiarity with tetrad evolution and constraint equation of GR as given by Ellis and van Elst (1999). The tetrad evolution and constraint equations of $f(R,T)$ gravity can be obtained by writing the effective total dynamical quantities μ^t, p^t, q^t_α and $\pi^t_{\alpha\beta}$ in place of μ, p, q_α and $\pi_{\alpha\beta}$ in tetrad equations of GR. We list the evolution and the constraint equations of $f(R,T)$ gravity in Appendix A (see (38)–(54)).

3 Metric and solutions

We consider rotating spatially homogeneous Bianchi type-IX universe, in a one-forms basis ω^a, given by

$$ds^2=-\left(\omega^0-a(t)v_\alpha\omega^\alpha\right)^2+a^2(t)k^2_\alpha\left(\omega^\alpha\right)^2, \tag{14}$$

with the following explicit realization (MacCallum 1979)

$$\begin{aligned}&\omega^0=dt,\\ &\omega^1=\cos y\cos z dx-\sin z dy,\\ &\omega^2=\cos y\sin z dx+\cos z dy,\\ &\omega^3=-\sin y dx+dz,\end{aligned} \tag{15}$$

where the scale factor a is only function of cosmic time t and k_α's are positive constant parameters. Now, we can write the ansatz for the line element (14)

$$ds^2=\eta_{ab}\sigma^a\sigma^b, \qquad \eta_{ab}=diag(-1,1,1,1), \tag{16}$$

choosing an orthonormal comoving ($e_0=\mathbf{u}$) tetrad frame with the following σ^a one-forms

$$\begin{aligned}&\sigma^0=\omega^0-a(t)v_1\omega^1-a(t)v_2\omega^2-a(t)v_3\omega^3,\\ &\sigma^1=a(t)k_1\omega^1, \qquad \sigma^2=a(t)k_2\omega^2,\\ &\sigma^3=a(t)k_3\omega^3,\end{aligned} \tag{17}$$

and the e_a basis vectors ($\sigma^a(e_b)=\delta^a_b$)

$$\begin{aligned}&e_0=\partial_t,\\ &e_1=\frac{v_1}{k_1}\partial_t+\frac{1}{k_1 a(t)}\left(\frac{\cos z}{\cos y}\partial_x-\sin z\partial_y\right.\\ &\qquad\left.+\frac{\cos z\sin y}{\cos y}\partial_z\right),\\ &e_2=\frac{v_2}{k_2}\partial_t+\frac{1}{k_2 a(t)}\left(\frac{\sin z}{\cos y}\partial_x+\cos z\partial_y\right.\\ &\qquad\left.+\frac{\sin y\sin z}{\cos y}\partial_z\right),\\ &e_3=\frac{v_3}{k_3}\partial_t+\frac{1}{k_3 a(t)}\partial_z.\end{aligned} \tag{18}$$

For such a frame, commutators of the basis vectors e_a are given by Ellis and van Elst (1999) in terms of the kinematic quantities of the fluid as measured with respect to e_0:

$$[e_0,e_\beta]=\dot{u}_\beta e_0+\left[\varepsilon^\gamma_{\beta\delta}\left(\Omega^\delta+\omega^\delta\right)-\sigma^\gamma_\beta-\frac{1}{3}\delta^\gamma_\beta\theta\right]e_\gamma, \tag{19a}$$

$$\begin{aligned}[e_\alpha,e_\beta]=&2\varepsilon_{\alpha\beta\gamma}\omega^\gamma e_0\\ &+\left(\varepsilon_{\alpha\beta\delta}n^{\gamma\delta}+a_\alpha\delta^\gamma_\beta-a_\beta\delta^\gamma_\alpha\right)e_\gamma,\end{aligned} \tag{19b}$$

where the kinematic quantities $\dot{u}_a$, θ, σ_{ab} ($\sigma^2=(1/2)\sigma_{ab}\times\sigma^{ab}$) and ω^a ($\omega^2=\omega_a\omega^a$) are acceleration, expansion, shear and vorticity, respectively, and $n_{\alpha\beta}$ and a_α are the commutation variables and Ω_α is the local angular velocity of the spatial triad $\{e_\alpha\}$.

Using the basis vectors (18), the commutation relations (19a) and (19b) give us the following kinematics for the Rotating Bianchi type-IX model:

$$\begin{aligned}
&\sigma_{\alpha\beta}=0, \qquad \theta=3\frac{\dot a}{a},\\
&\dot u_1=\frac{v_1}{k_1}\frac{\dot a}{a}=-a_1, \qquad \dot u_2=\frac{v_2}{k_2}\frac{\dot a}{a}=-a_2,\\
&\dot u_3=\frac{v_3}{k_3}\frac{\dot a}{a}=-a_3,\\
&\omega_1=\frac{v_1}{2k_2k_3}\frac{1}{a}=-\Omega_1, \qquad \omega_2=\frac{v_2}{2k_3k_1}\frac{1}{a}=-\Omega_2,\\
&\omega_3=\frac{v_3}{2k_1k_2}\frac{1}{a}=-\Omega_3,\\
&n_{11}=-\frac{k_1}{k_2k_3}\frac{1}{a}, \qquad n_{22}=-\frac{k_2}{k_3k_1}\frac{1}{a},\\
&n_{33}=-\frac{k_3}{k_1k_2}\frac{1}{a}, \qquad \text{others}=0.
\end{aligned} \tag{20}$$

Here and hereafter the dot $(\dot{})$ denotes derivative with respect to the cosmic time t.

As a first step we specialize to the case

$$v_1\neq 0, \quad v_2=0, \quad v_3=0. \tag{21}$$

Following the same procedure in Sofuoğlu and Mutuş (2014), using the basis vectors e_a given by (18) and the kinematic quantities (20) in the full set of the constraint equations (47)–(54) and the evolution Eq. (40) which converted into a constraint in the case of vanishing shear, we obtain the total effective dynamic quantities of the model. We list nontrivial of them for the case (21) in Appendix B (see (55)–(61)).

Our solution seeking for rotating Bianchi type-IX model in $f(R,T)$ gravity, is based on comparing the total effective dynamic quantities, which is given by the Eqs. (13a)–(13d) and (55)–(61). Then, comparing Eqs. (13c) with (57) and (13d) with (58)–(61), we have the following system of equations:

$$\frac{\ddot a}{a}-\frac{\dot a^2}{a^2}=\frac{k_1^2}{4k_2^2k_3^2}\frac{1}{a^2} \tag{22}$$

$$-4\frac{v_1^2}{k_1^2}\left(\frac{\ddot a}{a}-\frac{\dot a^2}{a^2}\right)+\left(\frac{2k_1^2-v_1^2}{k_2^2k_3^2}-\frac{k_2^2}{k_3^2k_1^2}-\frac{k_3^2}{k_1^2k_2^2}+\frac{2}{k_1^2}-\frac{1}{k_2^2}-\frac{1}{k_3^2}\right)\frac{1}{a^2}=0 \tag{23}$$

$$2\frac{v_1^2}{k_1^2}\left(\frac{\ddot a}{a}-\frac{\dot a^2}{a^2}\right)+\left(\frac{2k_2^2}{k_3^2k_1^2}-\frac{k_3^2}{k_1^2k_2^2}-\frac{2k_1^2-v_1^2}{2k_2^2k_3^2}+\frac{2}{k_2^2}-\frac{1}{k_3^2}-\frac{1}{k_1^2}\right)\frac{1}{a^2}=0 \tag{24}$$

$$2\frac{v_1^2}{k_1^2}\left(\frac{\ddot a}{a}-\frac{\dot a^2}{a^2}\right)+\left(\frac{2k_2^2}{k_3^2k_1^2}-\frac{k_3^2}{k_1^2k_2^2}-\frac{2k_1^2-v_1^2}{2k_2^2k_3^2}+\frac{2}{k_2^2}-\frac{1}{k_3^2}-\frac{1}{k_1^2}\right)\frac{1}{a^2}=0 \tag{25}$$

$$\frac{v_1(k_2^2-k_3^2)}{k_2k_3k_1^2}\frac{\dot a}{a^2}=0. \tag{26}$$

As it is seen immediately, the analysis of the Eqs. (22)–(26), shows that they are consistent under the conditions

$$k_2=k_3, \qquad v_1^2=k_1^2-k_2^2. \tag{27}$$

On the other hand, the second order differential equation (22) can be straightforwardly integrated as

$$a(t)=\frac{k_1}{2k_2^2c_1}\cosh(c_1t+c_2), \tag{28}$$

where c_1 and c_2 are constants of integration. It is worthwhile to note that this is the same solution found by Sofuoğlu and Mutuş (2014) and Obukhov et al. (2002) for GR in the case violated the shear-free conjecture condition $\mu^m+p^m\neq 0$.

Now, let us turn our analysis. Comparison the combinations of Eqs. (55) and (56) with of (13a) and (13b) by using (22) for $\frac{\ddot a}{a}-\frac{\dot a^2}{a^2}$, yields

$$0=\left[1+2f_T(T)\right]\left(\mu^m+p^m\right). \tag{29}$$

Since we have assumed that $\mu^m+p^m\neq 0$, we have from (29)

$$1+2f_T(T)=0, \tag{30}$$

which integrates to give

$$f(T)=-\frac{1}{2}T+c, \tag{31}$$

where c is an integration constant which can choose -2Λ, Λ being the cosmological constant, then (31) read

$$f(T)=-\frac{1}{2}T-2\Lambda. \tag{32}$$

Substituting the $f(T)$ function (31) into Eq. (8), with $c=-2\Lambda$, we have

$$f(R,T)=R-T-2\Lambda. \tag{33}$$

By this way, we have reconstructed simultaneously rotating and expanding $f(R,T)$ gravity model.

Up until now, we have not used the conservation equations. Inserting the expressions (55)–(61) into the required places of the total effective matter and momentum conservation equations given (43) and (44), respectively, coming from

$$\nabla^a G_{ab}=0 \quad\Rightarrow\quad \nabla^a T^t_{ab}=0, \tag{34}$$

we see that these equations are identically satisfied as other evolution equations (38), (39), (41)–(46). On the other hand, to be informed of μ^m and p^m, keeping in mind that, as it can be seen from (4), trace of the energy-momentum tensor is

$$T = -\mu^m + 3p^m, \tag{35}$$

which, using with (31) and (34), one can take the covariant derivative of (10) to give the following barotropic equation of state (EoS) $p^m = p^m(\mu^m)$:

$$p^m = \frac{1}{5}\mu^m + c_3, \tag{36}$$

where c_3 is an integration constant. Then using the consistency conditions (27), the $f(T)$ function (32), the solution (28) and the EoS (36), from comparison the expressions (13a) (or (13b)) with (55) (or (56)) we get

$$\Lambda = \frac{3}{2}c_1^2\frac{k_2^2}{k_1^2} - \frac{1}{4}c_3. \tag{37}$$

As it is easily seen from (36) and (37), for any standard matter energy density, provided that the natural assumption $\mu^m > 0$, there are cases that have positive pressure with positive or negative Λ. For instance, if $0 < c_3 < 6c_1^2k_2^2/k_1^2$, then we have $p^m > 0$ with $\Lambda > 0$; if $6c_1^2k_2^2/k_1^2 < c_3$, then we have again $p^m > 0$ but this time $\Lambda < 0$. Particularly, if we choose $c_3 = 0$ in (36), then we get the linear barotropic EoS $p^m = w\mu^m$ with $w = 1/5$ which corresponds to physical case $p^m > 0$ again.

4 Conclusions

In this paper, following the works of Sofuoğlu and Mutuş (2014), we have considered rotating Bianchi type-IX universe with a perfect fluid source in the framework of $f(R,T)$ theory of gravity to investigate the existence of shear-free, rotating and expanding perfect fluid solutions of the field equations of this modified theory.

By using tetrad equations, we obtain an exact solution of the field equations and we have reconstructed an $f(R,T)$ model for $f(R,T) = R + 2f(T)$ gravity such as $f(R,T) = R - T - 2\Lambda$. This shear-free Bianchi type-IX solution is the first one which can rotate and expand at the same time in $f(R,T)$ gravity for any matter content provided that $\mu^m > 0$ and $\mu^m + p^m \neq 0$. This suggests that the shear-free perfect fluid conjecture of GR do not have a counterpart in this theory, as in Newtonian and $f(R)$ theories of gravity.

It would be discussed here about energy conditions of this model. The energy conditions of $f(R,T)$ gravity are given by Sharif and Zubair (2012b) as, null energy condition (NEC): $\mu^t + p^t \geq 0$, weak energy condition (WEC): $\mu^t \geq 0$, $\mu^t + p^t \geq 0$, dominant energy condition (DEC): $\mu^t - p^t \geq 0$, $\mu^t + p^t \geq 0$, $\mu^t \geq 0$ and strong energy condition (SEC): $\mu^t + 3p^t \geq 0$. For the shear-free, rotating and expanding Bianchi type-IX model, using (13a), (13b), (32), (36) and (37) we get $\mu^t = 2c_3 + 3c_1^2k_2^2/k_1^2$ and $p^t = -2c_3 - 3c_1^2k_2^2/k_1^2$. It is straightforwardly seen that NEC is always trivially satisfied. WEC and DEC are satisfied except the interval $-3c_1^2k_2^2/2k_1^2 < c_3 < 0$. SEC is satisfied only if $-3c_1^2k_2^2/2k_1^2 \leq c_3 < 0$. The SEC implies that the expansion of the universe is decelerating conversely recent observational data indicating the accelerating universe. Then the SEC is violated on present cosmological scales (Visser 1997; Visser and Barcelo 2000). Thus, we can conclude that our model is substantially compatible with recent cosmological observations.

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Appendix A: Evolution and constraint equations of $f(R,T)$ gravity

A.1 Evolution equations

$$e_0\theta - e_\alpha\dot{u}^\alpha = -\frac{1}{3}\theta^2 + (\dot{u}_\alpha - 2a_\alpha)\dot{u}^\alpha - 2\sigma^2 + 2\omega^2 - \frac{1}{2}(\mu^t + 3p^t), \tag{38}$$

$$e_0\omega^\alpha - \frac{1}{2}\varepsilon^{\alpha\beta\gamma}e_\beta\dot{u}_\gamma = \varepsilon^{\alpha\beta\gamma}\left(\Omega_\beta\omega_\gamma - \frac{1}{2}a_\beta\dot{u}_\gamma\right) - \frac{1}{2}n^\alpha_\beta\dot{u}^\beta - \frac{2}{3}\theta\omega^\alpha + \sigma^\alpha_\beta\omega^\beta, \tag{39}$$

$$\begin{aligned} e_0\sigma_{\alpha\beta} - e_{\langle\alpha}\dot{u}_{\beta\rangle} &= \varepsilon_{\gamma\delta\langle\alpha}\left(2\Omega^\gamma\sigma_{\beta\rangle}{}^\delta - n_{\beta\rangle}{}^\gamma\dot{u}^\delta\right) + a_{\langle\alpha}\dot{u}_{\beta\rangle} + \dot{u}_{\langle\alpha}\dot{u}_{\beta\rangle} \\ &\quad - \frac{2}{3}\theta\sigma_{\alpha\beta} - \sigma_{\gamma\langle\alpha}\sigma_{\beta\rangle}{}^\gamma - \omega_{\langle\alpha}\omega_{\beta\rangle} \\ &\quad - \left(E_{\alpha\beta} - \frac{1}{2}\pi^t_{\alpha\beta}\right), \end{aligned} \tag{40}$$

$$\begin{aligned} e_0\left(E_{\alpha\beta} + \frac{1}{2}\pi^t_{\alpha\beta}\right) - \varepsilon^{\gamma\delta}{}_{\langle\alpha}e_{|\gamma|}H_{\beta\rangle\delta} + \frac{1}{2}e_{\langle\alpha}q^t_{\beta\rangle} &= -3n_{\gamma\langle\alpha}H_{\beta\rangle}{}^\gamma + \frac{1}{2}n^\gamma{}_\gamma H_{\alpha\beta} - \frac{1}{2}(a_{\langle\alpha} + 2\dot{u}_{\langle\alpha})q^t_{\beta\rangle} \\ &\quad - \frac{1}{2}(\mu^t + p^t)\sigma_{\alpha\beta} - \theta\left(E_{\alpha\beta} + \frac{1}{6}\pi^t_{\alpha\beta}\right) \\ &\quad + 3\sigma^\gamma{}_{\langle\alpha}\left(E_{\beta\rangle\gamma} - \frac{1}{6}\pi^t_{\beta\rangle\gamma}\right) \end{aligned}$$

$$+\varepsilon^{\gamma\delta}{}_{\langle\alpha}\bigg[(2\Omega_\gamma+\omega_\gamma)\bigg(E_{\beta\rangle\delta}+\frac{1}{2}\pi^t_{\beta\rangle\delta}\bigg)$$

$$+\frac{1}{2}n_{\beta\rangle\gamma}q^t_\delta+(2\dot{u}_\gamma-a_\gamma)H_{\beta\rangle\delta}\bigg], \tag{41}$$

$$e_0H_{\alpha\beta}+\varepsilon^{\gamma\delta}{}_{\langle\alpha}e_{|\gamma|}\bigg(E_{\beta\rangle\delta}-\frac{1}{2}\pi^t_{\beta\rangle\delta}\bigg)$$

$$=3n^\gamma{}_{\langle\alpha}\bigg(E_{\beta\rangle\gamma}-\frac{1}{2}\pi^t_{\beta\rangle\gamma}\bigg)-\frac{1}{2}n^\gamma{}_\gamma\bigg(E_{\alpha\beta}-\frac{1}{2}\pi^t_{\alpha\beta}\bigg)$$

$$-\theta H_{\alpha\beta}+3\sigma^\gamma{}_{\langle\alpha}H_{\beta\rangle\gamma}+\frac{3}{2}\omega_{\langle\alpha}q^t_{\beta\rangle}$$

$$+\varepsilon^{\gamma\delta}{}_{\langle\alpha}\bigg[(2\Omega_\gamma+\omega_\gamma)H_{\beta\rangle\delta}+a_\gamma\bigg(E_{\beta\rangle\delta}-\frac{1}{2}\pi^t_{\beta\rangle\delta}\bigg)$$

$$+\frac{1}{2}\sigma_{\beta\rangle\gamma}q^t_\delta-2\dot{u}_\gamma E_{\beta\rangle\delta}\bigg], \tag{42}$$

$$e_0\mu^t+\delta^{\alpha\beta}e_\beta q^t_\alpha+\big(\mu^t+p^t\big)\theta+\sigma^{\alpha\beta}\pi^t_{\alpha\beta}$$

$$+2(\dot{u}_\alpha-a_\alpha)q^t_\alpha=0, \tag{43}$$

$$e_0q^t_\alpha+e_\alpha p^t+\delta^{\beta\gamma}e_\gamma\pi^t_{\alpha\beta}+\big(\mu^t+p^t\big)\dot{u}_\alpha+\frac{4}{3}\theta q^t_\alpha$$

$$+\sigma^{\alpha\beta}q^t_\beta+\big(\dot{u}^\beta-3a^\beta\big)\pi^t_{\alpha\beta}$$

$$-\varepsilon_\alpha{}^{\beta\gamma}\big[(\Omega_\beta-\omega_\beta)q^t_\gamma+n^\delta_\beta\pi^t_{\delta\gamma}\big]=0, \tag{44}$$

$$e_0a_\alpha+\frac{1}{2}\varepsilon_{\alpha\beta}{}^\gamma(e_\gamma+\dot{u}_\gamma-2a_\gamma)\Omega^\beta+\frac{1}{2}\varepsilon_{\alpha\beta\gamma}\big(\dot{u}^\beta+a^\beta\big)\omega^\gamma$$

$$-\frac{1}{2}\sigma_\alpha{}^\beta(\dot{u}_\beta+a_\beta)+\frac{1}{3}(\dot{u}_\alpha+a_\alpha)\theta-\frac{1}{2}n_{\alpha\beta}\omega^\beta$$

$$+\frac{1}{2}\varepsilon_{\alpha\beta\lambda}\sigma^\beta{}_\gamma n^{\gamma\lambda}=0, \tag{45}$$

$$e_0n^{\alpha\beta}+\frac{1}{3}n^{\alpha\beta}\theta+\delta^{\alpha\beta}\big[(e_\gamma+\dot{u}_\gamma)\big(\Omega^\gamma+\omega^\gamma\big)\big]$$

$$-\delta^{\gamma(\beta}\big[(e_\gamma+\dot{u}_\gamma)\big(\Omega^{\alpha)}+\omega^{\alpha)}\big)\big]+\varepsilon^{\gamma\delta(\alpha}(e_\gamma+\dot{u}_\gamma)\sigma^{\beta)}{}_\delta$$

$$-2\sigma^{(\alpha}{}_\gamma n^{\beta)\gamma}+2\varepsilon^{\gamma\lambda(\alpha}n^{\beta)}{}_\gamma(\Omega_\lambda+\omega_\lambda)=0. \tag{46}$$

A.2 Constraint equations

$$e_\beta\sigma_\alpha{}^\beta-\frac{2}{3}e_\alpha\theta+\varepsilon_\alpha{}^{\beta\gamma}e_\beta\omega_\gamma-3a_\beta\sigma_\alpha{}^\beta-n_{\alpha\beta}\omega^\beta$$

$$-\varepsilon_\alpha{}^{\beta\gamma}\big[n_{\beta\delta}\sigma^\delta{}_\gamma+(a_\beta-2\dot{u}_\beta)\omega_\gamma\big]+q^t_\alpha=0, \tag{47}$$

$$e_\alpha\omega^\alpha-(2a_\alpha+\dot{u}_\alpha)\omega^\alpha=0, \tag{48}$$

$$H_{\alpha\beta}+e_{\langle\alpha}\omega_{\beta\rangle}-\varepsilon^{\gamma\delta}{}_{\langle\alpha}e_{|\gamma|}\sigma_{\beta\rangle\delta}+(2\dot{u}_{\langle\alpha}+a_{\langle\alpha})\omega_{\beta\rangle}$$

$$+3n_{\gamma\langle\alpha}\sigma_{\beta\rangle}{}^\gamma-\frac{1}{2}n^\gamma{}_\gamma\sigma_{\alpha\beta}$$

$$-\varepsilon^{\gamma\delta}{}_{\langle\alpha}(n_{\beta\rangle\gamma}\omega_\delta-a_\gamma\sigma_{\beta\rangle\delta})=0, \tag{49}$$

$$\delta^{\beta\gamma}e_\gamma\bigg(E_{\alpha\beta}+\frac{1}{2}\pi^t_{\alpha\beta}\bigg)-\frac{1}{3}e_\alpha\mu^t-3a^\beta\bigg(E_{\alpha\beta}+\frac{1}{2}\pi^t_{\alpha\beta}\bigg)$$

$$+\frac{1}{3}\theta q^t_\alpha-\frac{1}{2}\sigma^\beta_\alpha q^t_\beta-3\omega^\beta H_{\alpha\beta}-\varepsilon_\alpha{}^{\beta\gamma}\bigg[\sigma^\delta_\beta H_{\delta\gamma}-\frac{3}{2}\omega_\beta q^t_\gamma$$

$$+n_\beta{}^\delta\bigg(E_{\delta\gamma}+\frac{1}{2}\pi^t_{\delta\gamma}\bigg)\bigg]=0, \tag{50}$$

$$\delta^{\beta\gamma}e_\gamma H_{\alpha\beta}+\frac{1}{2}\varepsilon_\alpha{}^{\beta\gamma}e_\beta q^t_\gamma-3a_\beta H_\alpha{}^\beta+\big(\mu^t+p^t\big)\omega_\alpha$$

$$-\frac{1}{2}n_\alpha{}^\beta q^t_\beta+3\omega^\beta\bigg(E_{\alpha\beta}-\frac{1}{6}\pi^t_{\alpha\beta}\bigg)$$

$$+\varepsilon_\alpha{}^{\beta\gamma}\bigg[\sigma_\beta{}^\delta\bigg(E_{\delta\gamma}+\frac{1}{2}\pi^t_{\delta\gamma}\bigg)-\frac{1}{2}a_\beta q^t_\gamma-n_\beta{}^\delta H_{\delta\gamma}\bigg]=0, \tag{51}$$

$$e_\beta n^{\alpha\beta}+\varepsilon^{\alpha\beta\gamma}e_\beta a_\gamma-2\varepsilon^\alpha{}_{\beta\gamma}\omega^\beta\Omega^\gamma+2\sigma^\alpha{}_\beta\omega^\beta$$

$$+\frac{2}{3}\theta\omega^\alpha-2n^{\alpha\beta}a_\beta=0, \tag{52}$$

$$e_{\langle\alpha}a_{\beta\rangle}+b_{\langle\alpha\beta\rangle}+(e_\gamma-2a_\gamma)n^\lambda{}_{\langle\alpha}\varepsilon^\gamma{}_{\beta\rangle\lambda}$$

$$+\frac{1}{3}\theta\sigma_{\alpha\beta}-\sigma^\gamma{}_{\langle\alpha}\sigma_{\beta\rangle\gamma}+2\omega_{\langle\alpha}\Omega_{\beta\rangle}-\omega_{\langle\alpha}\omega_{\beta\rangle}$$

$$-\bigg(E_{\alpha\beta}+\frac{1}{2}\pi^t_{\alpha\beta}\bigg)=0, \tag{53}$$

$$4e_\alpha a^\alpha-6a_\alpha a^\alpha-n^{\alpha\gamma}n_{\alpha\gamma}+\frac{1}{2}n^\gamma{}_\gamma n^\alpha{}_\alpha-4\omega^\lambda\Omega_\lambda-2\mu^t$$

$$+\frac{2}{3}\theta^2+2\sigma^2+2\omega^2=0, \tag{54}$$

where E_{ab} and H_{ab} are the electric ($E_{ab}=C_{aebd}u^eu^d$) and the magnetic ($H_{ab}=(1/2)\varepsilon_a{}^{cd}C_{cdbe}u^e$) parts of the conformal Weyl curvature tensor C_{abcd}.

Appendix B: Dynamic quantities of the rotating Bianchi type-IX model

$$\mu^t=-\frac{v_1^2}{k_1^2}\bigg(2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}\bigg)+3\frac{\dot{a}^2}{a^2}-\frac{1}{4}\bigg[\frac{k_1^2-3v_1^2}{k_2^2k_3^2}+\frac{k_2^2}{k_1^2k_3^2}$$

$$+\frac{k_3^2}{k_1^2k_2^2}-\frac{2}{k_1^2}-\frac{2}{k_2^2}-\frac{2}{k_3^2})\bigg]\frac{1}{a^2}, \tag{55}$$

$$p^t=\frac{1}{3}\frac{v_1^2}{k_1^2}\bigg(4\frac{\ddot{a}}{a}+5\frac{\dot{a}^2}{a^2}\bigg)-2\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}+\frac{1}{12}\bigg[\frac{k_1^2+v_1^2}{k_2^2k_3^2}+\frac{k_2^2}{k_1^2k_3^2}$$

$$+\frac{k_3^2}{k_1^2k_2^2}-\frac{2}{k_1^2}-\frac{2}{k_2^2}-\frac{2}{k_3^2})\bigg]\frac{1}{a^2}, \tag{56}$$

$$q^t_1=2\frac{v_1}{k_1}\bigg(\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}\bigg)-\frac{k_1v_1}{2k_2^2k_3^2}\frac{1}{a^2}, \tag{57}$$

$$\pi_{11}^t = -\frac{4}{3}\frac{v_1^2}{k_1^2}\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) + \frac{1}{3}\left(\frac{2k_1^2 - v_1^2}{k_2^2 k_3^2} - \frac{k_2^2}{k_3^2 k_1^2} - \frac{k_3^2}{k_1^2 k_2^2} + \frac{2}{k_1^2} - \frac{1}{k_2^2} - \frac{1}{k_3^2}\right)\frac{1}{a^2}, \tag{58}$$

$$\pi_{22}^t = \frac{2}{3}\frac{v_1^2}{k_1^2}\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) + \frac{1}{3}\left(\frac{2k_2^2}{k_3^2 k_1^2} - \frac{k_3^2}{k_1^2 k_2^2} - \frac{2k_1^2 - v_1^2}{2k_2^2 k_3^2} + \frac{2}{k_2^2} - \frac{1}{k_3^2} - \frac{1}{k_1^2}\right)\frac{1}{a^2}, \tag{59}$$

$$\pi_{33}^t = \frac{2}{3}\frac{v_1^2}{k_1^2}\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) + \frac{1}{3}\left(\frac{2k_3^2}{k_1^2 k_2^2} - \frac{2k_1^2 - v_1^2}{2k_2^2 k_3^2} - \frac{k_2^2}{k_3^2 k_1^2} + \frac{2}{k_3^2} - \frac{1}{k_1^2} - \frac{1}{k_2^2}\right)\frac{1}{a^2}, \tag{60}$$

$$\pi_{23}^t = \frac{v_1(k_2^2 - k_3^2)}{k_2 k_3 k_1^2}\frac{\dot{a}}{a^2}. \tag{61}$$

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