

# FRW cosmological models in Brans-Dicke theory of gravity with variable $q$ and dynamical $\Lambda$ -term

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Received: 10 July 2015 / Accepted: 16 November 2015 / Published online: 27 January 2016  
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**Abstract** Exact solution of modified Einstein's field equations are considered within the scope of spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) space-time filled with perfect fluid in the frame work of Brans-Dicke scalar-tensor theory of gravity. In this paper we have investigated the flat, open and closed FRW models and the effect of dynamic cosmological term on the evolution of the universe. Two types of FRW cosmological models are obtained by setting the power law between the scalar field  $\phi$  and the scale factor  $a$  and deceleration parameter (DP)  $q$  as a time dependent. The concept of time dependent DP with some proper assumptions yield two type of the average scale factors (i)  $a(t) = [\sinh(\alpha t)]^{\frac{1}{n}}$  and (ii)  $a(t) = [t^\alpha e^t]^{\frac{1}{n}}$ ,  $\alpha$  and  $n \neq 0$  are arbitrary constants. In case (i), for  $0 < n \leq 1$ , it generates a class of accelerating models while for  $n > 1$ , the models of the universe exhibit phase transition from early decelerating to present accelerating phase and the transition redshift  $z_t$  has been calculated and found to be in good agreement with the results from recent astrophysical observations. In case (ii), for  $n \geq 2$  and  $\alpha = 1$ , we obtain a class of transit models of the universe from early decelerating to present accelerating phase. Taking into consideration the observational data, we conclude that the cosmological constant behaves as a positive decreasing function of time. The

physical and geometric properties of the models are also discussed with the help of graphical presentations.

**Keywords** Brans-Dicke gravity · Cosmological constant · Variable deceleration parameter

## 1 Introduction

The gravitational constant  $G$ , velocity of light  $c$  and cosmological constant  $\Lambda$  are all proper constants in Einstein's general theory of relativity. In 1961 Brans and Dicke contributed an interesting alternative to general relativity based on Mach's principle. To understand the reasons leading to their field equations, we first note that the concept of a variable mass. For how do we compare masses at two different points in space time? Masses are measured in certain units, such as masses of elementary particles, which are them self subject to change. We need an independent unit of mass against which an increase or decrease of a particle mass can be measured. Such a unit is provided by gravity, the so called Planck mass. Thus, if we insist on using mass unit that are the same everywhere, a change of dimensionless quantity  $\chi = m(\frac{G}{\hbar c})^{1/2}$  would tell us that  $G$  is changing. This is the conclusion Brans and Dicke arrived at in their approach to Mach's principle. They looked for a framework in which the gravitational constant  $G$  arises from the structure of the universe, so that a changing  $G$  could be looked upon as the Machian consequences of a changing universe.

These intuitive concepts are contained in the Brans-Dicke action principle, which may be written in the form

$$\mathcal{A} = \frac{c^3}{16\pi} \int_{\mathfrak{R}} (\phi R + \omega \phi^{-1} \phi^k \phi_k) \sqrt{-g} d^4x + \Lambda. \quad (1)$$

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Notice first that the coefficient of  $R$  is  $c^3\phi/16\pi$  instead of  $c^3\phi/16\pi G$  as in the Einstein-Hilbert action due to anticipated behaviour of  $G$ . The second term, with  $\phi_k \equiv \partial\phi/\partial x^k$ , ensures that  $\phi$  will satisfy a wave equation, while the third term includes, through a Lagrangian density  $L$ , all the matter and energy present in the space time region  $\mathfrak{R}$ . The energy momentum tensor  $T^{ik}$  is related to  $\Lambda$ ,  $\omega$  is coupling constant.

The variation of  $\mathcal{A}$  for small changes of  $g^{ik}$  leads to the field equations

$$R_{ik} - \frac{1}{2}g_{ik}R = -\frac{8\pi}{c^4\phi}T_{ik} - \frac{\omega}{\phi^2}\left(\phi_i\phi_k - \frac{1}{2}g_{ik}\phi^l\phi_l\right) - \frac{1}{\phi}(\phi_{;ik} - g_{ik}\square\phi), \quad (2)$$

where  $\square = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$ ,  $\nabla$  is Laplace'operator.

Similarly, the variation of  $\phi$  leads to the following equation for  $\phi$ :

$$2\phi\square\phi - \phi_k\phi^k = \frac{R}{\omega}\phi^2. \quad (3)$$

This latter equation can be simplified by substituting for  $R$  from the contracted form of Eq. (2). We finally get

$$\square\phi = \frac{8\pi}{(2\omega + 3)c^4}T, \quad (4)$$

where  $T$  is the trace of  $T_k^j$ . Equation (4) leads to the anticipated scalar wave equation for  $\phi$  with sources in matter. Because it contains a scalar field  $\phi$  in addition to the metric tensor  $g_{ik}$ , the Brans-Dicke (BD) is often referred to as *the scalar-tensor theory of gravitation*. BD theory is explained by a scalar function  $\phi$  and a constant coupling constant  $\omega$ , often known as the BD parameter. This can be obtained from general theory of relativity by letting  $\omega \rightarrow \infty$  and  $\phi = \text{constant}$  (Sahoo and Singh 2003). The BD theory of gravity is most promising one among all existing alternative theories of gravitation which has very effectively solved the problems of inflation and the early and late time behaviour of the universe (Linde 1990).

Recent measurements of redshift and luminosity-distance relations of type Ia Supernovae indicate that the expansion of the Universe is accelerating (Perlmutter et al. 1999; Riess et al. 1998). These observations give rise to the search for a matter field, which can be responsible for accelerated expansion. There are several proposals regarding this, *Cosmological Constant*, *Quintessence*, *Dark Energy* (Caldwell et al. 1998; Al-Rawaf and Taha 1996; Sahni and Starobinsky 2000; Padmanabhan 2003) being some of the competent candidates. Since the observed universe is almost homogeneous and isotropic, space-time is usually described by a

Friedman-Lemaitre-Robertson-Walker (FLRW) cosmology. However, most of these models fit only to spatially flat ( $k = 0$ ) FRW model (Banerjee and Pavon 2001), though a few models (Chimento et al. 2000) work for open universe ( $k = -1$ ) also.

The cosmological constant ( $\Lambda$ ) was introduced by Einstein in 1917 as the universal repulsion to make the Universe static in accordance with generally accepted picture of that time. In absence of matter described by the stress energy tensor  $T_{ij}$ ,  $\Lambda$  must be constant, since the Bianchi identities guarantee vanishing covariant divergence of the Einstein tensor,  $G_{;j}^{ij} = 0$ , while  $g_{;j}^{ij} = 0$  by definition. If Hubble parameter and age of the universe as measured from high red-shift would be found to satisfy the bound  $H_0t_0 > 1$  (index zero labels values today), it would require a term in the expansion rate equation that acts as a cosmological constant. Therefore the definitive measurement of  $H_0t_0 > 1$  and wide range of observations would necessitate a non-zero cosmological constant today or the abandonment of the standard big bang cosmology (Krauss and Turner 1995). However, a constant  $\Lambda$  cannot explain why the calculated value of vacuum energy density at Planck epoch following quantum field theory is 123 orders of magnitude larger than its value as observed or as predicted by standard cosmology at the present epoch (Weinberg 1989). In attempt to solve this problem, variable  $\Lambda$  was introduced such that  $\Lambda$  was larger in the early universe and then decayed with the evolution (Dolgov 1983). A dynamic cosmological term  $\Lambda(t)$  remains a focal point of interest in modern cosmological theories as it solves the cosmological constant problem in a natural way. In theories with a variable  $\Lambda$ -term, one either introduces new terms (involving scalar fields, for instance) into the left hand side of the Einstein's field equations to cancel the non-zero divergence of  $\Lambda g_{ij}$  (Bergmann 1968; Wagoner 1970) or interprets  $\Lambda$  as a matter source and moves it to the right hand side of the field equations (Zeldovich 1968), in which case energy momentum conservation is understood to mean  $T_{;j}^{*ij} = 0$ , where  $T_{ij}^* = T_{ij} - (\Lambda/8\pi G)g_{ij}$ . It is here that the first assumption that leads to the cosmological constant problem is made. It is that the vacuum has a non-zero energy density. If such a vacuum energy density exists, Lorentz invariance requires that it has the form  $\langle T_{\mu\nu} \rangle = -\langle \rho \rangle g_{\mu\nu}$ . This allows to define an effective cosmological constant and a total effective vacuum energy density  $\Lambda_{\text{eff}} = \Lambda + 8\pi G\langle \rho \rangle$  or  $\rho_{\text{vac}} = \langle \rho \rangle + \Lambda/8\pi G$ . Note at this point that only the effective cosmological constant,  $\Lambda_{\text{eff}}$ , is observable, not  $\Lambda$ , so the latter quantity may be referred to as a 'bare'. For detail discussions, the readers are advised to see the references (Peebles and Ratra 2003; Sahni and Starobinsky 2000; Padmanabhan 2003, 2008).

The deceleration parameter (DP) is the most important observational parameter in cosmology. The values of DP

separates decelerating ( $q > 0$ ) from accelerating ( $q < 0$ ) periods in the evolution of the universe. Determining the DP from the *Count-Magnitude Relation* for galaxies is a difficult task due to evolutionary effects. It has been shown by Riess et al. (1998) and Perlmutter et al. (1998) that the observed redshift-magnitude relation for supernovae of type Ia suggests that the DP  $q_0$  is negative. The present value  $q_0$  of DP obtained from observations (Schuecker et al. 1998) are  $-1.27 \leq q_0 \leq 2$ . Studies of galaxy counts from redshift surveys provide a value of  $q_0 = 0.1$ , with an upper limit of  $q_0 < 0.75$  (Schuecker et al. 1998). Recent observations (Riess et al. 1998; Perlmutter et al. 1998, 1999) show that the DP of the universe is in the range  $-1 \leq q \leq 0$ , and the present day universe is undergoing accelerated expansion. But of course these results do not exclude the existence of a decelerating phase in the early history of our universe. Mak and Harko (2002) obtained general solution of the gravitational field equations for a Bianchi type I space time with causal bulk viscous fluid for an arbitrary time dependent deceleration parameter  $q = q(t)$ . They also presented the general representation of the solution in terms of the DP. Pradhan and Otarod (2006, 2007) have also investigated a new class of universes with time dependent DP. Amirhashchi et al. (2011) have obtained two-fluid dark energy models in FRW universe with time dependent DP. Akarsu and Dereli (2012) have proposed a special law for DP which is linear in time with negative slope. This law covers the law of Berman (1983) and Berman and Gomide (1988). Recently, Ahmed and Pradhan (2014) and Pradhan et al. (2015) have obtained Bianchi type-V cosmology in  $f(R, T)$  gravity and Bianchi type-I transit cosmological models respectively by using a law of variation of scale factor which yields a time dependent DP.

The cosmological implications of the FRW models in Brans-Dicke theory of gravity with variable deceleration parameter and dynamic  $\Lambda$ -term will be discussed in detail in this paper. The out line of the paper is as follows: In Sect. 2, the metric and basic equations are described. Section 3 deals with the solutions of field equations by considering two types of scale factors. In Sect. 4, we discussed the physical and geometric properties of the two models depending on two different scale factors. Section 5 deals with the physical acceptability of the derived solutions. Finally, conclusions are summarized in the last Sect. 6.

## 2 Metric and field equations

In standard spherical coordinates  $(x^i) = (t, r, \theta, \phi)$ , a spatially homogeneous and isotropic FRW line-element has the form

$$ds^2 = -dt^2 + a^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right\}, \quad (5)$$

where  $a(t)$  is cosmic scale factor, which describes how these distances (scales) change in an expanding or contracting universe, and is related to redshift of 3-space;  $k$  is the curvature parameter, which describes geometry of the spatial section of space-time with closed, flat and open universes corresponding to  $k = 1, 0, -1$  respectively. The coordinates  $r, \theta$  and  $\phi$  in Eq. (5) are co-moving coordinates. FRW models have been remarkably successful in describing the observed nature of universe. The modified EFE in the frame work of Brans-Dicke gravity theory along cosmological constant may be written (in units  $G = c = 1$ ) as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} + \frac{\omega}{\phi^2} \left[ \phi_{;\mu}\phi_{;\nu} - \frac{1}{2}g_{\mu\nu}\phi_{;\eta}\phi^{;\eta} \right] + \frac{1}{\phi} [\phi_{\mu;\nu} - g_{\mu\nu}\square\phi] = \frac{8\pi}{\phi} T_{\mu\nu} \quad (6)$$

$$\square\phi = \phi_{;\mu}^{;\mu} = \frac{8\pi}{2\omega + 3} T, \quad (7)$$

where  $\phi$  is a scalar field,  $\omega$  is a dimensionless constant and  $T = T_{\mu}^{\mu}$  is the trace of the energy momentum tensor. Here a semicolon indicates covariant derivative and a comma stands for ordinary derivative with respect to  $x^k$ . The energy momentum tensor ( $T_{\mu\nu}$ ) for the cosmic fluid may be expressed as

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + g_{\mu\nu}p, \quad (8)$$

where  $\rho$  is the energy density,  $p$  is the isotropic pressure and  $u_{\mu}$  is four velocity vector. In a co-moving coordinate system  $u^{\mu} = (0, 0, 0, 1)$  with property  $u^{\mu}u_{\mu} = -v^{\mu}v_{\mu} = 1$ , and  $u^{\mu}v_{\mu} = 0$ . The Brans-Dicke field equations (6) and (7) for the FRW metric given by Eq. (5) with the help of Eq. (8) yield the following set of field equations:

$$\frac{3\dot{a}^2}{a^2} + 3\frac{\dot{a}\dot{\phi}}{a\phi} - \frac{\omega\dot{\phi}^2}{2\phi^2} + 3\frac{k}{a^2} = \frac{8\pi}{\phi}\rho + \Lambda, \quad (9)$$

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{\ddot{\phi}}{\phi} + 2\frac{\dot{a}\dot{\phi}}{a\phi} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{k}{a^2} = -\frac{8\pi}{\phi}p + \Lambda, \quad (10)$$

$$\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{a}\dot{\phi}}{a\phi} = \frac{8\pi(\rho - 3p)}{\phi(3 + 2\omega)} + \frac{2\Lambda}{3 + 2\omega}, \quad (11)$$

where an overhead dot denotes derivatives with respect to cosmic time  $t$ .

Let us introduce some physical parameters such as the spatial volume  $V$ , the expansion scalar  $\theta$ , the Hubble parameter  $H$  and red shift parameter  $z$  for the FRW metric (5)

$$V = a^3, \quad (12)$$

$$\theta = 3H = 3\frac{\dot{a}}{a}, \quad (13)$$

$$z = -1 + \frac{a_0}{a}, \quad (14)$$

where  $a_0$  is the present value of the cosmic scale factor  $a$ .

The deceleration parameter  $q$  in cosmology is the measure of the cosmic acceleration of the universe expansion and is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right). \quad (15)$$

### 3 Solution of field equations

The field equations (9)–(11) are a system of three independent equations along with five unknown parameters  $a$ ,  $\phi$ ,  $p$ ,  $\rho$  and  $\Lambda$ , therefore two more constraints are required to find the exact solution of this system of equations. For the exact solution of the set of above stated field equations we use the two constraints as follow.

(i) The deceleration parameter  $q$  is taken as a function of cosmic time ‘ $t$ ’ i.e.

$$q(t) = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \quad (16)$$

Mak and Harko (2002) have discussed to obtain general solution of field equations for  $q = q(t)$  as already mentioned in previous section of Introduction. The motivation to choose time dependent DP is behind the fundamental fact that the Universe has accelerated expansion at present and decelerated expansion in the early time. The time-dependent behaviour of  $q$  is supported by recent observations of SNe Ia (Perlmutter et al. 1999; Riess et al. 1998; Tonry et al. 2003; Clocchiatti et al. 2006) and CMB anisotropies (Bennett et al. 2003; de Bernardis et al. 2007; Hanany et al. 2000). These observations clearly argue an accelerating expansionary universe at present, which has been decelerating in past. In their preliminary analysis, it is found that supernovae (SNe) data favour recent acceleration ( $z < 0.5$ ) and past deceleration ( $z > 0.5$ ). Recently, High-Z Supernova Search (HZSNS) team have prevailed transition redshift  $z_t = 0.46 \pm 0.13$  at  $(1\sigma)$  c.l. (Riess et al. 2004) which has been further amended to  $z_t = 0.43 \pm 0.07$  at  $(1\sigma)$  c.l. (Riess et al. 2007). Supernova Legacy Survey (SNLS) (Astier et al. 2006), as well as the one recently compiled by Davis et al. (2003), bring forth  $z_t \sim 0.6(1\sigma)$  in better agreement with flat  $\Lambda$ CDM model ( $z_t = (2\Omega_\Lambda/\Omega_m)^{\frac{1}{3}} - 1 \sim 0.66$ ). Thus, the deceleration parameter (DP) which by definition is the rate with which the universe decelerates, must show signature flipping (Riess et al. 2001; Padmanabhan and Roychowdhury 2003; Amendola 2003) between positive and negative values. Several authors (Pradhan and Otarod 2006; Akarsu and Dereli 2012; Pradhan et al. 2012) have discussed models of the universe with variable DP in different context.

Equation (16) may be rewritten as

$$\frac{\ddot{a}}{a} + q\frac{\dot{a}^2}{a^2} = 0. \quad (17)$$

In order to solve the Eq. (17), we assume  $q = q(a)$ . It is important to note here that one can assume  $q = q(t) = q(a(t))$ , as  $a$  is also a time dependent function. It can be done only if there is a one to one correspondences between  $t$  and  $a$ . But this is only possible when one avoid singularity like big bang or big rip because both  $t$  and  $a$  are increasing functions.

The general solution of Eq. (17) with the assumption  $q = q(a)$ , is obtained as

$$\int e^{\int \frac{q}{a} da} da = t + k, \quad (18)$$

where  $k$  is an integrating constant. One cannot solve (18) in general as  $q$  is variable. So, in order to solve the problem completely, we have to choose  $\int \frac{q}{a} da$  in such a manner that (18) be integrable without any loss of generality. Hence we consider

$$\int \frac{q}{a} da = \ln f(a), \quad (19)$$

which does not affect the nature of generality of solution. Hence from (18) and (19), we obtain

$$\int f(a) da = t + k. \quad (20)$$

Of course the choice of  $f(a)$ , in (20), is quite arbitrary but, since we are looking for physically viable models of the Universe consistent with observations, we consider the value of function  $f(a)$  as

$$f(a) = \frac{na^{n-1}}{\alpha\sqrt{1+a^{2n}}}, \quad (21)$$

where  $\alpha$  and  $n > 0$  are arbitrary constants. In this case, on integrating equation (20) and neglecting the integration constant  $k$ , we obtain the exact solution as

$$a(t) = [\sinh(\alpha t)]^{\frac{1}{n}}. \quad (22)$$

Relation given by Eq. (22) is also recently used by Chawla et al. (2012) in studying the Bianchi type-I string cosmological models in Einstein’s field equations with variable gravitational and cosmological constants. This relation (22) generalizes the value of scale factor derived by Pradhan et al. (2012) in connection with the study of dark energy models in Bianchi type-VI<sub>0</sub> space-time. Recently, Pradhan (2014) used Eq. (22) to study two-fluid atmosphere from decelerating to accelerating FRW dark energy models.

Following Pradhan and Amirhashchi (2011), Yadav (2012) and Pradhan et al. (2013), we assume the following *ansatz* for the scale factor, where increase in terms of time evolution is

$$a = (t^\alpha e^t)^{\frac{1}{n}}, \quad (23)$$

where  $\alpha$  and  $n$  are positive constants. It is found that the *ansatz* (23) generalizes (Saha et al. 2012; Pradhan 2013; Pradhan et al. 2014). The choice of scale factor attracts a time-dependent deceleration parameter which brings that dark energy era, the solution gives inflation and radiation/matter dominance era with subsequent transition from deceleration to acceleration. This theme motivates to choose such scale factor (23) that yields time dependent deceleration parameter.

(ii) Secondly we consider a power law relation between scale factor  $a$  and BD scalar field  $\phi$ . We have reduced the cosmological equations to quadrature, assuming this relation. Brans-Dicke theory is a simple modification of Einstein general relativity where the purely metric coupling of matter with gravity is preserved, thus the universality of free fall (equivalence principle) is ensured (Liu and Zhang 2009). Here, the gravitational constant is replaced with the inverse of a time-dependent scalar field, namely,  $\phi(t) = 1/8\pi G$ , and this scalar field couples to gravity with a coupling constant  $\omega$ . It also passes the experimental tests from solar system (Bertotti et al. 2003) and is able to provide an explanation of the accelerated expansion of the universe (Mathiazhagan and Johri 1984). Landsberg and Bishop (1975) investigated Newtonian cosmology with  $G \propto a^\beta$  and an analogous procedure would lead to  $\phi \propto a^\beta$ . There is another investigation of cosmology presupposing  $\phi \propto a^\beta$  (Dehnen and Obregon 1971, 1972a, 1972b) in which the theory of gravity used in Brans-Dicke.

Since the field equations contain  $a$  and  $\phi$  and their derivatives, so without any loss of generality, we shall assume that the BD scalar field  $\phi$  is some power of the average scale factor. The power law relation between scale factor  $a$  and scalar field  $\phi$  has already been used by Johri and Desikan (1994) in the context of Robertson Walker Brans-Dicke models. Thus,

$$\phi = \phi_0 a^\beta, \tag{24}$$

where  $\beta$  is ordinary constant whereas  $\phi_0$  is the proportionality constant. The assumption of a power law between the scalar field  $\phi$  and the cosmological expansion factor  $a$ , it is possible to reduce the cosmological equations to quadrature for the scalar-tensor theory with cosmological constant (Pimentel 1985a, 1985b). Mach’s principle has many different formulations, the central idea being that the local properties of the Universe are connected to the overall distribution of matter in the Universe, represented by  $a$ . The simplest choice of function is  $\phi \propto a^\beta$ . In order to predict the past or future behaviour of the cosmological model the function  $\phi$  must be specific, and the constraint given by Eq. (24). A full discussion is given by Bishop (1976). Arik et al. (2011) have shown that if  $\phi$  is taken to be a complex scalar field, then an exact solution to the vacuum equations requires that the Friedmann equations possesses both

a constant term and one which is proportional to the inverse sixth power of the scale factor. Several authors have assumed the same law in previous papers in different scalar-tensor theory (Dehnen and Obregon 1971; Obregon and Pimentel 1978; Chauvel and Obregon 1979; Pimentel 1987; Ahmadi-Azar and Riazi 1995; Bahrehbakhsh et al. 2011; Shamir and Bhatti 2012).

Putting the value of  $\phi$  from Eq. (24) into field equations (9)–(11), we obtain the following set of field equations:

$$\left(\frac{6 + 6\beta - \omega\beta^2}{2}\right) \frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} = \frac{8\pi}{\phi_0 a^\beta} \rho + \Lambda, \tag{25}$$

$$(2 + \beta) \frac{\ddot{a}}{a} + \frac{2 + 2\beta + 2\beta^2 + \omega\beta^2}{2} \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{-8\pi}{\phi} p + \Lambda, \tag{26}$$

$$(3 + 2\omega) \left[ \beta \frac{\ddot{a}}{a} + \beta(\beta + 2) \frac{\dot{a}^2}{a^2} \right] = \frac{8\pi(\rho - 3p)}{\phi_0 a^\beta} + 2\Lambda. \tag{27}$$

A combination of Eqs. (25)–(27) leads to

$$2(3 - \omega\beta) \frac{\ddot{a}}{a} + (6 - 4\omega\beta - \omega\beta^2) \frac{\dot{a}^2}{a^2} + 6\frac{k}{a^2} = 2\Lambda. \tag{28}$$

In this paper we have constructed two models of the universe by applying two scale factors  $a(t) = [\sinh(\alpha t)]^{\frac{1}{n}}$  and  $a(t) = (t^\alpha e^t)^{\frac{1}{n}}$  as expressed by Eqs. (22) and (23) respectively.

## 4 Physical and geometric properties

### 4.1 Model 1: $a(t) = [\sinh(\alpha t)]^{\frac{1}{n}}$

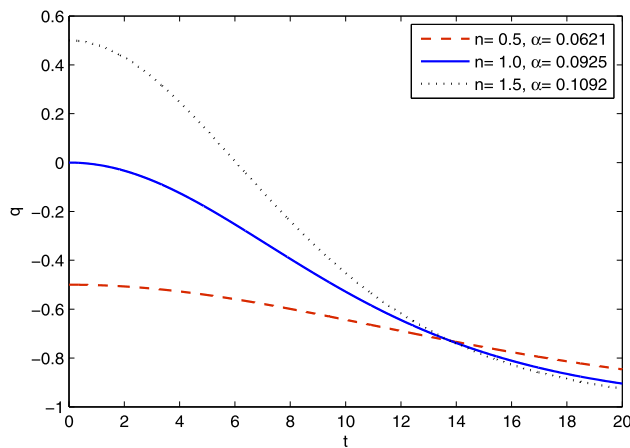
Using Eq. (22), the model given by Eq. (5) reduces to

$$ds^2 = -dt^2 + (\sinh(\alpha t))^{\frac{2}{n}} \times \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]. \tag{29}$$

Putting the value of  $a(t)$  into Eqs. (25)–(27), we get the following expressions for cosmological parameters such as  $\Lambda$ ,  $\rho$  and  $p$  for model (29)

$$2\Lambda = \frac{\alpha^2(6 - 3\omega\beta - 3n + \omega\beta - \frac{\omega\beta^2}{2})}{n^2} \coth^2(\alpha t) + n(3 - \omega\beta) + 3k(\operatorname{cosech}^2(\alpha t))^{\frac{1}{n}} \tag{30}$$

$$\rho = \frac{\alpha^2 \phi_0 (\sinh(\alpha t))^{\frac{\beta}{n}}}{8\pi} \times \left[ \frac{3\omega\beta + 3\beta - 3 + 3n - \omega\beta n}{n^2} \coth^2(\alpha t) + \frac{\omega\beta - 3}{n} \right] \tag{31}$$



**Fig. 1** Plot of deceleration parameter  $q$  versus time  $t$

$$p = \frac{\phi_0(\sinh(\alpha t))^{\frac{\beta}{n}}}{8\pi} \times \left[ \frac{\alpha^2(3 - 2\beta - \beta^2 - \omega\beta - \omega\beta^2)}{n^2} \coth^2(\alpha t) + \frac{\alpha(\omega\beta + \beta - 1)}{n} - 2k(\operatorname{cosech}(\alpha t))^{\frac{2}{n}} \right] \quad (32)$$

The expressions for kinematic parameters such as spatial volume ( $V$ ), Hubble parameter ( $H$ ), expansion scalar ( $\theta$ ) and deceleration parameter ( $q$ ) are obtain for the model (29) as:

$$V = a^3 = (\sinh(\alpha t))^{\frac{3}{n}} \quad (33)$$

$$\theta = 3H = \frac{3\alpha}{n} \coth(\alpha t) \quad (34)$$

$$q = -1 + n \operatorname{sech}^2(\alpha t) \quad (35)$$

From Eq. (35), we observe that  $q > 0$  for  $t < \frac{1}{\alpha} \tanh^{-1}(1 - \frac{1}{n})^{\frac{1}{2}}$  and  $q < 0$  for  $t > \frac{1}{\alpha} \tanh^{-1}(1 - \frac{1}{n})^{\frac{1}{2}}$ . It is also observed that for  $0 < n \leq 1$ , our model is in accelerating phase but for  $n > 1$ , our model is evolving from decelerating phase to accelerating phase. Also, recent observations of SNe Ia, expose that the present universe is accelerating and value of DP lies to some place in range  $-1 \leq q < 0$ . It follows that in our derived model, one can choose values of DP consistent with observations. Figure 1 depicts variation of deceleration parameter ( $q$ ) versus time ( $t$ ) which gives the behaviour of  $q$  for different values of  $n$ . It is also clear from Fig. 1 that for  $n \leq 1$ , the model is evolving only in accelerating phase whereas for  $n > 1$  the model is evolving from early decelerated phase to present accelerating phase.

Further, DP ( $q$ ) as a function of redshift parameter  $z = -1 + \frac{a_0}{a}$ , where  $a_0$  is the present value of the scale factor i.e.

at  $z = 0$ , is given by

$$q(z) = n - 1 - n \left[ \tanh \left( \sinh^{-1} \sqrt{\frac{n-1-q_0}{(q_0+1)(z+1)^{2n}}} \right) \right]^2 \quad (36)$$

Such type of relation obtained by us also provides a two-parameter ( $n, q_0$ ) parametrization just like a linear two-parameter expansion for  $q(z) = q_0 + q_1 z$  (Riess et al. 2004), where  $q_0$  is the present value of deceleration parameter and  $q_1$  is the deviation in the redshift evaluated at  $z = 0$ . If we set  $q_0 = -0.73$  (Cunha 2009), then a positive transition redshift ( $z_t$ ) may be obtained only for the positive values of  $q_1$  since  $q_0$  is negative and the dynamic transition (from deceleration to acceleration) occurs at  $q(z_t) = 0$ , or equivalently,  $z_t = -\frac{q_0}{q_1}$ . Another parametrization of considerable interest is  $q(z) = q_0 + q_1 z(1+z)^{-1}$  (Xu and Liu 2008) where the parameter  $q_1$  describes the total correction in the distant past ( $z \gg 0, q(z) = q_0 + q_1$ ). Moreover, a positive  $z_t$  may be obtained for the positive values of  $q_1 > |q_0|$  and the dynamic transition occurs at  $z_t = -\frac{q_0}{q_0+q_1}$ .

For the present Universe ( $t_0 = 13.75$  GYr) with  $q_0 = -0.73$  (Cunha 2009), Eq. (36) yields the following relationship between the constants  $n$  and  $\alpha$ :

$$\alpha = \frac{1}{13.75} \tanh^{-1} \left[ 1 - \frac{0.27}{n} \right]^{\frac{1}{2}} \quad (37)$$

It is self explanatory from the above relation that for the present Universe, the model is valid only for  $n > 0.27$ . Figure 5 depicts the variation of the DP ( $q$ ) versus time ( $t$ ) for different sets of the pair ( $n, \alpha$ ) satisfying the above relation (37). It is clearly observable from the figure that for  $0 < n \leq 1$ , our model is in accelerating phase but for  $n > 1$ , our model is evolving from decelerating phase to accelerating phase. It follows that in our derived model, one can choose the value of  $n$  which gives the physical behaviour of DP consistent with the observations.

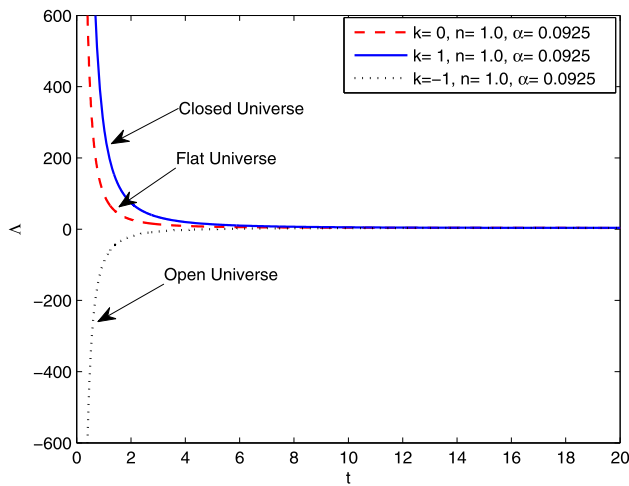
It is remarkable to add here that time dependent scale factor is stable under metric perturbation (Chen and Kao 2001). In term of redshift the above scale factor turns to

$$a = \frac{1}{1+z}, \quad z = \frac{1}{[\sinh(\alpha t)]^{\frac{1}{n}}} - 1. \quad (38)$$

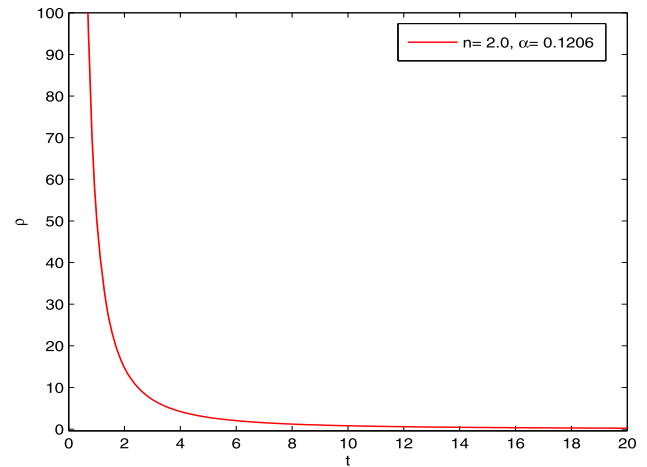
The red shift parameter  $z_1$  for model 1 may be expressed as

$$z_1 = -1 + [\sinh(\alpha t)]^{-\frac{1}{n}} \quad (39)$$

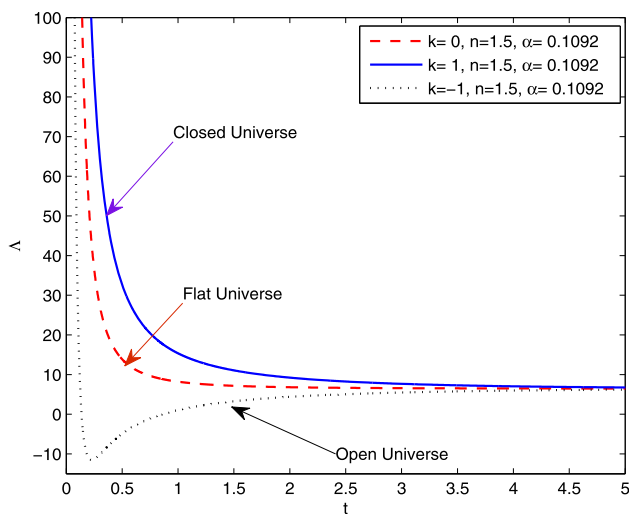
In Figs. 2 and 3, we have presented the behaviour of cosmological constant  $\Lambda$  with cosmic time ' $t$ ' for flat, open and closed FRW models for  $n = 1$  and  $n = 1.5$  respectively. Figure 2 indicates that for  $k = 0, 1$  cosmological constant



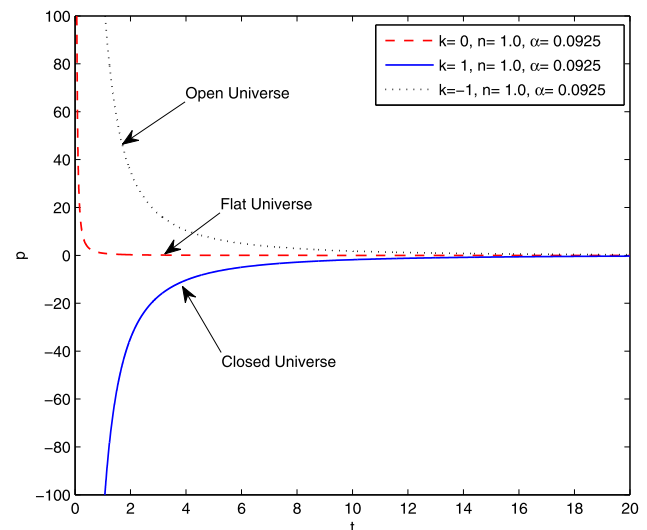
**Fig. 2** Plot of cosmological constant  $\Lambda$  versus time  $t$ . For  $\beta = 0.3$ ,  $\omega = 1$ ,  $\phi_0 = 1$



**Fig. 4** Plot of energy density  $\rho$  versus time  $t$ . For  $\beta = 0.3$ ,  $\omega = 1$ ,  $\phi_0 = 1$



**Fig. 3** Plot of cosmological constant  $\Lambda$  versus time  $t$ . For  $\beta = 0.3$ ,  $\omega = 1$ ,  $\phi_0 = 1$



**Fig. 5** Plot of pressure  $p$  versus time  $t$ . For  $\beta = 0.3$ ,  $\omega = 1$ ,  $\phi_0 = 1$

$\Lambda$  is decreasing with time and approaches to a small positive value at present epoch, but for open universe ( $k = -1$ ), the cosmological constant is found to be negative increasing behaviour with time and approaches to small positive value at present epoch. We also observe that flat model is sharply decreasing with time in comparison to closed model of universe. Figure 3 shows that  $\Lambda$  is decreasing with time for  $k = 0, 1$  but for  $k = -1$ ,  $\Lambda$  is decreasing sharply in early time from positive to negative value and then increasing with time and converges to a small constant value.

From Eq. (31), we observe that  $\rho$  is decreasing function of time and  $\rho > 0$  for all time if ( $n = 2, \alpha = 0.1206$ ) and for other values of  $n \geq 2$  and their corresponding values of  $\alpha$  satisfying Eq. (37). Figure 4 depicts the variation of  $\rho$  versus

time  $t$  by taking pair of ( $n, \alpha$ ) as ( $2, 0.1206$ ) as a toy model of the universe. From this figure, we observe that  $\rho$  is decreasing function of time and approaches to zero at present epoch which is consistent with observations. It is clear that  $\rho$  was very large at early time and approaches to zero at late time. Figure 5 shows the nature of pressure which decreases with time and approaches to zero at late time for flat and closed models and  $p > 0$  always but for open model pressure in negative increasing function of  $t$  and ultimately approaches to a small value near zero. Figure 6 shows the variation of redshift parameter versus  $t$ . The redshift parameter decreases with time and approaches to a small positive constant at present epoch.

From Eq. (33), it can be seen that spatial volume is zero at  $t = 0$  and it increases with time. This shows that the universe

starts evolving with zero volume at  $t = 0$  and expand with cosmic time. From this analysis we conclude that it is the choice of scale factor that makes the model inflationary at early stages of the universe and radiation/matter dominance phase before dynamic  $\Lambda$  (dark energy) dominated era. From Eq. (34), we observe that when  $t \rightarrow 0$ , expansion scalar  $\theta$  becomes infinity which indicates inflationary scenario. Also from Fig. 1, we observe that before  $t \approx 1$ ,  $q > 0$  indicating radiation/matter dominance era of the universe. However, after  $t \approx 1$ ,  $q < 0$  which indicates DE dominated era. The solution in our model does not blow up at any given epoch for the choice of *ansatz* Eq. (23). Hence our derived model is physically plausible.

From Table 1, we observe that all physical and geometric quantities of Model 1 lie in the range of observable universe. Hence our Model 1 is physically viable and consistent with  $\Lambda$ -CDM standard model of the universe as already discussed in Sect. 5.

### 4.2 Model 2: $a(t) = (t^\alpha e^t)^{\frac{1}{n}}$

Using Eq. (23), the model in Eq. (5) becomes

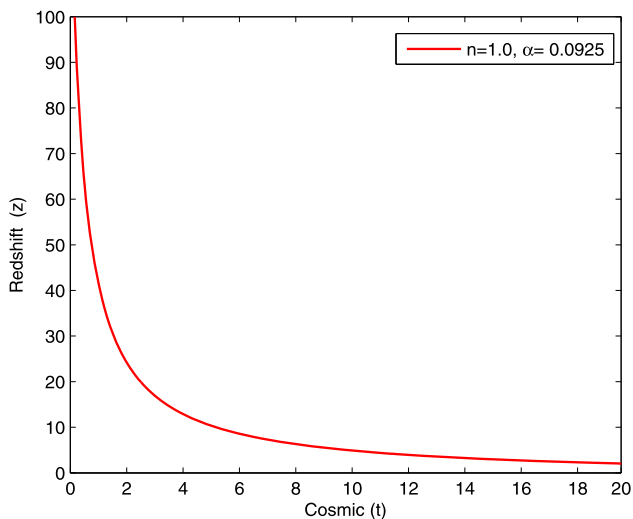


Fig. 6 Plot of red shift parameter  $z_1$  versus time  $t$

$$ds^2 = -dt^2 + (t^\alpha e^t)^{\frac{2}{n}} \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]. \tag{40}$$

Putting the value  $a(t)$  into Eqs. (25)–(27), we get the expressions for physical and kinematic parameters for model (40) as:

$$\Lambda = \frac{1}{2n^2} \left[ (12 - 6\omega\beta - \omega\beta^2) \left( 1 + \frac{\alpha}{t} \right)^2 - 2n\alpha(3 - \omega\beta) \frac{1}{t^2} \right] + \frac{3k}{(t^\alpha e^t)^{\frac{1}{n}}}, \tag{41}$$

$$\rho = \frac{\phi_0 (t^\alpha e^t)^{\frac{\beta}{n}}}{8\pi} \left[ \frac{3(\omega\beta + \beta - 1)}{n^2} \left( 1 + \frac{\alpha}{t} \right)^2 + \frac{\alpha(3 - \omega\beta)}{nt^2} \right], \tag{42}$$

$$p = \frac{-\phi_0 (t^\alpha e^t)^{\frac{\beta}{n}}}{8\pi} \left[ \frac{(\beta^2 + \omega\beta^2 + \omega\beta + 2\beta - 3)}{n^2} \left( 1 + \frac{\alpha}{t} \right)^2 - \frac{\alpha(\omega\beta + \beta - 1)}{nt^2} - 2k(t^\alpha e^t)^{\frac{-2}{n}} \right], \tag{43}$$

From Table 2, we observed that all physical and geometric quantities of Model 2 lie in the range of observable universe. Hence our model 2 is physically viable and consistent with  $\Lambda$ CDM standard model of the universe.

$$V = a^3 = (t^\alpha e^t)^{\frac{3}{n}}, \tag{44}$$

$$\theta = 3H = \frac{3}{n} \left[ 1 + \frac{\alpha}{t} \right], \tag{45}$$

$$q = -1 + \frac{n\alpha}{(\alpha + t)^2}. \tag{46}$$

The red shift parameter  $z_2$  for Model 2 may be expressed as

$$z_2 = -1 + [t^\alpha e^t]^{-\frac{1}{n}}. \tag{47}$$

From Eq. (46), we observe that  $q > 0$  for  $t < \sqrt{n\alpha} - \alpha$  and  $q < 0$  for  $t > \sqrt{n\alpha} - \alpha$ . It is observed that for  $n \geq 3$  and

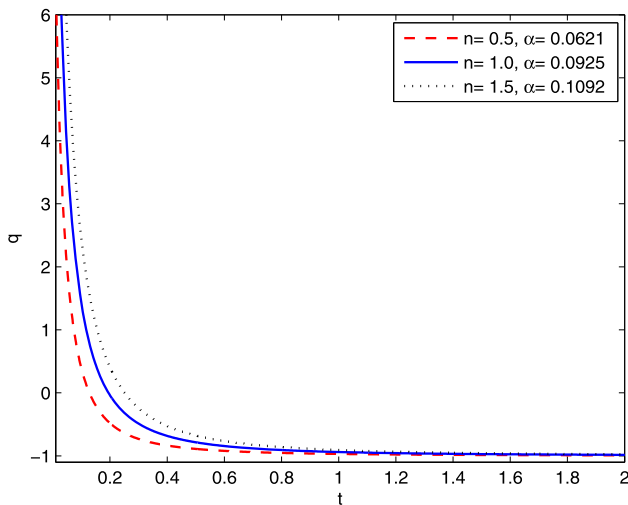
Table 1 Calculated values of cosmological parameters with help of Model 1. (Here  $t_0 = 13.75$  Gyr)

Cosmological parameters	$n$	$\alpha$	$q$	$\rho$	$\Lambda$	$r$	$s$
Expected/observable values	–	–	$-1 \leq q \leq 0.5$	+ve small	+ve small	$r = 1$	$s = 0$
Flat Model 1 ( $k = 0$ )	0.5	0.0621	-0.7402	$-1.560 \times 10^{-2}$	2.8238	0.4596	0.0732
	1.0	0.0925	-0.7298	$-6.400 \times 10^{-3}$	2.7276	0.7301	0.0366
	1.5	0.1092	<b>-0.7297</b>	$-3.501 \times 10^{-3}$	2.7055	<b>1.000</b>	<b>0.0000</b>
	2.0	0.1206	-0.7291	$4.200 \times 10^{-3}$	2.6991	1.2693	-0.0365



**Table 2** Calculated values of cosmological parameters with help of Model 2. (Here  $t_0 = 13.75$  Gyr)

Cosmological parameters	$n$	$\alpha$	$q$	$\rho$	$\Lambda$	$r$	$s$
Expected/observable values	–	–	$-1 \leq q \leq 0.5$	+ve small	+ve small	$r = 1$	$s = 0$
Flat Model 2 ( $k = 0$ )	0.5	0.0621	<b>-0.9998</b>	$-1.1663 \times 10^{-2}$	46.5352	<b>0.9995</b>	<b>0.0001</b>
	1.0	0.0925	-0.9995	$-91.9661 \times 10^{-1}$	11.6314	0.9986	0.0003
	1.5	0.1092	-0.9991	$-15.2661 \times 10^{-1}$	5.1660	0.9976	0.0005
	2.0	0.1206	-0.9987	$5.7016 \times 10^{-4}$	2.9016	0.9966	0.0007

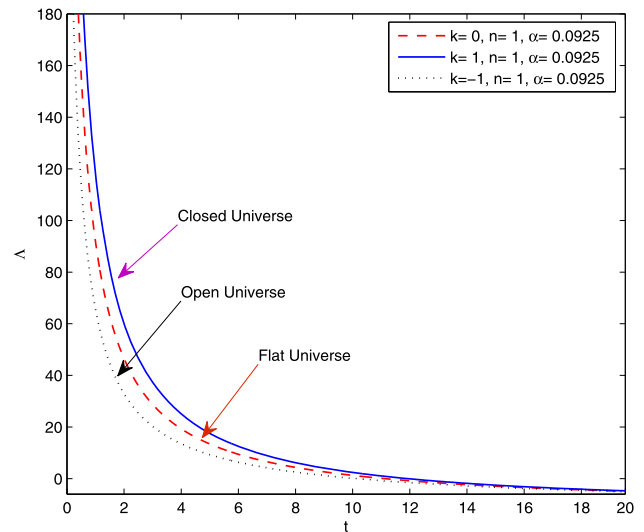


**Fig. 7** Plot of deceleration parameter  $q$  versus time  $t$

$\alpha = 1$ , our model is evolving from decelerating phase to accelerating phase. Also, recent observations of SNe Ia expose that the present universe is accelerating and the value of DP lies on some place in the range  $-1 < q < 0$ . It follows that in our derived model, one can choose the value of DP consistent with the observation. Figure 7 depicts the variation of deceleration parameter ( $q$ ) with cosmic time, giving the behaviour of  $q$  as in accelerating phase at present epoch for different values of  $(n, \alpha)$  which is consistent with recent observations of Type Ia supernovae (Riess et al. 1998; Perlmutter et al. 1999).

From Eqs. (44) and (45) we observe that the spatial volume is zero at  $t = 0$  and the expansion scalar is infinite, which show that the universe starts evolving with zero volume at  $t = 0$  which is big bang scenario. From Eq. (23), we observe that the spatial scale factor is zero at the initial epoch  $t = 0$  and hence the model has a point type singularity (MacCallum 1971). We observe that proper volume increases with time.

In Figs. 8 and 9, we have presented the behaviour of cosmological constant  $\Lambda$  with cosmic time ' $t$ ' for flat, open and closed FRW models for  $n = 1$  and  $n = 1.5$  respec-

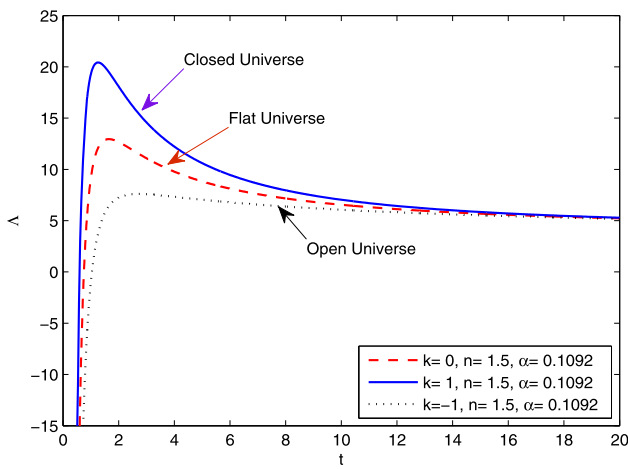


**Fig. 8** Plot of cosmological constant  $\Lambda$  versus time  $t$ . For  $\beta = 0.3$ ,  $\omega = 1$ ,  $\phi_0 = 1$

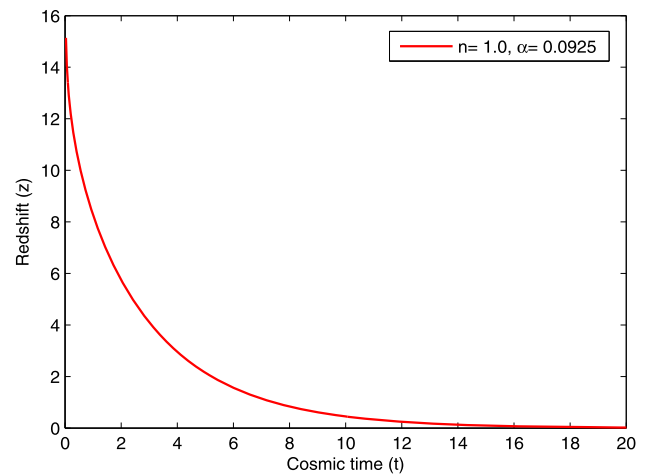
tively. Figure 8 indicates that for  $k = 0, 1, -1$  cosmological constant  $\Lambda$  is decreasing with time and approaches to a small positive value at present epoch. We also observe that open model ( $k = -1$ ) is decreasing sharply with time compared to flat ( $k = 0$ ) and closed ( $k = 1$ ) models of universe. Figure 9 shows that  $\Lambda$  is negative increasing function of time for all three types of open, flat and closed models and ultimately converge to a small constant value at present epoch.

From Eq. (42), we observe that  $\rho$  is decreasing function of time and  $\rho > 0$  for all time if  $(n = 2, \alpha = 0.1206)$  and for other values of  $n \geq 2$  and their corresponding values of  $\alpha$  satisfying Eq. (37). Figure 10 depicts the variation of  $\rho$  versus time  $t$  by taking pair of  $(n, \alpha)$  as  $(2, 0.1206)$  as a toy model of the universe. The nature of energy density is same as in Model 1.

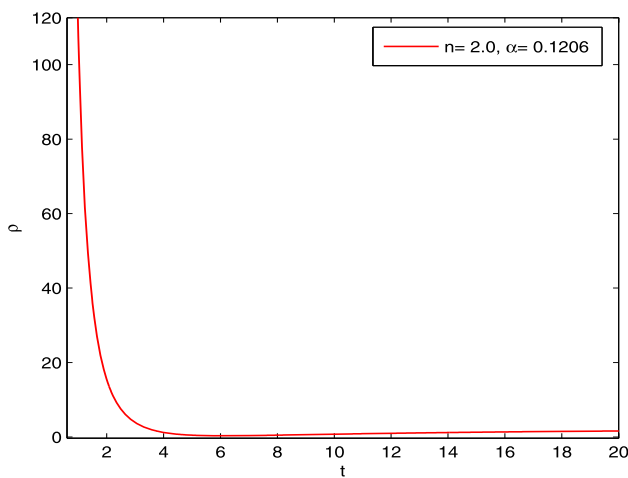
Figure 11 shows the variation of pressure  $p$  with cosmic time for all three flat, open and closed models. We observe that in all three models pressure is negative increasing function of time and they approach to zero. Figure 12 shows the variation of redshift parameter versus  $t$ . The redshift param-



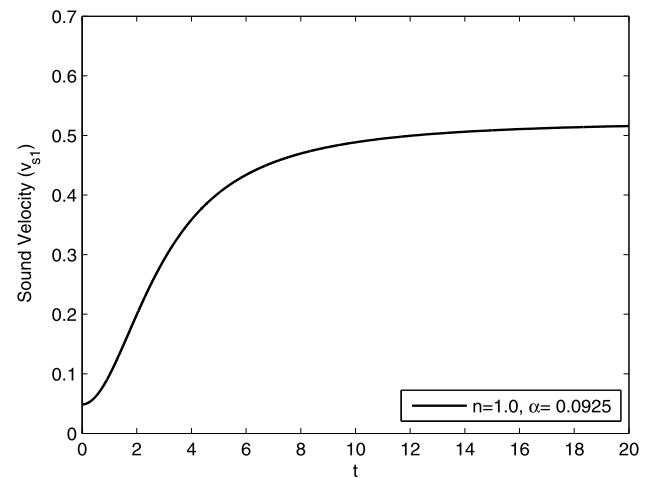
**Fig. 9** Plot of cosmological constant  $\Lambda$  versus time  $t$ . For  $\beta = 0.3$ ,  $\omega = 1$ ,  $\phi_0 = 1$



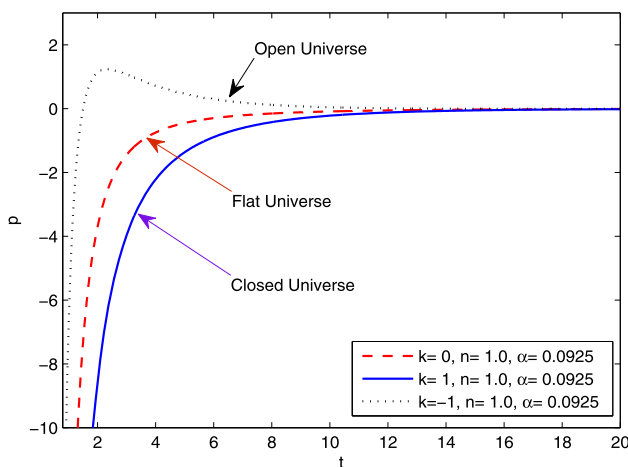
**Fig. 12** Plot of red shift parameter  $z_2$  versus time  $t$ .



**Fig. 10** Plot of energy density  $\rho$  versus time  $t$ . For  $\beta = 0.3$ ,  $\omega = 1$ ,  $\phi_0 = 1$



**Fig. 13** Plot of velocity of sound  $v_{s1}$  versus time  $t$



**Fig. 11** Plot of pressure  $p$  versus time  $t$ . For  $\beta = 0.3$ ,  $\omega = 1$ ,  $\phi_0 = 1$

eter decreases with time and approaches to a small positive constant at present epoch.

### 5 Physical acceptability of the solutions

For the stability of corresponding solutions, we should check that our model is physically acceptable.

#### Sound speed:

It is required that the velocity of sound  $v_s$  should be less than velocity of light ( $c$ ). As we are working in the gravitational units with unit speed of light, i.e. the velocity of sound exists within the range  $0 \leq v_s = \left(\frac{dp}{d\rho}\right) \leq 1$ .

We obtain the sound speed for Models I and II. The velocity of sound  $v_{s1}$  and  $v_{s2}$  for first and second models are respectively expressed as

$$v_{s_1} = \frac{\alpha^2\{(\beta - 2n) \coth^2(\alpha t) + 2n\}\{3 - 2\beta - \beta^2 - \omega\beta - \omega\beta^2\} + \alpha\beta n(\omega\beta - 1) + kn^2(1 - \beta) \times [\operatorname{csch}(\alpha t)]^{\frac{1}{n}}}{\alpha\{3\omega + 3\beta - 3 + 3n - \omega\beta n\}\{2\alpha + (\beta - 2\alpha n) \coth^2(\alpha t)\} + n\beta(\omega\beta - 3)} \tag{48}$$

and

$$v_{s_2} = \frac{(t + \alpha)\{\beta^2 + \omega\beta^2 + \omega\beta + 2\beta - 3\}\{(t + \alpha)^2 - 2\alpha n\} - \alpha\beta n(\omega\beta - 1) + 2\alpha n^2(\omega\beta - 1)}{(t + \alpha)\{3\omega\beta^2 + \beta^2 - \beta\}(t + \alpha)^2 + \alpha n(6 - 6\omega\beta - \omega\beta^2)\} - 2\alpha n^2(3 - \omega\beta)} \tag{49}$$

Here we observe that  $v_s < 1$ . Figure 13 depicts the plot of sound speed with time. We observe that  $v_s < 1$  throughout the evolution of the universe.

**Statefinder diagnostic:**

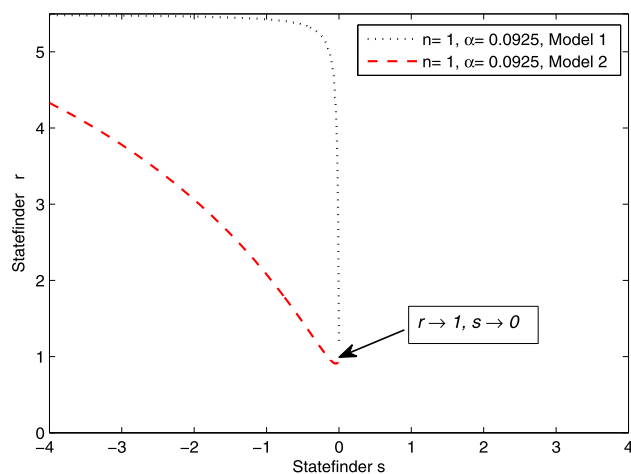
Sahni et al. (2003) have introduced a pair of parameters  $\{r, s\}$ , called Statefinder parameters. In fact, trajectories in the  $\{r, s\}$  plane corresponding to different cosmological models demonstrate qualitatively different behaviour. The statefinder parameters can effectively differentiate between different form of dark energy and provide simple diagnosis regarding whether a particular model fits into the basic observational data. The above statefinder diagnostic pair has the following form:

$$r = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \quad \text{and} \quad s = \frac{r - 1}{3(q - \frac{1}{2})}. \tag{50}$$

The state finder pairs  $(r_1, s_1)$  and  $(r_2, s_2)$  for the model 1 and model 2 respectively written as

$$r_1 = 1 + n(2n - 3) \operatorname{sech}^2(\alpha t) \tag{51}$$

$$s_1 = \frac{n(2n - 3)}{3\{2n - 3 \cosh^2(\alpha t)\}} \tag{52}$$



**Fig. 14** Plot of statefinder pair  $(r_1, s_1)$  and  $(r_2, s_2)$  for model 1 and model 2

$$r_2 = 1 - \frac{3\alpha n}{(t + \alpha)^2} + \frac{2\alpha n^2}{(t + \alpha)^3} \tag{53}$$

$$s_2 = \frac{2\alpha n\{3(t + \alpha) - 2n\}}{3(t + \alpha)\{3(t + \alpha)^2 - 2n\alpha\}} \tag{54}$$

From Fig. 14, we observe that  $s_1$  and  $s_2$  are negative when  $r_1 \geq 1$  and  $r_2 \geq 1$  respectively for both models. The figure shows that the universes start from an asymptotic Einstein static era ( $r \rightarrow \infty, s \rightarrow -\infty$ ) and go to the  $\Lambda$ CDM model ( $r = 1, s = 0$ ).

**6 Concluding remarks**

In this paper we have obtained two FRW cosmological models in the frame work of Brans-Dicke theory of gravitation with variable deceleration parameter ( $q$ ) and dynamic cosmological term  $\Lambda$ . We constructed two cosmological models by considering the power law relation of scalar field ( $\phi$ ) and time dependent DP ( $q$ ). The field equations have been solved exactly with suitable physical assumptions. It is to be noted that our methods of solving the field equations are different from technique of those authors who have solved field equations by considering the constant DP whereas we have considered time-dependent DP. As we have already mentioned in previous section that for a universe which has been decelerating in past and accelerating at current epoch, DP must show signature flipping. So, it is reasonable to consider time dependent DP.

Main features of Models 1 and 2 are as follows:

- The Models have a transition of the universe from early deceleration phase to recent accelerating phase which is in good agreement with recent observations (Caldwell et al. 2006).
- In Model 1, for different choice of  $n$ , we can generate a class of cosmological models in FRW universe. It is observed that such models are also in good harmony with current observations. It may also be observed that for  $n = \frac{3}{2}$ , one can obtain the expansion law for standard  $\Lambda$ CDM cosmology.
- In Model 2, for different values of  $\alpha$  and  $n$ , we can generate a class of models of the universe in FRW space-

time with time dependent DP ( $q$ ) and cosmological constant ( $\Lambda$ ). We observe that for  $n \geq 2$  and  $\alpha = 1$ , we obtain a class of transit models of the universe from early decelerated to present accelerating phase. For  $n \leq 1$  and  $\alpha = 1$ , we obtain accelerating models at present epoch.

- If we put  $n = 2$  in Eq. (23), we obtain  $a(t) = \sqrt{(t^\alpha e^t)}$ . In this case, one can obtain the expressions for different physical and geometric quantities.
- If we put  $n = 2$  and  $\alpha = 1$  in Eq. (23), we obtain  $a(t) = \sqrt{(te^t)}$ . In this case, one can obtain the expressions for different physical parameters and geometric quantities.
- If we put  $n = 3$  and  $\alpha = 1$  in Eq. (23), we obtain  $a(t) = (te^t)^{\frac{1}{3}}$ . In this case, we obtain the expressions for different physical parameters and geometric quantities as usual.
- If we put  $n = 1$  and  $\alpha = 1$  in Eq. (23), we obtain  $a(t) = te^t$ . In this case, we obtain the expressions for different physical parameters and geometric quantities as usual.

Thus, present works deal with most general cases of Model 1 and 2 in FRW space time in the frame work of BD theory with variable  $q$  and  $\Lambda$ . From these solutions we can generate a class of solutions which will be particular of our derived solutions.

**Acknowledgements** A. Chand thanks to the Director, SLIET, Longowal, Punjab for giving fellowship for research work. A. Chand and A. Pradhan would like to thank the IUCAA, Pune for providing facility and support during a visit where a part of this work is done. The authors would like to convey their sincere thanks and gratitude to the anonymous reviewer for his useful comments and suggestions for the improvement of the paper.

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